Multi-Objective Design of Antenna Structures Using Variable-Fidelity EM Simulations and Co-Kriging

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Abstract—**A methodology for low-cost multi-objective design of antenna structures is proposed. To reduce the computational effort of the design process the initial Pareto front is obtained by optimizing the response surface approximation (RSA) model obtained from low-fidelity EM simulations of the antenna structure of interest. The front is further refined by iterative incorporation of a limited number of high-fidelity training points into the RSA surrogate using co-kriging. Our considerations are illustrated using two examples of antenna structure.**

Keywords-Multi-objective design; EM simulation, co-kriging.

I. INTRODUCTION

Multi-objective design of antenna structures is a challenging process that aims at finding a set of designs corresponding to trade-offs between conflicting objectives (e.g., reflection, gain or size requirements), also referred to as a Pareto front [1]. Typically it is obtained by employing multi-objective evolutionary algorithms (MOEAs) [2], which requires considerable computational effort, and may be prohibitive when the antenna performance is evaluated using high-fidelity EM analysis.

Here, we propose a multi-objective design procedure that exploits variable-fidelity EM simulations and co-kriging [3] as a way to obtain the Pareto front at a low computational cost. In our approach, the initial Pareto front is obtained by multi-objective optimization of a cheap surrogate model created as kriging interpolation of low-fidelity EM simulations of the antenna structure of interest. The surrogate is subsequently enhanced with limited number of high-fidelity simulations evaluated at the selected designs allocated along the initial front and blended into the model using co-kriging [3]. The model enhancement and re-optimization is iterated until convergence. As demonstrated through examples, reliable representation of the high-fidelity Pareto front can be obtained at the cost corresponding to a few dozen on high-fidelity EM simulations of the antenna of interest.

II. DESIGN METHODOLOGY

A. Antenna Models

Let $\boldsymbol{R}_f(\boldsymbol{x})$ denote a computationally expensive EM-simulated high-fidelity model, which is an accurate representation of the antenna structure (e.g., $|S_{11}|$ over the frequency band of interest). Here, $\boldsymbol{x}$ is a vector of designable (e.g., geometry) parameters. We also consider an auxiliary (low-fidelity) model $\boldsymbol{R}_c$, usually evaluated using the same EM solver but with coarser discretization. $\boldsymbol{R}_c$ is much faster but less accurate than $\boldsymbol{R}_f$.

B. Kriging Interpolation

Kriging is a popular technique to interpolate deterministic noise-free data [4]. Let $X_{B.KR} = \{\boldsymbol{x}_{KR}^1, \boldsymbol{x}_{KR}^2, \ldots, \boldsymbol{x}_{KR}^{NKR}\} \subset X_R$ be the base (training) set and $\boldsymbol{R}_f(X_{B.KR})$ the associated fine model responses. Then, the kriging interpolant is derived as

$$\boldsymbol{R}_{s.KR}(\boldsymbol{x}) = M\alpha + r(\boldsymbol{x}) \cdot \Psi^{-1} \cdot (\boldsymbol{R}_f(X_{B.KR}) - F\alpha) \quad (1)$$

where M and F are Vandermonde matrices of the test point $\boldsymbol{x}$ and the base set $X_{B.KR}$, respectively. The coefficient vector α is determined by Generalized Least Squares (GLS). $r(\boldsymbol{x})$ is an $1\times N_{KR}$ vector of correlations between the point $\boldsymbol{x}$ and the base set $X_{B.KR}$, where the entries are $r_i(\boldsymbol{x}) = \psi(\boldsymbol{x},\boldsymbol{x}_{KR}^i)$, and Ψ is a $N_{KR}\times N_{KR}$ correlation matrix, with the entries given by $\Psi_{i,j} = \psi(\boldsymbol{x}_{KR}^i, \boldsymbol{x}_{KR}^j)$. In this work, the exponential correlation function is used, i.e., $\psi(\boldsymbol{x},\boldsymbol{y}) = \exp(\sum_{k=1,\ldots,n} -\theta_k|\boldsymbol{x}^k - \boldsymbol{y}^k|)$, where the parameters $\theta_1, \ldots, \theta_n$ are identified by Maximum Likelihood Estimation (MLE). The regression function is chosen constant, $F = [1 \ldots 1]^T$ and $M = (1)$.

