

TOWARDS THE UNDERSTANDING OF VARIABLE AMPLITUDE FATIGUE**Nahuel Micone¹, Wim De Waele¹, Saosometh Chhith¹***¹Department of mechanical, Construction and Production
Technologiepark Zwijnaarde 903, Belgium
Tel.: +32 9 331 0472, E-mail: nahuel.micone@ugent.be*

Abstract: Fatigue life estimation is of high importance during the design stage of a machine or component. Basic fatigue calculations are made based on the use of an S-N curve. As far as constant amplitude loads are applied, this approach works well. However, most part of components in service are subjected to a variable amplitude load spectrum. In this case, linear approaches for fatigue life estimation can lead to over conservative results, which in other words means a heavier and more expensive machine. To further investigate the effect of (complex) service spectra (measured or statistically calculated), simpler load variations must be studied. This paper aims to show the general trend of these events and suggest the underlying physical phenomena behind load and interaction effects. As it will be highlighted, overloads are frequent in a spectrum and they are believed to be responsible for retardation effects. The plasticity induced crack closure mechanism is the most profound explanation for them to occur.

Keywords: Variable Amplitude; Overload; Underload; Fatigue; Steel

INTRODUCTION

In almost every situation that involves fatigue, the nature of the loads to which a component is subjected, is time varying. The study of different “in service” time histograms has allowed engineers to statically describe them through loading spectrums. With the need for optimum light-weight design, originally the aircraft industry was the main driver for these efforts [1]. For the agricultural industry the transmission and drive axle seems to be of major concern, since most part of the actual literature focuses on these components. For this particular application, plowing speed and soil condition are some of the parameters of importance. These factors are combined and a linear damage rule (Miner’s rule) is normally used to calculate fatigue life. However, this approach has been reported to be (in certain situations) over or under conservative [2], which means that the variable amplitude loads could lead to a non-linear fatigue behaviour of the structure. The main reasons for this to occur are the load and interaction effects and are explained in the following.

Fatigue life is divided in three stages named, nucleation, initiation and crack propagation. These phases are loading dependent and therefore a change in the load level also means a change in the damage factor (D) which characterize the reduction (or extension) of the fatigue life. This is known as the load effect or load dependency. Fatigue damage variations are also dependent on the fatigue damage condition of the material as caused by previous cycles. In other words how the crack has grown from its initiation until its actual dimension. These effects are called interaction effects.

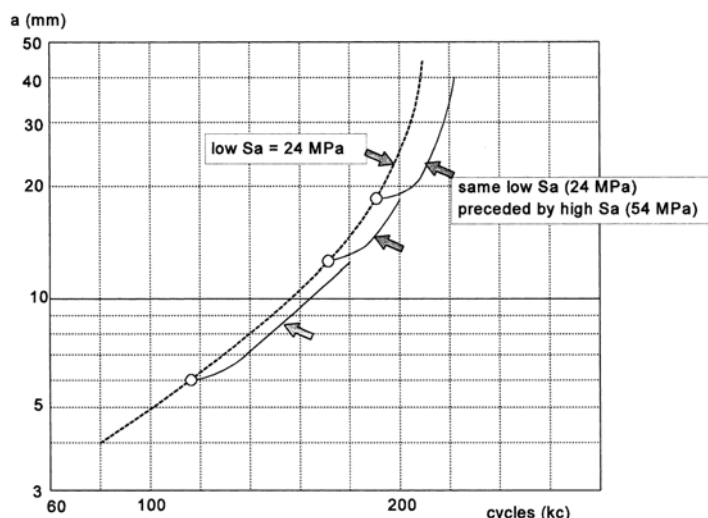


Fig.1 – Crack growth retardation after transitions from a higher to a lower amplitude level [4]

An example of load dependency can be seen in Fig. 2, where the comparison of the crack growth in a constant amplitude test and the introduction of three different moments of a secondary load block with a higher load range is displayed. After a number of cycles the first stress amplitude was restituted. Following the transition of a higher to a lower amplitude, the crack growth was retarded. After a certain amount of cycles, both curves were parallel to the original one and the crack growth rate was not delayed again.

