Practical Wide Frequency Approach for Calculating Eddy Current Losses in Transformer Windings

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Abstract - **A practical method for calculating eddy current losses in transformer windings is reported. The method improves the classical loss presentation by introducing a loss coefficient, called eddy current factor k_c. In this paper, eddy current losses in round conductors are discussed. A graphical approximation of k_c as a function of wire diameter, frequency, layer number, copper packing factors in the direction parallel and perpendicular to the layer is provided.   The graphs are obtained by analytical expressions compared with FEM simulations.   To unify the approach for different cases, a reference diameter, apparent and equivalent frequency are defined.   A few short examples for applying the method in transformer design are given.   The method is applicable for a variety of transformers with different frequencies, wire diameters and conductor fittings.**

The proposed method is verified by designing several transformers.   As an example, a 2.5 kW transformer is fully described.   The experiments show good matching with the calculations.

Key words: Magnetic components, Power transformers, Eddy current losses

I. INTRODUCTION

Eddy current effects are quite important in inductors and transformers for power electronics. Eddy current losses in round wires, including skin and proximity effects in transformers are discussed by Dowell [1] and many new papers [2], [3], related to some extent to Dowell's interpretation and results.   The 2-D analytical approximations are mainly focussed on uniform transverse fields (proximity losses) and also on the own field of a wire (skin-effect losses) [4], [5], [6]. The limitation of the conversion of round wires into rectangular ones is improved in [7], [8]. The optimal design and minimization of losses in Litz wire windings are provided in [9]. Full numerical methods can need up to 1 million of elements easily [10].   This affects calculation time and accuracy.

A conclusion can be made that on one hand, a lot of the design approaches are a compromise of the accuracy in eddy current losses estimation, and on the other hand, the calculation time and complicity.

In this paper a time-saving high accuracy method for eddy current loss calculation in transformer windings is introduced. The proposed approach is applicable for transformers in wide range of designs using round wires.

II. PROPOSED WIDE FREQUENCY APPROACH FOR CALCULATING EDDY CURRENT LOSSES IN TRANSFORMER WINDINGS

First, low frequency (LF) approximation is considered [11]. The LF approximation is the starting point of the proposed approach.

The LF approximation is applicable when the eddy currents, induced in the winding do not considerably change the applied field inside the conductor. The LF approximation is valid up $d \leq 1.6\,\delta$ [11], (d is the wire diameter and δ is the penetration depth $\delta = \sqrt{\dfrac{2\,\rho_c}{\omega\,\mu}}$, $\omega = 2\pi\,f$ is the frequency of the applied magnetic field; μ is the permeability of the material (for copper $\mu \cong \mu_o$), μ_o is the permeability of free space; $\mu_o = 1.25664 \times 10^{-6}$; ρ_c is the electrical resistivity of the conducting material (copper), for copper $\rho_c = 23 \times 10^{-9}\ \Omega\,m$ at 100°C; $\rho_c = 17 \times 10^{-9}\ \Omega\,m$ at 25°C.

The major part of the eddy current losses in LF approximation for round wires can be explained by the presence of a transverse magnetic field, which acts like an induction heating to the wire. To express the eddy current losses $P_{cu,eddy}(t)$, the following equation can be used:

$$P_{cu,eddy}(t) = \frac{\pi\,l_w\,d^4}{64\,\rho_c}\left(\frac{dB}{dt}\right)^2 \qquad (1)$$

where:
  d is the practical diameter of the copper wire of the considered winding;
  B is the induction, assumed perpendicular to the wire axis;
  l_w is the wire length of the winding

The equation (1) is quite general and applicable even for the cases with more complicated field distribution, e.g. for non-sinusoidal waveforms, [3].

A.   Eddy current losses factor k_c

To extend the validity of (1) for larger diameters, and a wide frequency range, the loss factor k_c is introduced. The factor k_c represents the ratio between the eddy current losses compared

to the losses in the ohmic resistance of the winding of the magnetic component.

