Quantifier Based Aggregation in LSP Suitability Map Construction

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***Abstract*—LSP suitability maps support geographic decision making by providing an efficient framework for evaluating multiple attributes, aggregating the evaluation results, and creating a map that graphically represents the overall suitability of locations for a given purpose. In this paper, we consider elementary LSP suitability map criteria for evaluating a distribution of points of interests (POIs) in a geographical area. More specifically, we study soft computing techniques for the specification and evaluation of criteria in cases where users want to express their preferences regarding the absolute or relative number of POIs that must satisfy the criterion. For that purpose, extended aggregation techniques, based on the use of an absolute or relative quantifier are presented and compared to existing approaches.**

I. Introduction

One of the aspects of geographic decision making is to evaluate and compare different geographic locations or areas in view of some predetermined objective(s). Such an objective could for example be the construction, buying or rental of a home or apartment, the planning of strategic locations for new services, advertisement or commercial activities, or the determination of safe and dangerous areas in case of disasters like flooding, forest fires, earthquakes, etc.

Geographic information systems (GIS), equipped with multicriteria decision methods (MDCM) are commonly used to support decision makers when performing multicriteria decision analysis [16], [17]. In general, the purpose of MCDM is to provide a criterion function

$$\sigma : \mathbb{R}^n \to [0, 1] \tag{1}$$

(where $\mathbb{R}$ denotes the set of real numbers) for computing an overall degree of suitability

$$S(x, y) = \sigma(a_1(x, y), \ldots, a_n(x, y)) \tag{2}$$

that reflects the suitability of the location with coordinates x, y with respect to the objective of the decision analysis. In essence, n elementary attributes $a_i(x, y)$, $i = 1, \ldots, n$ are evaluated, after which the resulting elementary suitability degrees $s_i(x, y)$, $i = 1, \ldots, n$, are aggregated to obtain the overall suitability $S(x, y)$. In soft computing approaches, the overall suitability is a matter of degree, i.e., $0 \leq S(x, y) \leq 1$, where 0 denotes a completely unsuitable location, and 1 denotes the highest level of suitability. The distribution of the overall suitability degrees $S(x, y)$ for a specific geographic region can be presented to users by means of map, which is called a *suitability map* [12].

Current computer systems allow to generate suitability maps on the fly, in a dynamic way. An example of such a suitability map, developed for supporting decisions about suitable locations for home rental is presented in Figure 1 (on top of Google maps [maps.google.com]). In this example the suitability map is a transparent layer on top of a Google map that shows suitability in selected area subdivided into square cells. Lighter colored areas are more suitable than areas denoted by darker color. The same map, without transparency and suitability scores is shown in Figure 2. The map user is advised to look for housing solutions in white areas only. In this example the suitability is computed with respect to the proximity to desirable points of interest (stores, restaurants, gyms, etc.).

In this paper, we focus on the specification and evaluation of elementary criteria which are meant to evaluate the location of a home/apartment with respect to its proximity to desired *points of interest* (POIs). A POI is a description of a geographic location or an entity at a geographic location. POIs exist for different purposes including denoting the localisation of entertainment (restaurants, bars, theatres,...), public services (hospitals, police stations, bus stations, schools,...), and infrastructure (speed control, bridges, tunnels,...). More specifically, we study the evaluation of criteria where users want to express their preferences regarding the absolute or relative number of POIs that must satisfy the criterion.

For example, consider a criterion, used for finding a good location for buying or renting a house, which specifies that there should be restaurants in the environment. In such a case, having only one restaurant in the neighbourhood, even it is next doors, would in general not be a satisfiable situation, as this restaurant might close in the near future, or might simply not offer satisfiable food. Therefore, it is more realistic for a decision maker to require that there are at least a given number of restaurants, for example five, in the environment. In the remainder of the paper, we study how criteria like these could be efficiently handled, hereby offering decision makers the flexibility to specify their needs by means of fuzzy quantifiers like ‘about five’, ‘at least five, but preferably more than ten’, and ‘most’.

Moreover, the investigated techniques also have to cope with the fact that some (types of) restaurants are more preferable than others. Hence, we must assign to them a kind of weight that reflects the relative importance of the POI compared to the others. For example, the user might have a strong preference for Italian food, hence Italian restaurants, in that case, should have a larger relative weight than other, non-Italian restaurants.