DESCRIPTION AND SEQUENCE TYPE

The simulation of spectrums derived from real structures appeared as a requirement to obtain realistic results. However their complexity encouraged researchers to create test programs that are as representative of the reality as possible but with high repeatability and a relatively simple mathematical description. Soon the results of these simplified programs showed a high variety of results depending on the load sequence. To better understand these differences, simplified sequences of loads have been defined (see table 1). These sequences can be split up in three main categories: random/load-service, simple VA load and block load [3]. Table 1 graphically shows the difference between these. Furthermore they are briefly described below.






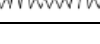




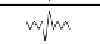



Loading type	Sequence type	Representation	Effect	Reported Materials	References
Random loading	Spectrum		Undefined	Ti and Al alloys	[4-6]
Simple loading	Single OL		Retardation	Ti and Al alloy, SS, structural and vessels steels	[9-14, 18, 20]
	Sequence of OL's		Retardation (higher than OL)	Al alloy, structural and vessels steels	[9, 12, 16, 22]
	Periodic sequence of OL's		Retardation or acceleration	Al alloy, structural and vessels steels	[22, 23, 36]
	Single UL		Acceleration	Al alloy, structural and vessels steels	[20, 24]
	Sequence of UL's		Retardation	Low carbon steel, Ti and Al alloys	[24]
	Periodic sequence of UL's		Acceleration	Al alloy, structural and vessels steels	[23, 27]
	OL-UL		Retardation	Ni-Cr and low carbon steel	[16, 29]
	UL-OL				
	Periodic OL-UL		Retardation	Low carbon steel	[16]
Periodic UL-OL					
Block loading	Low-high		Retardation or acceleration	Ti and Al alloys, Structural steel	[12, 22, 30]
	High-low			Stainless steel, Al alloy, structural and vessels steels	[10, 11, 18, 22, 30]
	Low-high-low			Al alloy	[9]

Table 1 – Overview of VA loading histories

RANDOM LOADING

In the next sections, the term spectrum is used as a generalisation for the power spectral density (PSD). The PSD describes how the power of a signal or time series is distributed over the different frequencies. The power is defined as the squared value of the signal. The integral of the PSD over a given frequency band computes the average power in the signal over that frequency band. In order to describe a function uniquely in the frequency domain, the amplitude and the phase versus the frequency are needed. By using only the PSD, the information of the phase is lost. A given time history has a unique power spectral density. On the other hand, a given power spectral density does not have a unique time history. The reason is that the phase angles are discarded in the power spectral density calculation. However in fatigue analysis, the phase of the signal is not of importance. Therefore, the PSD is a good and compact way to describe a certain characteristic load signal.

Service spectra in different sectors

A lot of research has been performed towards the simplifications that might be done to complex measured spectrums, without affecting the experimentally predicted fatigue life. Several standardised spectra have been

developed or proposed for different industrial sectors. Schijve [5-6] has done extensive research to the effect of a change in a flight-simulation load spectrum on the fatigue crack growth rate (FCGR). In these studies, the gust dominated spectrum TWIST and the manoeuvre-dominated spectrum FALSTAFF have been investigated. Additionally it has been found that the sequence of loads in a spectrum has only a small effect on the FCGR. Another standardised load spectrum called CARLOS describes the loadtime history for several automotive parts. FELIX is a standard used for the load spectrum of helicopter blades. FELIX/28 serves the same purpose, but the spectrum has been reduced for a more beneficial testing time. WISPER has been designed to describe the load spectrum of wind turbine blades. TRANSMISSION and AGRICULT. TRACTOR evaluate the torque and the bending and torsion moment in the tractor transmission and drive axle respectively. A summary of standardised spectra for multiple purposes can be found in [1]. Load spectra are often counted by different methods, such as the level crossing counting method or the rainflow method [7, 8] to evaluate linear damage accumulation. With this end, Miner's rule is applied. However a linear estimation is not always accurate since it does not consider load or interaction effects. For the better understanding of these non-linear effects, simpler loading events must be studied.

