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CALCULATION METHODS FOR THE BUCKLING BEHAVIOUR OF ARCH BRIDGES

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ABSTRACT

The structural behaviour of steel tied arch bridges is determined by the introduction of a large compressive force. As a consequence, slender steel arches are highly sensitive to in-plane as well as out-of-plane buckling. At present, no specific buckling curves for out-of-plane buckling exist for non-linear or curved elements in the international codes and calculation methods. Hence, the buckling curves for straight columns, as determined by ECCS are used, which leads to considerable inaccuracies in the assessment of the critical buckling load for arch bridges.

This paper presents two practical calculation methods to design for the buckling behaviour of slender steel arch bridges. The first one follows the calculation method of the Eurocodes, but proposes some augmented empirical formulas for the buckling length of the arches. This allows for a better representation of the out-of-plane stiffness of the arch cross section and of the wind bracings between both arches.

In addition a second method is proposed, based on the use of simplified finite element models to determine the relative slenderness of the structure. Both methods are validated using results from very detailed three dimensional finite element models. Finite element models of several tied arch bridges have been created. These models include variations of the bridge length, dimensions of the arch cross-section, boundary condition, load type, etc. The conclusion of these calculations is that for both of the proposed methods a higher buckling curve can be used than proposed by the code, thus resulting in a more slender bridge design.

1. INTRODUCTION

The fundamental behaviour of tied arches is based on the fact that a large compressive force is developed in the arch cross-section. Because of this, steel arches in particular can become highly sensitive to the out-of-plane buckling phenomenon. However, there is no clear and generally accepted calculation method to predict numerically this stability problem. On one hand, the buckling strength of a steel tied-arch bridge can be calculated by considering the non-linear elastic-plastic behaviour. As the imperfections of the arches highly influence the non-linear behaviour, these geometrical imperfections need to be known before starting this analysis. On the other hand, a linear calculation, resulting in an elastic buckling factor for the compression force, can be carried out. A multiplication factor for the occurring stresses can be found based on this calculation, using an adequate buckling curve, as mentioned for straight beams in (ECCS 1978, Eurocode 3, ECCS 1977). In this case as well, the arch imperfections should be known beforehand. However, the imperfections in slender steel arch bridges are not related to those of a straight beam or column which makes it fundamentally impossible, or at least overly safe, to use the standard buckling curves, derived for straight beams.

As the imperfections of arch bridges are smaller than can be expected for straight members, every detail of the bridge becomes important while determining the buckling load and may influence the results of the numerical simulations (Outtier 2006, Outtier 2009). Therefore, all details, such as diaphragms, connection plates, orthotropic plated bridge deck, bearing systems and arch springs, are modeled in a very exact manner for the calculations which are the basis of this research paper.

2. DESCRIPTION OF THE FINITE ELEMENT MODEL

A finite element model of the Albert Canal Bridge, , which can be seen in Fig. 1, is used as the basis for the research on the resistance to out-of-plane buckling of arches (Outtier et al. 2006, Outtier 2009, Van Bogaert et al. 2006). The Albert Canal Bridge built in 2004, in Belgium near the city of Antwerp, as part of the new high speed railway between Antwerp and Amsterdam. The bridge span equals 115 m, which is quite larger than the Albert Canal itself. However, this bridge span has been chosen in view of the further widening of the canal and the increasing of fluvial traffic on the canal towards the Port of Antwerp. The two arches of this steel tied arch bridge are connected to the lower chord members by sixteen inclined hangers. The upper bracing consists of three tubes of large diameters spread along the length of the arch. The arch springs are tied by the lower chord, consisting of an orthotropic steel deck plate. The bridge is supported by neoprene bearing systems.

Fig. 1. The Albert Canal Bridge, Antwerp, Belgium

The finite element model, developed for this bridge using the SAMCEF software can be seen in Fig. 2. To obtain a model which is as accurate as possible, special attention is given to all details of the bridge, as is being described further, especially those which might introduce asymmetry into the model or might influence the buckling behaviour of the bridge. Since an out-of-plane movement of the bridge is mainly resisted by the wind bracings and by lateral clamping of the arch springs, it was important to model these as accurate as possible.

They are the only part of the construction, connecting both bridges and resisting lateral forces. Several possible shapes of out-of-plane imperfections, such as sinusoidal, double sinusoidal and random shapes are then superposed on the model of the bridge, to assess their influence on the buckling behaviour.

