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## REFERENCES

- [1] M. De Bock, A. Babaie-Fishani, and P. Rombouts, "Improved offline calibration for DAC mismatch in low OSR Sigma Delta ADCs with distributed feedback," in *Proc. IEEE Int. Symp. Circuits and Syst. (ISCAS)*, 2014, pp. 938–941.

# Improved offline calibration for DAC mismatch in low OSR $\Sigma\Delta$ ADCs with distributed feedback

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**Abstract**—We present an offline calibration method to correct the non-linearity due to DAC element mismatch in distributed feedback SigmaDelta-modulation A/D-converters. The improvement over previous methods is that not only the first feedback DAC is calibrated, but also the DACs that are coupled to later stages can be calibrated as well. This is needed in the case of Sigma Delta modulators with a low OSR, where the contribution of the second feedback DAC should not be neglected. The technique is based on a calibration measurement with a two-tone input signal.

## I. INTRODUCTION

The continuous evolution toward higher bandwidth has increased the interest for  $\Sigma\Delta$ -modulators with a low oversampling ratio (OSR). In this case multibit quantization is almost unavoidable. It is well known that the resolution is then limited by the linearity of the feedback digital-to-analog converter (DAC) [1]. This way, some kind of linearization scheme is needed if linearity beyond the intrinsic device matching is required. For this purpose, in the past, dynamic element matching (DEM) techniques have been successfully exploited [1]. With such techniques the DAC mismatch errors are spectrally shaped and as a result greatly reduced within the low-frequency signal band as well. But DEM techniques inherently become less efficient when the OSR is reduced and a larger fraction of the Nyquist band is occupied by the signal band. This is particularly true for the popular Data weighted averaging (DWA)[2] which achieves only first order shaping of the DAC mismatch errors. Moreover, such DEM-technique must act upon the feedback DAC, which is inside the loop. This way all DEM-techniques put stress on the latency budget and become problematic at high clock speeds.

A solution for these problems is to compensate the DAC mismatch errors in the digital domain. This can be done outside the modulator loop and as such doesn't limit the modulator speed. The implementation of this is based on a look-up table (LUT) in which a digital estimation of the DAC mismatch errors is stored. This LUT is then used to correct the digital output of the modulator during normal operation. Obviously, the performance of such a LUT-based compensation scheme depends on the accuracy of the digital estimation of the mismatch errors. Therefore, the actual calibration, i.e. the process of determining the mismatch errors, is essential. In a related work [3] a background calibration scheme for this was presented. But this increases the modulator complexity and also adds some blocks inside the loop, which might be a problem in high speed modulators. An alternative is the offline calibration of [4]. It is based on a single measurement with a spectrally pure sine wave, from which the DAC mismatch er-

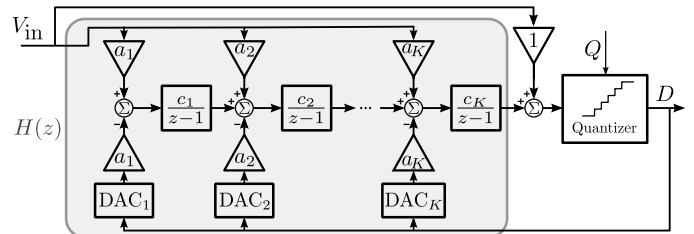


Fig. 1. A CIFB  $\Sigma\Delta$ -modulator structure.

rors are then estimated. This work was focused on modulators where only one global feedback DAC needed to be calibrated. However, in a distributed feedback  $\Sigma\Delta$ -modulator, multiple feedback DACs coupling to the successive integrator stages, are used (see fig. 1). Usually the first DAC will dominate the overall distortion and by neglecting the errors of the other DACs we can use the technique of [4]. However, as will be shown in section II, at low OSR also the contribution of the subsequent DACs (DAC<sub>2</sub>, DAC<sub>3</sub>, ...) will become significant and limit the performance.