SIMPLE LOADING

Single OL

A single overload is reported to cause a retardation (or even arrest) of the crack growth. The description of this process can be characterized by the following parameters (see Fig. 2):

- N_D : delayed number of cycles. It can be divided into N_{D1} and N_{D2} (real and total number of cycles respectively)
- a_D : delayed distance (mm)
- Δa_{OL} : OL-affected crack growth increment (mm)

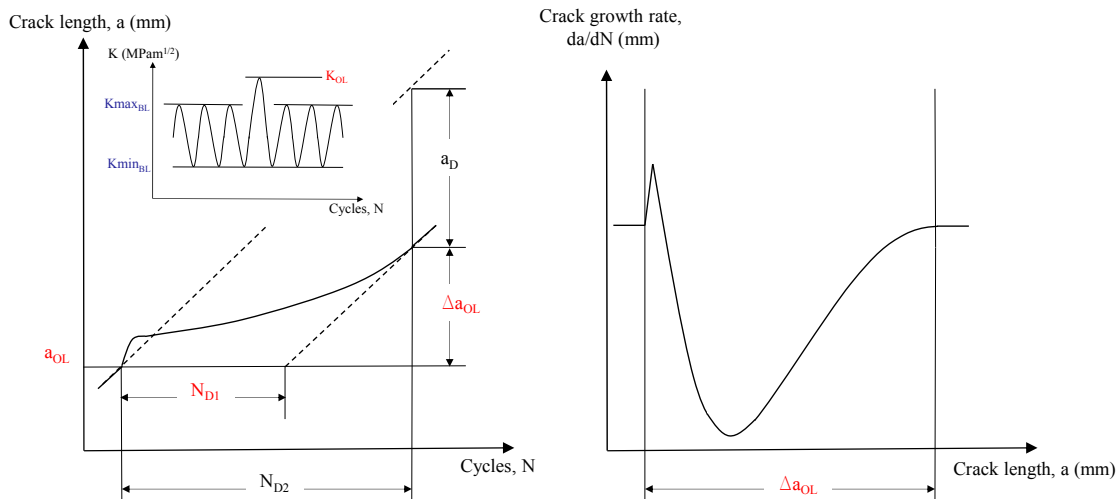


Fig. 2 - Schematic of delayed retardation of crack growth following a single OL in a K-controlled test, [2]

The main parameter influencing the FCGR (or da/dN) is the Over Load Ratio (OLR) defined as:

$$OLR = \frac{K_{OL} - K_{minBL}}{\Delta K_{BL}} \quad (1)$$

Increasing the OLR value will result in an increment of previous defined parameters, and in a lower minimum da/dN level as reported in [2, 9- 21]. A second parameter influencing the FCGR is the R-ratio. When the R-ratio is increased, the retardation effect will diminish [2, 9-17]. Sander [9] investigated more deeply the effect of the base-line loading. He discovered that with an increasing baseline-level loading the retardation effect decreased. The effect of most of the different parameters for a single OL event is summarised on the diagram represented in Fig. 3.

When the baseline-level loading drops below ΔK_{th} , no crack growth is observed. An unstable crack growth occurs when the OL exceeds the fracture toughness K_{IC} leading to overload failure. Depending on the R-ratio and the OLR, either crack arrest (crack stops growing) or retarded crack growth (crack growth rate is lower than predicted for the CA case) is observed. The curve which separates these two regions is strongly influenced by R-ratio.

