A Physics Based Rank Revealing Multiplication Scheme for the Fast Analysis of Photonic Crystal Devices

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A novel rank-revealing multiplication scheme that accelerates the iterative integral equation-based analysis of wave propagation in photonic crystal devices is presented. The proposed multiplication scheme exploits the bandgap character of the background photonic crystal to achieve both rapid convergence of the iterative solver as well as a low matrix-vector multiplication cost. The versatility and computational efficiency of the shielded-block preconditioner are demonstrated by its application to the analysis of wave propagation out of an photonic crystal horn antenna array.

1 Introduction

Recently, two-dimensional photonic crystals (PhCs) become more and more studied because of their promising application to various optoelectronic devices [1]. The PhCs studied in this paper comprise parallel, homogeneous, dielectric/magnetic cylinders that snap to a periodic lattice and that reside in a homogeneous background medium. The underlying periodic structure gives rise to the appearance of frequency ranges - the so-called *photonic or electromagnetic bandgaps* - for which no electromagnetic fields can propagate through the crystal. In turn, this property can be used to create PhC waveguides, for example, by removing/adding cylinders from/to the otherwise defect-less PhC. Various applications of this phenomenon have been demonstrated, such as waveguides with sharp bends [2], multiplexers [3], superprisms [4], etc.

As these structures are often electromagnetically large, usage of computationally efficient methods is imperative to their analysis. The multiple scattering technique (MST) [5] is a popular method for analyzing PhC devices. This frequency domain technique solves integral equations in terms of equivalent currents residing on the PhC cylinders' surfaces. Often, the MST exploits the cylinders' circular nature by expanding surface currents in angular Fourier series, which permits their fields to be cast in terms of Bessel/Hankel functions. With this method, high accuracy can be obtained with only a few unknowns per cylinder. The MST's principal disadvantage is that it requires the solution of a dense linear system of equations

$$\boldsymbol{Z}\boldsymbol{I} = \boldsymbol{E}, \tag{1}$$

whose dimension scales linearly with the number of cylinders. Moreover, numerical experiments show that when solving this system for realistic PhCs iteratively, a high number of iterations is required. This observation necessitates the use of preconditioners.

In this paper, a fast iterative solution method for the analysis of finite PhC devices that explicitly exploits both the bandgap character of the PhC and a (shielded-block) preconditioner, will be described. The use of the

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shielded-block preconditioner automatically leads (i) to fast convergence of the iterative solver and (ii) to fast matrix-vector multiplications following a rank revealing scheme. In this sense, the proposed scheme contrasts itself to classical fast schemes in which the acceleration and the preconditioning are treated separately.

2 Shielded-block Preconditioner

This section details the proposed rank-revealing shielded-block preconditioner. Instead of solving (1) directly, an equivalent system of equations

$$\boldsymbol{MZI} = \boldsymbol{ME}, \tag{2}$$

where the preconditioner $\boldsymbol{M}$ approximates $\boldsymbol{Z}^{-1}$, is constructed and solved iteratively. In what follows $\tilde{\boldsymbol{Z}}$ denotes the preconditioned interaction matrix $\boldsymbol{MZ}$.

2.1 Construction of the shielded-block preconditioner

The proposed shielded-block preconditioner can be seen as an improvement on a classical block-diagonal preconditioner. To construct the latter, the PhC device first is subdivided into N_g contiguous blocks of cylinders. Next, for each block j, the submatrix $\boldsymbol{Z}_j$ of $\boldsymbol{Z}$ describing all of its cylinders' self- and mutual-interactions is inverted. Finally, all resulting inverses are collected into a block-diagonal matrix to form an approximate inverse $\boldsymbol{M}$ of $\boldsymbol{Z}$. The construction of the shielded-block preconditioner proceeds only slightly differently. First, for each block the submatrix $\hat{\boldsymbol{Z}}_j$ describing self- and mutual-interactions of its cylinders plus those residing in a shield around it, viz. a physical jacket of preset thickness that surrounds the block and comprises cylinders present in the block's immediate environment, is inverted. Next, from $\hat{\boldsymbol{Z}}_j^{-1}$, only the rectangular submatrix $\boldsymbol{Y}_j$ comprising rows pointing to variables inside block j are retained. Finally, all $\boldsymbol{Y}_j$ are arranged into a new approximate inverse $\boldsymbol{M}$ of $\boldsymbol{Z}$; this new $\boldsymbol{M}$ is no longer block-diagonal as its block constituents overlap along the column index.