As mentioned before, the possible out-of-plane imperfections are superposed on the actual arch geometry. All of the enforced imperfections show a single wave sinus curve, with maximum amplitude of 115 mm. This amplitude equals 1/1000 fraction of the total the arch span, which is the value recommended by the buckling curves from Eurocode (Eurocode 1993). The influence of the geometrical out-of-plane imperfections of both arches is also investigated. These calculations are performed, once for the case of both arches having identical imperfections in the same direction and once with both arches having imperfections with identical amplitudes, but in opposite directions.

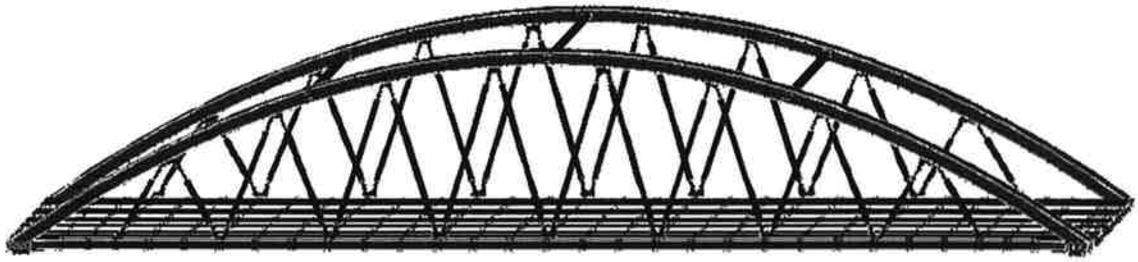

Fig. 2. Finite element model of the Albert Canal Bridge

A subsequent calculation is of the elastic-plastic type, using plastic material behaviour laws for the steel parts of the structure. The definition of this material law complies with Eurocode guidelines for the finite element modeling of plastic materials. The load acting on the bridge is increased linearly and the calculation is stopped, if the displacements of the bridge becomes as large, any further increase would inevitably resulting in an infinite increase of displacements. This calculation starts while having the dead load of the structure. In the following time steps, the live load consisting of sixteen heavy lorries is placed on the bridge deck and is being increased stepwise. Starting from time step 2, the weight of these 16 lorries is increased linearly until the end of the calculation is reached by divergence of the finite element calculation criterion. The buckling reduction factor can be calculated based on the maximum value $N_{FE,pl}$ of the normal force in the finite element model of the arch bridge before buckling of the arch.

[1] $$\chi_{FE} = \frac{N_{FE,pl}}{A \cdot f_y}$$

By modelling several variations of this bridge, as well as a number of other recently built bridges, it became possible to assemble a database of the actual buckling behaviour of more than 50 steel tied arch bridges with span length varying between 45 and 200m. The variations not only include geometrical parameters, but also other parameters, e.g. the influence of residual weld stresses. This database is then used to validate both of the design methods described in the following paragraphs.

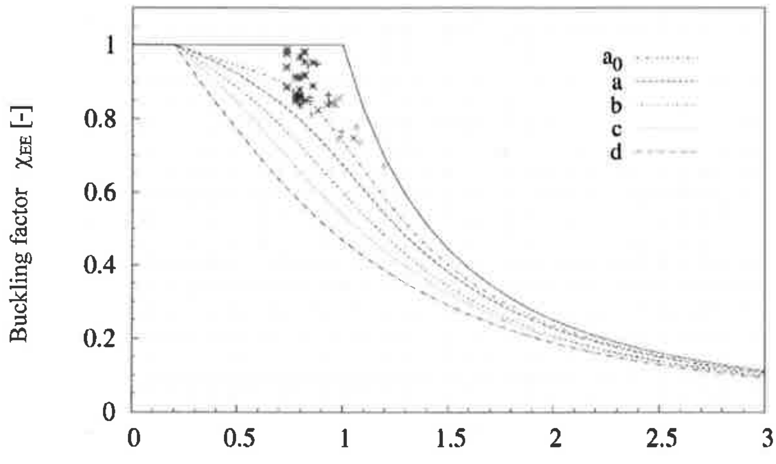

Fig. 3. Results of finite element buckling calculations
in comparison to the Eurocode buckling curves

If the buckling reduction factor, as determined by equation one is plotted on the same diagram as the buckling curves, it can be seen that almost all of the arch bridges end up well above buckling curve a. Most of them are even situated above bucking curve a_0. Based on these results, shown in Fig. 3, it would seem that buckling curve a would be a safe and economical choice for the design of the out-of-plane buckling behaviour of steel tied-arch bridges. However, most codes advice the use of buckling curve b at present. The curves shown in Fig. 3, represent the Eurocode buckling curves. The lowest one is the a_0 buckling curve, followed by the a, b, c and d curve as well as the upper buckling limit.

