Cover page

Title: Practical reliability-based calculation tool for the post-fire assessment of concrete beams

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ABSTRACT

If a structure has been able to maintain stability during fire exposure, the residual load-bearing capacity of the structural elements after fire should be determined when deciding upon the further use of the structure. Since adequate safety is a primary requirement for all structures and since many uncertainties are associated with the post-fire assessment, only a reliability-based assessment can be acceptable for real-life applications. In this contribution an easy-to-use reliability-based tool is presented for the post-fire assessment of the load-bearing capacity of concrete beams.

INTRODUCTION

Although fire is a very severe load condition for structures, concrete elements generally have a good fire resistance and rarely collapse during fire [1]. Consequently, after fire exposure the question of the residual load-bearing capacity arises: can the structure be used without repair or rehabilitation, or should the structure be demolished or repaired?

As exposure to elevated temperatures may result in permanent damage to the concrete and reinforcement ([1]-[3]), the maximum service load may be significantly reduced. Current practice focusses on destructive and non-destructive testing to assess the concrete degradation due to high temperatures [4]. However, test results indicate a significant scatter of the residual mechanical properties for a given maximum temperature [1], and even prior to fire large uncertainties may exist with respect to the strength characteristics and geometry (e.g. concrete cover) of concrete elements.

For the design of new structures according to the Eurocodes, these uncertainties are taken into account through a semi-probabilistic methodology where characteristic values for the mechanical properties are combined with partial safety factors to provide an adequate level of safety [5]. In EN 1990 [6] the target reliability index β_t for normal structures is 3.8 (for a 50 year reference period).

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A similar reliability-based approach should be used when determining the maximum allowable service load after fire exposure in order to ensure that the structure has the same structural reliability for continued use as a new structure. One possible approach would be to perform a probabilistic evaluation using for example the assessment method presented in [7]. However, in practice these fully probabilistic calculations are too complex and time-consuming for most projects. In this contribution these difficulties are overcome by introducing a simplified reliability-based assessment method for determining the maximum service load for concrete beams after fire exposure. The methodology presented here is an extension and improvement of a concept initially presented by the authors in [8].

THE ASSESSMENT INTERACTION DIAGRAM (AID)

The simplified methodology is based on the application of what the authors call an 'assessment interaction diagrams' (AID). The AID gives a visual representation of the maximum allowable load ratio χ which corresponds with a specific target reliability index β_t, where χ is defined by equation (1) with Q_k the characteristic value of the imposed load effect and G_k the characteristic value of the permanent load effect. In most situations the permanent load G_k can easily be determined and can be considered unaffected by the fire exposure. Consequently, assessing the maximum allowable load after fire exposure comes down to calculating the maximum allowable characteristic value of the imposed load effect $Q_{k,max}$. The AID corresponding with β_t = 3.8 (50 year reference period) is given in Figure 1.