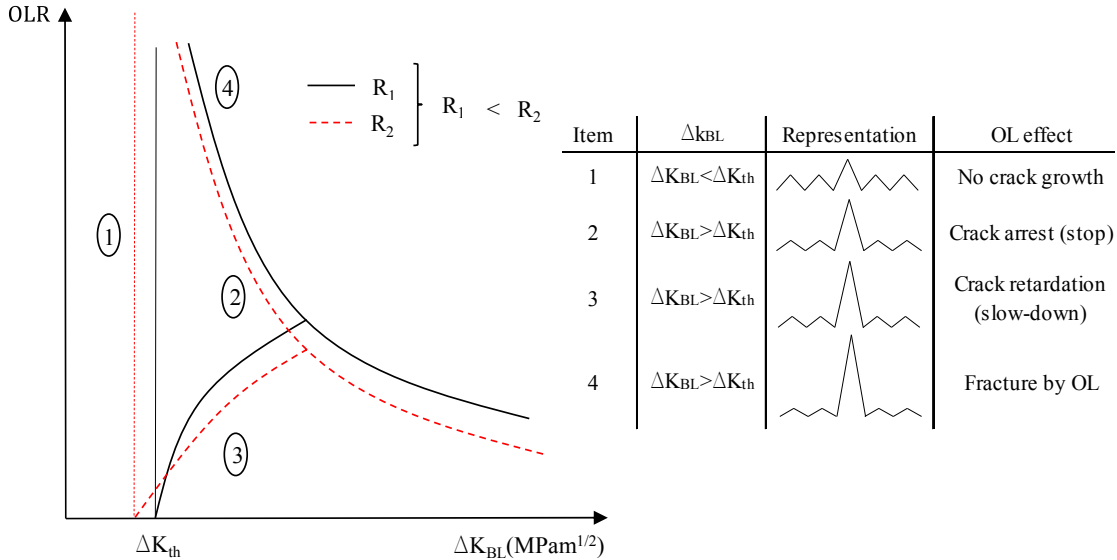


Fig.3 – Influence of single overloads on the crack growth rate [modified from 9]

In general it can be stated that the retardation effect due to an overload is a beneficial effect regarding the lifetime of components. This has also been proven by more recent studies [9, 11, 13-17]. The retardation effect occurs in three stages as can be seen in Fig. 2. First, there is a small acceleration in fatigue crack growth rate. This acceleration is however swiftly followed by the main effect, the retardation. After the minimum crack growth rate has been obtained, thus the maximum retardation, the crack growth rate starts to accelerate again and becomes equal to the rate which would have been obtained if no overload would have been present.

Sequence of OLS

The difference between a sequence of OLS and block loading is the amount of OLS. In a sequence, only a few OLS are present, whereas in a block loading the amount of OLS is so large that they are in a regime. The retardation effect is more pronounced for a sequence of OLS than for a single overload [2, 12, 22]. Just as with a single OL, the retardation becomes more severe when increasing the OLR value [2]. An increase in the amount of OLS within the sequence will increase the retardation effect. However, the Δa_{OL} remains the same as for a single OL [2, 18]. The higher the R-ratio, the less the retardation [2, 9] which is also observed for a single OL.

Periodic sequence of OLS

For this sequence, there is not a common agreement. The result seems to depend on the combination of the number of overloads and base load cycles. If periodic single OLS are applied frequently, an accelerated fatigue crack growth might be observed for structural steels [2]. This is opposite to the observations for a single OL and a sequence of OLS. There are also several studies where the observations are in line with the ones of single OL and sequence of OLS [2, 23]. Here the main parameter controlling the retardation is the number of base cycles between the OLS. A longer interval between OLS results in a more extreme retardation [2, 22, 23].

Single UL

When applying a single UL, several authors agree that an acceleration in the FCGR takes place [2, 15, 16, 20, 24] but the rate at which the acceleration occurs is highly scattered [2]. From all analysed literature on this topic it has been concluded that overloads have more influence in the retardation effect than underloads in the acceleration effect.

Sequence of UL

Not much information is available for this load combination. According to [25], a loading change from a sequence of ULs to a base line block causes a retardation transient.