The criteria specification and evaluation under consideration is studied within the context of *LSP suitability maps* [7]. Logic scoring of preference (LSP) is a MDCM based on generalised conjunction/disjunction (GCD) aggregation operators [5], [6] and used for the purpose of suitability map construction [7]. In what follows, we present an extension of LSP so that it can efficiently and flexibly cope with (absolute and relative) quantifiers in (elementary) criteria specification and evaluation.

The remainder of the paper is organised as follows. In the next Section II some preliminaries and background information on POIs and LSP are presented. The specification of quantifier based elementary criteria is described in Section III. In Section IV the evaluation of quantifier based criteria is discussed. A comparison with alternative, related approaches is presented in Section V. Finally, in Section VI, the results of this work are summarised and some conclusions are stated.

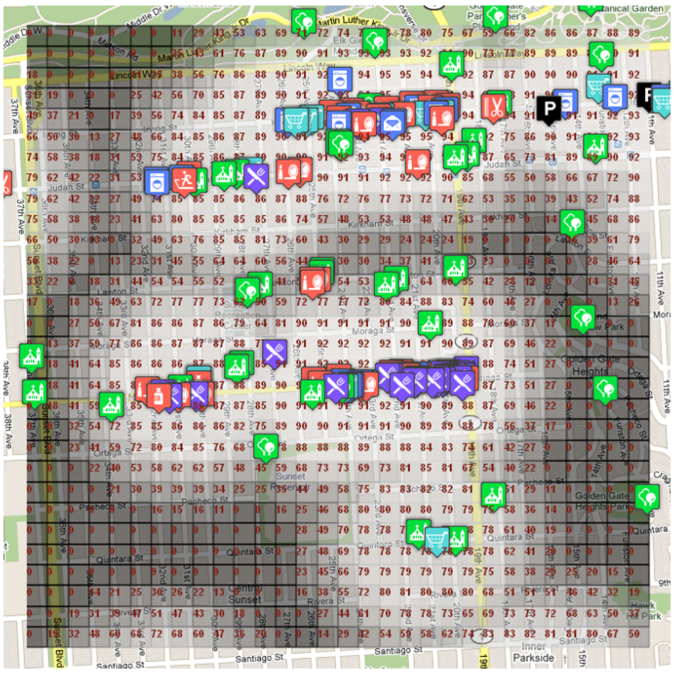

Fig. 1. A sample transparent suitability map with points of interests and cell structure with cell suitability scores.

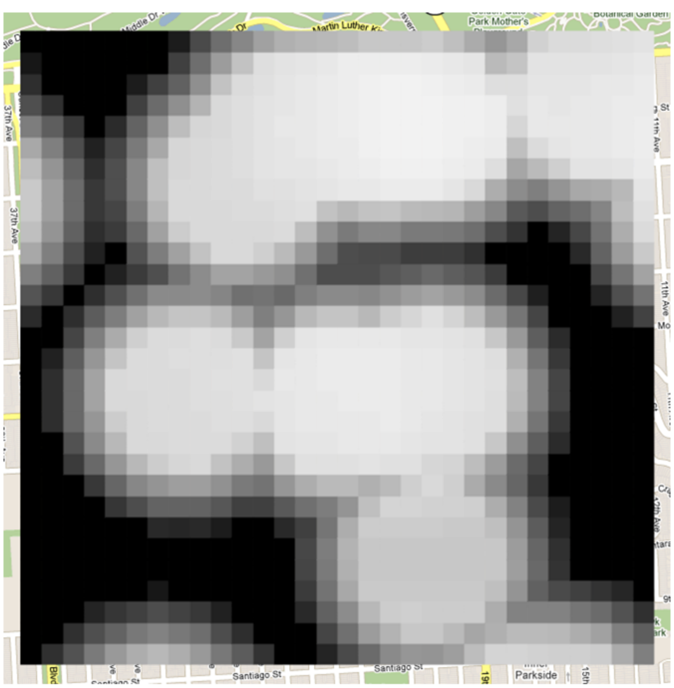

Fig. 2. A sample cell-structured suitability map without transparency, points of interests, and suitability scores.

II. Preliminaries

A. Geographic data and POIs

GIS systems usually contain large amounts of geographic data (e.g., definitions of geographic entities like locations, borderlines, areas, rivers,...) and related metadata, describing what is located or has been observed at a geographic entity (e.g., buildings, land-use, temperature,...).

A specific kind of data are *points of interest* (POIs), which are descriptions that denote geographical locations or entities at geographic locations that might be of interest for some user purposes. Examples of POIs are objects that describe historical buildings, public services, hotels, restaurants and bars, panoramic views, interesting places to visit, etc. Usually, POIs contain information about location (coordinates) and a short textual description, but also other information such as the category the POI belongs to, multimedia like pictures and video and metadata like the creator’s name, the time stamp of creation, the file size, etc. can be provided.

