XFEM VS FEM:

APPLICTION TO FRETTING FATIGIE 2D CRACK PROPAGATION

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Abstract: Fretting fatigue is a combination of two complex and serious mechanical phenomena, namely fretting and fatigue. The combination of these two phenomena can cause sudden fracture of components that are subjected to the oscillatory motions (fatigue) and at the same time are in contact with each other (fretting). Fretting fatigue lifetime can be divided to two different portions, namely crack initiation and crack propagation. In this study a modified fretting fatigue contact model is used to compare conventional Finite Element Method (FEM) and eXtended Finite Element Method (XFEM) techniques for 2D fretting fatigue crack propagation model. For this purpose, Python programming language along with ABAQUS software is used to implement FEM and XFEM to fretting fatigue crack propagation. In the first step, a Double Edge Notch Tension (DENT) specimen is used to validate both FEM and XFEM results with the analytical solution. Then, the validated codes are applied to modified fretting fatigue contact model to compare the extracted Stress Intensity Factors (SIFs) at crack tips after each increments of crack propagation. Results show good correlation between FEM model using re-meshing technique and XFEM with single-mesh technique for crack propagation. Moreover, by comparing the numerical results with analytical solutions it can be concluded that the analytical results cannot predict the exact value of SIFs. However, far from the contact region the results converge to numerical ones. As a conclusion, implementing numerical methods such as FEM and XFEM to fretting fatigue crack propagation, found to be really accurate and robust.

Keywords: Fretting fatigue; Crack propagation; FEM; XFEM

1   INTRODUCTION

Fretting occurs due to oscillatory relative displacement between two components that are in contact together, which results in damage at the contact interface. Once these components face cyclic fatigue load at the same time, fretting fatigue occurs. Fretting fatigue is a serious phenomenon, which leads to reduction of fatigue lifetime of component compared to pure fatigue case. The schematic view of experimental setup of fretting fatigue is illustrated in Figure 1 In this setup, two identical fretting pads are pushed against the fatigue specimen using constant load (F), which is called normal load and at the same time the specimen is subjected to oscillatory fatigue load. Therefore, at presence of these two loads fretting fatigue failure occurs.
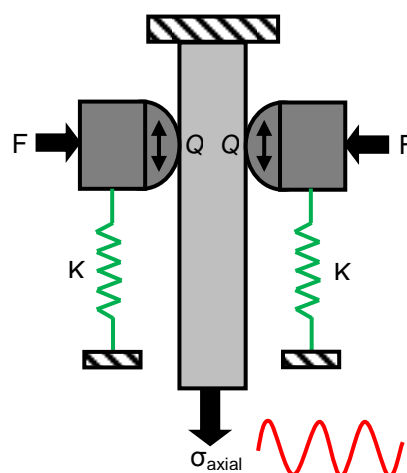

Figure 1. Schematic view of fretting fatigue experimental setup

Fretting fatigue failure process can be divided into two main portions, namely crack initiation and crack propagation. In term of crack propagation there are numerous studies [1-9] that have used fracture

mechanics approach to calculate fretting fatigue crack propagation lifetime. The fracture mechanics approach is based on calculating Stress Intensity Factors (SIFs) at the crack tip either in cracked or un-cracked fatigue specimen. Rooke and Jones [1] used Green's function which is purely analytical formula for calculating SIFs at the crack tip. Some researchers used combination of FE methods and analytical formula such as Weight functions [3, 4, 7] to calculate SIFs for the cracks normal to the contact line or an arbitrary path of crack inside the un-cracked fatigue specimen. Also, FEA method has been used widely to calculate SIFs at the crack tip in pre-cracked specimen, more information can be found in [2, 5, 6, 8, 10]. Recently a new FE formulation, i.e. eXtended Finite Element Method (XFEM) [11], became more popular due to its capability to model the crack inside the mesh without any re-meshing technique. All of these studies have used SIFs at the crack tip with assumption of Linear Elastic Fracture Mechanics (LEFM) to calculate number of cycles of crack propagation from a certain crack length up to final rupture.

In this study, a modified fretting fatigue contact model is used to compare conventional Finite Element Method (FEM) and eXtended Finite Element Method (XFEM) techniques for 2-D fretting fatigue crack propagation model. For this purpose, Python programming language along with ABAQUS$^®$ software were used to implement the application of XFEM. In order to validate the accuracy of extracted SIFs, the crack propagation behaviour of Double Edge Notch Tension (DENT) is modelled using both FEM and XFEM, then verified by analytical solution. Finally, the fretting fatigue crack propagation behaviour using FEM, XFEM and analytical solution are compared to each other.

