UNIVERSITEIT GENT

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Weak heat transfer coefficient dependency of thermal spreading resistance in convectively cooled substrates

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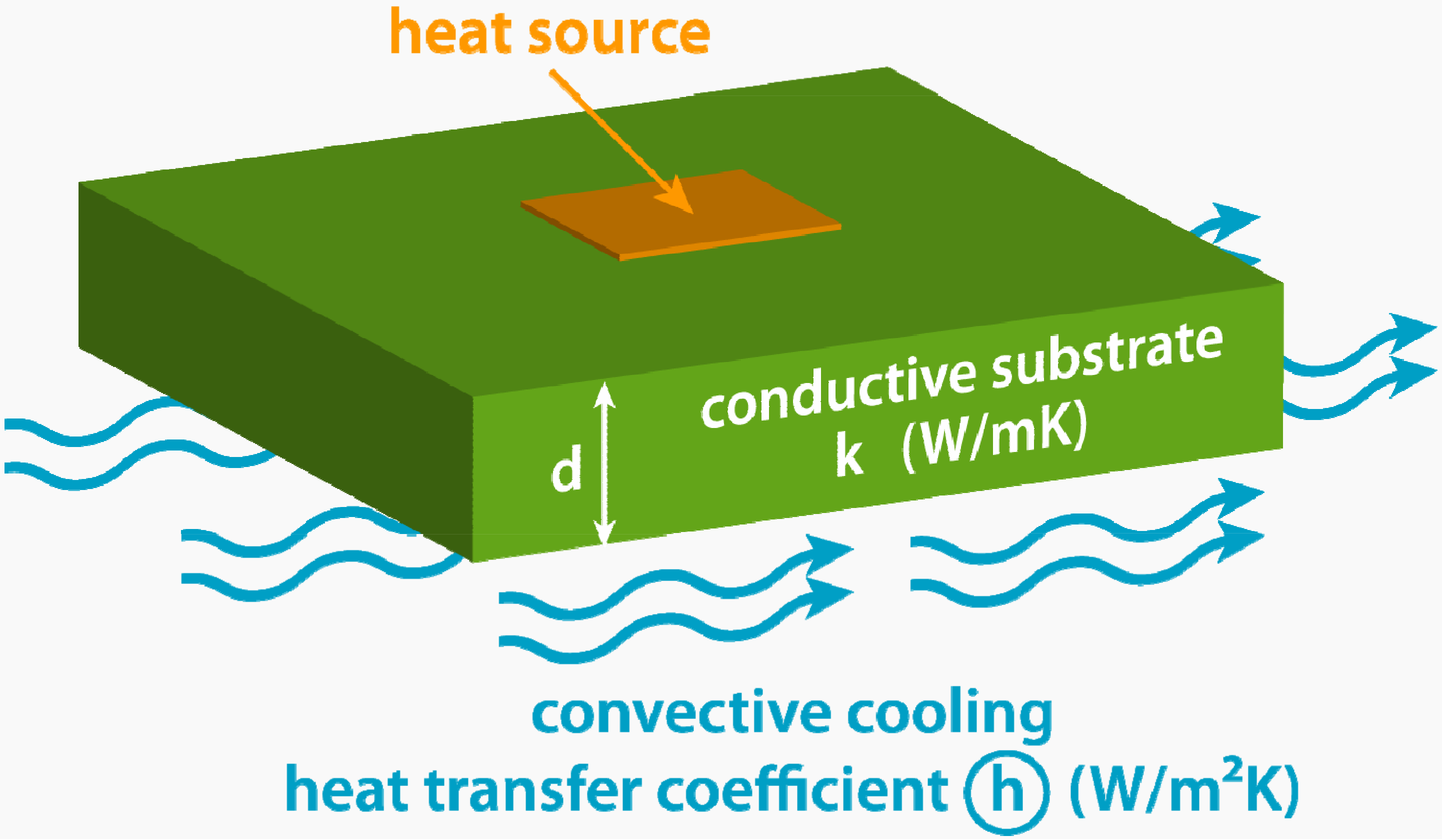
Outline

- **Introduction**
- Exact calculations
- Approximate model
- Discussion
- Conclusions

Introduction

Problem formulation



- calculation of thermal resistance (maximum temperature used)

$$R_{th} = \frac{T_{source}}{P}$$

Introduction

Earlier works in literature (1)

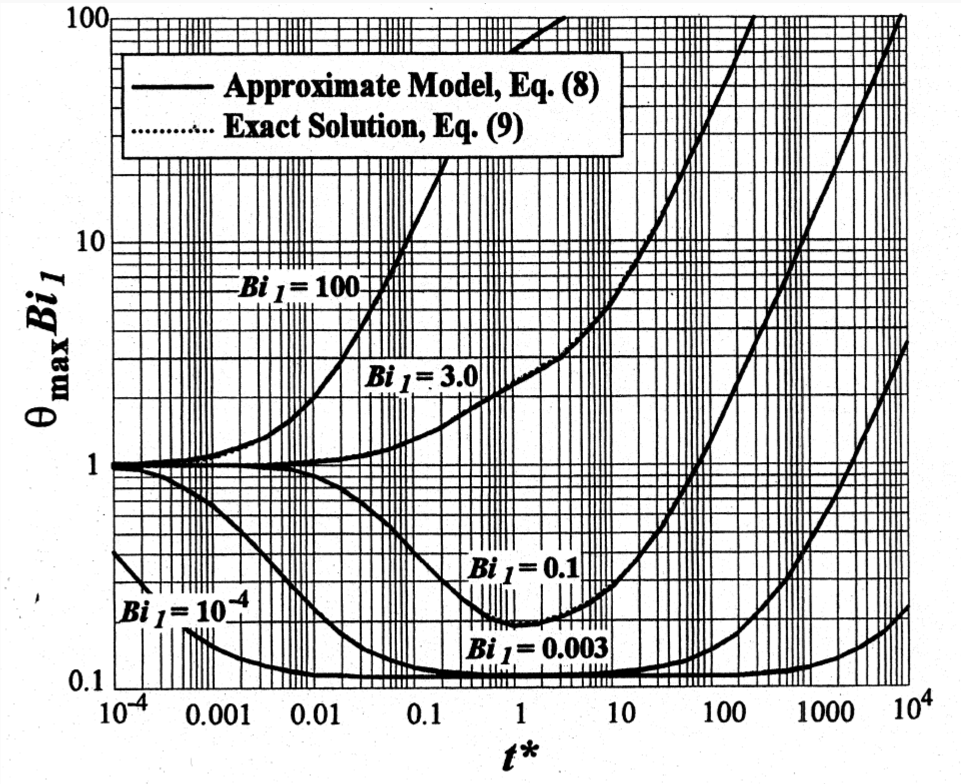


Fig. 3 Bottom-side convection. Nondimensional maximum temperature as a function of substrate thickness t^* and bottom-side Biot number Bi_1.

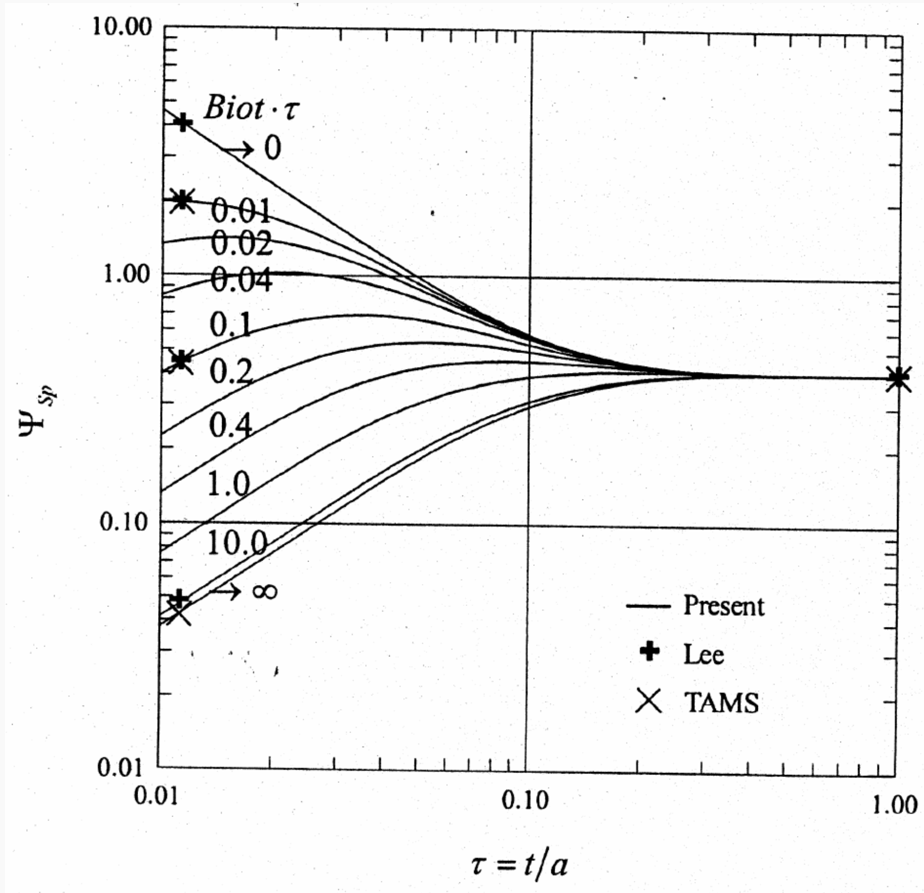


Fig. 4. Spreading resistance for $\Delta x/a = 0.25$, $\Delta y/a = 0.25$.