In this paper, we present an improved offline calibration method and compensation scheme that corrects the error contribution of these subsequent DACs as well. For this, we start with an exact analysis of the non-linearity of the overall distributed feedback modulator due to DAC element mismatch in section II. The results of this analysis are then used in sections III and IV where the improved calibration method and compensation scheme are derived respectively. Section V presents the measurement results on an integrated prototype where the presented method is used for DAC mismatch compensation.

## II. EXACT ANALYSIS OF NON-LINEARITY DUE TO ELEMENT MISMATCH

Fig. 1 shows a  $\Sigma\Delta$ -modulator with a distributed feedback topology (CIFB as in [1]). A quantizer is embedded in a control loop with loop filter  $H(z)$ . The loop filter  $H(z)$  is of  $K^{\text{th}}$ -order and consists of a cascade of  $K$  integrators, each with a gain  $c_k$ . The digital output  $D$  of the multibit quantizer is fed back toward every integrator stage of the loop filter using  $K$  feedback paths. In every feedback path the DAC converts the digital output  $d$  into an analog signal  $v_k$  to which a gain  $a_k$  is applied. The additional feed-in paths of the input signal  $V_{\text{in}}$  are added to create what is commonly known as a low-distortion topology [5]. A common analysis for this system models the quantizer as an additive white noise contribution  $Q$  and all DAC <sub>$k$</sub>  in the feedback path are assumed to be ideal.

The digital output  $D(z)$  of the complete system can then be written in the  $Z$ -domain as:

$$D(z) = \underbrace{\left( \frac{H(z)}{1+H(z)} + 1 \right)}_{STF(z)} V_{in}(z) + \underbrace{\frac{1}{1+H(z)}}_{NTF(z)} Q(z) \quad (1)$$

From eq. (1) it is readily seen that  $STF(z) = 1$ . The quantization noise transfer function ( $NTF$ ) can in general be written as:

$$NTF(z) = \frac{\prod_{k=1}^K (z - z_k)}{P_K(z)} \quad (2)$$

Here,  $P_K(z)$  is an  $K^{\text{th}}$ -order polynomial describing the pole placement of the  $NTF$ . The zeros  $z_i$  can be optimized by applying local feedback around the integrator stages [6] and as such spreading them over the low pass signal band. For the remainder of this paper however, we will assume that all zeros  $z_k = 1$ . This will ease the equations, but it should be noted that the analysis also holds for the general case where the  $NTF$ -zeros are spread over the signal bandwidth.

In practice, all  $DAC_k$  will have some mismatch. We will assume that each DAC has a similar topology and consists of  $N$  levels. To analyze the effect of this mismatch, first we introduce the DAC-level selection signals  $x_i$ , which are defined as:

$$x_i(n) = \begin{cases} +1, & \text{if the } i\text{th DAC-level is selected} \\ 0, & \text{else} \end{cases} \quad (3)$$

This way, there are  $N$  selection signals, and there is always exactly one selection signal high at the same time. We can now write each feedback signal  $v_k$  as

$$v_k(n) = \sum_{i=1}^N x_i(n) \cdot C_{k,i} \quad (4)$$

where  $C_{k,i}$  corresponds to the  $i$ th DAC-level of the  $k$ th feedback DAC. The DAC non-linearity manifests itself in the sense that the actual DAC levels  $C_{k,i}$  deviate from their nominal values  $C_{nm,k,i}$  by a mismatch error  $\varepsilon_{k,i}$ . If we assume all DACs to be equal, we can assume  $C_{nm,k,i} = C_{nm,i}$ . Each DAC output  $v_k$  can now be written as

$$v_k(n) = \sum_{i=1}^N x_i(n) \cdot (C_{nm,i} + \varepsilon_{k,i}) = v_{nm}(n) + \underbrace{\sum_{i=1}^N x_i(n) \cdot \varepsilon_{k,i}}_{e_{DAC,k}(n)} \quad (5)$$

where we have introduced  $e_{DAC,k}$ , the error signal of the  $i$ th DAC. Every feedback signal can thus be written as a nominal feedback signal  $v_{nm}$  and an error contribution  $e_{DAC,k}$ .