3. EUROCODE DESIGN OF ARCH BUCKLING

A detailed analytic calculation method has been developed in the Eurocodes, which allows for the determination of the buckling strength of steel members. The first step of this method is the determination of a dimensionless slenderness parameter, λ, for the cross-section of the considered member. Based on this slenderness factor and a correct choice for the buckling curve, a reduction factor, χ, can be directly determined. This clearly defines maximal normal force in the cross-section before buckling occurs as a reduction of the plastic carrying capacity.
For arches, the slenderness λ, can be calculated using the following formula:

[2] $$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{cr}}}$$

where A is the area of the arch cross-section, f_y the yield strength of steel and N_{cr} the critical elastic normal force of the arch. The critical elastic normal force for out-of-plane buckling of steel tied-arch bridges can be determined using:

[3] $$N_{cr} = \left(\frac{\pi}{\beta l}\right)^2 EI_z$$

where l is the bridge span, EI_z the out-of-plane bending stiffness of the arch and β the buckling length factor. The buckling length factor can be found in diagrams in Eurocode 3, based on the span to height ratio of the arch, the out-of-plane bending stiffness and the hanger configuration of the arch.
The reduction factor for the buckling force can be determined based on the slenderness, using the following formula:

[4] $$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 + \bar{\lambda}^2}}$$

[5] $$\Phi = 0{,}5\left(1 + \alpha(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2\right)$$

where the parameter α determines the choice of the buckling curve and equals 0.34 for buckling curve b.

4. PRACTICAL DESIGN METHODS FOR ARCH BUCKLING

4.1 Introduction

Using finite element models as detailed as the one in Fig. 2 is not really acceptable. Still detailed finite element modelling indicates that the analytical design rules in the Eurocodes might be too conservative. Therefore an alternative design method is necessary.

4.2 Design Method Using a Simplified Finite Element Model

A simplified finite element model, using only 1-dimensional beam elements can suffice to get an accurate estimation of the buckling strength of an arch bridge. This finite element model does not include the influence of residual weld stresses, nor geometrical imperfections. It is a model of a "perfect" version of the arch bridge upon which a stability analysis can be performed. A stability analysis consists in solving an eigenvalue problem of the form:

[6] $K\,x = \lambda\,S\,x$

where K represents the structural stiffness matrix, S the geometric stiffness matrix in stability, x one of the buckling modes and λ the associated buckling load. The components of vector x are the structure's degrees of freedom, usually displacements (translations and rotations). The buckling load must be interpreted as the factor by which the external loads must be multiplied for the structure to become unstable. This calculation method is much easier and faster than the one described in previous paragraphs.

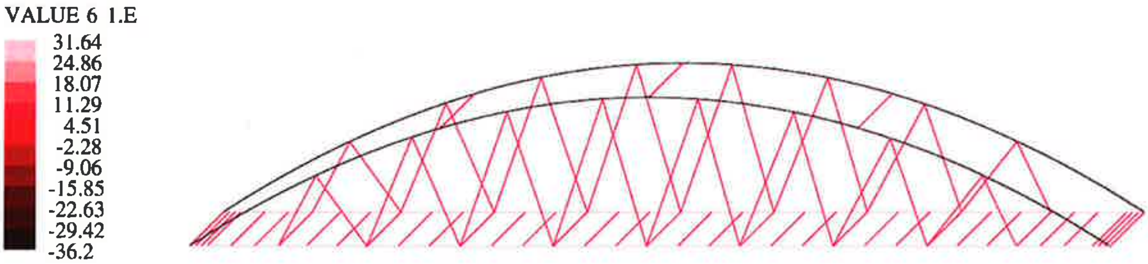

Fig. 4. Normal force in the arch when buckling occurs (N)

The slenderness λ can thus be calculated using equation [7] based on the maximum value $N_{FE,el}$ of the normal force in the finite element model of the arch bridge before buckling of the arch in a calculation using only linear elastic material models. This normal force is illustrated in Fig. 4 showing the simplified version of the finite element model shown in Fig. 2. The most critical buckling mode, also shown in Fig. 5, appears to be a sinusoidal deformation of both arches.