2   FRETTING FATIGUE MODIFIED CONTACT MODEL

To solve the fretting fatigue contact model shown in Figure 1, only half the experimental setup needs to be modelled using FE technique, because the experimental setup is ideally symmetric about the axial centreline of the specimen. As depicted in Figure 2, the specimen was restricted from vertical movement along its bottom surface and free to roll in the x-direction and along its bottom edge. The length of the specimen, width of the specimen and the radius of pad were selected as L=20 mm, b=10 mm and R= 101.6 mm, respectively. Both the fretting pad and the fatigue specimen had a unit depth. Both sides of cylindrical pad were restricted to move just in vertical direction. The Multi-Point Constraint (MPC) was also applied at the top of pad in order to avoid it from rotating due to the application of loads. A two-dimensional, 4-node (bilinear), plane strain quadrilateral, reduced integration element (CPE4R) was used. The mesh size of 5 μm × 5 μm was considered at contact interface and decreased gradually far from the contact region for all models. This mesh size was gained by mesh convergence study which was achieved in previous study [12]. The contact between the fretting pad and the fatigue specimen was defined using the master-slave algorithm in ABAQUS$^®$ for contact between two surfaces. The circular surface of the pad was defined as a slave surface and top surface of the specimen was defined as a master surface. Al 7075-T6 was selected for both the pad and the specimen with Modulus of Elasticity of 71 GPa and a Poisson's of 0.33. A Coefficient of Friction (COF) of 0.75 was used in this study. The normal load, tangential load and maximum axial stress were considered as F= 60 N, Q= 60 N, $\sigma_{axial} = 80$ MPa, respectively.
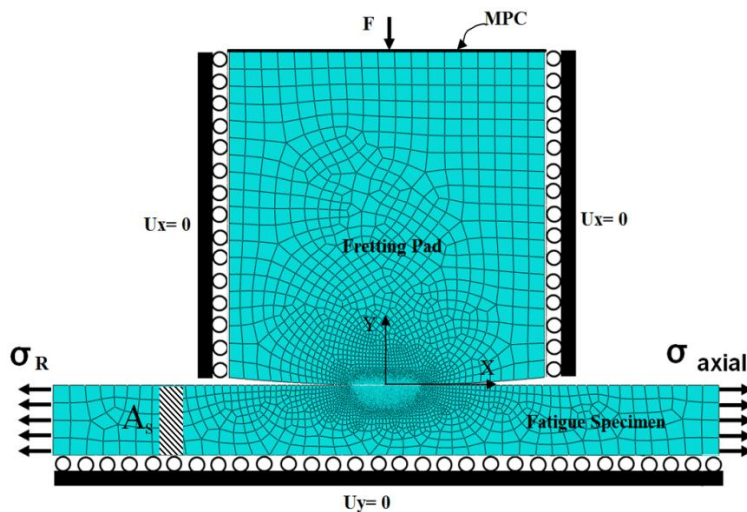

Figure 2. Modified FE model of fretting fatigue

In order to model the effect of attached spring to the fretting pad for generating the tangential load (Q), the reaction stress (σ_R) can be calculated based on $\sigma_R = \sigma_{axial} - Q/A_s$, where A_s is cross section area of

specimen as shown in Figure 2. Therefore in the modified model, in the second step, the maximum axial stress σ_{axial} and the reaction stress σ_R were applied at the same time at the right and left sides of specimen, respectively, to match the experimental maximum cyclic loading condition. The accuracy of proposed model has been compared with several fretting fatigue FE models that were available in literature in ref. [12].

3   BASIC CONCEPT OF CRACK PROPAGATION USING FEM AND XFEM

Modelling stationary cracks using FEM needs the geometry of cracked body to be matched with the mesh. Therefore, in order to capture singularity at the crack tip, the mesh around the crack tip is needed to be considerably refined. Moreover, modelling crack propagation using mesh refinement techniques are really cumbersome, especially in 3-D and complex models. Recently, XFEM decreases inadequacy associated with re-meshing of the crack tip [11, 13]. The XFEM is the extended version of FEM, which is based on concept of partition of unity method introduced by Melenk and Babuska [14]. It allows local enrichment functions to be easily incorporated into a finite element approximation. For the purpose of fracture mechanics analysis, the enrichment functions typically consist of the near-tip asymptotic functions that capture the singularity around the crack tip and a discontinuous function that represents the jump in displacement across the crack line (in case of 2-D). The approximation for a displacement vector function $\boldsymbol{u}^h(x)$ with the partition of unity enrichment is:

$$\boldsymbol{u}^h(x) = \sum_{\forall I} N_I(x)\boldsymbol{u}_I + \sum_{j \in S_H} N_j(x)H(x)\boldsymbol{q}_j^0 + \sum_j \sum_{k \in S_c} N_k(x)F_j(x)\boldsymbol{q}_k^j \tag{1}$$