Fig. 2 then shows the model of the CIFB-structure where the mismatch signals  $e_{DAC,k}$  are added to the system. Each feedback  $DAC_k$  is modeled as an ideal DAC with output  $v_{nm}$  with added mismatch contribution  $e_{DAC,k}$ . If we now analyze the digital output  $D$  of this modulator model, we find:

$$D(z) = V_{in}(z) + NTF(z)Q(z) - \sum_{k=1}^K ETF_k(z)E_{DAC,k}(z) \quad (6)$$

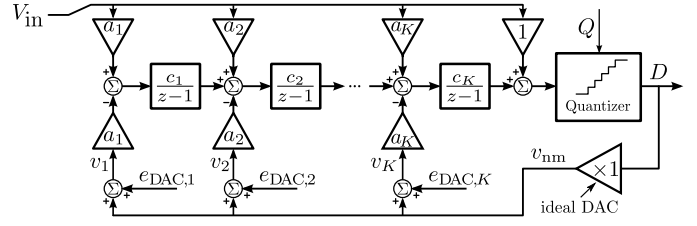


Fig. 2. The CIFB  $\Sigma\Delta$ -modulator model where the DACs are replaced by an ideal DAC and additional error signals  $e_{DAC,k}$ .

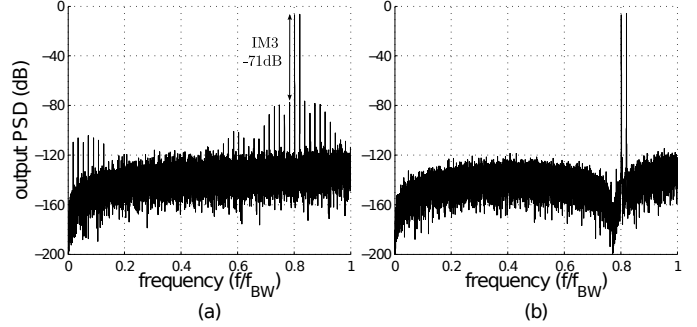


Fig. 3. The output PSD of  $\Sigma\Delta$ -modulator with only mismatch in  $DAC_2$  (a) without and (b) with the proposed compensation method (eq. (19)).

Here, we have introduced the error transfer function ( $ETF$ ) for each error signal  $e_{DAC,k}$ , which denotes the contribution of that error signal to the output  $D$ . Each  $ETF_k$  can be written as:

$$ETF_k(z) = NTF(z) a_k \prod_{j=k}^K \frac{c_j}{z-1} \quad (7)$$

If we look at the contribution of the mismatch of the first DAC ( $e_{DAC,1}$ ) and using (2), we find:

$$ETF_1(z) = NTF(z) \frac{a_1}{(z-1)^K} \prod_{j=1}^K c_j = \frac{a_1}{P_K(z)} \prod_{j=1}^K c_j \quad (8)$$

We can write the contribution of subsequent DAC errors ( $e_{DAC,2}, e_{DAC,3}, \dots, e_{DAC,K}$ ) in terms of  $ETF_1(z)$ :

$$ETF_k(z) = \frac{a_k}{a_1} \frac{(z-1)^{k-1}}{\prod_{j=1}^{k-1} c_j} ETF_1(z) \quad (9)$$

To show that the contribution of the subsequent DACs can indeed be significant, fig. 3(a) shows the base-band output power spectral density (PSD) of a 3<sup>rd</sup>-order  $\Sigma\Delta$ -modulator (the  $NTF$  is designed according to [6] with  $\|H\|_\infty = 2.4$  and  $OSR=16$ ) where only  $DAC_2$  has mismatch. The modulator uses 32 unit elements, each with a normally distributed random mismatch with  $\sigma = 1\%$ . The linearity of this two-tone measurement is limited by the 3<sup>rd</sup>-order intermodulation product (IM3) at  $-71$  dB.