Fisher et al. (Trans ASME 1996)

Ellison (IEEE Trans CPT 2003)

Introduction

Earlier works in literature (2)

$$\theta_{max} = \left\{ \left[\frac{t^{*2} e^{-\lambda t^*}(Bi_1 - \lambda)}{2\alpha\beta} + \frac{(1 + Bi_1 t^*)(e^{-\lambda t^*} - 1)}{\lambda\alpha\beta} - \frac{\delta}{\phi} \right] \times \frac{1}{I_0(m) - \frac{m}{M} I_1(m) \frac{I_0(M)K_1(Mb^*) + I_1(Mb^*)K_0(M)}{I_1(M)K_1(Mb^*) - I_1(Mb^*)K_1(M)}} + \frac{\delta}{\phi} \right\} \times \left(1 - \frac{Bi_1}{\alpha}\right) + \frac{1 + Bi_1 t^*}{\alpha} \quad (8)$$

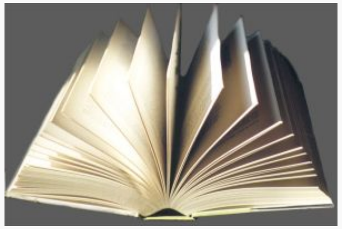
T.S. Fisher, F.A. Zell, K.K. Sikka & K.E. Torrance:
Efficient heat transfer approximation for the chip-on-substrate problem
Transactions of the ASME – Journal of Electronic Packaging **118** pp. 271-279, 1996.

Table 2 Terms appearing in the approximate solution

α	$e^{-\lambda t^*}(Bi_1 - \lambda) + \lambda(1 + Bi_1 t^*)$
β	$t^* - t^{*2}\lambda Bi_1/(2\alpha) + Bi_1(e^{-\lambda t^*} - 1)/\lambda\alpha$
δ	$1 - Bi_1 e^{-\lambda t^*}(1 + \lambda t^*)/\alpha$
ϕ	$Bi_1[1 - Bi_1(\lambda t^* + e^{-\lambda t^*})/\alpha]$
ϵ	$\dfrac{Bi_1 + Bi_2 + Bi_1 Bi_2 t^*}{Bi_2 + \lambda(1 - e^{-\lambda t^*}) + Bi_1 t^*(Bi_2 + \lambda) + Bi_1 e^{-\lambda t^*}}$
η	$t^* + t^{*2}[Bi_2 - \epsilon(Bi_2 + \lambda)]/2 + \epsilon(e^{-\lambda t^*} - 1)/\lambda$
μ	$Bi_1\{1 + t^*[Bi_2 - \epsilon(Bi_2 + \lambda)] - \epsilon e^{-\lambda t^*}\} + Bi_2(1 - \epsilon)$
m	$(\phi/\beta)^{1/2}$
M	$(\mu/\eta)^{1/2}$

Introduction

New approach



- R_{th} vs. normalized **substrate thickness** with Biot number (Bi = hd/k) as a parameter
- **approximate solution** accurate but still very **complicated**, with lot of variables

- thickness not a real design parameter but determined by technology (e.g. Si: 300µm)

  → R_{th} vs. **heat transfer coefficient**

- **simple model** to provide insight

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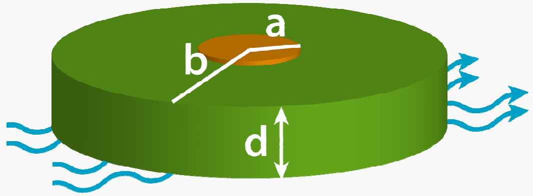
Exact calculations

Infinite series solution

- Lee, Song & Moran (ASME conference 1995)



$$b = 10^5 a$$

$$T(r,z) = \frac{pa}{k}\left[\varepsilon\left(\frac{1}{Bi} + \zeta\right) + 2\sum_{n=1}^{\infty}\frac{J_1(\lambda_n\varepsilon)}{\lambda_n^2 J_0^2(\lambda_n)}\cdot\frac{\cosh(\lambda_n\zeta)}{\cosh(\lambda_n\tau)}\cdot\frac{\tanh(\lambda_n\zeta) + \frac{\lambda_n}{Bi}}{1 + \frac{\lambda_n}{Bi}\tanh(\lambda_n\tau)}\right]$$

$$\varepsilon = a/b\,,\ \tau = t/b\,,\ \zeta = z/b\,,\ \gamma = r/b\,,\ Bi = \frac{hb}{k}$$

$$J_1(\lambda_n) = 0$$

EASY

- Carslaw & Jaeger (*Heat in solids*, Oxford Press)

$$G(\vec{r}|\vec{r}';t) = \frac{\exp\left(-\frac{C_v\left[(x-x')^2+(y-y')^2\right]}{4kt}\right)}{4\pi kt}\sum_{n=1}^{\infty}\frac{\left[\alpha_n\cos(\alpha_n z) + \frac{h}{k}\sin(\alpha_n z)\right]\cdot\left[\alpha_n\cos(\alpha_n d) + \frac{h}{k}\sin(\alpha_n d)\right]}{2d\left(\alpha_n^2 + \frac{h^2}{k^2}\right) + 2\frac{h}{k}}\exp\left(-\frac{\alpha_n kt}{C_v}\right)$$

steady state:

$$G_{DC}(r|r') = \int_{t=0}^{\infty} G(r|r';t)\,dt$$

$$= \sum_{n=1}^{\infty} C_n K_0\left(\alpha_n\sqrt{(x-x')^2 + (y-y')^2}\right)$$

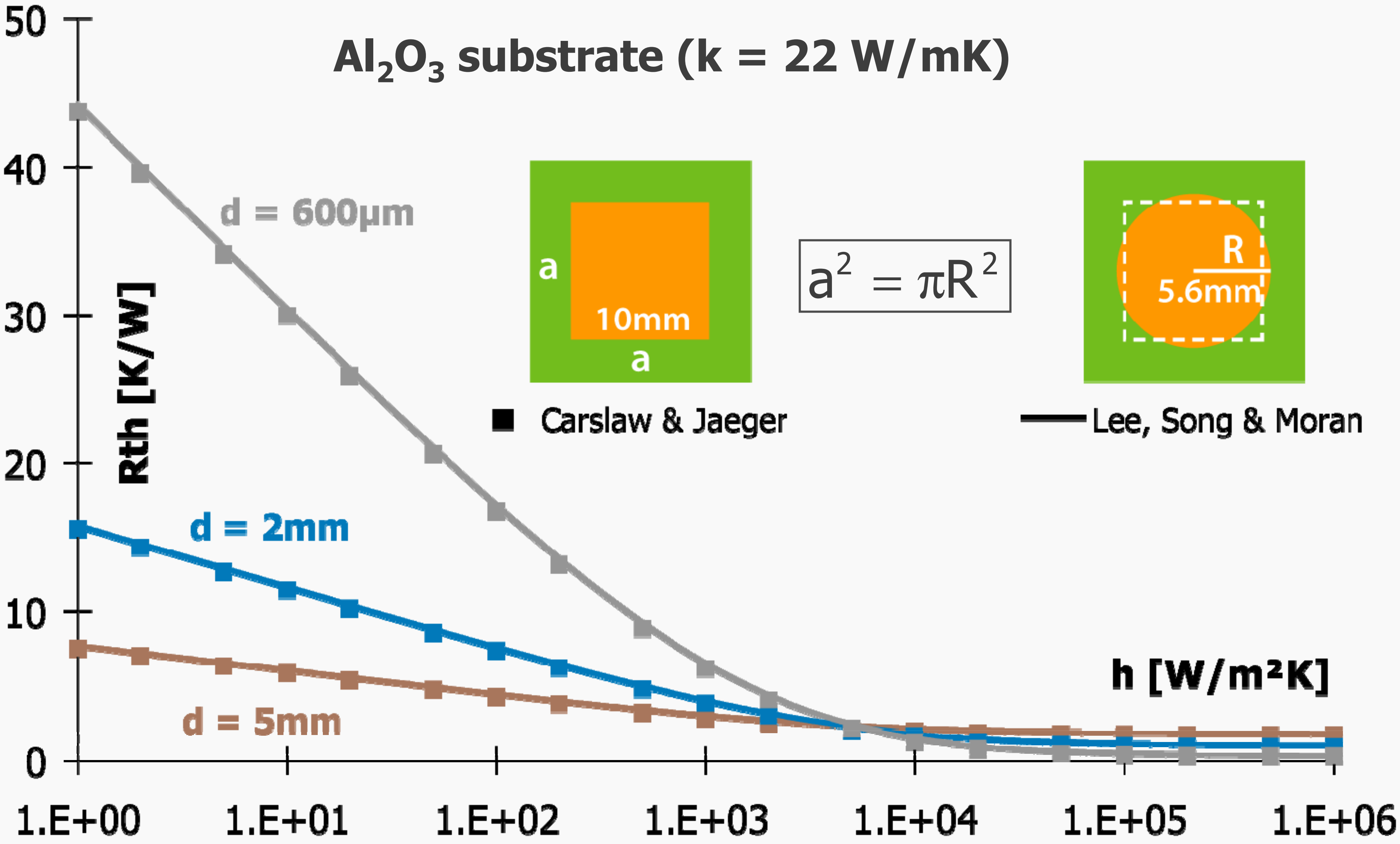
$$\tan(2\alpha_n d) = \frac{2\alpha_n\frac{h}{k}}{\alpha_n^2 - \frac{h^2}{k^2}}$$

HARD

$$T(x,y,z) = \iint_{source} G_{DC}(\vec{r}|\vec{r}')\,dx'\,dy'$$

Exact calculations

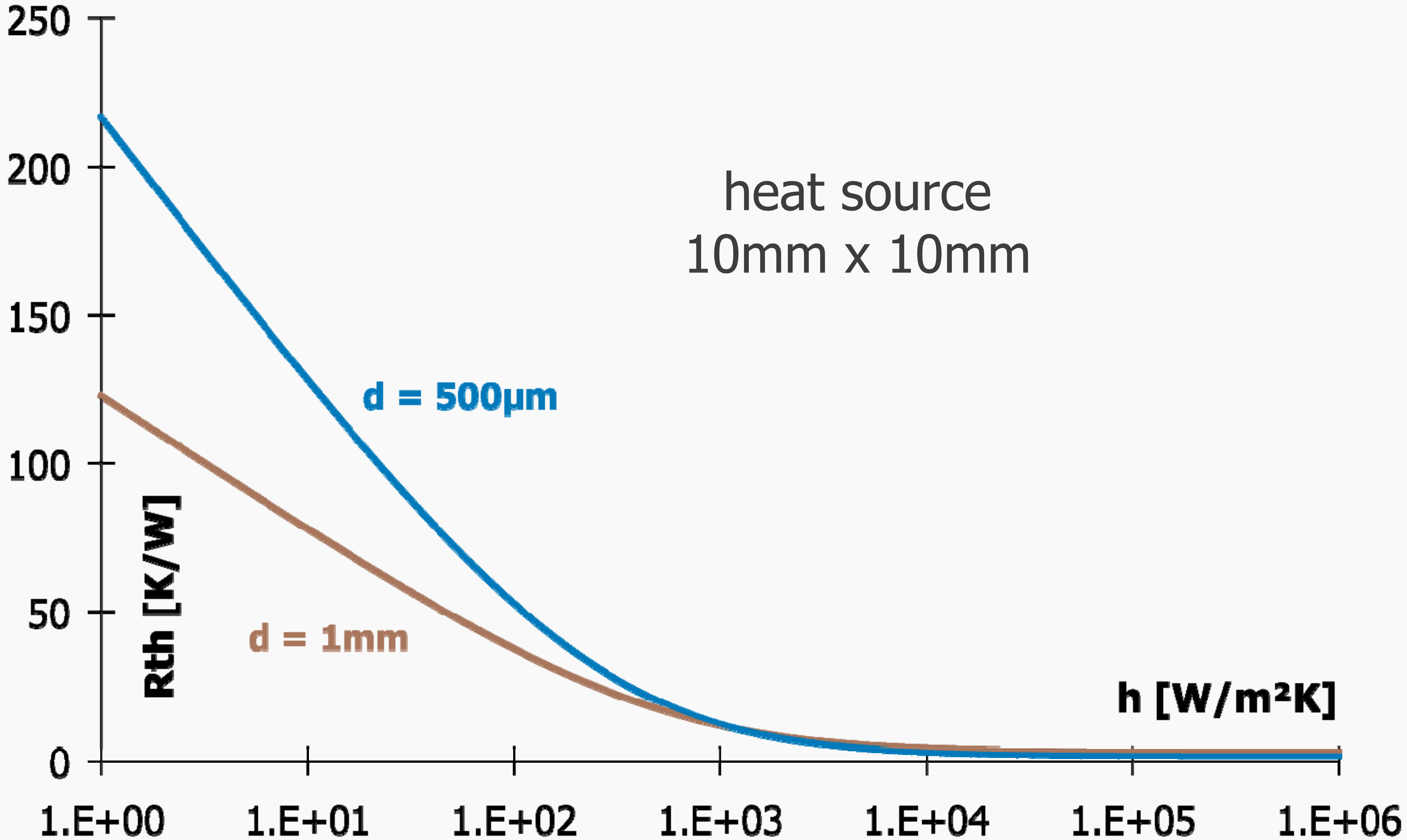
Method comparison



Weak h-dependency of spreading R_{th} in convectively cooled substrates

Exact calculations

LTCC [4 W/mK]