[7] $\overline{\lambda_{FE}} = \sqrt{\frac{A \cdot f_y}{N_{FE,el}}}$

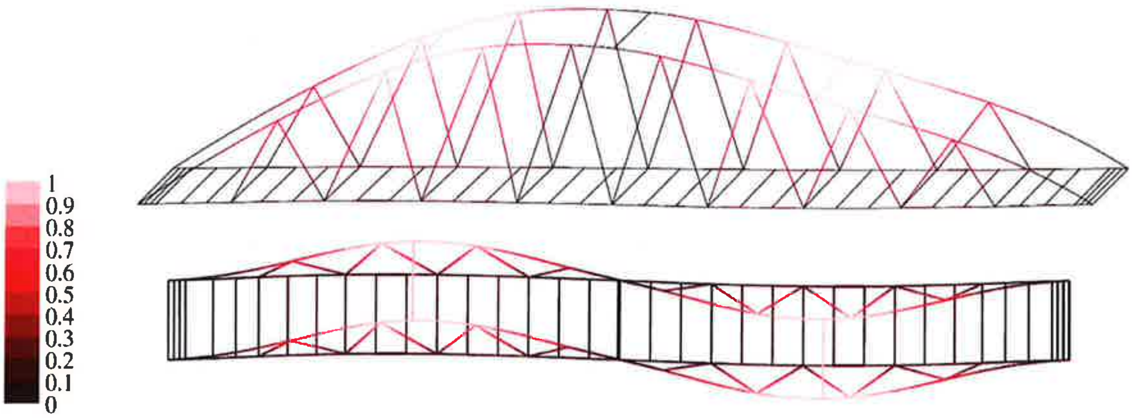

Fig. 5. Buckling shape of the arch bridge

When using the slenderness as defined by equation [7], it becomes possible to calculate the reduction factor χ for buckling, using equations [4] and [5]. Still, the choice remains whether to use buckling curve a or b for this calculation step. Fig. 6 shows a comparison of the buckling reduction factor, once determined using buckling curve b and once based on the elastic-plastic finite element calculations described in the previous paragraphs. It is once again obvious that the calculation using buckling curve b underestimates the actual buckling capacity of the bridge. It seems advisable to use buckling curve a for the design of steel tied-arch bridges, having a welded box section as arch cross-section and with span lengths of about 50 tot 200m.

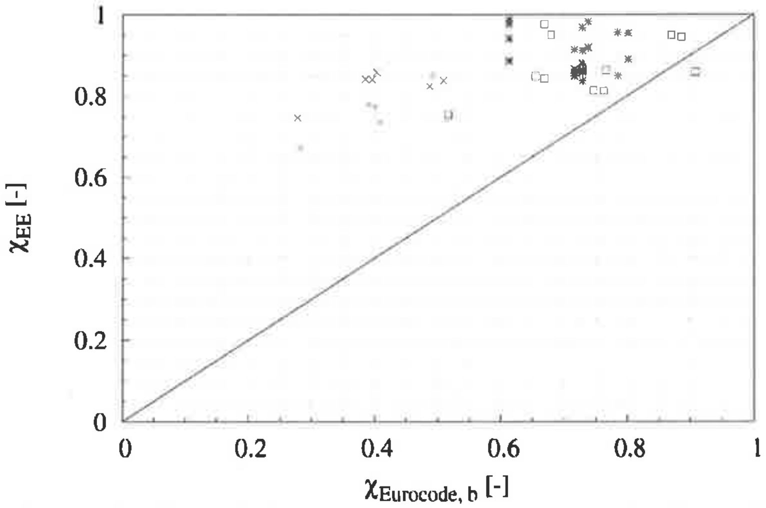

Fig. 6. Comparison of a buckling reduction factor as determined by Eurocode using buckling curve b, with results from finite element modelling

4.2 Design Method Using an Alternative Buckling Length Factor

Using equation [3] and the results of the simplified finite element model discussed in paragraph 4.1, it is possible to calculate the actual values for the buckling length factor β based on the maximum value $N_{FE,el}$ of the normal force in the finite element model. This value is shown on the horizontal axis of the diagram in Fig. 7. The vertical axis represents the value of the buckling length factor β for the same fifty arch bridges, but calculated using the guidelines of the Eurocodes. It seems that the variation of the buckling length factor is in reality much larger than assumed in the Eurocodes.

In addition, the values for bridges with larger span lengths, longer than 200m, are situated at the right side of diagram. At the moment, the Eurocode mainly bases the buckling reduction factor on the ratio between the bridge height and the bridge span, f/l. All of the considered bridges have f/l-ratios varying between 0.16 and 0.19 which gives buckling length factors varying between 0.39 and 0.42 according to Annex D of the Eurocodes.