### III. ESTIMATION OF MISMATCH ERRORS

The estimation of the mismatch errors occurs in an off-line procedure prior to the normal operation. For this we apply a

signal  $s(t)$  to the  $\Sigma\Delta$  ADC. Based on (6), we can write the output  $d(n)$  of the modulator as :

$$d(n) = s(n) + ntf(n) \star q(n) - \sum_{k=1}^N etf_k(n) \star e_{\text{DAC},k}(n) \quad (10)$$

We will use a digital decimation low-pass filter  $L(z)$  (with corresponding impulse response  $l(n)$ ) to remove the quantization noise contribution:

$$d_{\text{LP}}(n) = s_{\text{LP}}(n) + e_{\text{noise,LP}}(n) - \sum_{k=1}^K l(n) \star etf_k(n) \star e_{\text{DAC},k}(n) \quad (11)$$

Here,  $e_{\text{noise,LP}}$  contains the electrical noise as well as the residual quantization noise that may still be present. Applying the definition of the DAC error signals  $e_{\text{DAC},k}$  of eq. (5) we find:

$$d_{\text{LP}}(n) = s_{\text{LP}}(n) + e_{\text{noise,LP}}(n) - \sum_{k=1}^K \sum_{i=1}^N l(n) \star etf_k(n) \star x_i(n) \cdot \varepsilon_{k,i} \quad (12)$$

Now, we introduce the filtered selection signals  $x_{\text{EL},k,i} = l(n) \star etf_k(n) \star x_i(n)$  and the residue signal  $r(n)$ , which is equal to:

$$r(n) = d_{\text{LP}}(n) - s_{\text{LP}}(n) \quad (13)$$

If the applied signal  $s(t)$  is known, then also its digitized version  $s_{\text{LP}}(n)$  is known. E.g. if  $s(t)$  is a combination of spectrally pure sine waves,  $s_{\text{LP}}(n)$  can easily be estimated using standard curve fitting algorithms such as IEEE-STD-1057 [7], [8]. The residue signal  $r(n)$  is then evaluated as the remaining curve fit error after elimination of the different sine waves and hence can be considered to be known. We now find:

$$r(n) = e_{\text{noise,LP}}(n) - \sum_{k=1}^K \sum_{i=1}^N x_{\text{EL},k,i}(n) \cdot \varepsilon_{k,i} \quad (14)$$

From this equation, we observe that we can obtain an estimation  $\hat{\varepsilon}_{i,j}$  of the different DAC errors by performing a least mean square minimization (LMS):

$$\begin{aligned} E \left\{ \left( r(n) + \sum_{k=1}^K \sum_{i=1}^N x_{\text{EL},k,i}(n) \cdot \hat{\varepsilon}_{k,i} \right)^2 \right\} \\ = \underbrace{E \{ e_{\text{noise,LP}}^2 \}}_{\text{noise variance}} \\ + \underbrace{E \left\{ \left( \sum_{k=1}^K \sum_{i=1}^N x_{\text{EL},k,i}(n) \cdot (\hat{\varepsilon}_{k,i} - \varepsilon_{k,i}) \right)^2 \right\}}_{\text{calibration error variance}} \quad (15) \end{aligned}$$

Observing the calibration error variance, we find that the LMS optimization corresponds to a correct estimation of  $\varepsilon_{i,j}$ . In practice, we will use a finite measurement interval. As a result, (15) must be approximated as the LMS over a finite data set

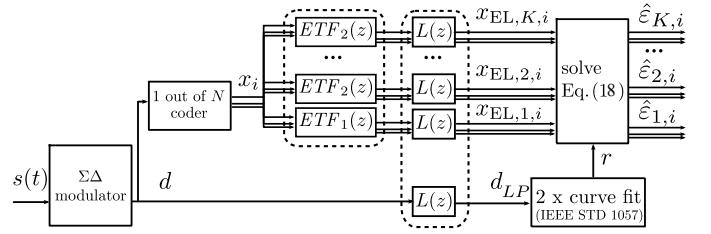


Fig. 4. The calibration setup with a two-tone test signal  $s(t)$ .