Using the above introduced loss factor k_c, the eddy current losses are given by the following equation

$$P_{eddy} = \left(R_0 \, I_{ac}^2\right) k_c \qquad (2)$$

where:

k_c is the loss factor, depends on the operating frequency f, the wire diameter d and the distance between the conductors, presented by the parameter η and the distance between the layers presented by the parameter λ;

I_{ac} is the AC current component;

R_0 is the ohmic resistance of the winding.

The *copper fill factor in the direction of the layer* η can be defined as $\eta = n\,d\,/\,w)$, where d is the wire diameter; n in the number of turns in a layer; w is the winding width.

The *copper fill factor in the direction perpendicular to the layer* λ can be defined as $\lambda = m_E\,d\,/\,h$, where h is the window height (for m_E see below).

To allow an easy use of k_c, we provide two sets of graphs shown in Fig.1 and Fig.2, giving the additional coefficient k_{tf}. For the transformers the value of the coefficient k_c is:

$$k_c \approx m_E^2 \; k_{tf} \qquad (3)$$

where the value of k_{tf} is found using Fig.1 and Fig.2.

The parameter m_E (equivalent number of layers) is defined as the number of layers going from no field to the maximum field.

For p parallel wires a number of wires is present in the same layer, which will become an equivalent number of wires in a layer: $m_E = m$.

For *Litz wire* one cannot count exactly the number of individual wires in a layer. We distribute the equivalent turns in both directions and then we have: $m_E = m\sqrt{p}$, where p is the number of paralleled strands.

For interleaved windings in transformers (and no parallel wires) $m_E = m\,/\,K$, where K is the field symmetry factor [11]. This presentation is also necessary for so called 'half layers'.

It is not recommended to use partially filled layers in transformer designs. If anyhow partially filled layers are used, in the proposed method, the wires should be equally spread. The effect of the partially filled layers is reduced at high values of m_E.

The graphs are obtained by approximating the exact solution for eddy current losses for different cases, found by using software package MathCad [11].
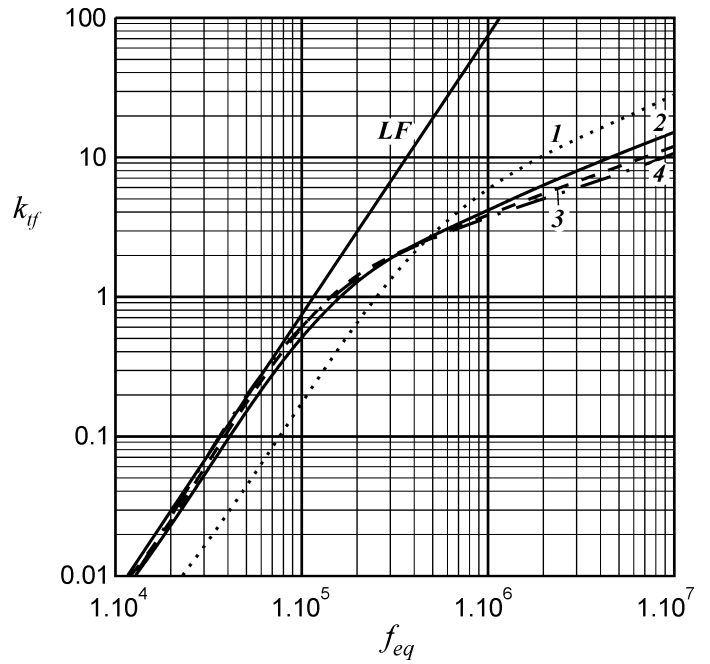