From a formal point of view, a POI can be axiomatically understood as a piece of data that describes an entity in the real world. POIs are assumed to be described in a structured way.

An example of a POI structure with three components is:

$$POI(loc : pos(lat : real, lon : real),$$
$$descr : text, cat : text)$$

The first component ‘loc’ denotes the location of the POI, which is modelled by two real numbers that respectively express the latitude (‘lat’) and longitude (‘lon’) of the POI in decimal degrees (where 0.000001 degrees corresponds to 0.111 metre). The second component 'descr' denotes a free description, provided by the user and modelled by full text, whereas the third component 'cat' denotes the category with which the POI is labeled. It is assumed that this label is chosen from a given fixed list. Examples of POIs with this structure are:

$$POI_1(loc : LOC_{POI_1}(lat : 51.056934, lon : 3.727112), \quad descr : \text{“Het Pand, Ghent”}, cat : \text{“restaurant”})$$

Because information gathering to set up a POI data source is usually a resource consuming task, the user community is often involved in this task. Hence, POI data usually originates from different data sources which are often maintained by user community and can be combined and loaded into a GIS. An example of such data sources are the free Google POI files that can be automatically accessed using a Google-supplied API. If POIs are maintained by a user community, then taking care of data consistency and correctness needs special attention. Indeed, user data is extremely vulnerable to errors, which might among others be due to uncertainty, imprecision, vagueness or missing information. A problem that seriously decreases data quality occurs when different POIs are entered in the GIS to denote the same geographic location or entity at a geographic location. Such POIs are called *coreferent POIs*. Coreferent POIs not only introduce uncertainty and inconsistency in the data, but moreover lead to incorrect query results, inefficient storage use and data processing overhead [10].

It is therefore important to develop techniques to *detect* coreferent POIs and, once detected, to solve the problem of coreference by *merging* all detected coreferent POIs into one single, consistent POI [3], [4]. In the remainder of this paper we assume that POI data have been preprocessed so that they are consistent and do not contain coreferent data.

B. LSP suitability map construction

Suitability maps introduced in GIS literature are based on a variety of multicriteria decision making techniques including simple additive scoring (SAS, a.k.a. SAW, simple additive weighting [23]) [18], [13], [11], the multi-attribute value technique (MAVT) [26], [15], the multi-attribute utility technique (MAUT) [9], [15], the analytic hierarchy process (AHP) [20], [1], [21], ordered weighted average (OWA) [22], [19], outranking methods [2], [14], and logic scoring of preference (LSP) [5], [7]. The framework of LSP, and hence LSP suitability maps form the starting point for this work.

Suitability map construction with LSP consists of a unique configuration task and a repetitive evaluation task. This is illustrated in Figure 3. The main steps of the configuration task are:

1) *Creation of a system attribute tree*. The main purpose of this task is identifying the different properties of a competitive location or object at a competitive location on which the decision making process is based. Usually properties can be decomposed in sub properties which on their turn could also be decomposed and so on. Hence, properties are generally hierarchically structured, hereby reflecting the way how decision makers analyse the input parameters of the decision making process.
The leaf nodes of the resulting hierarchical tree structure represent the elementary (cell) attributes $(a_1, \ldots, a_n)$ (with $n \in \mathbb{N}$) of the LSP system (see Figure 3, number 1). An example of a system attribute tree for decision making with respect to apartment location is presented in Figure 4. Basic properties being considered are, e.g., accessibility, entertainment and recreation, population density, healthcare, and employment and education. Accessibility is, for example, determined by proximity of bus stops, railway stations and main roads and for entertainment and recreation, one has to check sport facilities, libraries, theatres and restaurants, which on their turn have to be checked by making a distinction between Chinese, French and Italian restaurants. Elementary attributes are underlined.
Within the context of a GIS, each elementary cell attribute a_i will be characterized by an associated data