$$\chi = \frac{Q_k}{Q_k + G_k} \tag{1}$$

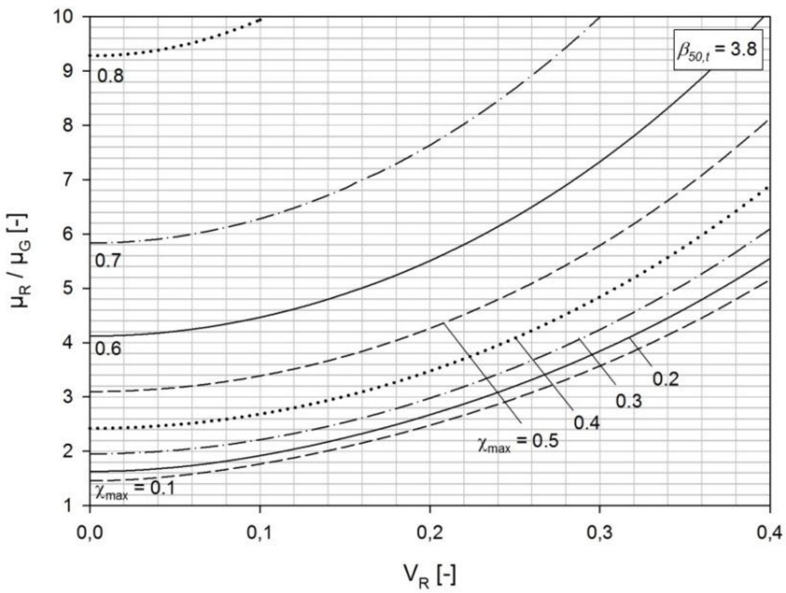

Figure 1. Assessment interaction diagram for β_t = 3.8 (50 year reference).

The AID given by Figure 1 is based on equation (2), considering a Gumbel distribution for the imposed load Q, a normal distribution for the permanent load G, lognormal distributions for the model uncertainties K_R and K_E, and a lognormal distribution for the resistance effect R. All distributions have been chosen in accordance with [9].

$$Z = K_R R - K_E (G + Q) \tag{2}$$

For given combinations of μ_R / μ_G and V_R the AID provides the maximum allowable load ratio χ_{max}, with V_R the coefficient of variation of the resistance effect, μ_R the mean resistance effect, and μ_G the mean value of the permanent load (which can be assumed equal to G_k in accordance with [8] and can easily be determined). As the permanent load effect G_k is assumed to be known, the maximum allowable imposed load Q_k is given by equation (3):

$$Q_{k,\max} = \frac{\chi_{\max}}{1 - \chi_{\max}} G_k \tag{3}$$

ANALYTICAL FORMULAS FOR μ_R AND V_R

In order to apply the AID for the post-fire assessment of concrete beams, the mean value μ_R and coefficient of variation V_R of the resistance effect R have to be determined. A common method to evaluate the response of concrete structures exposed to fire is to neglect the strength loss of the concrete below 500°C and to assume complete loss of strength of concrete above 500°C. This simplified method is allowed by EN 1992-1-2 [10] for the design of concrete structures exposed to fire (i.e. during fire) and has been applied by Kodur et al. [7] for the post-fire assessment of concrete columns. The concept of this isotherm method is illustrated by Figure 2.

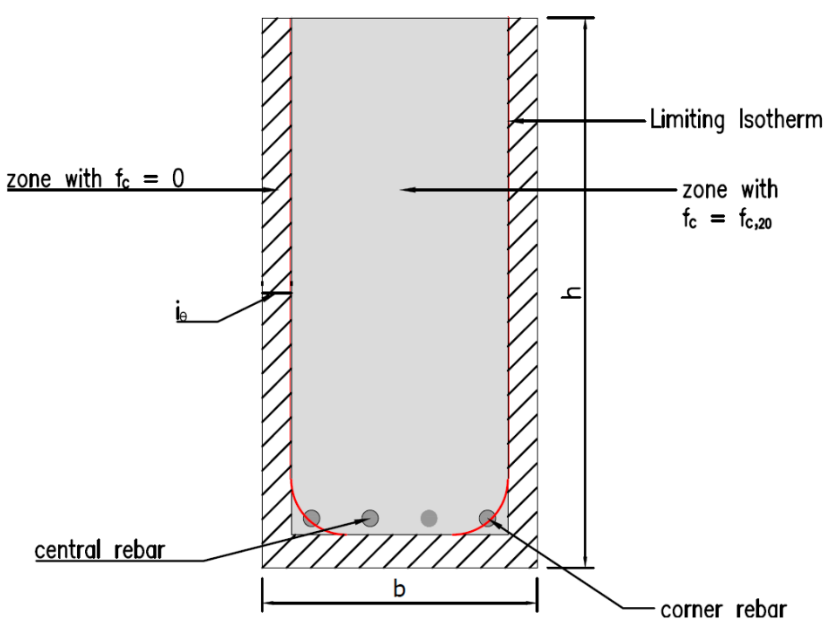

Figure 2. Conceptual visualization of the limiting isotherm method for the concrete compressive strength