C. Co-Kriging Modeling

Co-kriging [3] is a type of kriging where the $\boldsymbol{R}_f$ and $\boldsymbol{R}_c$ model data are combined to enhance the prediction accuracy. Co-kriging is a two-steps process: first a kriging model $\boldsymbol{R}_{s.KRc}$ of the coarse data $(X_{B.KRc},\boldsymbol{R}_c(X_{B.KRc}))$ is constructed and on the residuals of the fine data $(X_{B.KRf},\boldsymbol{R}_d)$ a second kriging model $\boldsymbol{R}_{s.KRd}$ is applied, where $\boldsymbol{R}_d = \boldsymbol{R}_f(X_{B.KRf}) - \rho\cdot\boldsymbol{R}_c(X_{B.KRf})$. The parameter ρ is included in the MLE. Note that if the response values $\boldsymbol{R}_c(X_{B.KRf})$ are not available, they can be approximated by using the first kriging model $\boldsymbol{R}_{s.KRc}$, namely, $\boldsymbol{R}_c(X_{B.KRf}) \approx \boldsymbol{R}_{s.KRc}(X_{B.KRf})$. The resulting co-kriging interpolant is defined as

$$\boldsymbol{R}_{s.CO}(\boldsymbol{x}) = M\alpha + r(\boldsymbol{x}) \cdot \Psi^{-1} \cdot (\boldsymbol{R}_d - F\alpha) \quad (2)$$

where the block matrices M, F, $r(\boldsymbol{x})$ and Ψ can be written in function of the two separate kriging models $\boldsymbol{R}_{s.KRc}$ and $\boldsymbol{R}_{s.KRd}$:

$$r(\boldsymbol{x}) = [\rho\cdot\sigma_c^2\cdot r_c(\boldsymbol{x}), \rho^2\cdot\sigma_c^2\cdot r_c(\boldsymbol{x}, X_{B.KR_f}) + \sigma_d^2\cdot r_d(\boldsymbol{x})]$$

$$\Psi = \begin{bmatrix} \sigma_c^2\Psi_c & \rho\cdot\sigma_c^2\cdot\Psi_c(X_{B.KR_c}, X_{B.KR_f}) \\ 0 & \rho^2\cdot\sigma_c^2\cdot\Psi_c(X_{B.KR_f}, X_{B.KR_f}) + \sigma_d^2\cdot\Psi_d \end{bmatrix} \quad (3)$$

$$F = \begin{bmatrix} F_c & 0 \\ \rho\cdot F_d & F_d \end{bmatrix}, \qquad M = [\rho\cdot M_c \quad M_d]$$

where $(F_c,\sigma_c,\Psi_c,M_c)$ and $(F_d,\sigma_d,\Psi_d,M_d)$ are matrices obtained from the kriging models $\boldsymbol{R}_{s.KRc}$ and $\boldsymbol{R}_{s.KRd}$, respectively; σ_c^2 and σ_d^2 are process variances, while $\Psi_c(\cdot,\cdot)$ and $\Psi_d(\cdot,\cdot)$ denote correlation matrices of two datasets with the optimized $\theta_1, \ldots, \theta_n$ parameters and correlation function of the kriging models $\boldsymbol{R}_{s.KRc}$ and $\boldsymbol{R}_{s.KRd}$, respectively.

