MULTIVARIATE TEXTURE DISCRIMINATION USING A PRINCIPAL GEODESIC CLASSIFIER

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ABSTRACT

A new texture discrimination method is presented for classification and retrieval of colored textures represented in the wavelet domain. The interband correlation structure is modeled by multivariate probability models which constitute a Riemannian manifold. The presented method considers the shape of the class on the manifold by determining the principal geodesic of each class. The method, which we call *principal geodesic classification*, then determines the shortest distance from a test texture to the principal geodesic of each class. We use the Rao geodesic distance (GD) for calculating distances on the manifold. We compare the performance of the proposed method with distance-to-centroid and *k*-nearest neighbor classifiers and of the GD with the Euclidean distance. The principal geodesic classifier coupled with the GD yields better results, indicating the usefulness of effectively and concisely quantifying the variability of the classes in the probabilistic feature space.

Index Terms— Texture classification, principal geodesic analysis, geodesic distance

1. INTRODUCTION

Several texture discrimination techniques have shown the wavelet representation to be a well suited domain for characterizing textures [1,2,3]. Hence, wavelet decomposition is often conducted for the generation of a set of features (*signature*) that accurately characterize the texture image. In many discrimination methods, each wavelet subband is modelled by a probability density function (PDF). The distribution parameters are estimated, composing the signature of the texture. The next step entails the use of an appropriate similarity measure for assessing the similarity of two textures based on their respective signatures.

The Euclidean distance (ED) and the Kullback-Leibler divergence (KLD) between probability distributions have yielded acceptable performances in various texture retrieval contexts [1,2]. However, the ED is not a natural similarity measure between probability distributions and the KLD is in fact not even a true distance measure. The Rao geodesic distance (GD) derived from the Fisher information has outperformed KLD and Euclidean in many contexts [2,3]. Therefore, in this work, the GD between multivariate probability distributions has been used, as it provides a natural similarity measure between PDFs.

Numerous univariate models, such as the generalised Gaussian [1] and Weibull [4], have been proposed for characterizing wavelet subbands. However, these models are inadequate for modelling the correlation between color bands and thus do not completely capture the rich texture information. In this work, we employ the multivariate Laplacian and Gaussian probability distributions for joint modeling of the spectral bands, while assuming independence amongst the wavelet subbands corresponding to the same color.

Texture retrieval techniques frequently compute the distance between the unlabelled (query) texture image and the nearest texture in the training set [1,2,5], seldom taking into account the underlying shape and variability of the class. In this paper, we present a new scheme for texture discrimination based on the calculation of the minimum geodesic distance between the unlabelled texture and the principal geodesic (principal direction) for each class. The principal direction, also called the first 'principal component', of the class is the direction in which the class members exhibit most variance.

For data lying in Euclidean space, principal component analysis (PCA) [6] provides an efficient parameterization of class variability. It yields the principal components of the data corresponding to the eigenvectors of the data covariance matrix. However, in our proposed scheme the texture signatures are parameters of PDFs and are no longer elements of a Euclidean space but in fact constitute a Riemannian manifold. Hence, PCA, being a standard linear technique, cannot be applied to textures. Therefore we employ principal geodesic analysis (PGA) [7] to each class for determining the direction with the greatest variability on the manifold. PGA is a generalisation of PCA for the manifold setting.

Further, we compare the performance of our proposed scheme with the performance of the GD-based k-nearest neighbour (kNN) [2] and distance-to-centroid classifiers [3] on the manifold. We also evaluate the outcome of the techniques when they operate with the ED as the underlying distance measure.

The rest of the paper is organised as follows. Section 2 summarises the statistical models and the Rao geodesic distance. Section 3 presents our proposed principal geodesic classifier. Section 4 outlines the experimental setup and presents the attained classification results. Finally, Section 5 concludes the paper.

2. MULTIVARIATE TEXTURE MODELLING

2.1 The multivariate Laplace distribution

The multivariate Laplace distribution is a particular case of the multivariate generalized Gaussian distribution (MGGD) that has been introduced in [2] and [8] for modeling the wavelet detail coefficients for color images. The MGGD is defined in [2] as:

$$f(\boldsymbol{X}|\Sigma,\beta) = \frac{\Gamma\frac{m}{2}}{\pi^{\frac{m}{2}}\Gamma\left(\frac{m}{2\beta}\right)2^{\frac{m}{2\beta}}}\frac{\beta}{|\Sigma|^{\frac{1}{2}}} \times \exp\left\{-\frac{1}{2}[\boldsymbol{X}'\Sigma^{-1}\boldsymbol{X}]^{\beta}\right\},$$

where $\Gamma(.)$ denotes the Gamma function and Σ is the dispersion matrix. β is the shape parameter and controls the fall-off rate of the distribution. Also, m is the dimensionality of the probability space, and is equal to 3 in our case of RGB colored images. The distribution reduces to a multivariate Gaussian case for β = 1 and to a multivariate Laplace case for β = 0.5. The parameters of the probability models are estimated via the method of moments followed by an optimization through maximum likelihood estimation [2].