of  $T$  data points. This way, we will find the DAC errors  $\varepsilon_{k,i}$  by minimizing

$$\sum_{n=1}^T \left( r(n) + \sum_{k=1}^K \sum_{i=1}^N x_{\text{EL},k,i}(n) \cdot \hat{\varepsilon}_{k,i} \right)^2 \quad (16)$$

with regard to  $\hat{\varepsilon}_{k,i}$ . This can be expanded into

$$\frac{\partial}{\partial \hat{\varepsilon}_{j,m}} \sum_{n=1}^T \left( r(n) + \sum_{k=1}^K \sum_{i=1}^N x_{\text{EL},k,i}(n) \cdot \hat{\varepsilon}_{k,i} \right)^2 = 0, \forall j, m \quad (17)$$

with  $j = 1..K$  and  $m = 1..N$ . This yields a system of  $K \times N$  equations ( $N$  mismatch errors for  $K$  feedback DACs) for the  $K \times N$  unknown DAC errors  $\hat{\varepsilon}_{k,i}$ :

$$\begin{aligned} \sum_{n=1}^T r(n) x_{\text{EL},j,m}(n) = \\ \sum_{n=1}^T \sum_{k=1}^K \sum_{i=1}^N x_{\text{EL},j,m}(n) x_{\text{EL},k,i}(n) \cdot \hat{\varepsilon}_{k,i}, \forall j, m \quad (18) \end{aligned}$$

All coefficients in this system can easily be evaluated and solving this system is trivial. Fig. 4 then shows the calibration measurement setup. As shown in eq. (9), the contribution of the subsequent DACs (DAC<sub>2</sub>, DAC<sub>3</sub>, ... DAC<sub>K</sub>) is differentiated. It is however important that the applied calibration signal  $s(t)$  results in a non-linear contribution in the output that is very sensitive to the contribution of the error signals of the subsequent DACs ( $e_{\text{DAC},2}$ ,  $e_{\text{DAC},3}$ , ...  $e_{\text{DAC},K}$ ). Therefore, the calibration setup must be a two-tone test with frequencies at the edge of the signal band where the differentiation factor  $(z - 1)$  is at its maximum. Eliminating the two sine waves from the output signal is trivial using IEEE-STD-1057, and the remaining residue of the double sine fit is  $r(n)$  as in eq. (13).

## IV. LUT-BASED COMPENSATION

### A. General compensation scheme

After estimating the DAC mismatch errors  $\hat{\varepsilon}_{k,i}$  in an offline calibration cycle, they are permanently stored in a look-up table (LUT). Fig. 5 then shows the modulator during normal operation. The corrected output  $d_{\text{cal}}(n)$  is then calculated as:

$$d_{\text{cal}}(n) = d(n) + \sum_{i=1}^N \sum_{k=1}^K etf_k(n) \star x_i(n) \cdot \hat{\varepsilon}_{k,i} \quad (19)$$

Fig. 3(b) shows the simulated output spectrum of the same  $\Sigma\Delta$ -modulator as in fig. 3(a), but this time using the proposed compensation method (eq. (19)). All distortion tones are now reduced to beneath the quantization noise floor.

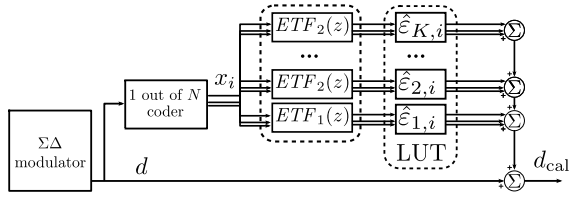


Fig. 5. The  $\Sigma\Delta$ -modulator during normal operation with DAC mismatch compensation.

## B. Discussion

The filters  $ETF_k(z)$  are of  $K^{\text{th}}$ -order (the order of the modulator) and the coefficients of  $P_K$  are in general non integer and as such require considerable calculation effort. This is not a problem during the one time offline calibration cycle where the mismatch errors are determined, but applying these filters to the selection signals during normal operation might add considerable effort. The compensation scheme of fig. 5 can be simplified by reducing the complexity of the  $ETF_k(z)$ . Eq. (8) can be approximated to 1 for signals at DC ( $z \approx 1$ ):

$$\lim_{z \rightarrow 1} ETF_1(z) = 1 \quad (20)$$

Using this, also the higher order  $ETF_k(z)$  complexity can be further reduced, as (9) can now be approximated as:

$$ETF_k(z) = \frac{a_k (z - 1)^{k-1}}{a_1 \prod_{j=1}^{k-1} c_j} \quad (21)$$

The differentiation factor  $(z - 1)^{k-1}$  is easily calculated and the gain factor  $a_k / (a_1 \prod_{j=1}^{k-1} c_j)$  in general can be expected to be close to integer.