Where, $N_I(x)$ are the usual nodal shape functions for conventional finite element formulation. The first term on the right-hand side of equation 1, u_I, is the usual nodal displacement vector associated with the continuous part of the finite element solution. The second term is the product of the nodal enriched degree of freedom vector, q_j^0, and the associated discontinuous jump function $H(x)$ across the crack line. The third term is the product of the nodal enriched degree of freedom vector, q_k^j, and the related elastic asymptotic crack-tip functions, $F_j(x)$. The usual nodal displacement vector, u_I, is implemented to all the nodes in the FEA model. The second term, $N_j(x)H(x)q_j^0$, is valid for nodes whose shape function support is cut by the crack. The third term i.e. $N_k(x)F_j(x)q_k^j$, is used only for nodes whose shape function support is cut by the crack tip. Figure 3 shows the discontinuous jump function across the crack surfaces, $H(x)$, which is defined by:

$$H(x) = \begin{cases} 1 & \text{for } (\boldsymbol{x} - \boldsymbol{x}^*).\mathbf{n} \geq 0 \\ -1 & \text{else,} \end{cases} \tag{2}$$

Where x is a sample integration (Gauss) point, x^* is the point on the crack closest to x, and $\boldsymbol{n}$ is the unit outward normal to the crack at $\boldsymbol{x}^*$. Figure 3, also depicts the asymptotic crack tip functions in an isotropic elastic material, $F_1(r, \theta)$, which are given by:

$$\{F_1(r,\theta)\}_{I=1}^4 = \left\{ \sqrt{r} \sin\left(\frac{\theta}{2}\right), \sqrt{r} \cos\left(\frac{\theta}{2}\right), \sqrt{r} \sin\left(\frac{\theta}{2}\right) \sin(\theta), \sqrt{r} \cos\left(\frac{\theta}{2}\right) \sin(\theta) \right\} \tag{3}$$

Where (r, θ), is a polar coordinate system with its origin at the crack tip and $\theta = 0$ is tangent to the crack faces. These functions span the asymptotic crack-tip function of elasto-statics.

■ Nodes with Jump Function Enrichment (S_H)

▲ Nodes with Crack Tip Enrichment (S_C)
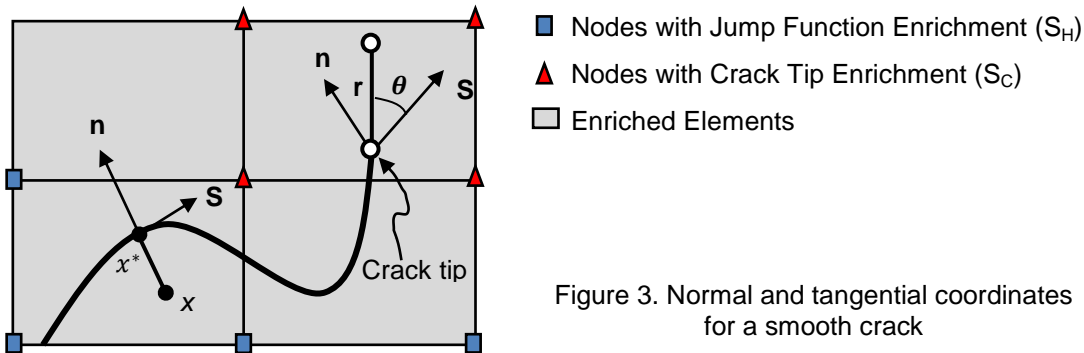
▢ Enriched Elements

Figure 3. Normal and tangential coordinates for a smooth crack

There are a lot of studies, which are aimed to implement XFEM feature in conjunction with conventional FE software. For instance, Giner *et. al.* [6] have carried out a two-dimensional implementation of XFEM within the FE software ABAQUS® by means of user subroutines. Since after ABAQUS 6.9® the XFEM feature is added with some limitations by developers, in this study this capability was used to model the 2-D fretting fatigue crack propagation. One of the limitations that should be solved is extracting the SIFs at the crack tip for a 2-D stationary crack. This will be elaborated on later.