2.2 Geodesic distance

The Rao geodesic distance (GD) between two multivariate Laplace or two multivariate Gaussian distributions denoted by (β,Σ_1) and (β,Σ_2) is given as:

$$GD = \left[\left(3b_h - \frac{1}{4}\right)\sum_i \left(r_2^i\right)^2 + 2\left(b_h - \frac{1}{4}\right)\sum_{i<j} r_2^i r_2^j\right]^{\frac{1}{2}}.$$

Here, $r_2^i \equiv \ln \lambda_2^i$ and $\lambda_2^i, i = 1,\dots,m,$ are the m eigenvalues of $\Sigma_1^{-1}\,\Sigma_2$. Also, b_h is defined by

$$b_h \equiv \frac{1}{4}\frac{m+2\beta}{m+2}.$$

3. PRINCIPAL GEODESIC CLASSIFICATION

A geodesic curve on a connected and complete manifold M is locally the shortest path between points. Essentially, a geodesic is a generalization of a straight line. Hence, a geodesic curve on the manifold is a natural analog of the first principal direction yielded by PCA. This is shown in Figure 1.

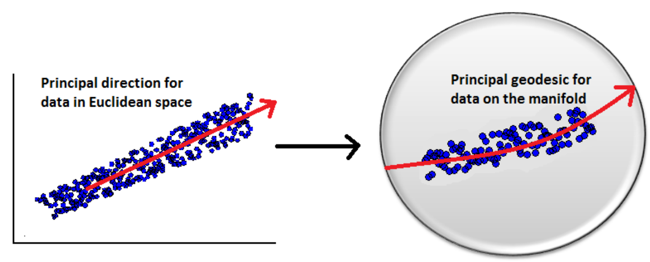

Figure 1: The principal geodesic on a manifold is an analog of the principal component direction in the Euclidean space.

PGA is outlined as follows:

- The class mean is computed for each class on the manifold. This entails the minimization of the sum of squared distance functions f for the class members $y_1, \dots y_N \in M$.

$$f(x) = \frac{1}{2N}\sum_{i=1}^{N} d(y, y_i)^2$$

  This is achieved via a gradient descent algorithm first proposed by Pennec [9].

- The class members are now projected onto the tangent space $T_\mu M$ of the manifold M at the class mean μ. The transformation to the tangent space is done through a *logarithmic map*:

$$log_\mu \colon y \epsilon M \rightarrow log_\mu(y) = \overrightarrow{\mu y}, \overrightarrow{\mu y} \epsilon T_\mu M.$$

- PCA is conducted on the class members in the tangent space for obtaining the principal component directions (eigenvectors).
- The eigenvector corresponding to the first principal component is projected onto the manifold using the *exponential map*:

$$exp_\mu \colon \overrightarrow{\mu y} \epsilon T_\mu M \rightarrow exp_\mu(\overrightarrow{\mu y}) = y, y \epsilon M$$

  This results in a point on the principal geodesic on the manifold.

The workflow of principal geodesic classification (PGC) is given in Figure 2.

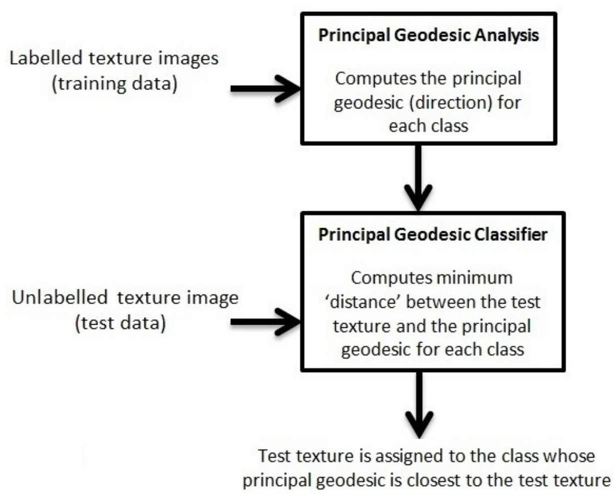

Figure 2: Workflow of principal geodesic classification

In the training phase of the principal geodesic classifier, the principal geodesic is obtained for each texture class. In the testing phase, the distance of the test texture to the closest point on the principal geodesic is obtained via optimization (gradient descent) as shown in Figure 3. The test texture is assigned to the class whose principal geodesic is nearest to the test texture. Computationally, the advantage of this scheme is that only a few distances need to be evaluated in the gradient descent algorithm to find the distance to a specific class. This is opposed to e.g. kNN, which has to calculate distances to each sample in the database.