Fig. 1 Transformer cases, typical transformer factor k_{tf} for d=0.5mm, η=0.9, ρ=23×10⁻⁹ and λ=0.5, 1) dotted line: half layer, m_E=0.5; 2) solid line: single layer, m_E=1; 3) dashed: two layers, m_E=2; 4) dash-dot: three or more layers, m_E>2. *LF* – low frequency approximation.
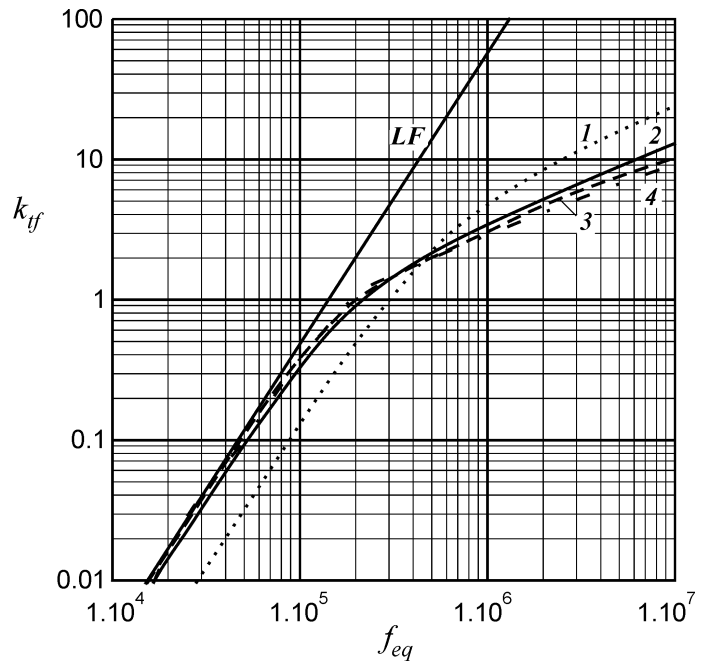

Fig. 2 Transformer cases, typical transformer factor k_{tf} for d=0.5mm, η=0.7, ρ=23×10⁻⁹ and λ=0.5, 1) dotted line: half layer, m_E=0.5; 2) solid line: single layer, m_E=1; 3) dashed: two layers, m_E=2; 4) dash-dot: three or more layers, m_E>2. *LF* – low frequency approximation.

To unify the use of the graphs, we introduce a reference diameter, an apparent frequency and an equivalent frequency.

B. Apparent frequency calculation

In (2), the apparent frequency should be used.

1. In general, one has to sum the contribution of each current harmonic in order to calculate losses. In the presented method, for sinusoidal currents no corrections have to be made and the apparent frequency f_{ap} is the real one:

$$f_{ap} = f \tag{4}$$

2. For a symmetrical triangular current waveform, in the low frequency approximation we obtain the following apparent frequency f_{ap}:

$$f_{ap} = f \frac{2\sqrt{3}}{\pi} \approx 1.10 f \tag{5}$$

3. At high frequency, the losses tend to increase with the root of frequency for a given current. In that case, the contribution of harmonics in the current is low and one can use the rms value of the current instead of summing over all harmonics. For a symmetrical triangular current waveform, this results in an apparent frequency f_{ap}:

$$f_{ap} \approx 1.025 f \tag{6}$$

C. Reference wire diameter

The choice of 0.5mm as a *reference wire diameter* is done in order to use a typical wire diameter for power electronics. The frequency, for which the penetration depth is equal to the reference diameter $d=\delta$, is 20kHz. The limit of the 'low frequency (LF) approximation' for the reference diameter d=0.5mm is 50kHz, thus LF can be applied below 50kHz for that wire diameter. These values are easy to remember. The diameters of wires in adjacent layers are taken equal and in a square fitting. This is the worst-case design, as a hexagonal fitting usually reduces the losses.

D. Equivalent frequency calculation

To use the provided graphs (Fig.1, Fig.2) for any frequency, wire diameter and conductor resistivity, *the equivalent frequency* of the considered case should be first found:

$$f_{eq} = f_{ap} \left(\frac{d}{0.5} \right)^2 \left(\frac{23 \times 10^{-9}}{\rho_c} \right) \tag{7}$$

where:
f_{ap} is the apparent frequency;
d is the wire diameter in [mm];
ρ_c is the conductor resistivity in [Ωm].

