

STRUCTURAL HEALTH MONITORING USING TRANSMISSIBILITY COHERENCE

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Abstract: Transmissibility has been commonly used in structural health monitoring, during the last decades; transmissibility has been both put forward in theory and applications. However, the use of transmissibility is still a profound and uncompleted work. In this study, a general summary of transmissibility is given, and in addition, transmissibility coherence is put forward to system identification for extracting natural frequencies in theory with recalling the transmissibility for system identification.

Keywords: Transmissibility; Structural health monitoring; Transmissibility coherence; system identification

1 INTRODUCTION

In the past decades, structural health monitoring (SHM) has become a multidisciplinary research focus to the scientific communities that attracts a lot of attention, due to the fact that the engineering structures are commonly designed with more complexity and more sophisticated newly invented material productions, and within daily use the structures are usually and generally applied with higher operational loads and unexpected loadings, and are demanding for longer lifecycle periods. Hence, numerous mechanical, civil and aerospace engineering researchers extensively developed vast of approaches for analysing the structural states that means to evaluate whether the structure is damaged or not, in order to prevent the anticipated damage, which might cause a vast loss in human daily lives in an inevitable way. Various categories of SHM procedures have been developed for analyzing the structural states. Vibration-based, strain-based, Electrical Impedance-based, probability-based, statistical based methods and so on have been studied and published a quantity of papers, reports and books. Literature review about the SHM can be found in [1].

For SHM, methodologies can be divided into two categories: physical model and data model/statistical model. For physical model, normally finite element analysis (FEA) is undertaken and different levels and patterns, such as fatigue in adhesively bonded joints [2-3]; crack initiation [4]; fretting wear [5], are numerically analyzed in order to provide a pre-design assessment as a reference for further analysis, especially in fatigue life-cycle prediction [4]. Generally, FEA analysis is validated with experimental results. And model updating intends to minimize the differences between FEA and experimental responses [6-8] by optimizing the FEA model. In this direction, due to the low cost and good performance in analyzing real engineering problems, other numerical techniques have been developed, e.g. boundary element methods (BEM), mesh free approaches, extended finite element methods (XFEM), isogeometric analysis (IGA).

On the other hand, for data model, the traditional modal testing is quite commonly used in structural dynamic analysis [9], like experimental modal analysis (EMA). In modal analysis, the mode shape derivatives, for instance, first derivative (rotations), second derivative (curvatures) and third and higher derivatives were utilized for damage localization. Frequency response function (FRF) is another parameter commonly used in EMA. In addition, the strain before and after damage in the structure is also a direction in SHM. However, EMA requires the measurement of excitation while this is arduous in real engineering as normally the engineering structures are commonly subjected to complex loading and environmental uncertainties. Then, new methodology is pursued due to the demanding from engineering application.

In SHM, transmissibility has been widely studied in the past decades, and it has been used for damage detection, localization, quantification and so on [10-17]. On the other hand, it has also been used for FRF estimation [18], force reconstruction [19], system identification [20-21] and so on.

This study tries to extend the transmissibility coherence (TC) for natural frequency extraction, and henceforth, to give a general summary in transmissibility estimation.

2 THREITICAL DERIVATION

In structural dynamics, for a linear multiple-degree-of-freedom (MDOF) system shown in Figure 1, the dynamic equilibrium equation can be written by the well-known second order differential equation,

$$\mathbf{M}\ddot{\mathbf{x}}(\mathbf{t}) + \mathbf{C}\dot{\mathbf{x}}(\mathbf{t}) + \mathbf{K}\mathbf{x}(\mathbf{t}) = \mathbf{f}(\mathbf{t}) \quad (1)$$

where \mathbf{M} , \mathbf{C} and \mathbf{K} are the mass, damping, and stiffness matrices of the system, respectively, $\mathbf{f}(\mathbf{t})$ is the input force vector and $\mathbf{x}(\mathbf{t})$ contains the responses of each degree-of-freedom (DOF) of the system.

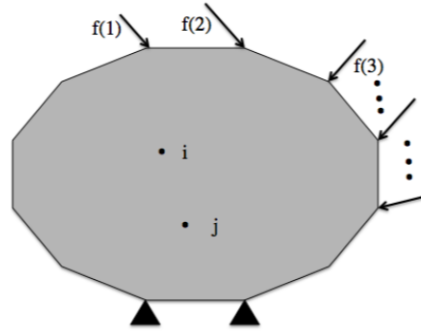


Figure 1. A linear multiple-degree-of-freedom system.