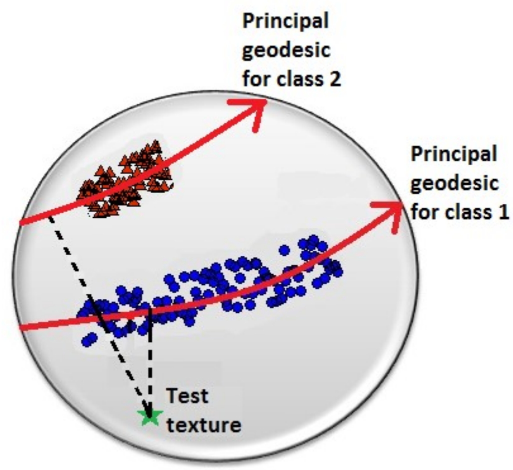

Figure 3: Illustration of classification of a test texture by PGC. The distance of the test texture to the closest point on the principal geodesic is calculated for each class.

3. CLASSIFICATION EXPERIMENTS

3.1. Experimental setup

We carried out our experiments with 40 colored texture classes from the MIT Vision Texture (VisTex) database [10]. The database consists of glimpses of different natural scenes possessing sufficient homogeneity and having a 512×512 image size. From each of these texture images, 16 non-overlapping subimages of size 128 x 128 are created. This leads to a database of 640 subimages. Each subimage is expressed in the RGB color space. Further, every color component of each subimage is individually normalized to zero mean and unit variance resulting in the subimages from the same original image not generally lying in the same range. This renders the classification task even more challenging. Following this, a discrete wavelet transform with one level is applied individually on every component using Daubechies filter of length eight. The wavelet detail coefficients of every subband over the three color components are then modeled by a multivariate Gaussian or Laplacian distribution. These estimated parameters constitute the feature set for a single subimage. The dimensionality of the complete manifold is given by the number of independent entries in the dispersion matrices (6 for three-band color images), multiplied by the number of wavelet subbands.

In the training phase of the principal geodesic classifier, the principal geodesic for each class is computed assuming that the label for each texture image is known. 640 subimages are each used as a test texture once and their minimum distance to the principal geodesic of each class is calculated. Texture classification is also carried out using a distance-to-centroid classifier and kNN, to provide a reference for comparison with our proposed method. In the training phase of distance-to-centroid classifier, the centroid for each class is calculated. The test texture is assigned to the class whose centroid has the minimum distance to the test texture. Likewise, in kNN the test texture is assigned to the class most common amongst its fifteen nearest neighbours. The choice of $k = 15$ is driven by the hypothesis that the 15 nearest neighbours of the test texture should naturally be the 15 subimages originating from the same class to which the test texture belonged. Each subimage is treated as a test texture once, both in the distance-to-centroid classifier and kNN.

The correct classification success rate for each classifier is then evaluated by calculating the ratio of textures that are correctly classified to the total number of textures.

The experiments are conducted with the GD as a distance measure and then also using the Euclidean distance (ED). This enables a comparison of the GD as a similarity measure between probability distributions to the ED.

3.1. Results

The results of the classification experiments on the VisTex database are presented in Table 1. The highest classification accuracy is achieved with our proposed principal geodesic classifier based on the GD, compared to distance-to-centroid and kNN. This indicates that accomodating the geometrical variability of the textures in the feature space can potentially lead to a performance improvement. PGA is essentially a dimensionality reduction procedure on the manifold, expressing each 6-dimensional texture image class by a single principal geodesic. This reduces the dimensionality of each wavelet subband to 1, yielding effective and concise image features. As mentioned before, PGC also offers a significant computational advantage over kNN. In addition, the superior performance of the classifiers with GD as a distance measure, compared to the Euclidean distance, further substantiates the superiority of the GD as a well-suited distance measure for probability distributions on a manifold. Finally, the Laplace distribution appears to be a better model than the Gaussian, though the differences in classification rates are marginal. On the other hand, it has been shown empirically in [2] that in retrieval applications the advantage of a Laplacian distribution can become more important. At this point it should be noted that, to the best of our knowledge, no analytic expression for the KLD between multivariate Laplace distributions has been found so far, as opposed to the GD.

Classifier	Measure	Model	SR (%)
Principal geodesic	GD	Gauss	99.06
		Laplace	99.22
	ED	Gauss	71.25
		Laplace	75.00
Distance-to-centroid	GD	Gauss	95.94
		Laplace	95.78
	ED	Gauss	71.72
		Laplace	70.31
k-nearest neighbour	GD	Gauss	94.53
		Laplace	95.31
	ED	Gauss	69.06
		Laplace	69.53

Table 1: Correct classification success rates (SR), based on Laplace and Gaussian models for one wavelet scale, using principal geodesic, distance-to-centroid and *k*-nearest neighbor classifiers.

5. CONCLUSION

In this work, we have presented a new texture discrimination method and demonstrated its classification performance on a database of 640 textured images. The presented principal geodesic classifier performs better than distance-to-centroid and *k*-nearest neighbor classifiers, making use of a highly optimized set of features on a probabilistic manifold. Further, we have shown the superior classification performance of the GD versus Euclidean distance in all our experiments.

Investigating the performance of our proposed classifier on other data sets and applications will be a subject of future work.

6. REFERENCES

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