For fast calculation, the waveforms and resistivity could be neglected, but the diameter effect has still to be taken into account by the following simplified expression:

$$f_{eq} \approx f \left(\frac{d}{0.5\,\text{mm}} \right)^2 \tag{8}$$

III. SHORT EXAMPLES OF APPLYING THE PROPOSED APPROACH

The graphs shown in Fig.1 and Fig.2 concern a design example with a typical wire diameter of 0.5mm and the usual frequency range for power electronics: 10kHz to 10MHz. For more than two layers (m_E>2), the result is almost independent of the number of layers. The usual values of η in transformers are between 0.7 (typical for thin wires and Litz wire) and 0.9 (typical for d>0.5mm). For other values of η, a linear interpolation between Fig.1 and Fig.2 can be done. The additional error due to that interpolation is below 2%.

A few short examples of transformer designs show the use of the graphs in Fig.1 and Fig.2.

1) *A single layer winding of a transformer uses a wire diameter of 0.9 mm and an outer diameter of 1mm, the frequency is 30kHz, the copper resistivity is ρ =23×10^{-9} Ω m.*

We have η = 0.9mm/1mm = 0.9 , so we use Fig.1. We have to keep the same diameter/penetration depth ratio, i.e. to find the equivalent frequency f_{eq} = 30kHz×(0.9/0.5)^2 = 97.2 kHz.

Reading from Fig.1 gives k_{tf}=0.473. It is a single layer transformer, so $m_E = 1$ and we obtain k_c=k_{tf}=0.473.

2) *The same wire diameters and resistivity as in 1), but a three-layer transformer winding, using a three times smaller winding width, the same turn number.*

We use the same values η = 0.9 and f_{eq}=97.2 kHz. It is a three-layer transformer, so $m_E = 3$. For 3 layers and more, we read k_{tf}=0.575 from Fig.1. Then we have $k_c = 3^2 k_{tf} = 5.17$.

3) *The same wire diameters and resistivity as above , but a half layer transformer design (the considered single layer secondary is sandwiched between two primary windings).*

Using the same values η = 0.9 and f_{eq}=97.2 kHz, we read from Fig.1 k_{tf} =0.166. It is a half-layer transformer, so $m_E = 0.5$ and we have $k_c = 0.5^2 k_{tf} = 0.0415$. This value is much lower than the cases 1) and 2) above. The reason is that in this design case, the transverse field is zero and only local fields are present. In practical realizations, the half layer solutions indeed perform well, but a non-careful winding (e.g. non-equal winding width of the layers) generates parasitic transverse fields that increase the losses considerably.

4) *A two layer transformer winding of 0.5 mm copper diameter at 50 kHz and packing factor in the direction of the layer η=0.8, copper resistivity ρ_c=23×10^{-9} Ωm.*

We have the same diameter as the graphs, so the equivalent frequency is equal to the applied frequency. We obtain $k_{tf,0.9} = 0.170$ from Fig.1 and $k_{tf,0.7} = 0.104$ form Fig.2. To find $k_{tf,0.8}$ (for η=0.8), we take the average of the two values $k_{tf,0.8} = (0.104 + 0.170)/2 = 0.137$. Then we find $k_c = 2^2 k_{tf,0.8} = 0.549$.

IV. PRACTICAL IMPLEMENTATION OF THE PROPOSED APPROACH

A. Realized Transformer Design

A welding transformer is designed using the "fast design approach", proposed in [12]. The method, presented here, is applied for calculating the eddy current losses of the primary and secondary windings of that transformer.

The transformer contains 2 identical coils connected in parallel. Each coil contains a primary winding with number of turns N=20. Each primary winding includes two windings in parallel, Fig.3. Secondary winding of each coil (5 turns, 2 conductors in parallel) is sandwiched between the two primary windings of each coil.

The arrangement of the windings is shown in Fig.3.

The realized transformer is pictured in Fig.4.