Periodic Sequence UL

It has been observed by [2, 15, 26] that the FCGR under periodic ULs goes significantly faster than the results obtained from a CA test. Yet for single periodic ULs there is some disagreement whether or not the acceleration factor reaches a maximum or not when plotted in function of NBL [23].

An important tool to measure the effect of ULs sequences is the acceleration factor defined as:

$$AF = \frac{\left(\frac{da}{dN}\right)_{VA}}{\left(\frac{da}{dN}\right)_{CA}} \quad (2)$$

Where:

- $(da/dN)_{VA}$: measured crack growth rate per sequence
- $(da/dN)_{CA}$: the predicted growth rate per sequence by a linear summation

The Under Load Ratio (ULR) is defined as:

$$ULR = \frac{K_{maxBL} - K_{UL}}{\Delta K_{BL}} \quad (3)$$

The FCGR response to single periodic ULs depends on the material and is strongly affected by the R-ratio [2, 27]. The ULR value however has no effect at all [2]. Whilst mostly the acceleration effect of the FCGR has been noted for periodic UL sequences, some researchers found some interaction effects [26, 28] and even retardation induced by the UL [2].

Combined overload/underload events

OL-UL or UL-OL

There is in both cases (OL followed by UL and UL followed by OL) a retardation effect. When an UL is applied immediately after an OL, it reduces the post-OL retardation more significantly than when an UL immediately precedes an OL. The retardation effect increases with increasing length of the OL sequence. Yet it has a smaller retardation effect than a pure OL sequence. The difference in effect of the UL compared to an OL-UL and OL-UL block is very small [2, 16].

In more recent studies [29], it has also been concluded that the retardation effect of an OL is very sensitive to the subsequent UL. When the UL becomes bigger, the number of delay cycles N_D decrease. The crack length affected by the delay is not dependent on the magnitude of the UL. The minimum crack length associated to the minimum FCGR varies with the UL. When the UL becomes bigger, the minimum FCGR occurs later [29]. In [16] it has been concluded that the influence of an UL can make the retardation effect due to an OL go away.

Periodic OL-UL or Periodic UL-OL

A periodic behaviour of an OL immediately followed by an UL or vice versa will most likely cause a retardation in crack growth rate. There exists a maximum in retardation for a certain period of applied BLs and the effect of change in FCGR eventually disappears when the period becomes very large [16].

BLOCK LOADING

In general, block loadings can be categorised in low-high, high-low or combinations of these sequences.

The main parameters to define the blocks are (see Fig. 4a):

- R
- ΔK
- K_{min}

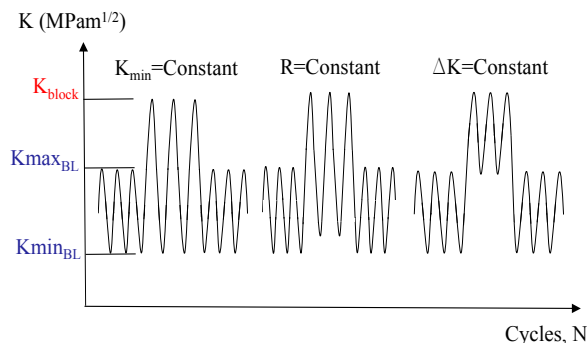


Fig. 4a – Characteristic parameters in a L-H-L block loading system [9]

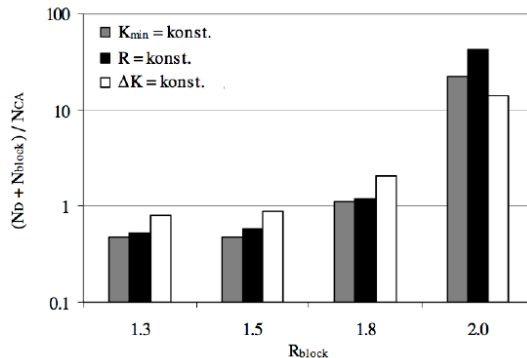


Fig. 4b – Total lifetime vs. block loading ratio for 3 different loading configurations [9]