In reality, a variation between 0.20 and 0.55 is found, which suggests that a number of other parameters influence the buckling length factor. Because of this, an alternative determination formula for the buckling length factor is proposed, usable for bridges with a height to span ratio between 0.15 and 0.20. This alternative method assumes the buckling length factor to be mainly determined by the bridge span, l, and also influenced by the bending stiffness of the arch cross-section for out-of-plane bending, I_z. This is illustrated in Fig. 8. Bridges with comparable bridge spans are drawn using the same symbol, indicating that longer bridges are grouped more to the top of the diagram. Bridges with the same bridge span apparently show a linear connection between the buckling length factor and I_z.

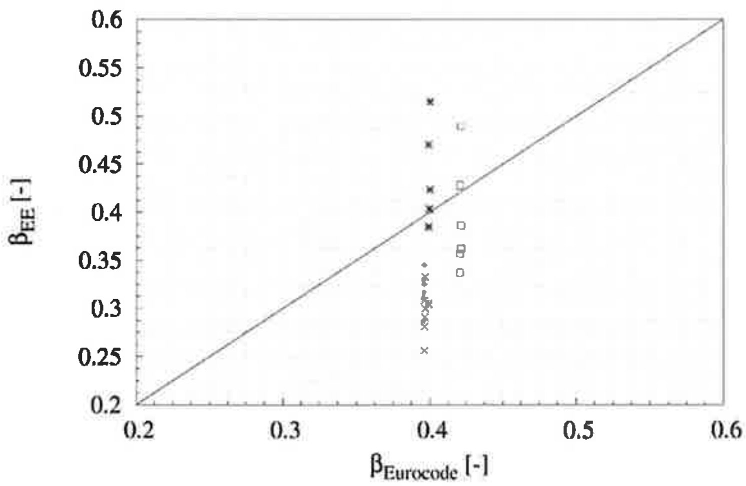

Fig. 7. Comparison of buckling length factors as determined by Eurocode and from finite element modelling

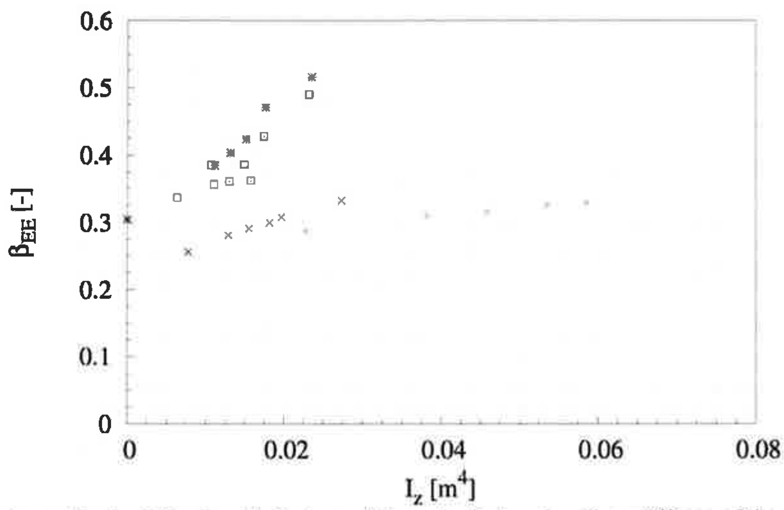

Fig. 8. Relation between the buckling length factor and the out-of-plane bending stiffness of the arch cross-section

Based on these observations, the following formula can be proposed for the determination of the buckling length factor:

[8] $\beta_{al} = \beta_A + I_z(\beta_B - 1\beta_C)$

Based on the parametric finite element analyses, the coefficients of equation [8] are determined to be: $\beta_A = 0.255$; $\beta_B = 16.939$ and $\beta_C = 0.114$. The buckling length factor for all fifty considered arch bridges as determined using equation [8] is compared with the actual buckling length factor in Fig. 8. When comparing Fig. 8 with Fig. 9 it is immediately obvious that the alternative method for calculating the buckling length factor is a much better representation of the actual buckling behaviour, since all values are situated in a narrow zone close to the bisector line of the diagram.