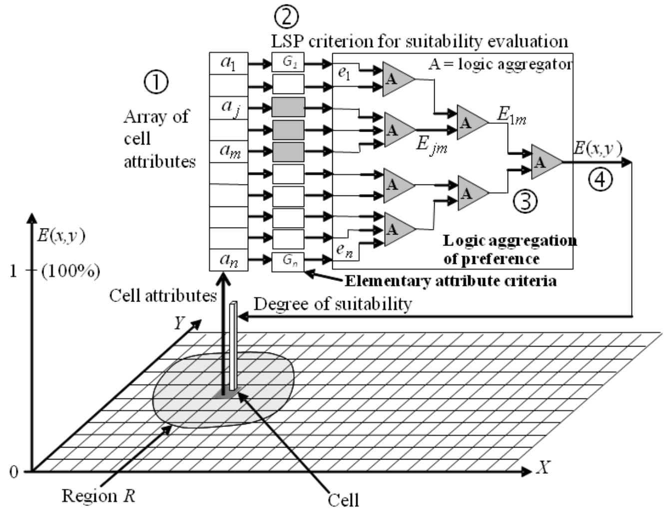

Fig. 3. LSP suitability map construction.

Fig. 4. Example of a system attribute tree.

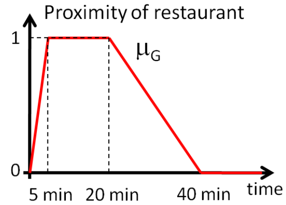

Fig. 5. Example of an elementary criterion.

type t_i, which defines the domain dom_{a_i} of allowed attribute values for a_i.

2) *Definition of elementary criteria*. For each elementary (cell) attribute a_i of the system attribute tree an adequate evaluation criterion G_i has to be specified. In order to be suitable, the criterion has to accurately reflect the decision maker's preferences with respect to the acceptable and unacceptable values from the domain of the data type of the attribute under consideration. For that purpose LSP uses some basic criterion modelling technique that is based on soft computing. Because being acceptable or not, is considered to be a matter of degree, a regular fuzzy set that is defined over the set of valid domain values can be used to represent the preferences of the decision maker. Hereby a membership degree 0 means not acceptable at all and a degree 1 reflects fully acceptable. This is illustrated in Figure 5.
   In the example, preferences related to the proximity of a restaurant expressed in time distance are modelled by the fuzzy set G with membership function μ_G. This fuzzy set denotes that walking distances between 5 and 20 minutes are fully acceptable, whereas distances longer than 40 minutes are completely unacceptable. For distances between 0 and 5 minutes acceptability grows linearly (expressing that being too close to a restaurant brings some inconveniences like shortage of parking and noise annoyance). Likewise, for distances between 20 and 40 minutes, acceptability decreases linearly (expressing that longer walking times are less preferred).
3) *Creation of an aggregation structure*. The last configuration step concerns the specification of appropriate aggregation operators for aggregating elementary and intermediate evaluation results (see Figure 3, number 3). This aggregation structure has to be consistent with the structure of the system attribute tree and reflect the decision maker's preferences regarding the combination of the different properties and sub properties of the decision making process. It will be used to compute an overall suitability degree [8].
   The aggregation operators in LSP are all based on the generalised conjunction/disjunction function [6] which is implemented as a weighted power mean:

$$M(x_1, \ldots, x_n; W_1, \ldots, W_n; r) = (W_1 x_1^r + \cdots + W_n x_n^r)^{1/r} \quad (3)$$

   where $W_1, \ldots, W_n$ are weight parameters used to model the relative importance of their associated input values (i.e., satisfaction degrees), and the parameter $-\infty \leq r \leq +\infty$ is used to set the logic properties of the operator. With $r = +\infty$, resp. $r = -\infty$, the operator behaves like pure disjunction, resp. pure conjunction, whereas the value $r = 1$ corresponds to the (weighted) arithmetic mean. Other values can be used to model partial conjunction ($-\infty < r < 1$) and partial disjunction ($1 < r < +\infty$). LSP supports simple, compound and advanced compound aggregation operators [5].

The evaluation task is executed for each cell (x, y) on the map and consists of the following main tasks:

1) *Evaluation of elementary criteria*. Each elementary criterion G_i has to be evaluated using the actual value of the attribute a_i for the location (x, y) under consideration (see Figure 3, number 2). For the evaluation, the membership grade of the attribute value in the fuzzy set that expresses the criterion is computed. This membership grade is then interpreted as the elementary satisfaction degree e_i of the value for the criterion. For example, for the criterion G, proximity of restaurant, as presented in Figure 5, a walking distance of 30 min would result in an elementary satisfaction degree $e = \mu_G(30) = 0.5$.
2) *Aggregation of elementary satisfaction degrees*. The elementary satisfaction degrees e_i obtained from the evaluation of elementary criteria are aggregated using the aggregation structure that has been set up during configuration. Finally, an overall degree of satisfaction $E(x, y)$ is obtained (see Figure 3, number 4). This degree reflects the overall suitability of the location (x, y).