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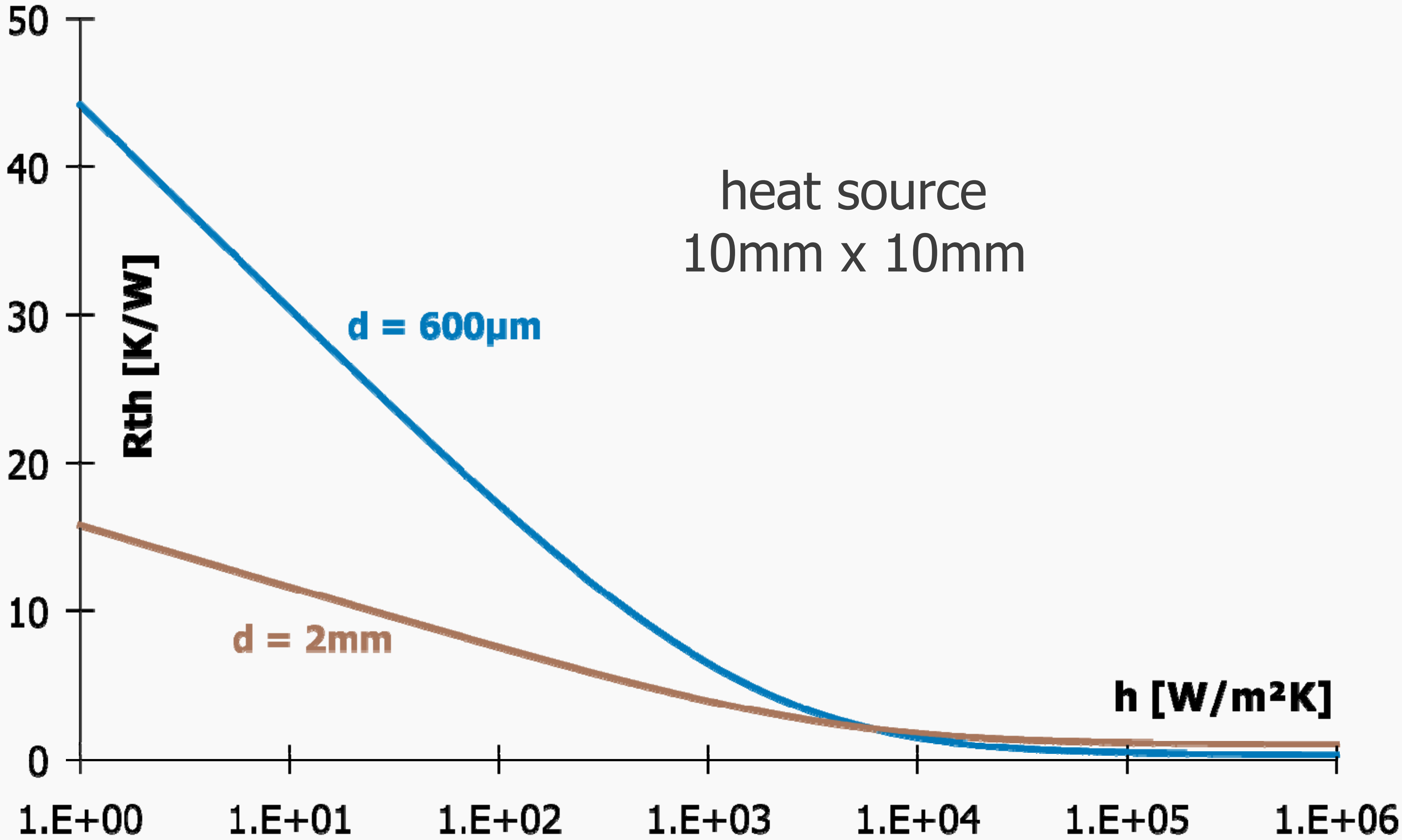
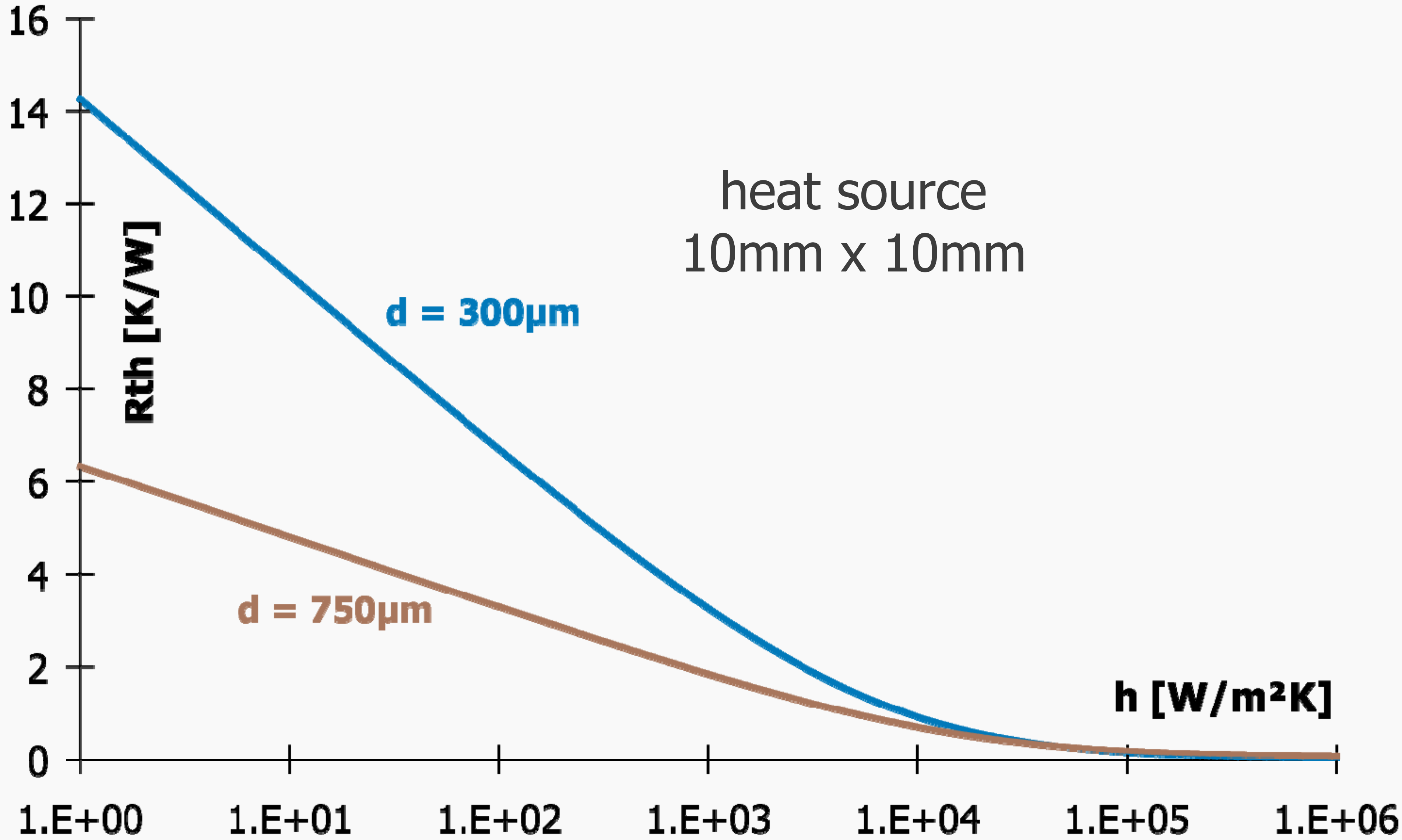
Exact calculations

Al2O3 [22 W/mK]



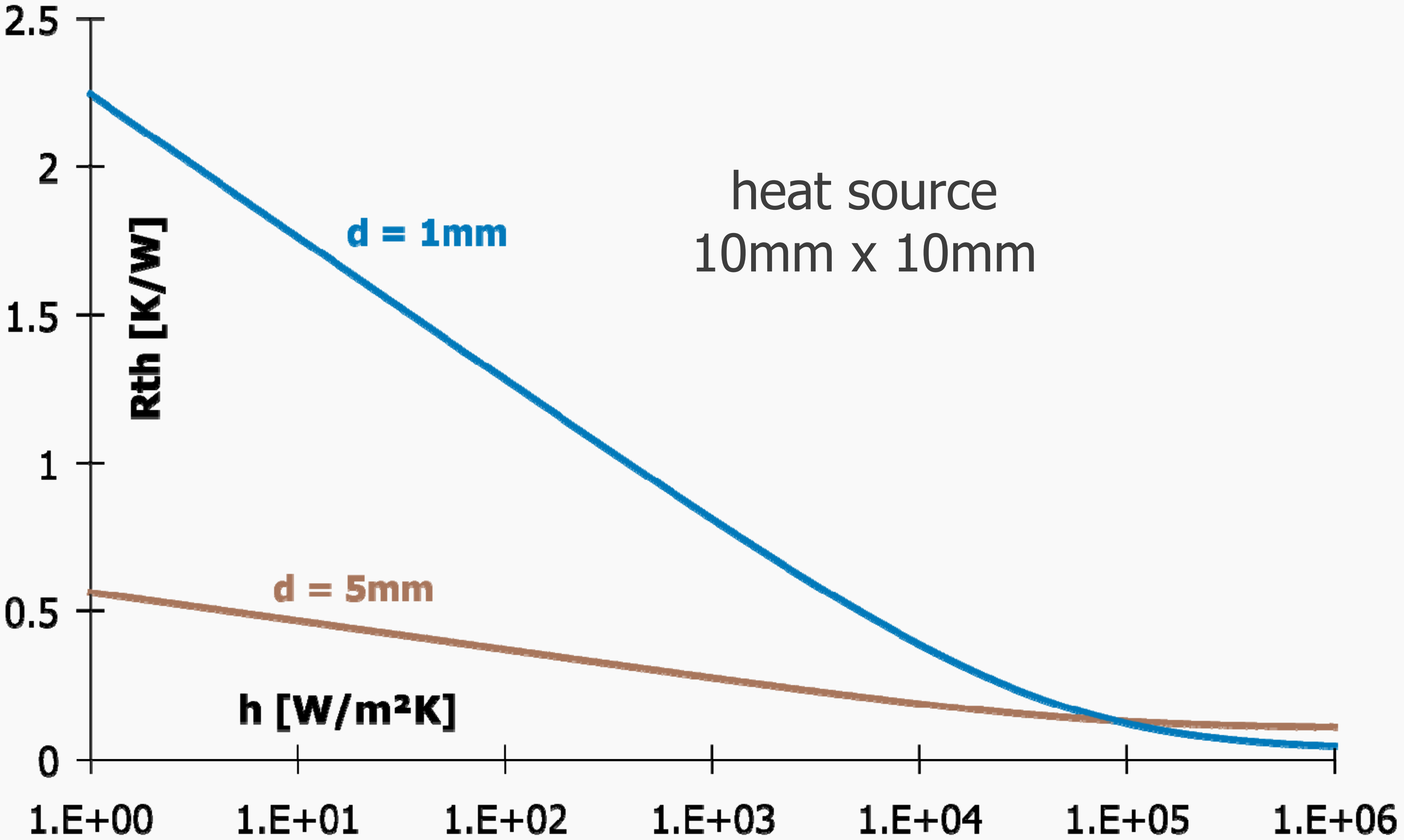
Exact calculations

Si [160 W/mK]