As expected, we find that the contribution of each  $e_{\text{DAC},k}$  is  $(k - 1)$ -times differentiated. For high OSR,  $z - 1 \approx 0$  for signals within the signal band and the contribution of all but the first DAC can be neglected when looking at the non-linearity of the overall modulator. However, when targeting high accuracy  $\Sigma\Delta$ -modulators with low OSRs, the assumption  $z - 1 \approx 0$  does not hold anymore, and the contribution of the other DACs (in practice  $\text{DAC}_2$ ) can not be neglected. The mismatch errors in the subsequent DACs are also increased as subsequent integrator stages are in general scaled down to reduce power consumption. As a result, these DACs will be made with smaller devices which will increase their mismatch.

## V. EXPERIMENTAL RESULTS

The proposed calibration procedure and compensation scheme are tested on an integrated switched-capacitor  $\Sigma\Delta$ -modulator prototype with 3<sup>rd</sup>-order noise shaping and a signal bandwidth of 1.25 MHz for an OSR of 12. This is not a state-of-the art  $\Sigma\Delta$ -modulator in terms of signal bandwidth, but it is here only used to illustrate the proposed calibration procedure. The quantizer has 9 quantization levels. The mismatch errors of the first two DACs ( $\text{DAC}_1$  and  $\text{DAC}_2$ ) are determined during a calibration cycle with a two-tone signal at frequencies 975 kHz and 1025 kHz and at  $-10$  dBFS amplitude using the calibration setup as shown in fig. 4. The  $ETF_k(z)$  are approximated as in equations (20) and (21). Fig. 6(a) shows the measured compensated output spectrum when applying a two-tone signal at frequencies 1000 kHz and 1050 kHz with  $-9.5$  dBFS when

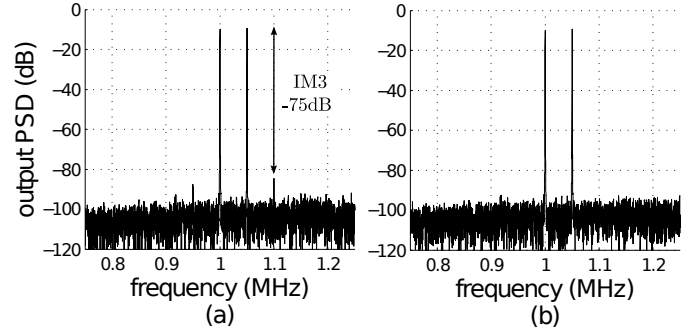


Fig. 6. The measured output PSD for a two-tone test on a distributed feedback  $\Sigma\Delta$ -modulator with 1.25 MHz bandwidth (OSR=12) when the mismatch errors of (a) only  $\text{DAC}_1$  are compensated and (b) both  $\text{DAC}_1$  and  $\text{DAC}_2$  are compensated.

only the mismatch errors of  $\text{DAC}_1$  are compensated. We see that the IM3 is at  $-75$  dB. When the mismatch errors of  $\text{DAC}_2$  are also compensated, the IM3 is further reduced to beneath the noise floor as shown in fig. 6(b).

## VI. CONCLUSION

An improved offline calibration method for DAC mismatch errors in low oversampling multibit SigmaDelta ADCs has been presented. It allows to calibrate every DAC in a distributed feedback structure. The technique uses a single two-tone test and calculates the DAC mismatch errors from the resulting digital output signal. The mismatch errors are then stored in a LUT from which the compensated digital output signal is calculated during normal operation.

## ACKNOWLEDGMENT

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