In the remainder of the paper we will focus on the definition and evaluation of more advanced elementary criteria (Step 2 of the configuration task and Step 1 of the evaluation task) for POIs, which additionally allow to specify user preferences regarding the number of POIs that should satisfy the criterion.

III. Quantifier based LSP criterion specification

Consider the situation where a decision maker needs to find an area suitable for home or apartment location and has the preference that at least four restaurants should be in the environment, but preferably there should be six or more restaurants that are close enough. Such kind of criteria are called *quantifier based criteria*, because a quantifier is used to specify how many cases ('at least four, preferably six or more') should satisfy the basic criterion ('proximity'). An example of such a situation is depicted in Figure 6. The location under investigation is represented by a dot, whereas POIs denoting restaurants are represented by black circles with numbers.

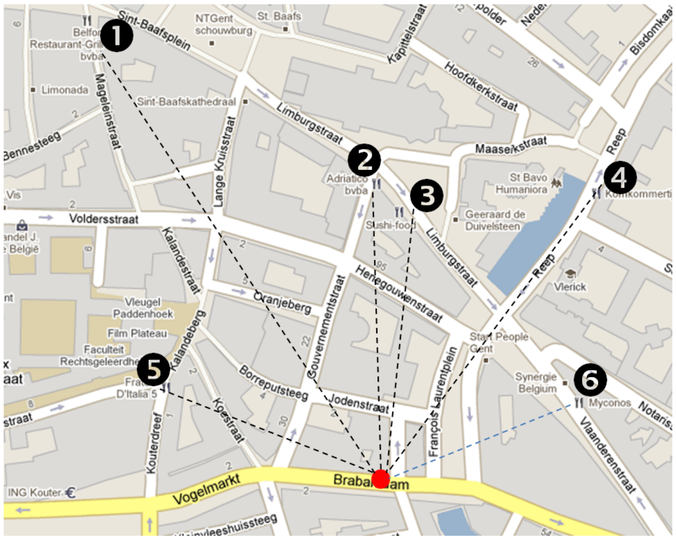

Fig. 6. Example of location of restaurants.

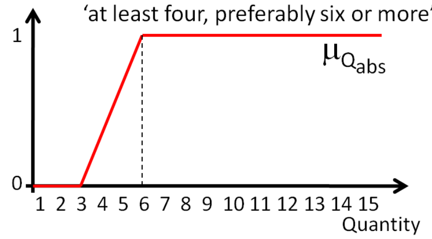

Fig. 7. Soft modeling of an absolute quantifier.

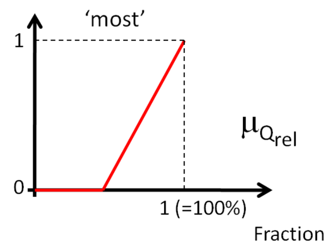

Fig. 8. Soft modeling of a relative quantifier.

The basic LSP criterion modeling technique does only support the handling of elementary criteria of the form:

$$a \text{ IS } G \tag{4}$$

where a is an elementary system attribute and G is an elementary criterion defined for that attribute, e.g., specified by means of a fuzzy set with membership function $\mu_G : dom_a \rightarrow [0,1]$. What is needed is support of criteria of the form:

$$Q(a \text{ IS } G) \tag{5}$$

where Q is a soft quantifier.

Fuzzy sets allow it to define a variety of soft quantifiers [25]. Indeed, beside the universal quantifier ($\forall$) and the existential quantifier ($\exists$), we can consider quantifiers that are described by linguistic terms. A distinction is made between *absolute quantifiers* which denote a number or quantity like, e.g., 'around twelve', 'around six', etc. and *relative quantifier* which refer to a total number and denote a fraction of this total like, e.g. 'most', 'a small number', etc.

An absolute quantifier Q_{abs} can be modelled by means of a membership function

$$\mu_{Q_{abs}} : \mathbb{N} \rightarrow [0,1] \tag{6}$$

or

$$\mu_{Q_{abs}} : \mathbb{R} \rightarrow [0,1] \tag{7}$$

whereas a relative quantifier can be modelled by a membership function

$$\mu_{Q_{rel}} : [0,1] \rightarrow [0,1] \tag{8}$$

Hereby, the value $\mu_{Q_{abs}}(n)$ expresses the extent to which the number n corresponds to the quantifier; analogously, the value $\mu_{Q_{rel}}(p)$ expresses the extent to which the fraction p corresponds to the quantifier. For example, the absolute quantifier corresponding to 'at least four, preferably six or more' can be modelled as depicted in Figure 7. An example of a relative quantifier that can be used to model 'most' is given in Figure 8. Hereby it has been assumed that most cases are satisfied if at least 80%, but preferably more than 95% of the cases are satisfied. Hence, by providing the decision maker with the extra facility to specify a soft quantifier, the LSP model can be meaningfully extended with quantifier based aggregation.