The beam in Figure 2 is assumed to be exposed to fire from three sides. Applying the concept of a limiting isotherm for the concrete compressive strength f_c, the residual bending capacity after fire exposure is given by equation (4), with $F_{s,res}$ the sum of the residual yield force of the tensile reinforcement bars, h the beam height, c the concrete cover, Ø the reinforcement bar diameter, and i_θ the depth of the limiting isotherm.

$$M_{R,res} = F_{s,res}\left(\left(h - c - \frac{\varnothing}{2}\right) - 0.5\frac{F_{s,res}}{\left(b - 2i_\theta\right) f_{c,20}}\right) \tag{4}$$

If all reinforcement bars have the same diameter and can be considered to have attained the same maximum temperature θ_{max}, or if an averaged residual yield stress is applied, $F_{s,res}$ is given by equation (5), with A_s the total reinforcement area, $k_{fy,res}$ the reduction factor for the residual reinforcement yield stress, and $f_{y,20}$ the initial 20°C reinforcement yield stress.

$$F_{s,res} = A_s k_{fy,res} f_{y,20} \tag{5}$$

Applying Taylor approximations, the mean value μ_R can be approximated by (6), while the standard deviation σ_R is approximated by (7), and V_R is given by σ_R / μ_R. The constituents S_1 to S_9 contributing to σ_R are given by equations (8) to (16). In the equations below μ_{KT} and σ_{KT} are the mean value and standard deviation of the total model uncertainty K_T, defining the resistance R of the limit state equation as $K_T{\cdot}M_{R,res}$.

$$\mu_R \approx R\left(\bar{\mu}\right) = \mu_{KT}\mu_{As}\mu_{kfy,res}\mu_{fy,20}\left(\mu_h - \mu_c - \frac{\mu_\varnothing}{2} - 0.5\frac{\mu_{As}\mu_{kfy,res}\mu_{fy,20}}{\left(\mu_b - 2\mu_{i\theta}\right)\mu_{fc,20}}\right) \tag{6}$$

$$\sigma_R \approx \sqrt{\sum_{X_i}\left(\frac{\partial R\left(\bar{\mu}\right)}{\partial X_i}\right)^2 \sigma_{X_i}^2} = \sqrt{\sum_{i=1}^{9} S_i} \tag{7}$$

$$S_1 = \left(\mu_{As}\mu_{kfy,res}\mu_{fy,20}\left(\mu_h - \mu_c - \frac{\mu_\varnothing}{2}\right) - 0.5\frac{\mu_{As}^2\mu_{kfy,res}^2\mu_{fy,20}^2}{\left(\mu_b - 2\mu_{i\theta}\right)\mu_{fc,20}}\right)^2 \sigma_{KT}^2 \tag{8}$$

$$S_2 = \left(\mu_{KT}\mu_{kfy,res}\mu_{fy,20}\left(\mu_h - \mu_c - \frac{\mu_\varnothing}{2}\right) - \mu_{KT}\frac{\mu_{As}\mu_{kfy,res}^2\mu_{fy,20}^2}{\left(\mu_b - 2\mu_{i\theta}\right)\mu_{fc,20}}\right)^2 \sigma_{As}^2 \tag{9}$$

$$S_3 = \left(\mu_{KT}\mu_{As}\mu_{kfy,res}\mu_{fy,20}\right)^2 \sigma_h^2 \tag{10}$$

$$S_4 = \left(-\mu_{KT}\mu_{As}\mu_{kfy,res}\mu_{fy,20}\right)^2 \sigma_c^2 \tag{11}$$

$$S_5 = \left(\mu_{KT}\mu_{As}\mu_{fy,20}\left(\mu_h - \mu_c - \frac{\mu_\varnothing}{2}\right) - \mu_{KT}\frac{\mu_{As}^2\mu_{kfy,res}\mu_{fy,20}^2}{\left(\mu_b - 2\mu_{i\theta}\right)\mu_{fc,20}}\right)^2 \sigma_{kfy,res}^2 \tag{12}$$