In Fig. 4b the lifetime normalized to the CA fatigue life is illustrated depending on the block loading ratio and the block loading category is defined. The CA reference fatigue loading corresponds to the base loading. The lifetime due to high-low block loading increases exponentially with increasing block loading ratio [9]. In [30], high-low and low-high blocks were tested with constant ΔK . The effect of a high-low block has the same trend as a single OL (see Fig. 1). However, for a high-low load block, the retardation always occurs immediate and is not preceded by the acceleration phase [10, 18]. The effect of crack retardation is much higher for the high-low block than for an equivalent single tensile overload [11, 22, 30]. The low-high block results in an acceleration of FCGR. This behaviour is identical to that generally observed following an UL [2, 12, 22]. Also the effect of the R-ratio has been investigated by Borrego [30]. A significant reduction in delay cycles when R is increased has been observed. This is a similar behaviour as in single OL [10, 11, 14].

OVERVIEW OF CRACK PROPAGATION MECHANISMS

Notwithstanding retardation and acceleration effects have been mentioned, nothing about the physics behind these effect has been said.

Crack tip blunting is one of the phenomena to explain the retardation [3]. Due to an overload, the crack tip is blunted, creating a new initiation site. Before the crack can propagate further at a normal ratio, it has to be reinitiated which causes retardation.

According to [3], an OL induces compressive residual stresses ahead of the crack tip. By superimposing these stresses on the applied stresses, the local stress ratio reduces. Thus the compressive residual stresses induced by an OL can also be seen as a reason for FCGR retardation due to an OL. Elber [31] identified the phenomenon of plasticity-induced crack closure (PICC) and was able to fully explain all stages of retardation. Due to the OL, large tensile deformations are induced in the material ahead of the crack tip. This zone affected by the OL, is called the OL plastic zone.

An increase in residual wake of plastic deformation is left on the crack flanks when the crack advances through this zone. This increase causes the crack to stop at high tensile loads. The PICC models predict delayed retardation. If the OL caused direct crack branching, immediate retardation is to be expected.

However when the crack after the OL first starts to grow in a normal way and branches afterwards, delayed retardation occurs. This study was later confirmed by [16]. Jones [32] made a suggestion that the high plastic strains induced by the OL harden the region ahead of the crack tip and cause the retardation. In general, the PICC phenomenon has been most prominent to account for crack retardation in a single OL. It has been confirmed by several researchers [33-35, 22] and numerical analysis [10, 36]. Also the effect of the OLR and the R-ratio are in accordance with the PICC arguments [10].

The residual stress concept has been used to prove the acceleration effect upon a single UL [3, 16, 33-35, 22] as the UL induces tensile residual stresses ahead of the crack tip. Also the PICC mechanism is able to prove the acceleration. An UL reduces the height of plastically deformed material in the area behind the crack by compressive yielding [3].

Mechanism	OL	UL
Crack tip blunting	x	
Compressive residual stresses	x	
Tensile residual stresses		x
PICC	x	x

Table 2 – Overview of crack propagation mechanisms

CONCLUSIONS

There has been done a lot of research to describe load spectra of different applications. Notwithstanding numerous research efforts, the effects of VA loads are still not fully understood. Most research has been done on the effect of a single OL. It can be concluded that an OL and all of its variations have a retardation of the crack growth as a consequence. ULs however have an acceleration of crack growth as a consequence. When an OL and an UL are applied after each other, retardation is most likely to occur, but the UL has a strong influence on the amount of retardation. For block loading, the effect highly depends on the parameters defining the shape of the block. The plasticity induced crack closure (PICC) mechanism has been proven useful to explain retardation and acceleration effects.