IV. Quantifier based criterion evaluation in LSP

Different quantifier based criterion evaluation approaches exist. The purpose of such approaches is always to determine to which extent it is the case that Q POIs satisfy condition C, or in other words to determine to which extent the number of POIs, resp. the fraction of POIs, that satisfy C is compatible with Q.

A. Basic approach

Perhaps the simplest evaluation technique is the one that is based on Zadeh's original concepts of fuzzy cardinality [24] and fuzzy quantifiers [25]. In case of an absolute quantifier the elementary satisfaction degree e is then computed by

$$e = Q_{abs}\left(\sum_{POI \in U} \mu_G(POI[a])\right). \tag{9}$$

Hereby, $POI[a]$ denotes the value of attribute a in POI and U denotes the universe of all POIs under consideration (i.e., all POIs satisfying the other preferences of the decision maker). In the restaurant example, U can be the set of all POIs with category label 'restaurant'. All POIs of U are evaluated using the POI criterion G for their attribute a, then all resulting satisfaction degrees are summed. Finally, the sum is evaluated using the absolute quantifier Q_{abs}.

POI	$time$	$\mu_G(POI[time])$
1	38 min	0.1
2	34 min	0.3
3	30 min	0.5
4	34 min	0.3
5	18 min	1
6	24 min	0.8

TABLE I
PROXIMITY OF RESTAURANTS.

When using a relative quantifier the evaluation of the criterion can be done by

$$e = Q_{rel}\left(\frac{\sum_{POI \in U} \mu_G(POI[a])}{|U|}\right). \tag{10}$$

Hereby, the sum of all satisfaction degrees is divided by the number of POIs of the category under consideration, i.e., the cardinality of U. This reflects the fraction of the POIs that satisfy the condition. This fraction is then evaluated using the relative quantifier Q_{rel}.

For example, consider the restaurant POIs depicted in Figure 6 and the condition, 'at least four, preferably six or more' (cf. Figure 7) 'nearby restaurants' (cf. Figure 5) to be checked for the location denoted by the dot in Figure 6. Furthermore, assume that the proximity of the restaurants is as given in Table I. With these data, the evaluation of the criterion becomes $e = Q_{abs}(3) = 0$ with the basic approach.

B. Approach for monotonic increasing quantifiers

In the basic approach, the number of POIs satisfying the elementary criterion G is approximated by the fuzzy cardinality of G, i.e., the sum of the membership degrees of all elements of G. Such an approximation might be inaccurate as there is no way to distinguish for example between the case where only one POI fully satisfies G and the case where five POIs all satisfy G with satisfaction degree 0.2.

If the used quantifier is modelled by a monotonic increasing membership function, a more adequate approach can be obtained by carefully trying to balance satisfaction of G and preference of quantity, hereby considering all POIs in U. In case of using an absolute quantifier Q_{abs} this balance can be determined by considering all possible levels α of criterion satisfaction, i.e., considering all $\alpha \in [0, 1]$.

For each level α we consider the crisp set G_α of all POIs that satisfy criterion G to at least an extent α. Because G is an elementary criterion that is specified by a fuzzy set with membership function $\mu_G : dom_a \to [0, 1]$, the α-cut function for fuzzy sets can be used for this purpose, i.e.,

$$G_\alpha = \{POI|POI \in U \wedge \mu_G(POI[a]) \geq \alpha\} \tag{11}$$

For example, considering the data given in Table I, $G_{0.2} = \{5, 6, 3, 2, 4\}$ as all POIs except the POI with number 1 have a query satisfaction degree $\mu_G(POI[time])$ that is greater than or equal to 0.2.

Next, considering all POIs in G_α, a trade-off is made between criterion satisfaction and preference of quantity. This is done as follows:

1) *Determining criterion satisfaction.* The extent to which all POIs in G_α satisfy G can be expressed by

$$\min_{POI \in G_\alpha} \mu_G(POI[a]). \tag{12}$$

For the set $G_{0.2}$ of the previous example this extent is 0.3 as this is the largest extent to which *all* POIs of $G_{0.2}$ satisfy G.