$$S_6 = \left(\mu_{KT}\mu_{As}\mu_{kfy,res}\left(\mu_h - \mu_c - \frac{\mu_{\varnothing}}{2} \right) - \mu_{KT}\frac{\mu_{As}^2\mu_{kfy,res}^2\mu_{fy,20}}{\left(\mu_b - 2\mu_{i\theta}\right)\mu_{fc,20}} \right)^2 \sigma_{fy,20}^2 \tag{13}$$

$$S_7 = \left(0.5\mu_{KT}\frac{\mu_{As}^2\mu_{kfy,res}^2\mu_{fy,20}^2}{\left(\mu_b - 2\mu_{i\theta}\right)\mu_{fc,20}^2} \right)^2 \sigma_{fc,20}^2 \tag{14}$$

$$S_8 = \left(0.5\mu_{KT}\frac{\mu_{As}^2\mu_{kfy,res}^2\mu_{fy,20}^2}{\mu_b^2\mu_{fc,20}} \right)^2 \sigma_b^2 \tag{15}$$

$$S_9 = \left(-0.25\mu_{KT}\frac{\mu_{As}^2\mu_{kfy,res}^2\mu_{fy,20}^2}{\mu_{i\theta}^2\mu_{fc,20}} \right)^2 \sigma_{i\theta}^2 \tag{16}$$

EVALUATION OF THE BASIC VARIABLES

The mean value μ and standard deviation σ of the basic variables can be determined using data from inspections or can be based on literature data. An overview of standard values for the mean μ and standard deviation σ, or coefficient of variation V are listed in Table 1, based on [9], with the subscript *nom* indicating the nominal design value. The model uncertainty K_T has been calibrated comparing the results of the simplified formula (4) with numerical calculations based on [11] and also incorporates K_R and K_E. In case tests are performed to estimate for example the initial (20°C) concrete compressive strength, the sample mean and standard deviation are an estimation of $\mu_{fc,20}$ and $\sigma_{fc,20}$ respectively, and these values should be used instead of the default literature data given in Table I.

The depth i_θ of the limiting isotherm can be estimated directly using test results (for example using the methodology described in [4]), or can be based on an estimation of the fire severity by a fire expert. The latter method has the advantage that no tests are required which can be very valuable for an early preliminary evaluation of the safety of the structure immediately after the fire. When the fire expert assigns different probabilities p_{tE} to different fire severities t_E, the depth of the limiting isotherm $i_{\theta,tE}$ can be evaluated for each of these fire severities using a simple thermal calculation tool. If this thermal calculation is considered too complex or time-consuming, the fire expert can assign probabilities to equivalent ISO 834 fire durations for which temperature diagrams are listed in EN 1992-1-2 [10], allowing for an easy evaluation of the associated depth $i_{\theta,tE}$. Once both the probabilities p_{tE} and the depths $i_{\theta,tE}$ are evaluated, $\mu_{i\theta}$ and $\sigma_{i\theta}$ are given by equations (17) and (18). Note that it is possible to assign a probability of 1 to a single conservatively assessed fire severity t_E. This results in $\mu_{i\theta} = i_{\theta,tE}$ and $\sigma_{i\theta} = 0$.

$$\mu_{i\theta} = \sum_{t_E} i_{\theta,tE}\, p_{tE} \tag{17}$$

$$\sigma_{i\theta} = \sqrt{\sum_{tE}\left(i_{\theta,tE} - \mu_{i\theta}\right)^2 p_{tE}} \tag{18}$$

TABLE I. PROBABILISTIC MODELS FOR BASIC VARIABLES, BASED ON [9].