The specifications of the designed transformer are:

U_{pri}=300V; P_{out}=2.5kW (continuous power)

Core: U 80/40/25; Primary Winding: N=20, d=1.18 [mm]; Secondary Winding: N=5, d=2 [mm]
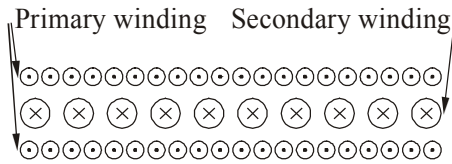

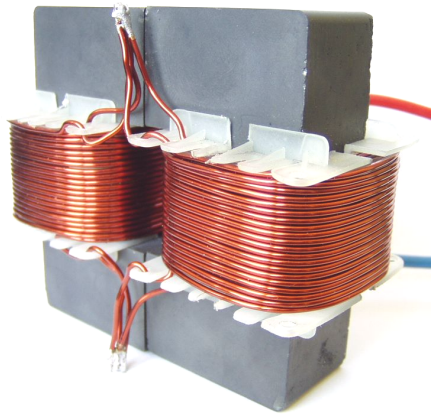Fig. 3. Designed welding transformer - winding arrangement.

Fig. 4. Realized welding transformer, UU core, N87 ferrite.

B. Winding Loss Calculations

Primary Winding -A single layer winding, wire diameter of 1.18 mm, the frequency is 60kHz, the copper resistivity is ρ=21×10^{-9} Ω m.

We have $\eta = d\,N\,p/w = 1.18\,mm \times 20 \times 1/26\,mm = 0.908$. We have to keep the same diameter/penetration depth ratio, i.e. we have to find the equivalent frequency. We use (7)

$$f_{eq,1} = 60\,\text{kHz}\left(\frac{1.18}{0.5}\right)^2\left(\frac{23}{21}\right) = 366\,\text{kHz} \qquad (9)$$

We read $k_{tf,0.9}$=2.15 from Fig. 1 for η=0.9. The actual value η_2=0.908 is very close to η=0.9. It is a single layer transformer, so $m_E = 1$ and we obtain $k_c = k_{tf}$=2.15.

Then, using (2) we obtain the eddy current losses of the primary winding. The total losses in the primary winding are the sum of ohmic and eddy current losses.

Secondary Winding – A half layer transformer design $m_E = 0.5$ (the considered single layer secondary is sandwiched between two primaries), 2 wires in parallel, wire diameter of 2 mm, the frequency is 60kHz, the copper resistivity is ρ =21×10^{-9} Ω m.

We have $\eta = d\,N\,p/w = 2\,mm \times 5 \times 2/26\,mm = 0.769$. The equivalent frequency is

$$f_{eq,1} = 60\,\text{kHz}\left(\frac{2}{0.5}\right)^2\left(\frac{23}{21}\right) = 1.051\,\text{MHz} \qquad (10)$$

We find $k_{tf,0.9}$=6 from Fig. 1 for η=0.9 and $k_{tf,0.7}$=4.7 form Fig. 2 for η=0.7. The actual value of η for the secondary is η_2=0.769. Using the found two values and interpolating for η_2=0.769, we obtain k_{tf}≈5.149. It is a half layer transformer, so m_E=0.5 and we obtain $k_c = 0.5^2 k_{tf} = 1.287$.

The calculated results for the total losses of both primary and secondary windings are tabulated in Table 1.

TABLE 1 CALCULATED RESULTS FOR THE EXPERIMENTAL TRANSFORMER

	k_{tf}	k_c	P_{ohm} [W]	P_{eddy} [W]	$P_{cu,i}$ [W]
primary	2.15	2.15	1.455	3.129	4.584
secondary	5.149	1.287	2.059	2.65	4.709

The total losses in primary and secondary windings are:

$$P_{cu} = P_{cu,1} + P_{cu,2} = 4.584 + 4.709 = 9.293 \qquad (11)$$

C. Measured Winding Losses

We measured the winding losses P_{meas} of the realized transformer (short circuit test, primary current 10A) and the results are shown in Table 2, together with the calculated results $P_{total,cu}$.