2.2 Effect on preconditioning and iteration counts

This section compares the performance of the shielded-block preconditioner to that of diagonal and block-diagonal preconditioners via their application to a simple PhC device, namely a PhC waveguide formed by removing one row of cylinders from a defect-less PhC. The defect-less PhC is composed of dielectric cylinders with constitutive parameters $(\epsilon_2, \mu_2) = (11.56\epsilon_0, \mu_0)$ and radius $r = 0.18a$, that are spaced by a from center to center along the x and y directions -a is called the lattice constant- and that are residing in free space, viz. $(\epsilon_1, \mu_1) = (\epsilon_0, \mu_0)$. The structure comprise 40 rows of 25 cylinders, i.e., a total of $N_c = 1000$ cylinders, which translates into 3000 unknowns. For illustrative purposes, the block-diagonal and shielded-block preconditioners are constructed by subdividing the PhC devices into blocks of 5 by 5 cylinders. Figure 1 shows the evolution of the residual error versus the iteration count for this PhC device when solving (2) with the quasi-minimal residual (QMR) routine [6] and this for the three different types of preconditioners. When using only a diagonal preconditioner, the residual error quasi-stagnates during many iterations. The block-diagonal and the shielded-block preconditioner entirely alleviate this phenomenon. Moreover, for both PhC configurations studied, application of the shielded-block preconditioner leads to a smaller iteration count than the block-diagonal preconditioner.

2.3 Effect on interaction ranks and matrix-vector multiplication speed

This section details the effect of the shielded-block preconditioner on the numerical ranks of submatrices of the preconditioned interaction matrix. Specifically, it shows that left-multiplication of the unpreconditioned interaction matrix $\boldsymbol{Z}$ with the shielded-block preconditioner automatically leads to a fast matrix-vector multiplication scheme. First note that, by it very definition, $\tilde{\boldsymbol{Z}} = \boldsymbol{1} + \tilde{\boldsymbol{Z}}^{\mathrm{nf}} + \tilde{\boldsymbol{Z}}^{\mathrm{ff}}$, where $\boldsymbol{1}$ stands for the identity matrix, $\tilde{\boldsymbol{Z}}^{\mathrm{nf}}$ comprises all *near field* interactions between adjacent blocks, and $\tilde{\boldsymbol{Z}}^{\mathrm{ff}}$ comprises all *far field* interactions between two non-touching blocks. Let $\tilde{\boldsymbol{Z}}^{\mathrm{ff}}_{ij}$ denote the submatrix of $\tilde{\boldsymbol{Z}}^{\mathrm{ff}}$ describing interactions between non-touching blocks i and j. Because ω lies within the PhC bandgap, the rank of $\tilde{\boldsymbol{Z}}^{\mathrm{ff}}_{ij}$ typically is very low. The rank deficiency of submatrices of $\tilde{\boldsymbol{Z}}^{\mathrm{ff}}$ can be traced to the bandgap character of the PhC: for frequencies inside the electromagnetic bandgap no fields can propagate through a defect-less PhC. As a result, upon preconditioning, blocks no longer

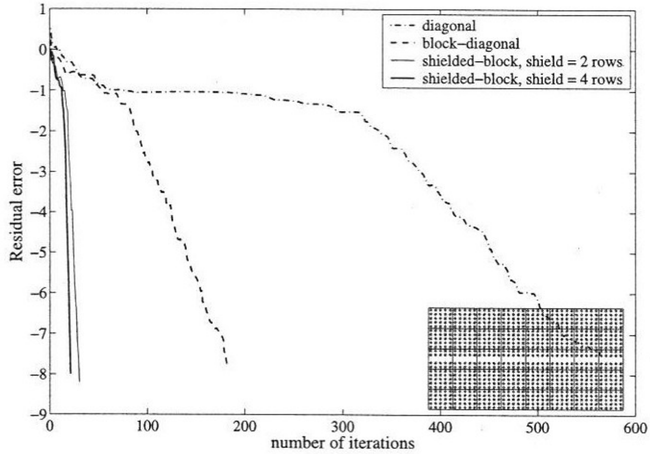

Fig. 1 Residual error vs iteration number for an EC waveguide.

interact through PhC-immersed boundaries, but only through intended interaction ports or PhC interfaces to the homogeneous background medium. This rank deficiency implies that the matrix-vector product $\tilde{\boldsymbol{Z}}^{\mathrm{ff}}\boldsymbol{A}$ can be evaluated fast by approximating $\tilde{\boldsymbol{Z}}_{ij}^{\mathrm{ff}}$ by its truncated singular value decomposition, viz. by discarding all singular vectors with singular values below a set tolerance/noise floor:

$$\tilde{\boldsymbol{Z}}_{ij}^{\mathrm{ff}}\boldsymbol{A}_j \approx \tilde{\boldsymbol{U}}_{ij}\tilde{\boldsymbol{S}}_{ij}\tilde{\boldsymbol{V}}_{ij}^{H}\boldsymbol{A}_j. \tag{3}$$