Exact calculations

Cu [380 W/mK]



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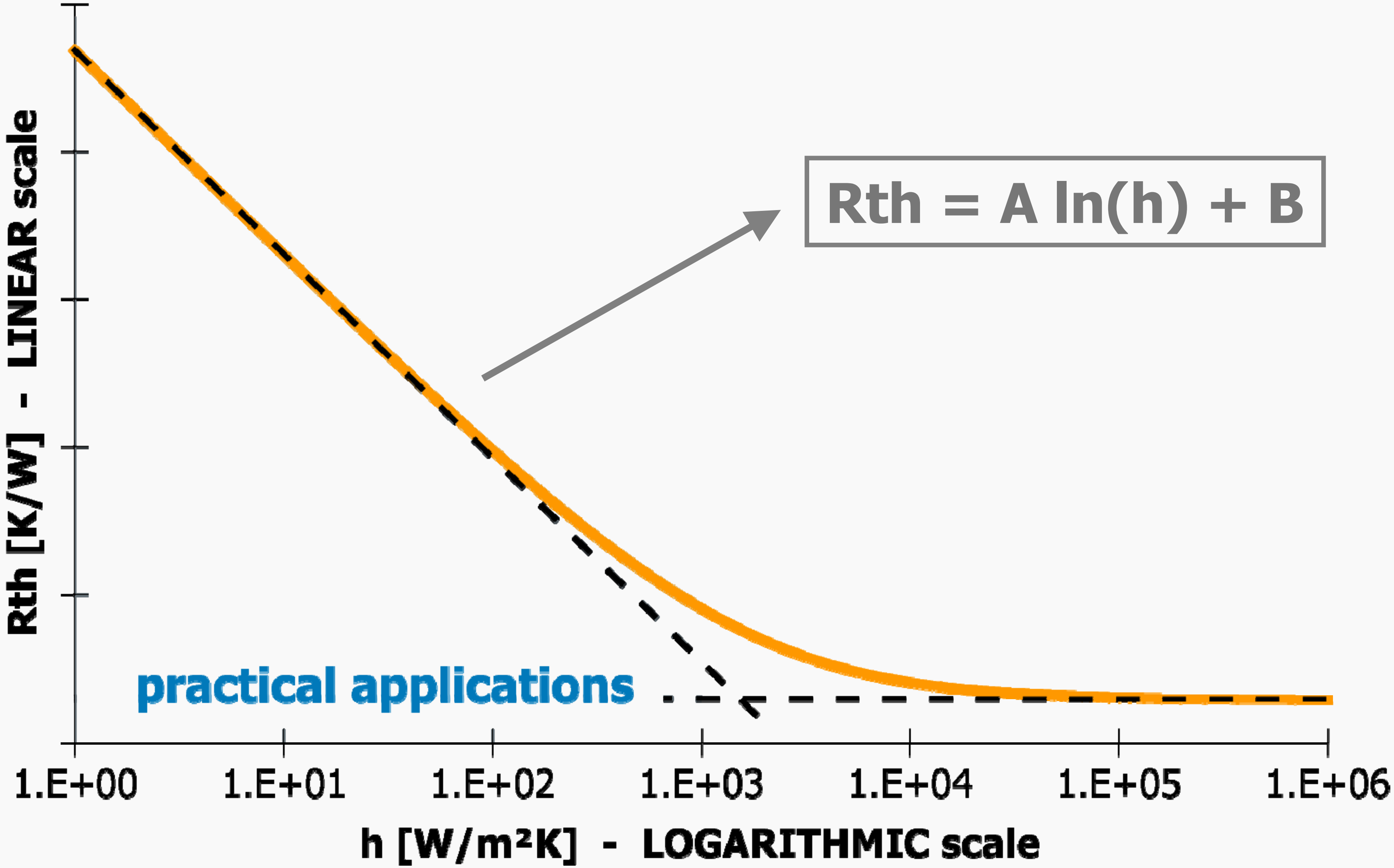
Exact calculations

General tendency



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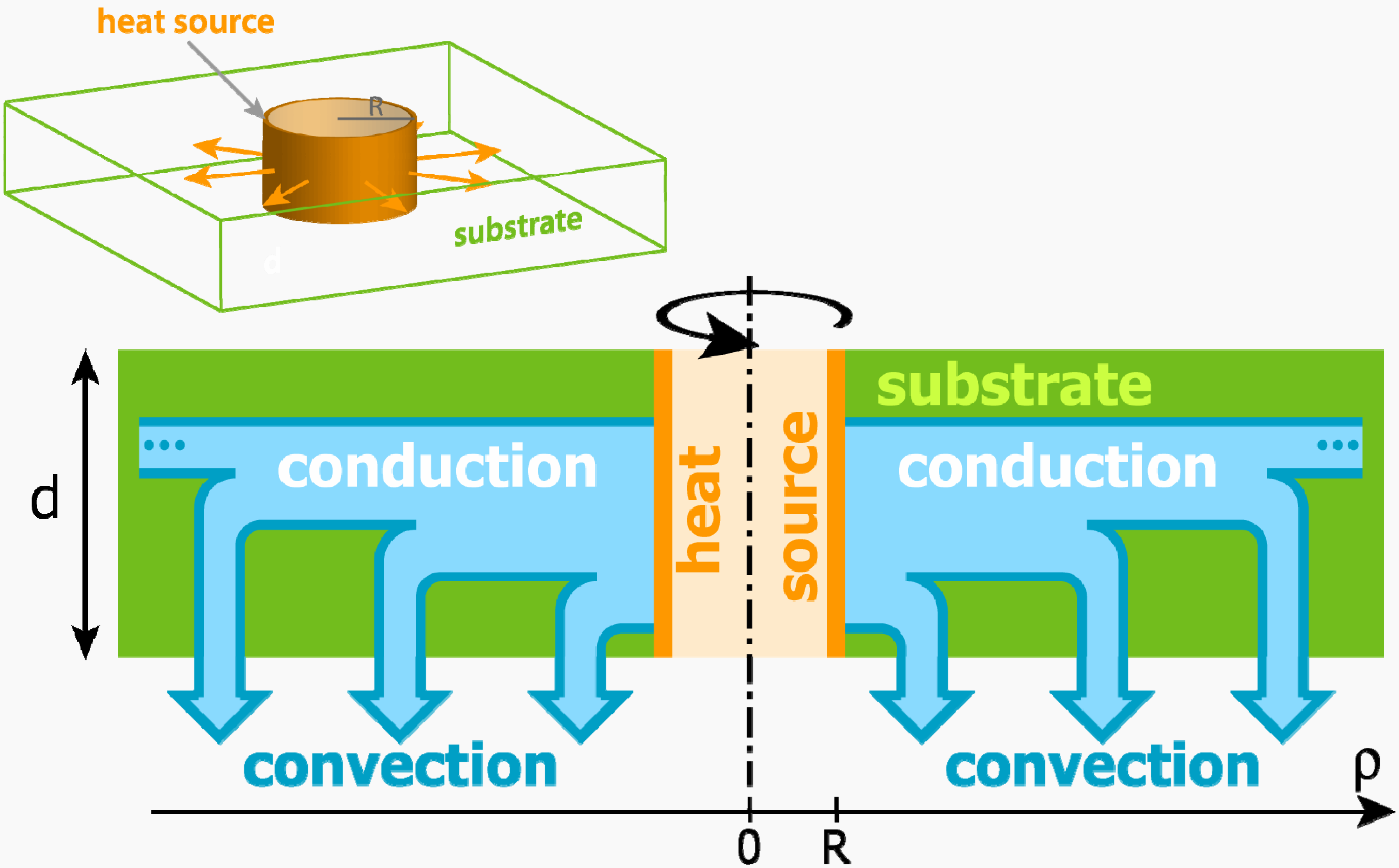
Approximate model