REFERENCES

- [1] – P. Heuler, T. Bruder, H. Klatschke - Standardised load-time histories – a contribution to durability issues under spectrum loading – *Mat.-wiss. U. Werkstofftech.* 2005, 36, No. 11 – DOI: 10.1002/mawe.200500936.
- [2] - M. Skorupa, “Load interaction effects during fatigue crack growth under variable amplitude loading - a literature review. Part I: empirical trends,” *Fatigue & Fracture of Engineering Materials & Structures*, vol. 21, no. 8, pp. 987–1006, 1998.
- [3] - M. Skorupa, “Load interaction effects during fatigue crack growth under variable amplitude loading - a literature review. Part II: qualitative interpretation,” *Fatigue & Fracture of Engineering Materials & Structures*, vol. 22, no. 10, pp. 905–926, 1999.
- [4] - J. Schijve - *Fatigue of structures and materials* - ISBN-13:978-1-4020-6807-2 – 2003.
- [5] - J. Schijve, “The significance of flight simulation fatigue tests.,” *Durability and Damage Tolerance in Aircraft Design, Proc. 13th ICAF Symp., Pisa* (Edited by A. Salvetti and G. Cavallini), EMAS, Warley, pp. 71 – 170, 1985.
- [6] - J. Siegl and J. J. Schijve, “Fractographic observations on fatigue crack growth under minitwist flight simulation-loading.,” LR-631, Delft University of Technology, The Netherlands., 1990.
- [7] - J.P. Amzallag, C. Gerey, J.L. Robert, and J. Bahuaud, “Standardization of the rainflow counting method for fatigue analysis” *Fatigue*, vol. 16, pp. 287 – 293, 1994.
- [8] - A. K. Khosrovaneh, “Fatigue loading history reconstruction based on the rainflow technique”. *International Journal of Fatigue*, vol. 12, pp. 99 – 106, 1990.
- [9] - M. Sander and H.A. Richard, “Fatigue crack growth under variable amplitude loading. part I: Experimental investigations.,” *Institute of Applied Mechanics (FAM), University of Paderborn, Pohlweg 47-49, D-33098 Paderborn, Germany*, 2005.
- [10] - C. S. Shin and S. H. Hsu, “On the mechanisms and behaviour of overload retardation in AISI 304 stainless steel” *Int. J. Fatigue*, vol. 15, pp. 181– 192, 1993.
- [11] - L.P. Borrego, J.M. Ferreira, and J.M. Costa, “Evaluation of overload effects on fatigue crack growth and closure” *Eng Fract Mech*, vol. 70, pp. 1379– 1397, 2003.
- [12] - C. M. Ward-Close, A. F. Blom, and R.O. Ritchie, “Mechanisms associated with transient fatigue crack growth under variable-amplitude loading: An experimental and numerical study,” *Engineering Fracture Mech.* 32, pp. 613– 638, 1989.
- [13] - L.P. Borrego, J.M. Ferreira, and J.M. Costa, “Fatigue crack growth and crack closure in an AlMgSi alloy” *Fatigue Fract Eng Mater Struct*, vol. 24, pp. 255 – 266, 2001.
- [14] - X.P. Huang, J.B Zhang, W.C. Cui, and J.X. Leng, “Fatigue crack growth with overload under spectrum loading” *Theor Appl Mech*, vol. 44, pp. 105 – 115, 2005.
- [15] - V. Tvergaard, “Effect of underloads or overloads in fatigue crack growth by crack tip blunting” *Eng Fract Mech*, vol. 73, pp. 869 – 879, 2006.
- [16] - F. Romeiro and M. de Freitas, “The effect of overloads and underloads on fatigue crack growth” *Anales de Mecanica de la fractura*, vol. 18, pp. 79 – 85, 2001.
- [17] - R. Seifi, M. Eshraghi, “Effects of mixed-mode overloading on the mixed-mode I+II fatigue crack growth”, *Arch. Appl. Mech.*, Vol. 83, pp. 987–1000, 2013.
- [18] - G. L. Chen and R. Roberts, “Delay effects in AISI 1035 steel” *Engng Fracture Mech.*, vol. 22, pp. 201– 212, 1985.
- [19] - R.L. Carlson, G.A. Kardomateas and P.R. Bates, “The effects of overloads in fatigue crack growth”. *International Journal of Fatigue*, Vol. 13, No. 6, pp. 453 – 460, 1991.
- [20] - K. Sadananda, A.K. Vasudevan, R.L. Holtz and E.U. Lee, “Analysis of overload effects and related phenomena”. *International Journal of Fatigue*, Vol. 21, pp. 233 – 246, 1999.
- [21] - B. B. Verma and R. K. Pandey, “The effects of loading variables on overload induced fatigue crack growth retardation parameters”. *Journal of material science*, Vol. 34, pp. 4867– 4871, 1999.
- [22] - M. Skorupa, J. Schijve, A. Skorupa and T. Machniewicz, “Fatigue crack growth in a structural steel under single and multiple periodic overload cycles”, *Fatigue Fract Eng Mater Struct*, Vol. 22, pp. 879 – 887, 1999.
- [23] - N. A. Fleck, “Fatigue crack growth due to periodic underloads and overloads”. *Acta Metall.*, vol. 33, pp. 1339 – 1354, 1985.
- [24] - F.S. Silva, “Fatigue crack propagation after overloading and underloading at negative stress ratios”, *International Journal of Fatigue*, vol. 29, pp. 1757 – 1771, 2007.
- [25] - D.J.T. Carter, “Estimation of wave spectra from wave height and period”. *Institution of oceanographic sciences, I.O.S. Report No. 135*, 1982.
- [26] - R. L. Carlson and G. A. Kardomateas, “Effects of compressive load excursions on fatigue crack growth”. *Fatigue*, vol. 16, pp. 141 – 146, 1994.
- [27] - T.H. Topper and M.T. Yu, “The effect of overloads on threshold and crack closure”. *Int. J. Fatigue*, vol. 3, pp. 159 – 164, 1985.