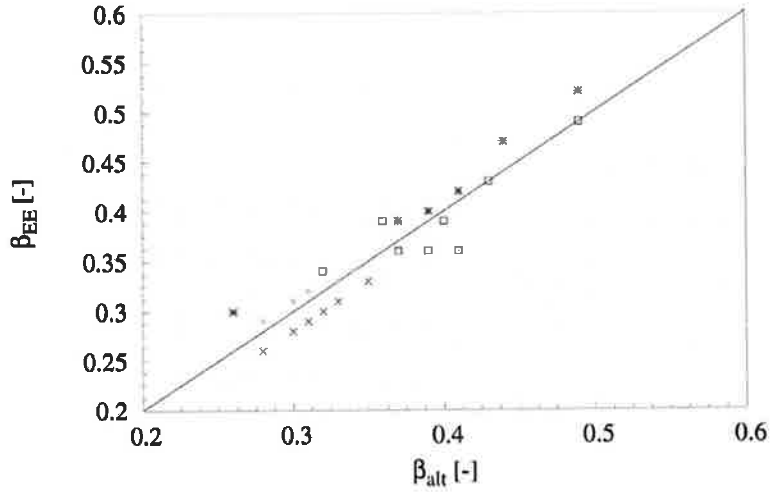

Fig. 9. Comparison of buckling length factors as determined by the alternative formula and from finite element modelling

Using the alternative formula for the buckling length factor it is possible to also calculate an alternative dimensionless slenderness λ, using equations [2] and [3]. Afterwards, equations [4] and [5] and the assumption that the use of buckling curve a is allowable gives an alternative buckling reduction factor $\chi_{alt,a}$. The comparison of this analytically determined alternative buckling reduction factor with the one based on detailed finite element modelling is shown in Fig. 10.

All of the data points are still situated above the bisector line, indicating that the proposed calculation method still errs on the safe side. However, the calculated values are situated much closer to the actual values than when the Eurocode formula for the buckling length factor was used.

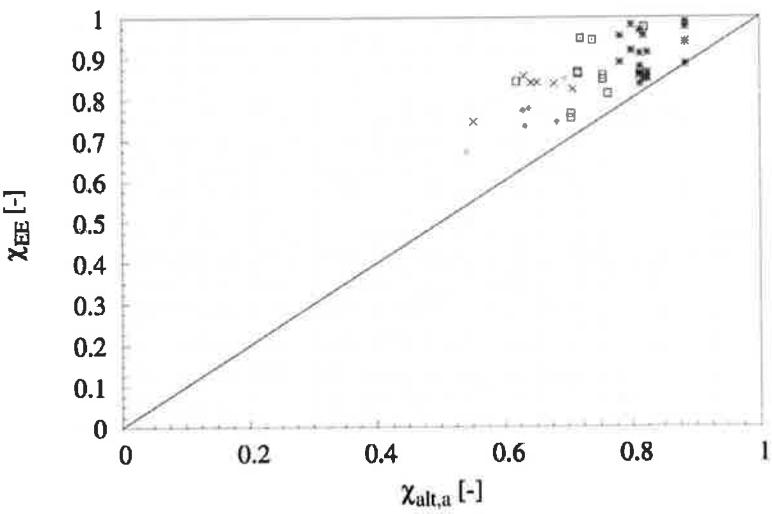

Fig. 10. Comparison of reduction factor for buckling as determined by the alternative formula for the buckling length factor and from finite element modelling

5. CONCLUSIONS

An elaborate and detailed finite element model of a steel tied arch bridge is used to calculate a large number of geometric variations of the six variants of tied-arch bridges. Variations include the size of the arch box section, type of bearing system, load type, hanger configuration and amplitude and size of the assumed imperfections.

The results of these calculations are then used to determine the resulting relative slenderness and buckling reduction factor. When comparing these values with those determined using the Eurocodes, the results seem quite favourable. While the Eurocode dictates the use of buckling curve b for the design of arch bridges, all calculation results are situated well above buckling curve a. Finally, two possible design methods are presented, which will result in a less conservative design. For the first method, the elastic buckling force is determined using a simple finite element model of a perfect arch bridge. The reduction factor can then be determined using buckling curve a. The second method follows the guidelines of Annex D of the Eurocode, but proposes an alternative value for the buckling length factor, based on the stiffness of the arch.

The Eurocode design codes are proven to be quite conservative, but if used with an adapted buckling length factor, can allow for using buckling curve a for the design of arch bridges. Although the calculation method using an alternative formula for the buckling length factor seems to be pretty straightforward and easy, the advantages of using finite element modelling for buckling design cannot be underestimated since they will always lead to a more accurate design. After all, even the simplified finite element models allow for taking account of the influence of the exact geometry of the arch section, the bending stiffness of the cross-section, the hangers and wind bracings, etc.

6. REFERENCES

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