Layout



Approximate model

Calculations (1)

- Heat equation $\nabla^2 T(\rho) - \frac{h}{kd} T(\rho) = 0 \quad \frac{1}{L^2} \quad \rho \geq R$
- Boundary conditions $-(2\pi R d)\cdot k \left.\frac{dT}{d\rho}\right|_{\rho=R} = P \quad , \quad T(\rho \to \infty)$ is finite
- Solution

$$T(\rho) = \frac{P \cdot L}{2\pi k \cdot R \cdot d} \cdot \frac{K_0(\rho / L)}{K_1(\rho / L)}$$

$$R_{th} = \frac{T(R)}{P} = \frac{1}{2\pi k d} \cdot \frac{K_0\left(\frac{R}{L}\right)}{\frac{R}{L} K_1\left(\frac{R}{L}\right)}$$

Approximate model

Calculations (2)

$$\frac{R}{L} << 1 \qquad L = \sqrt{\frac{kd}{h}} \qquad \begin{cases} R \text{ small} & d \text{ large} \\ k \text{ large} & \textbf{h small} \end{cases}$$

For small arguments:

$$K_0(x) \approx \ln(2) - \ln(x) - \gamma + O(x^2)$$

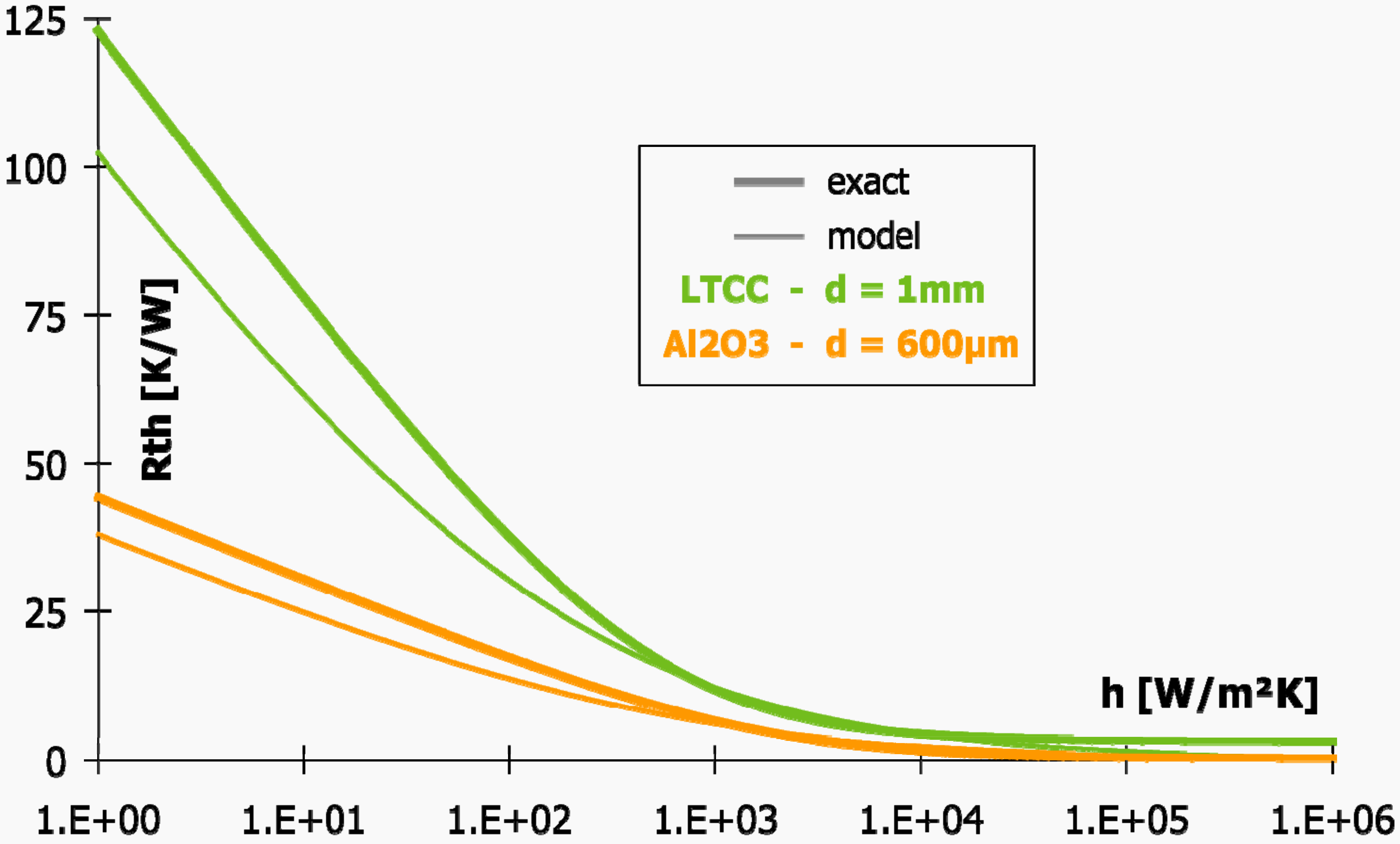
$$x \cdot K_1(x) \approx 1 + O(x^2)$$

$$R_{th} \approx -\frac{1}{4\pi kd}\ln(h) + \frac{\ln(2) - \ln\left(\frac{R}{\sqrt{kd}}\right) - \gamma}{2\pi kd}$$

$$R_{th} \approx A\ln(h) + B$$

Approximate model

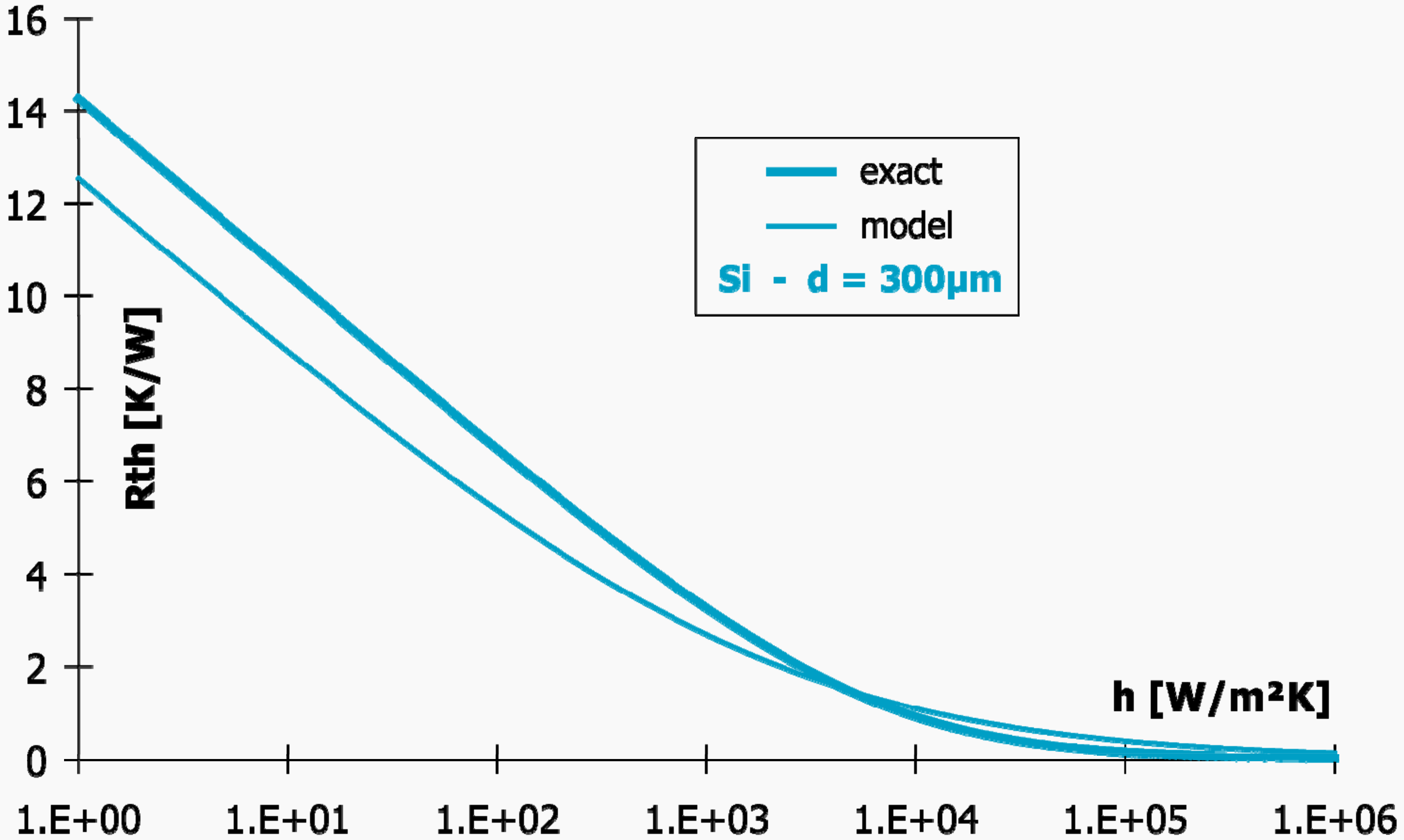
Results (1)