2) *Determining the preference of quantity.* The number of POIs that satisfy G to at least degree α is equal to the cardinality $|G_\alpha|$ of G_α. In our example, $|G_{0.2}| = 5$. To determine the preference of this quantity, we have to check to what extent $|G_\alpha|$ corresponds to the specified quantifier Q_{abs}. This is done, using the membership function of Q_{abs}, i.e.,

$$Q_{abs}(|G_\alpha|). \tag{13}$$

For the example under consideration, using the quantifier depicted in Figure 7 we obtain that $Q_{abs}(|G_{0.2}|) = Q_{abs}(5) = 0.67$.

3) *Determining the trade-off between criterion satisfaction and preference of quantity.* This trade-off should reflect how good all POIs in G_α satisfy the full quantifier based criterion. In other words, this trade-off should reflect the extent to which both the criterion and the quantifier are satisfied. For reason we apply the conjunctive minimum operator, i.e.,

$$\min\left(\min_{POI \in G_\alpha} \mu_G(POI[a]), Q_{abs}(|G_\alpha|)\right). \tag{14}$$

Finally, to evaluate all POIs in U, the trade-offs between criterion satisfaction and preference of quantity (as explained above) are considered for each α-level $\alpha \in [0, 1]$. The best trade-off, corresponding to one or more specific subsets of U, is then chosen as the measure for the overall satisfaction of the POIs in U. For that purpose, the disjunctive maximum operator is used. This leads to the following evaluation function:

$$e = \max_{\alpha \in [0,1]}\left[\min\left(\min_{POI \in G_\alpha} \mu_G(POI[a]), Q_{abs}(|G_\alpha|)\right)\right]. \tag{15}$$

For a monotonic increasing relative quantifier Q_{rel}, the equation becomes

$$e = \max_{\alpha \in [0,1]}\left[\min\left(\min_{POI \in G_\alpha} \mu_G(POI[a]), Q_{rel}\left(\frac{|G_\alpha|}{|G_0|}\right)\right)\right]. \tag{16}$$

For example, reconsider the restaurant POIs depicted in Figure 6 and the condition, 'at least four, preferably six or more' (cf. Figure 7) 'nearby restaurants' (cf. Figure 5) where the proximity of the restaurants is as given in Table I. Now, with the new approach, the evaluation of the condition yields

$$\begin{aligned} e = \max(&\min(1, 0), \min(0.8, 0), \min(0.5, 0), \\ &\min(0.3, 0.7), \min(0.1, 1)) = 0.3. \end{aligned}$$

Hence, compared to the basic approach, the novel approach allows to balance proximity and quantity in a more natural way. Moreover, compared to basic approaches based on fuzzy cardinality, the new approach presented in this paper better reflects the way how human experts make their decisions and can therefore be considered as being more consistent with human reasoning. Indeed, instead of aggregating fuzzy cardinalities and throwing away important quantitative information, subgroups of relevant POIs are considered and evaluated independently. The subgroups that best satisfy the quantifier based criterion are then used to compute the satisfaction degree.

C. Approach for monotonic decreasing quantifiers

If the used absolute quantifier Q_{abs} is modelled by a monotonic decreasing membership function, the following equation can be used:

$$e = 1 - \max_{\alpha\in[0,1]}[\min(\min_{POI\in G_\alpha} \mu_G(POI[a]), 1 - Q_{abs}(|G_\alpha|))] \tag{17}$$

or equivalently

$$e = \min_{\alpha\in[0,1]}[\max(\max_{POI\in G_\alpha} (1 - \mu_G(POI[a])), Q_{abs}(|G_\alpha|))]. \tag{18}$$

Here, the underlying idea is that the complement $1 - Q_{abs}$ is a monotonic increasing function and hence can be handled based on Equation (15). However, as this complement reflects the opposite of the quantifier in the original criterion, the complement of Equation (15) is used to compute the resulting satisfaction degree e.

