Nonlinear vibrations phenomena of a tunable nonlinear spring characteristic

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1 Introduction

Nonlinear vibrations occur in systems that have a nonlinear spring force. This system in the undamped case is:

$$m\ddot{x} + z(x) = F(t) \tag{1}$$

Previous research has focussed on spring forces expressed as polynomials, $z(x) = a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3 + \ldots$. If $a_1 >> a_i$, the spring is called a weakly nonlinear. Semi-analytic techniques have shown the existence of sub and superharmonic resonances, as well bifurcations near these resonances ([1]). Other research focuses on $z(x) = ax^p$ for $p > 1$, often called strong nonlinear springs. Dependence of the free vibration frequency on initial energy is shown in ([2]). When harmonically excited, strongly nonlinear systems can vibrate significantly at any frequency, as long as the initial energy is above a certain threshold ([2]). The current study present a tunable spring element, first introduced in ([3]), for which $z(x)$ can be approximated as a polynomial in x through Taylor expansion. The polynomial coefficients are tunable to $a_1 >> a_i$ but also $a_1 \geq a_i$, increasing nonlinearity. With simulations, it is checked to what extend the approximated polynomial can predict nonlinear phenomena of the exact model.

2 Concept

The spring force is generated by the proposed string-pulley system, fig. 1a. Pulleys P1 and P3 are fixed, but P2 can move in the direction perpendicular to x. As the string is pulled at point M in the positive x direction, the tension in the string pushes down pulley $P2$ and the linear spring, which has stiffness k. A negative tension (pushing) will loosen the string from the pulleys, such that a positive displacement/force is required. The tension T in the string in function of x is:

$$T(x) = \frac{1}{2}k \cdot \frac{(h_0 - \sqrt{(L_0 - \frac{x}{2})^2 - l^2})(L_0 - \frac{x}{2})}{\sqrt{(L_0 - \frac{x}{2})^2 - l^2}} \tag{2}$$

with dimensions L_0, h_0 and l as indicated on fig. 1a.
To agree with the constraint of positive tension, two setups are concieved. In the first setup, fig. 1b, a mass m is attached to the string and moves forward and backward around a chosen working point (x_0, F_0). The second setup employs two spring elements, fig. 1c. By imposing an initial displacement x_0 on both elements, the tension in both strings is the same but opposite, creating an equilibrium position. A periodic force then acts on the mass. Series expansion of (2) reveals that the ratio L/h around an x_0 highly influences the coefficients of higher order terms for both setups. As the coefficients are tuned, different type of nonlinear phenomena occur for the approximate series, which are found in simulations of the exact system as well.

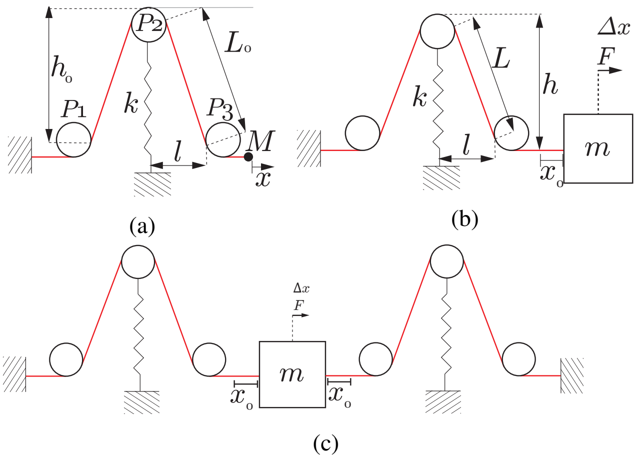

Figure 1: Nonlinear spring element in rest, (a) and with a connected mass and an initial displacement/force (b). Two elements allow for vibration around equilibrium Δx ([3])

3 Conclusion

A nonlinear spring element is proposed which can be tuned to exhibit both weakly and strongly nonlinear vibration phenomena. In the future works, the role of the spring element on vibration absorption of a main system will be investigated.

References

[1] A. H. Nayfeh and D. T. Mook, "Nonlinear oscillations," WHILEY-VCH, 1995.

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[3] G. Tionietti et al., "Design and control of a variable stiffness actuator for safe and fast physical human/robot interaction", IEEE proceedings, 2005.