Symbol	Dim.	μ	σ	V
$f_{c,20}$	MPa	$\frac{f_{ck}}{1-2V_{fc}}$	-	0.15
$f_{y,20}$	MPa	$\frac{f_{yk}}{1-2V_{fy}}$	-	0.07
A_s	mm²	$A_{s,nom}$	-	0.02
c	mm	c_{nom}	5	-
h	mm	h_{nom}	5	-
b	mm	b_{nom}	5	-
K_T	-	1.06	-	0.07

The mean value $\mu_{kfy,res}$ of the reduction factor $k_{fy,res}$ for the reinforcement yield stress is given by equations (19) to (21). First $k_{fy,res}$ is evaluated for each of the reinforcement bars for different fire severities t_E and a discrete set of possible positions x_i, y_i. The considered positions x_i, y_i are given in Table II for corner reinforcement bars and central reinforcement bars together with their associated occurrence probabilities based on a Beta distribution of the concrete cover. Subsequently, these values for $k_{fy,res}$ are combined by equation (19) across the different fire severities t_E, after which equation (20) integrates across the different positions x_i, y_i. Finally, equation (21) takes the average of the different rebars to obtain an average $k_{fy,res}$ as in equation (5). Note that for many practical situations it suffices to evaluate $k_{fy,res,j}$ for a single corner rebar and a single central rebar and apply these values for other rebars.

For the standard deviation $\sigma_{kfy,res}$ a conservative assessment is made by considering only the corner rebar as this rebar experiences the highest variability of the reduction factor $k_{fy,res}$. The final value for $\sigma_{kfy,res}$ is calculated through equations (22) and (23). The model for $k_{fy,res}$ as a function of the maximum attained reinforcement temperature θ is based on [1] and [3] and is illustrated by Figure 3, as introduced in detail in [8].

A more straightforward but more conservative alternative method is to evaluate both $\mu_{kfy,res}$ and $\sigma_{kfy,res}$ for the corner rebar considering a single conservative axis position $\{\mu_c - 2\sigma_c + Ø/2; \mu_c - 2\sigma_c + Ø/2\}$. This further conservative simplification can be partially compensated by a change of the mean value of the model uncertainty K_T.

$$\mu_{kfy,res,j}\left(x_i, y_i\right) = \sum_{t_E} k_{fy,res}\left(t_E, x_i, y_i\right) p_{tE} \tag{19}$$

$$\mu_{kfy,res,j} = \sum_{xiyi} \mu_{kfy,res,j}\left(x_i, y_i\right) p_{xiyi} \tag{20}$$

$$\mu_{kfy,res} = \frac{\sum_j \mu_{kfy,res,j}}{n} \tag{21}$$

$$\sigma_{kfy,res}\left(x_i, y_i\right) = \sqrt{\begin{array}{l}\sum_{tE}\left(k_{fy,res}\left(t_{E,i}, x_i, y_i\right) - \mu_{kfy,res}\left(x_i, y_i\right)\right)^2 p_{tE,i} \\ +\sum_{tE} \sigma^2_{kfy,res}\left(t_{E,i}, x_i, y_i\right) p_{tE,i}\end{array}} \tag{22}$$

$$\sigma_{kfy,res} = \sqrt{\begin{array}{l}\sum_{xiyi}\left(\left(\mu_{kfy,res}\left(x_i, y_i\right) - \mu_{kfy,res,j}\right)^2 p_{xiyi}\right) \\ + \sum_{xiyi} \sigma^2_{kfy,res}\left(x_i, y_i\right) p_{xiyi}\end{array}} \tag{23}$$

TABLE II. POSITIONS (x_i, y_i) AND ASSOCIATED PROBABILITY p_{xiyi} FOR CORNER AND CENTRAL REINFORCEMENT

x_i [mm]	y_i [mm]	p_{xiyi}
CORNER REINFORCEMENT		
$\mu_c - 2\sigma_c + Ø/2$	$\mu_c - 2\sigma_c + Ø/2$	0.03
$\mu_c - 2\sigma_c + Ø/2$	$\mu_c + 2\sigma_c + Ø/2$	0.03
$\mu_c - 2\sigma_c + Ø/2$	$\mu_c + Ø/2$	0.11
$\mu_c + Ø/2$	$\mu_c - 2\sigma_c + Ø/2$	0.11
$\mu_c + 2\sigma_c + Ø/2$	$\mu_c - 2\sigma_c + Ø/2$	0.03
$\mu_c + Ø/2$	$\mu_c + Ø/2$	0.69
CENTRAL REINFORCEMENT		
$\mu_c + Ø/2$	$\mu_c - 2\sigma_c + Ø/2$	0.17
$\mu_c + Ø/2$	$\mu_c + Ø/2$	0.83