For a monotonic decreasing relative quantifier Q_{rel}, the equation becomes

$$e = 1 - \max_{\alpha\in[0,1]}[\min(\min_{POI\in G_\alpha} \mu_G(POI[a]), 1 - Q_{rel}(\frac{|G_\alpha|}{|G_0|}))]. \tag{19}$$

If we reconsider the restaurant POIs depicted in Figure 6, but now with the condition, ‘preferably less than four, at most five’ (which is modelled by the complement of the membership function $\mu_{Q_{abs}}$ that is depicted in Figure 7). Assuming the same interpretation of proximity as given in Table I. The evaluation of the condition ‘preferably less than four, at most five nearby restaurants’ for the location represented by the dot in Figure 6 yields

$$\begin{aligned} e = 1 - \max(&\min(1, 0), \min(0.8, 0), \min(0.5, 0),\\ &\min(0.3, 0.7), \min(0.1, 1)) = 1 - 0.3 = 0.7. \end{aligned}$$

V. Comparison with other approaches

With the presented approach, elementary quantifier based criteria for LSP are modelled by means of absolute (or relative) fuzzy quantifiers. The main advantage of such an approach is that a decision maker can straightforwardly specify his preferences regarding the number of POIs that should satisfy the criterion by means of a (simple) membership function.

Alternative soft computing techniques, based on OWA operators [22], also support the modelling quantifiers by means of an adequate list of weights. Satisfaction degrees of elementary criteria are then ordered, after which the best degree is associated with the first weight in the list, the second best degree is associated with the second weight and so on. Application of an OWA operator for the handling of a quantifier in an elementary criterion would require the extra specification of weights.

Such a specification of extra weights and the specification of a parameter to set the logic properties of the aggregation operator would also be required if the generalized conjunction/disjunction (GCD) aggregation operators [6] would be used for the modelling of quantifiers in elementary criteria. By using fuzzy quantifiers, we explicitly choose to avoid overloading the LSP model with extra weights to model elementary criteria. Consequently, we make a clear decision to use weights exclusively at the level of the LSP aggregation structure. Weights then always denote the relative importance of the (sub)criteria in the evaluation of a more general criterion (cf. Equation (3)).

Such an approach provides LSP with the facilities to support complex criteria. An example is the implementation of a complex criterion ‘Restaurant’ as given in Figure 4, which could be specified as ‘the proximity of at least one Chinese restaurant and two Italian restaurants is mandatory; preferably more than four Italian restaurants should be nearby; the proximity of a French restaurant is desired’. Modelling of such a criterion will result in three sub criteria: two mandatory criteria ‘at least one Chinese restaurant’ and ‘at least two, preferably more than four Italian restaurants’, and a desired criterion ‘at least one French restaurant’. In the LSP aggregation structure a conjunctive GCD operator can then first be used to aggregate the two mandatory criteria, after which a mandatory/desired aggregation operator, like conjunctive partial absorption, can be used to combine the mandatory and desired criteria [8].

Using regular approaches (which are not based on soft computing) for modelling quantifiers restricts the capability of the presented construction technique to efficiently reflect the decision maker’s preferences. Indeed, in such a regular approach either the universal quantifier (∀), the existential quantifier (∃), or a crisp quantification has to be used. The use of the universal quantifier puts the (too) stringent constraint that all relevant POIs must satisfy the criterion. E.g., all relevant restaurants must be in the environment of the location under consideration. Using the existential quantifier implies that it is sufficient that only one of the relevant POIs satisfies the criterion. This situation is similar to the case where no quantifiers are supported. Crisp quantification implies that the number of relevant POIs satisfying the criterion must be strictly equal to, lower than (or equal to), or greater than (or equal to) a crisp number specified by the decision maker. Because crisp quantifiers can also be modelled as special cases of fuzzy quantifiers and because one needs to cope with partial criterion satisfaction, such approaches can be handled with the techniques presented in this paper, but are clearly less flexible than the presented approach.

VI. Conclusions

In this paper we studied the specification and evaluation of quantifier based elementary criteria in LSP. Such criteria are for example required for the evaluation of proximity of POIs in suitability map construction, a technique that is used to support geographic decision making. Two novel techniques for evaluating monotonic increasing and monotonic decreasing quantifiers have been presented and discussed. These techniques allow to better balance satisfaction of the basic criterion, e.g., proximity, and satisfaction of the quantifier. Illustrating examples have been provided. Furthermore, we described and illustrated how the introduced techniques can be integrated in LSP and combined with GCD aggregation operators in the construction of the LSP aggregation structure.

In the paper, we assumed that the POI database is consistent and complete and contains no coreferent POIs. In practice this is however not often the case, as POI database are usually the result of data integration processes and user involvement. As future work, we explicitly want to cope with such data imperfections and plan to integrate the presented techniques with approaches that allow detect and solve coreference of POIs. Another planned research topic concerns performance issues: efficient algorithms, implementing the presented techniques, are required if the operators have to be included in real-time dynamic suitability map construction software.

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