D. Multi-Objective Design Problem and Solution Approach

Let $F_k(\boldsymbol{x})$, $k = 1, \ldots, N_{obj}$, be a kth design objective (e.g., $|S_{11}| < -10$dB over certain frequency band of interest, or minimization of the antenna size defined in a convenient way). The goal of multi-objective optimization is to find a representation of a so-called Pareto front X_P of the design space X, such that for any $\boldsymbol{x} \in X_P$, there is no $\boldsymbol{y} \in X$ for which $\boldsymbol{y} \prec \boldsymbol{x}$, where $\prec$ is a dominance relation defined for the two designs $\boldsymbol{x}$ and $\boldsymbol{y}$ as follwos: $\boldsymbol{x} \prec \boldsymbol{y}$ ($\boldsymbol{x}$ dominates $\boldsymbol{y}$) if $F_k(\boldsymbol{x}) < F_k(\boldsymbol{y})$ for all $k = 1, \ldots, N_{obj}$ [2].

The proposed design approach can be summarized as follows:

1. Sample the design space and acquire the $\boldsymbol{R}_c$ data;
2. Construct the kriging interpolation model $\boldsymbol{R}_{s.KR}$ (cf. (1));
3. Obtain Pareto front by optimizing $\boldsymbol{R}_{s.KR}$ using MOEA;
4. Evaluate high-fidelity model $\boldsymbol{R}_f$ at selected locations along the front obtained in 3;
5. Update the co-kriging surrogate $\boldsymbol{R}_{s.CO}$ (cf. (2));
6. Update Pareto front by optimizing $\boldsymbol{R}_{s.CO}$ using MOEA;
7. If termination condition is not satisfied go to 4; else END

Here, the surrogate is created at the level of objectives, which are easier to model than antenna reflection/gain responses. We use a multi-objective EA with fitness sharing, Pareto-dominance tournament selection and mating restrictions [2]. Typically, ~10 high-fidelity model evaluations are used in Step 4, and the number of iterations necessary to converge is 2 to 3. Our convergence criterion is the maximum distance between the Pareto front estimated in 6 and the sampled $\boldsymbol{R}_f$ data (here, we use 0.5 dB for reflection objective).

III. VERIFICATION EXAMPLES

A. UWB Monopole

Consider a UWB monopole shown in Fig. 1. The antenna is energized through a 50 ohm coaxial input. No extra matching circuitry is used here. Design variables are $\boldsymbol{x} = [z_1\ z_2\ r_1]^T$ mm. The design objectives are: (i) to minimize antenna reflection over the frequency band 3 GHz to 10 GHz, and (ii) to minimize antenna size defined as the maximum dimension out of vertical and lateral ones, $S(\boldsymbol{x}) = \max\{2r_2, z_1 + z_2 + r_2\}$, where $r_2 = (r_1^2 - (z_1 + z_2)^2)^{1/2}$ is the radius of the hemisphere terminating the monopole. The high-fidelity model of the antenna is simulated in CST Microwave Studio [5] (~1,400,000 mesh cells, evaluation time 23 minutes). The coarse-discretization model $\boldsymbol{R}_{cd}$ is also simulated in CST (~33,000 mesh cells, 33 s). Figure 2 shows the initial and final Pareto front obtained using the methodology of Section II, as well as its verification using several high-fidelity model designs. The total multi-objective design cost corresponds to about 44 evaluations of the high-fidelity model ($600 \times \boldsymbol{R}_c \approx 14 \times \boldsymbol{R}_f$ to construct the initial kriging surrogate, and $30 \times \boldsymbol{R}_f$ for three iterations of the surrogate enhancement).

B. Planar Yagi Antenna

The Yagi antenna of interest (layout shown in Fig. 3) comprises a driven element fed by a coplanar strip-line, director, and microstrip balun. The substrate is a 0.635 mm thick Rogers RT6010. The antenna is fed with 50 ohm microstrip. Design variables are $\boldsymbol{x} = [s_1\ s_2\ v_1\ v_2\ u_1\ u_2\ u_3\ u_4]^T$. Both the high-fidelity model $\boldsymbol{R}_f$ (~1,400,000 mesh cells, simulation time 36 min) and the low-fidelity model $\boldsymbol{R}_{c,a}$ (~100,000 mesh cells, simulation time 90 s) are evaluated in CST [5]. There are two design objectives: (i) minimization of antenna reflection, and (ii) maximization of average end-fire gain, both within 10-11 GHz bandwidth. Figure 4 shows the initial and final Pareto front obtained using the methodology of Section II, as well as its verification using several high-fidelity model designs. The total multi-objective design cost corresponds to about 41 evaluations of the high-fidelity model ($500 \times \boldsymbol{R}_c \approx 21 \times \boldsymbol{R}_f$ to construct the initial kriging surrogate, and $20 \times \boldsymbol{R}_f$ for two iterations of the surrogate enhancement).