Weak h-dependency of spreading R_{th} in convectively cooled substrates

Approximate model

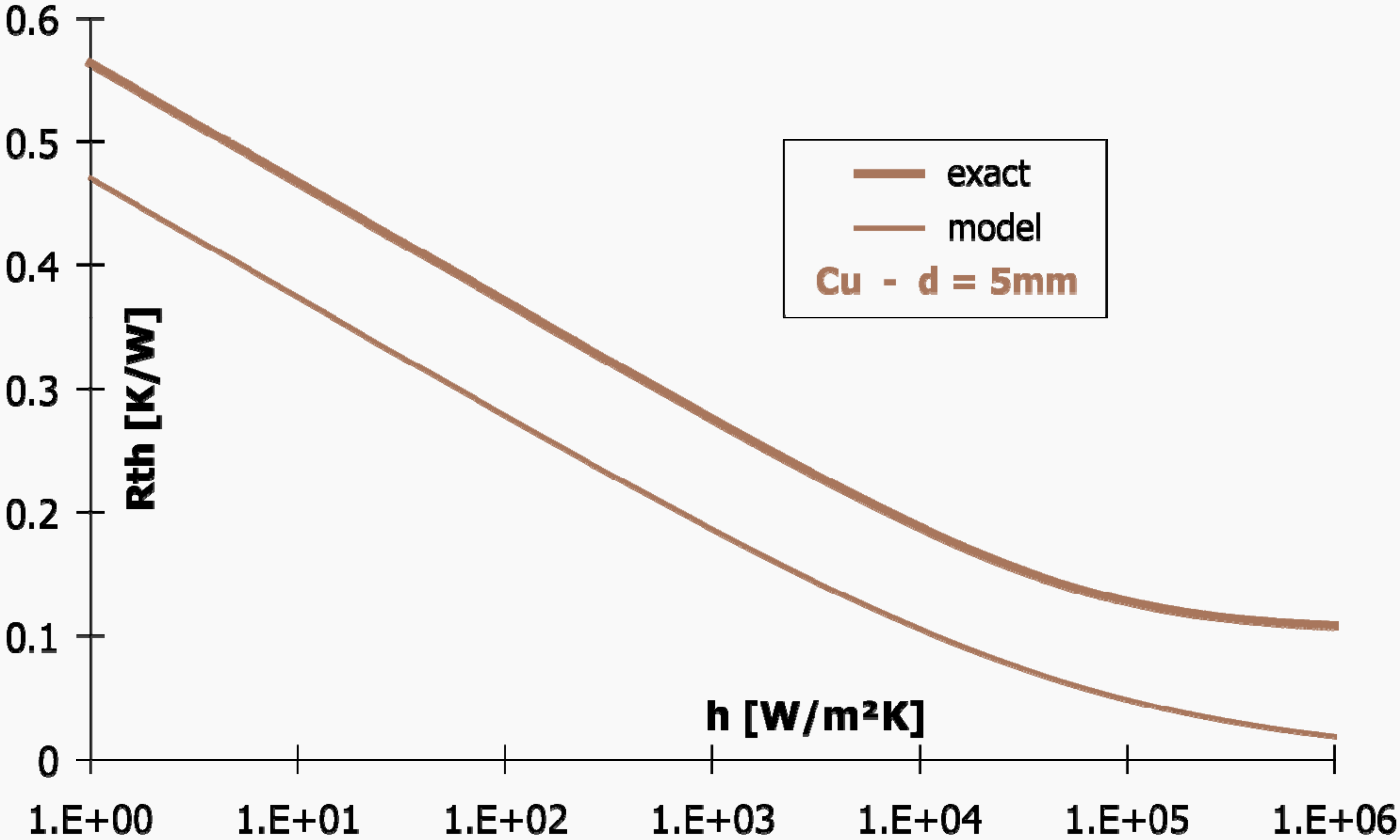
Results (2)



Weak h-dependency of spreading R_{th} in convectively cooled substrates

Approximate model

Results (3)



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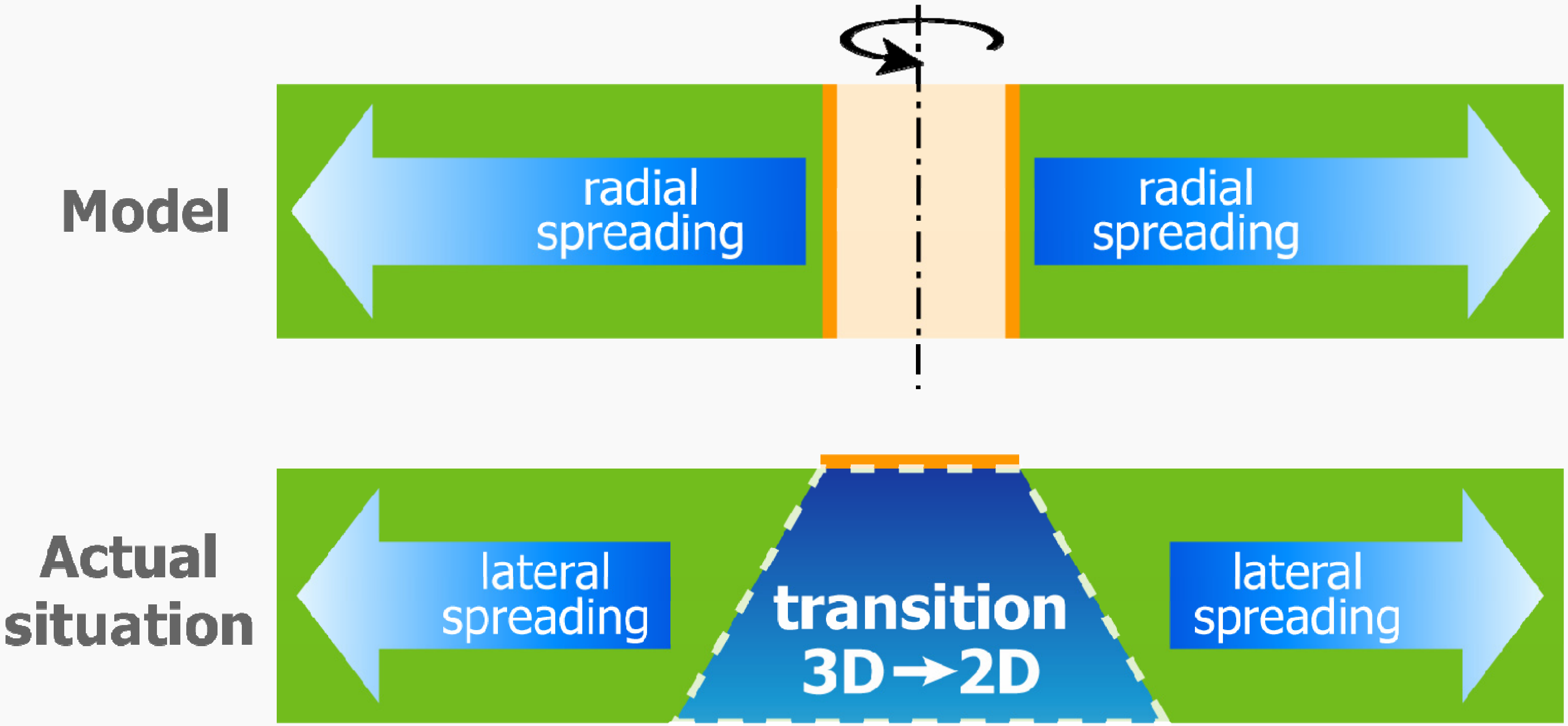
Discussion