Here, $\boldsymbol{A}_j$ denotes a vector comprising all unknowns of block j, $\tilde{\boldsymbol{S}}_{ij}$ is a $(R \times R)$ diagonal matrix containing the R largest singular values of $\tilde{\boldsymbol{Z}}_{ij}^{\mathrm{ff}}$, the matrices $\tilde{\boldsymbol{U}}_{ij}$ and $\tilde{\boldsymbol{V}}_{ij}$ contain $\tilde{\boldsymbol{Z}}_{ij}^{\mathrm{ff}}$'s left and right singular vectors, and $(\cdot)^H$ denotes the Hermitian conjugate. (In practice, rank revealing submatrix decompositions other than the SVD are used as they serve the same purpose while being cheaper to compute.)

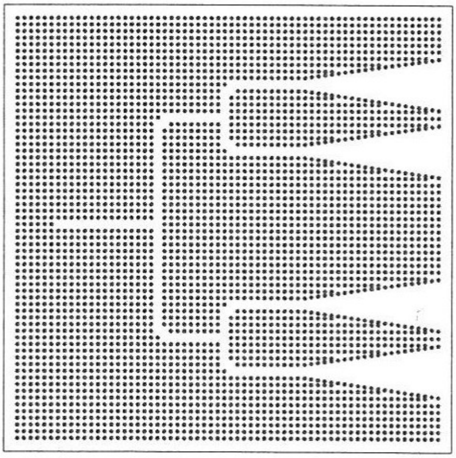

Fig. 2 EC horn antenna array.

3 Example: PhC horn antenna array

Consider the PhC horn antenna array depicted in Figure 2, comprising four basic PhC horn antennas connected to a feed network of PhC waveguides. The basic PhC horn antenna was presented in [7] and was analyzed there using the finite difference time domain method. Its taper length and taper angle are $21a$ and $\arctan(\frac{4}{21})$, respectively.

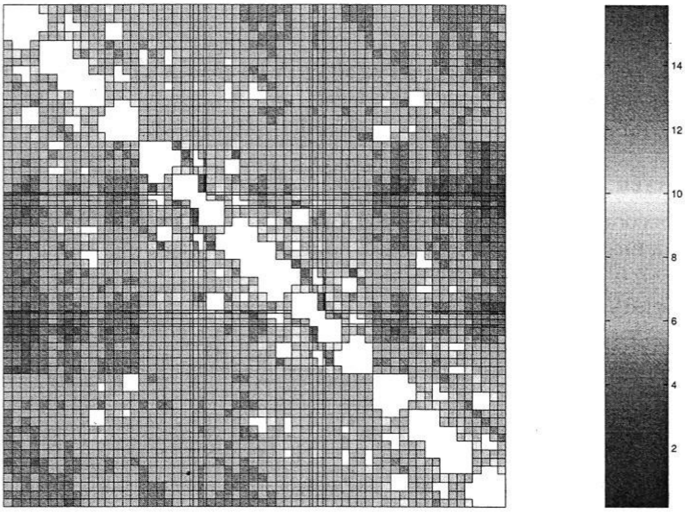

Fig. 3 Rank map of the shielded-block preconditioned interaction matrix for the EC horn array device. Every small rectangle corresponds with a submatrix $\tilde{Z}_{ij}^{\mathrm{ff}}$ and its color indicates the number of singular values larger than 10^{-6}.

The complete PhC horn antenna array comprises 3600 cylinders, which corresponds to 10800 unknowns. For this example, the blocks to form M are squares of 8 by 8 lattice constants. The shield thickness also is 8 lattice constants. In this case, $\tilde{Z}^{\mathrm{nf}}$ reduces to a zero matrix. Figure 3 shows the "rank map" of the (shielded-block) preconditioned interaction matrix for this PhC horn antenna array when the noise floor below which all singular values are discarded is set to 10^{-6}. With this shielded-block preconditioner, the iterative solution of the system of equations (1) with transposed-free QMR to a tolerance of 10^{-5} requires only 17 iterations and using the proposed scheme this takes 3.49 s on a dual AMD Opteron 244, 1.8GHz PC. Using the multilevel fast multipole algorithm in combination with the same preconditioner, the iterative solution takes 29.2 s. Applying a block-diagonal preconditioner (with the same blocks), the iterative solution of the system of equations (1) requires 354 iterations, which takes more than 9 minutes using the multilevel fast multipole algorithm [8]. It is seen that the single-level preconditioner outperforms the multilevel fast multipole algorithm.

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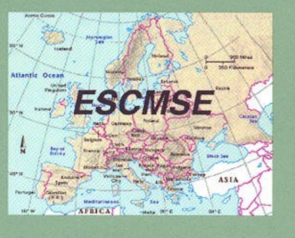

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