TABLE 2 MEASURED WINDING LOSSES P_{MEAS} UNDER SHORT CIRCUIT TEST, 10 A

Measured results	Calculated results		
P_{meas} [W]	P_{cu} [W]	P_{end} [W]	$P_{total,cu}$ [W]
9.31	9.293	0.11	9.403

Remark: In Table 2 P_{end} are the losses due to the ends of the secondary winding.

The calculated values for the total copper losses P_{total}=9.403W are very close to the measured values P_{meas}=9.31W. Nevertheless, we expect a general accuracy of 3% on eddy current losses. The tolerances on dimensions of the wires and other dimensions can also affect the practical accuracy as well.

V. Conclusion

The advantages of the proposed approach are the fast and straight calculation combined with good accuracy and wide application. The method is applicable for a wide variety of transformers with different frequencies, wire diameters and conductor arrangements.

The proposed method is used in designing several welding transformers. The target power of the transformers is 2.5 kW. The experiments show good matching with the calculations.

Acknowledgment

References

[1] P.L. Dowell, "Effects of eddy currents in transformer windings", *Proc. Inst. Elect. Eng.*, vol. 113, No.8, August 1966, pp.1387-1394.

[2] P.N. Murgatroyd, 'Calculation of proximity losses in multistrand conductor bunches', *IEE Proceedings - B*, 1992, vol.139, No 1, pp. 21-28.

[3] C.R. Sullivan, "Computationally Efficient Winding Loss Calculation with Multiple Windings, Arbitrary Waveforms, and Two-Dimensional or Three-Dimensional Field Geometry", *IEEE Transactions on Power Electronics,* vol. 16, No 4, January 2001, pp. 142-150.

[4] P. Wallmeier, N. Frohleke, H. Grotstollen, "Improved analytical modeling of conductive losses in gapped high-frequency inductors", *IEEE-IAS Annual Meeting*, 1998, pp. 913-920.

[5] C.R. Sullivan, "Winding loss calculation with multiple windings, arbitrary waveforms and 2-D field geometry', *IEEE IAS Annual Meeting,*1999, pp. 2093-2099.

[4] W.G. Hurley, E. Gath and J. G. Breslin, "Optimized Transformer Design: Inclusive of High-Frequency Effects", *IEEE Transactions on Power Electronics,* vol. 13, No 4, Yuly 1998, pp. 651-658.

[5] W.G. Hurley, E. Gath and J. G. Breslin, "Optimising the AC Resistance of Multilayer Transformer Windings with Arbitrary Current Waveforms", *IEEE Transactions on Power Electronics,* vol. 15, No 2, March 2000, pp. 369-376.

[6] R. Petkov, "Optimum Design of a High-Power, High-Frequency Transformer", *IEEE Transactions on Power Electronics,* vol. 11, No 1, January 1996, pp. 33-42.

[7] J. Ferreira, 'Improved analytical modelling of conductive losses in magnetic components', *IEEE Transactions on Power Electronics,* vol. 9, No 1, 1994, pp. 127-131.

[8] J. Ferreira, 'Analytical computation of AC resistance of round and rectangular litz wire windings' *IEE Proceedings - B,* vol. 139, No 1, 1992, pp. 21-25.

[9] C.R. Sullivan, "Cost-constrained selection of strand diameter and number in a litz-wire transformer", *IEEE Transactions on Power Electronics,* vol. 16, No 2, March 2001, pp. 281-288.

[10] J.D Lavers. and V. Bolborici "Loss Comparison in the Design of High Frequency Inductors and Transformers", *IEEE Transactions on Magnetics,* vol. 35, No 5, September 1999, pp. 3541-3542.

[11] Alex Van den Bossche, Vencislav Valchev, *'Inductors and Transformers for Power Electronics'*, , CRC Press, Boca Raton, Florida, USA. 2005, 480 pp.

[12] V.C. Valchev, A.Van den Bossche, 'Design method for power electronic magnetic components including eddy current losses – Part 2', *1st International Congress, MEET-MARIND*, Varna, Bulgaria, 7-11 October, 2002, pp. 317-325.