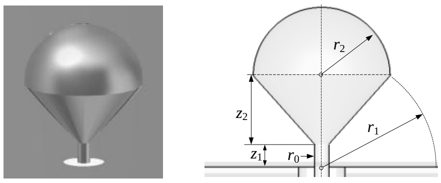

Fig. 1. UWB monopole: 3D view and the cut view.

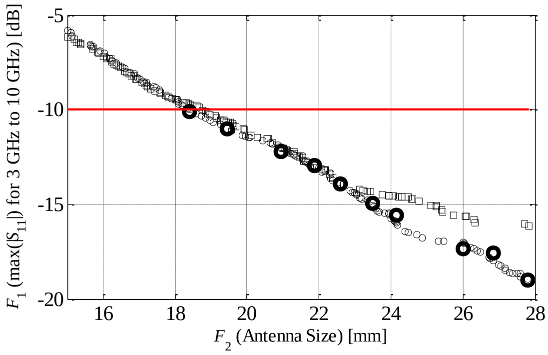

Fig. 2. UWB monopole: initial Pareto front (□), final Pareto front obtained after 3 iterations of the proposed methodology (o), selected high-fidelity model designs (**o**) evaluated and plotted for verification.

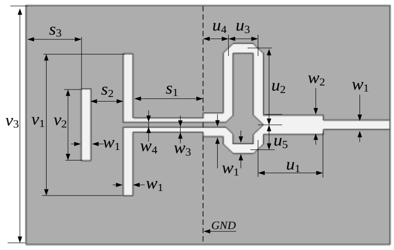

Fig. 3. Planar Yagi antenna: layout.

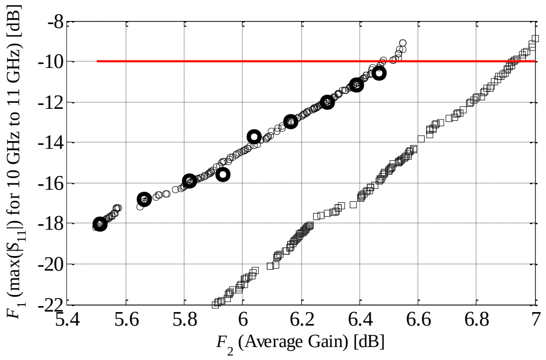

Fig. 4. Planar Yagi: initial Pareto front (□), final Pareto front obtained after 2 iterations of the proposed methodology (o), selected high-fidelity model designs (**o**) evaluated and plotted for verification.

REFERENCES

[1] B. Aljibouri, E.G. Lim, H. Evans, A. Sambell, "Multiobjective genetic algorithm approach for a dual-feed circular polarised patch antenna design," *Electronic Letters*, vol. 36, no. 12, pp. 1005-1006, 2000.

[2] C.M. Fonseca, „Multiobjective genetic algorithms with applications to control engineering problems," PhD thesis, Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, UK, 1995.

[3] M.C. Kennedy and A. O'Hagan, "Predicting the output from complex computer code when fast approximations are available", *Biometrika*, vol. 87, pp. 1-13, 2000.

[4] A.I.J. Forrester and A.J. Keane, "Recent advances in surrogate-based optimization," *Prog. in Aerospace Sc.*, vol. 45, no. 1-3, pp. 50-79, 2009.

[5] CST Microwave Studio, ver. 2012, CST AG, Bad Nauheimer Str. 19, D-64289 Darmstadt, Germany, 2012.