Herein, for a harmonic applied force at a given coordinate, the transmissibility between point i and a reference point j can be defined as

$$T_{(i,j)}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} \quad (2)$$

where X_i and X_j are the complex amplitudes of the system responses, $x_i(t)$ and $x_j(t)$, respectively, and ω is the frequency.

In order to calculate the transmissibility, no matter in real engineering or experiment analysis, apart from its direct extracting from the two responses, it can be derived in several ways, for instance:

$$T_{(i,j)}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{X_i(\omega) \cdot X_i(\omega)}{X_j(\omega) \cdot X_i(\omega)} = \frac{G_{(i,i)}(\omega)}{X_{(j,i)}(\omega)} \quad (3)$$

$$T_{(i,j)}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{X_i(\omega) \cdot X_j(\omega)}{X_j(\omega) \cdot X_j(\omega)} = \frac{G_{(i,j)}(\omega)}{X_{(j,j)}(\omega)} \quad (4)$$

where G means the auto- or cross- spectrum. Herein, Equation (3) and (4) can be compared with the FRF estimation for avoiding noise influence, then transmissibility coherence can be drawn out. Detailed analysis about it will be given in next section.

Besides, for a chosen reference P , when the variable approaches system's v^{th} pole, denoted by λ_v , the following equation is verified with Laplace transform [21] and Fourier transform [20] as

$$\lim_{s \rightarrow \lambda_v} T_{(i,j)}^P(\omega) = \frac{\hat{f}_{(i,v)}}{\hat{f}_{(j,v)}} \quad (5)$$

And its inverse [20, 21], also called inverse transmissibility subtraction function (ITSF) [21] is as

$$D^{-1} T_{(i,j)}^{P_1 P_2} = \frac{1}{D T_{(i,j)}^{P_1 P_2}} = \frac{1}{T_{(i,j)}^{P_1} - T_{(i,j)}^{P_2}} = \frac{G_{(i,P_1)} G_{(j,P_2)}}{G_{(i,P_1)} G_{(j,P_2)} - G_{(i,P_2)} G_{(j,P_1)}} \quad (6)$$

Herein, through the equation above one can identify the natural frequencies via peak picking method. Note that the denominator of the equation above is result of a subtraction, which might cause singularity if the reference is not well chosen or the transform is not well chosen and made. Meanwhile, it can yield more roots than the system real roots, which requires further work in validating the corresponding frequencies.

Thirdly, all the references like j and P (P_1, P_2, \dots) should be paid more attention, otherwise it would be possible to miss some system roots. One possible solution is to use average normalization ITSF [21], or to take all the ITSFs into consideration directly.

On the other hand, if transmissibility is directly estimated using two outputs, i.e. not taking the FRFs into account, referring to the conception of coherence, TC can be also derived solely by using the auto- and cross- spectrum of the two responses signals [12, 15]. And TC will be expressed as

$$g_{TC}^2 = \frac{T_{1(i,j)}(W)}{T_{2(i,j)}(W)} = \frac{G_{(i,j)}(W)}{G_{(j,i)}(W)} / \frac{G_{(i,i)}(W)}{G_{(j,i)}(W)} = \frac{G_{(i,j)}(W)G_{(j,i)}(W)}{G_{(j,i)}(W)G_{(i,i)}(W)} = \frac{|G_{(i,j)}(W)|^2}{G_{(j,i)}(W)G_{(i,i)}(W)} \quad (7)$$

As the coherence is a squared magnitude, TC is higher than zero. And basically, TC reveals the coherence of two outputs, i.e. it indicates the interrelation of the dynamic characteristics of two outputs.

Herein, note that the TC might be used for system identification, i.e. to identify the resonant frequencies. Recalling the Equation (6), and by introducing TC into it, one can get

$$D^{-1}T_{(i,j)}^{i,j} = \frac{1}{T_{(i,j)}^i - T_{(i,j)}^j} = \frac{1}{T_{2(i,j)} - T_{1(i,j)}} = \frac{1}{T_{2(i,j)}} \times \frac{1}{1 - \frac{T_{1(i,j)}}{T_{2(i,j)}}} = \frac{1}{T_{2(i,j)}} \times \frac{1}{1 - g_{TC}^2} \quad (8)$$

Herein, one might use TC in the resonant frequencies estimation; however, further investigation should be conducted for a better understanding.

3 CONCLUSIONS

In this paper, transmissibility estimation flowchart is given, and transmissibility is put forward to system identification in theory. We presented a general summary of transmissibility, and transmissibility coherence and their applications to system identification for extracting natural frequencies. However, further investigation is needed to have a better analyzing of their performance.

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