Multipoint Model Order Reduction using Reflective Exploration

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Summary. Reduced order models obtained by model order reduction methods must be accurate over the whole frequency range of interest. Multipoint reduction algorithms allow to generate accurate reduced models. In this paper, we propose the use of reflective exploration technique for obtaining the expansion points adaptively for the reduction algorithm. At each expansion point the corresponding projection matrix is computed. Then, the projection matrices are merged and truncated based on their singular values to obtain a compact reduced order model.

1 Introduction

Electromagnetic (EM) methods are used for the analysis of complex high-speed systems and usually generate very large systems of equations. Therefore, model order reduction (MOR) techniques are crucial to reduce the complexity of large scale models and the computational cost of the simulations, while retaining the important physical features of the original system [1–3]. Multipoint MOR methods have been developed over the years [1,4,5], which allows to generate accurate reduced models over the whole frequency range of interest. In this paper, the expansion points are selected adaptively using a reflective exploration technique. It is a selective sampling algorithm, where the model is improved incrementally using the best possible data at each iteration, allowing it to propose candidate exploration points [6]. An error-based exploration is performed to find the expansion points. After obtaining the expansion points, the corresponding projection matrices are computed using any of the Krylov based MOR techniques. The projection matrices are then merged and truncated based on their singular values to obtain a compact reduced order model. Then the reduced order models are obtained by congruence transformation using the truncated projection matrix. The technique is validated using a multiconductor transmission line example.

2 Projection Matrix

For this paper, the PRIMA algorithm [3] has been used for obtaining the projection matrices at the expansion points.

For n expansion points we obtain the corresponding projection matrices V_{q_i} for $i = 1, 2, \ldots, n$, then the common projection matrix is defined as:

$$V_{comm} = [V_{q_1}\ V_{q_2}\ \ldots\ V_{q_n}]. \quad (1)$$

The common projection matrix is not truncated using its singular values during the iterative procedure of the adaptive reflective exploration. It is truncated after all the expansion points have been adaptively chosen using reflective exploration.

3 Reflective Exploration

The reflective exploration requires a reflective function to select the expansion points. The reflective function used for the proposed algorithm is the root mean square (RMS) (2) error between the obtained best models:

$$Err_{est}^{(I)} = \sqrt{\frac{\sum_{k=1}^{K_s} \sum_{i=1}^{P_{in}} \sum_{j=1}^{P_{out}} \frac{\left|H_{I,(ij)}(s_k) - H_{I-1,(ij)}(s_k)\right|^2}{|H_{I,(ij)}(s_k)|^2}}{P_{in} P_{out} K_s}} \quad (2)$$

where, K_s, P_{in} and P_{out} are the number of frequency samples considered on a dense grid, input and output ports of the system, respectively. The exploration consists of an adaptive modeling loop and an adaptive sampling loop.

1. Adaptive Modeling Loop: The algorithm starts with two expansion points selected at $[\omega_{min}, \omega_{max}]$ of the frequency range of interest. The reduced order q at these points is equal to the number of ports of the system. Then with a common projection matrix as explained in Section 2, the reduced model is obtained. Then in the next iteration again the projection matrix is computed for a reduced order equal to two times the port of the system. If the RMS error between the two best models (i.e., the model obtained in the I^{th} and the $(I-1)^{th}$ iteartion) exceeds a certain threshold, then the reduced order q is again increased by the number of ports for the respective expansion points.
2. Adaptive Sampling Loop: When the difference in RMS error between the I^{th} and $(I-1)^{th}$, is less than 10%, a new expansion point is selected. For selecting the new expansion point the error per frequency is computed by taking the norm L2, of

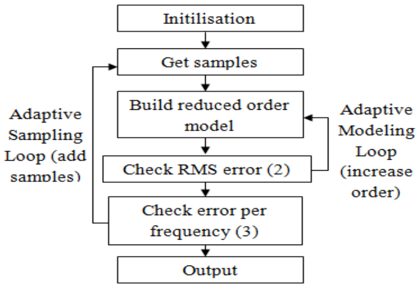

Fig. 1. Flowchart 1: Reflective Exploration.

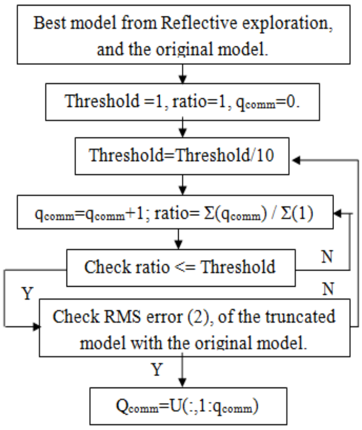

Fig. 2. Flowchart 2:Truncation of the projection matrix.

the frequency response of the best model (H_I) and the original model (H_{act}):

$$Err_{s_k} = norm(H_{act,(ij)}(s_k) - H_{I,(ij)}(s_k)); \quad k = 1, \ldots, K_s, \quad (3)$$

and the frequency at which Err_{s_k} is maximum is considered as the new expansion point.

This process is iteratively repeated until the RMS error between the original frequency response and the reduced model is 0.001. Figure 1 shows the reflective exploration algorithm.

4 Model compacting

After obtaining the best reduced order model from the iterative procedure, it might be possible to further compact the model with the information obtained from the singular values Σ of V_{comm} (1). Figure 2 shows the flowchart for the truncation of the singular values. The projection matrix Q_{comm} with congruence transformation gives the reduced state-space matrices of order q_{comm}.

5 Numerical Results

A multiconductor transmission line described by an original state-space of order 1202 and 4 ports is con-

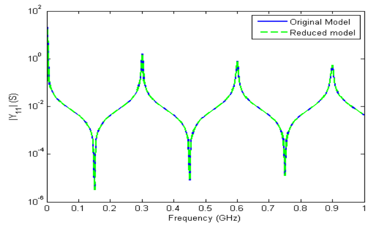

Fig. 3. Magnitude of Y_{11}.

sidered. As described in Section 3, 4 expansion points are chosen adaptively. Table 1 gives the dimension of

Table 1. Dimension of the Original and Reduced Model.

Models	Dimension
Original	1202
Model after reflective exploration	64
Model after compacting	42

the reduced models. The reduced model after truncation has an RMS error of $4.128e-4$ when evaluted over a dense grid of $K_s = 200$ frequency samples as shown in Figure 3.

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