EXAMPLE APPLICATION

After a severe office fire, a conservative assessment by a fire expert indicates an ISO 834 standard fire duration of 90 minutes. Simply supported beams with a height of 800 mm, width of 500 mm, and span of 8 m support the ceiling (i.e. the floor of the story above). Table IV gives an overview of the calculated and assessed values for the basic variables. Applying equations (6)-(16), μ_R = 1065 kNm and V_R = 0.20. Considering the layout of the structure the bending moment induced by the permanent load (including self-weight of the beam) is 408 kNm, and therefore μ_R / μ_G = 2.61. Applying the AID of Figure 1, χ_{max} is 0.17, and consequently $M_{Qk,max}$ = 69.4 kNm. For the specific building 8 m of ceiling width is transferred to the beam, and therefore the maximum allowable characteristic value of the imposed load on the floor above is 1.08 kN/m². If the required value of the imposed load on the floor above the fire compartment is larger than 1.08 kN/m², the beams should be strengthened.

TABLE IV. PARAMETERS FOR THE INVESTIGATED BEAM (t_E = 90 min)

Symbol	Dim.	µ	σ
$f_{c,20}$	MPa	57.1	8.6
$f_{y,20}$	MPa	581.4	40.7
A_s	mm²	2513	50
c_v	mm	20	5
c_h	mm	30	5
h	mm	800	5
b	mm	500	5
i_{500}	mm	29	0
$k_{fy,res}$	-	0.92	0.08
K_T	-	1.06	0.07

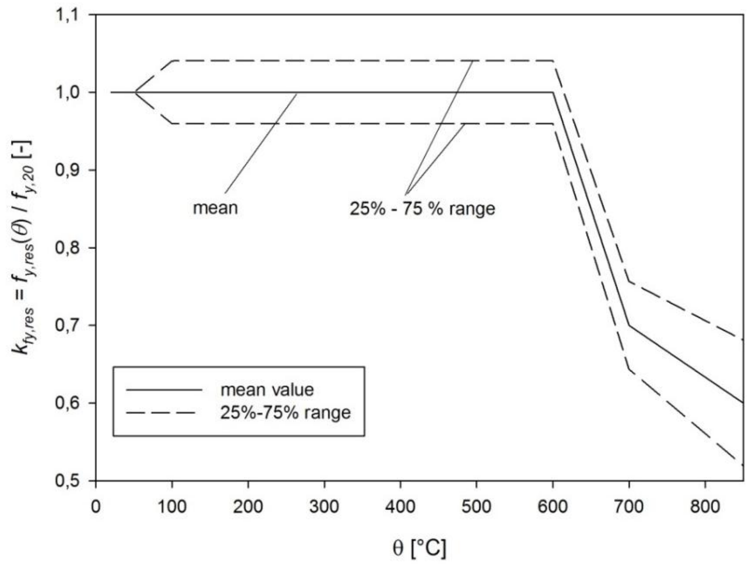

Figure 3. Residual reinforcement yield stress ratio $k_{fy,res} = f_{y,res} / f_{y,20}$ as a function of θ.

CONCLUSIONS

A reliability-based methodology for determining the maximum allowable imposed load on a concrete beam after fire exposure has been presented. The methodology is easy-to-use for practitioners as only simple analytical formulas have to be evaluated. Subsequently, the maximum allowable load is determined by applying pre-calculated graphs, called 'assessment interaction diagrams' (AID). While the method can easily be implemented in normal spreadsheet software its reliability-based background ensures a rational answer to the question if the concrete beam is safe enough for continued use, or whether strengthening is necessary.

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