Model discrepancy

- correct slope but **underestimates** R_{th}
- reason: heat source is on top surface, not in substrate



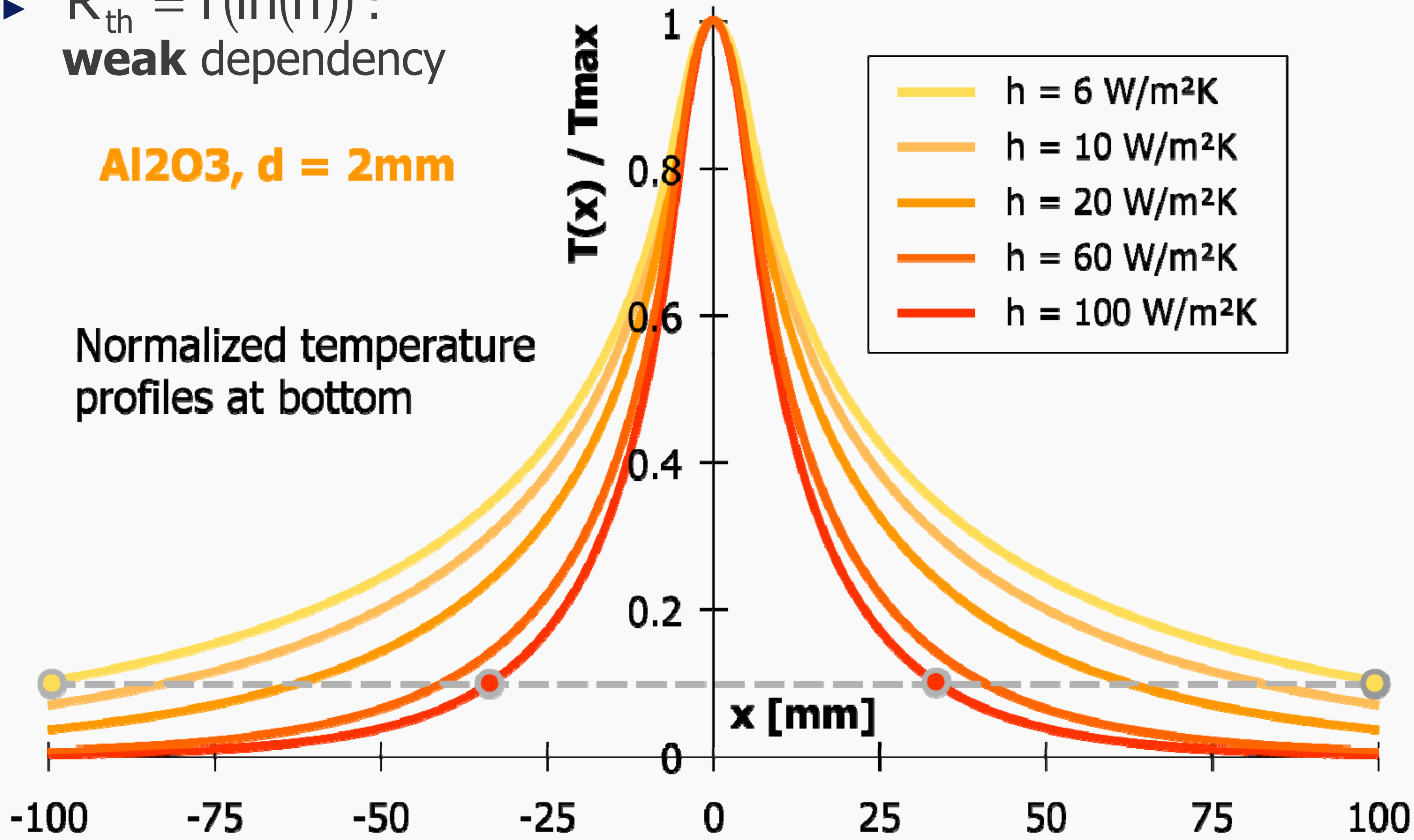
Discussion

Rth vs. h relation

- $R_{th} = f(\ln(h))$:
weak dependency

Al2O3, d = 2mm

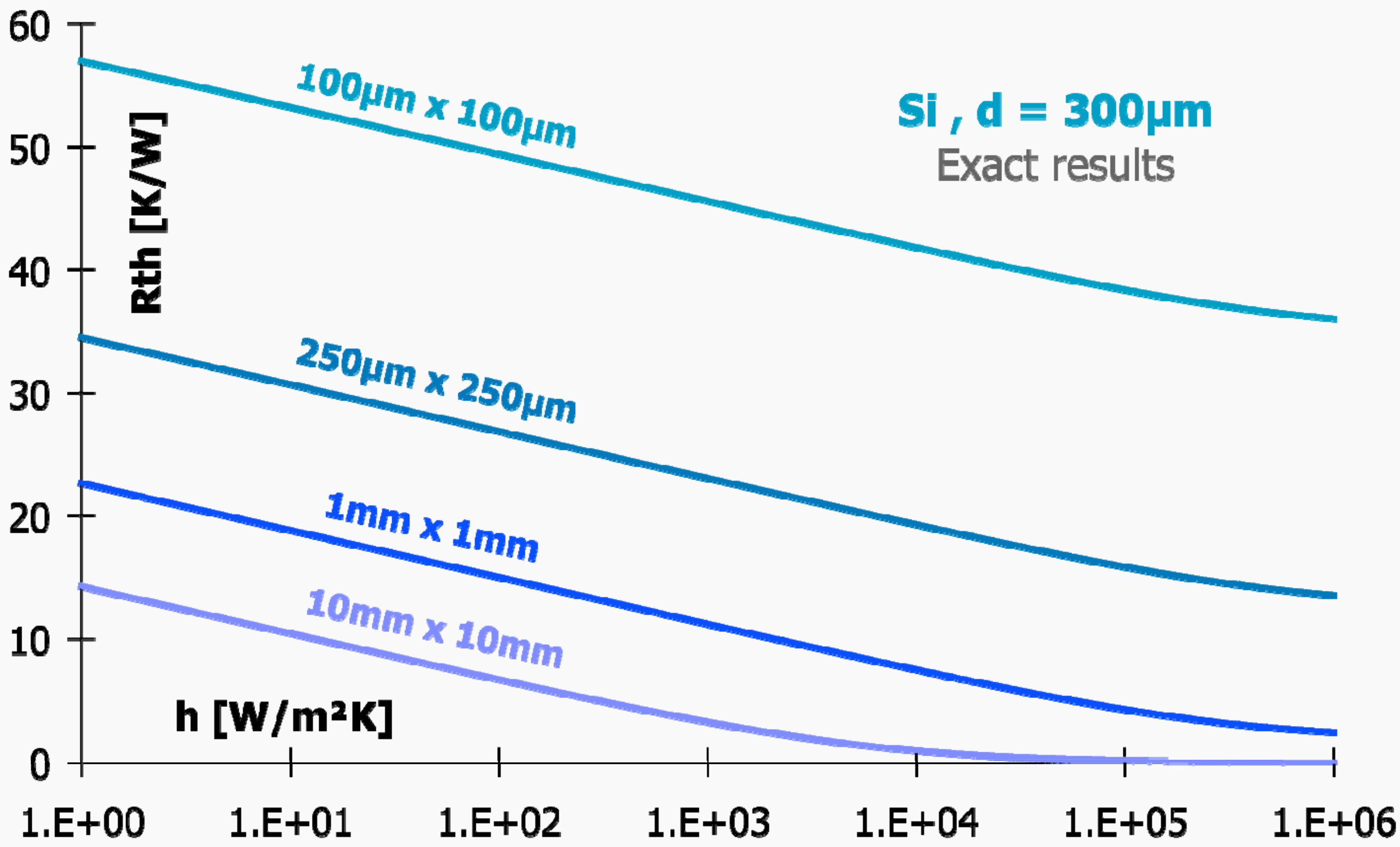
Normalized temperature profiles at bottom



Discussion

Slope

- $A = -\dfrac{1}{4\pi kd}$ ⟶ independent of source dimension



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Conclusions

- surface heat source on substrate with bottom-side convective cooling
- R_{th} vs. h
- for wide variety of k and d: $R_{th} \approx A \cdot \ln(h) + B$

  in range of at least 3 decades (h = 1 – 1000 W/m²K)
- explanation with simple model
- weak ln(h) dependency due to compensation (less cooling = more spreading)

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