- [28] - R. Yang, "Prediction of crack growth under complex loading cycles". *Int. J. Fatigue*, vol. 16, pp. 397 – 402, 1994.
- [29] - A. Bacila, X. Decoopman, G. Mesmacque, M. Voda, and V.A. Serban, "Study of underload effects on the delay induced by an overload in fatigue crack propagation". *International Journal of Fatigue*, vol. 29, pp. 1781 – 1787, 2007.
- [30] - L.P. Borrego, J.M. Ferreira, and J.M. Costa, "Partial crack closure under block loading". *International Journal of Fatigue*, vol. 30, pp. 1787 – 1796, 2008.
- [31] - W. Elber, "Fatigue crack closure under cyclic tension". *Engineering Fracture Mechanics*, vol. 2, pp. 37–45, 1970.
- [32] - R. E. Jones, "Fatigue crack growth retardation after single cycle peak overload in Ti6Al4v titanium alloy" *Engng Fracture Mech.*, vol. 5, pp. 585 – 604, 1973.
- [33] - M. A. Meggiolaro and J. T. Pinho de Castro, "On the dominant role of crack closure on fatigue crack growth modeling". *International Journal of Fatigue*, Vol. 25, pp 843 – 854, 2003.
- [34] - P. Dai, S. Li and Z. Li, "The effects of overload on the fatigue crack growth in ductile materials predicted by plasticity-corrected stress intensity factor". *Engineering fracture mechanics*, Vol. 11, pp. 26 – 37, 2013.
- [35] - T. Manson, H. Öberg and F. Nilsson, "Closure effects on fatigue crack growth rates at constant and variable amplitude loading". *Engineering fracture mechanics*, Vol. 71, pp. 1273 – 1288, 2004.
- [36] - X. Zhang, A. S. L. Chan, and G. A. O. Davies, "Numerical simulation of fatigue crack growth under complex loading sequences". *Engng Fracture Mech.*, vol. 42, pp. 305 – 321, 1992.
- [37] - M. Skorupa and A. Skorupa, "Experimental results and predictions on fatigue crack growth in structural steel", *International Journal of Fatigue*, vol. 27, pp. 1016 – 1028, 2005.