4   CRACK PROPAGATION MODEL FOR DENT SPECIMEN

In order to study the fretting fatigue crack propagation behaviour a python script was written in conjunction with ABAQUS® software for both FE and XFEM models. Before implementing crack propagation code to fretting fatigue problem, the code was applied to a DENT specimen for both cases of FEM using re-meshing and XFEM techniques. Then, the results were compared with analytical solution to validate the accuracy of the models. In this study, a DENT was considered as cracked rectangular linear elastic plate with width w=10 mm and length C= 20 mm. The initial crack length and crack propagation increment was considered as 0.2 mm and 0.1 mm, respectively. A uniform tension stress (σ_{axial} = 80 MPa) was applied at both sides of the specimen at the same time. The *SIF*s were extracted step by step during crack propagation.

For modelling the crack propagation using conventional FEM, A two-dimensional, 8-node, plane strain quadrilateral, reduced integration element (CPE8R) was used. The crack-tip singularity was modelled using the same element type. The mesh was collapsed at crack tip. Therefore, all the nodes at the crack tip had the same geometry location. Also, the mid-side nodes on the sides connected to the crack tip were moved to the quarter point nearest the crack tip. The SIFs were evaluated based on domain integral over an area contained within a contour surrounding the crack tip.

In terms of using XFEM the CPE4R element are used for whole model. As mentioned above one of the main restrictions of ABAQUS's XFEM capability up to date (ABAQUS® 6.11) is that it is not possible to extract the SIFs for a 2-D stationary crack. After modelling cracked fatigue specimen based on the stress and displacement fields at the crack tip, J integral approach was used to calculate the SIFs using LEFM assumption. The original form of the J-integral for a line contour surrounding the crack tip can be written as:

$$ J = \oint_\Gamma W \ dy - \oint_\Gamma (t_x \frac{\partial u_x}{\partial x} + t_y \frac{\partial u_y}{\partial y}) \ ds \tag{4} $$

In which, $W = \sigma_{ij} \ d\varepsilon_{ij}$, is the strain energy density (σ_{ij} and ε_{ij} are stress and strain tensors), t_x and t_y are the components of the traction vector, which acts on the contour, u_x and u_y are the displacement components, and *ds* is a length increment along the contour Γ. In case of LEFM, the J-integral is equal to energy release rate G ($J = G = \frac{K_I^2}{E'}$), with $E' = E/(1 - \vartheta^2)$ for plane strain problem.

5   FRETTIING FATIGUE CRACK PROPAGATION

After validating the extracted SIFs for both FEM using re-meshing technique and XFEM crack propagation models by analytical solution. The same procedure is implemented to fretting fatigue problem. In this study, for fretting fatigue crack propagation problem, initial crack inserted at x/a=1, where 'x' is the contact distant and 'a' is the semi contact width in the FE model configuration, which is shown in Figure 2, as it is proven by experimental observation [15]. The crack considered to propagate perpendicular to the fatigue specimen in order to compare the results of FEM and XFEM models with analytical solution, which will be elaborated later on. An initial crack of length, l_0=50 μm, is introduced in the contact interface. Also, crack length increment of Δl=50 μm is considered for crack propagation. The loading and boundary conditions are the same as used for contact model. The stress ratio of zero was selected for both cases. Finally, the calculated SIFs are compared with analytical solution, which is based on weight function method [16]. In the weight function (WF) method, the stress distribution along the hypothetical crack location calculated from FEA contact model is combined with a weight function W specifically derived for a given geometry [16] to calculate the K_I as:

$$ K_I^{WF} = \sqrt{\frac{2}{\pi}} \int_0^{l_c} \sigma_{xx} (x, y) W(y) \ dy \tag{5} $$

For a crack normal to the surface of fatigue specimen, the tangential stress σ_{xx} is extracted from the FE contact model at location of hypothetical crack (*x/a=1*). In this work, the weight functions W for both DENT and SENT specimens in a strip of finite width were used [16].

6   RESULTS AND DISCUSSION

In order to verify the contact model, two assumptions were taken into account. The first one was the elastic behaviour of material, while the second was the half space assumption. Thus, the boundaries can be considered infinite if one half of the fretting specimen width, b, is equal to or greater than ten times the contact half width, a, or in other words $b/a > 10$. In this study, the analytical and FE contact half widths were $a_{analytical} = 441$ μm and $a_{FEM} = 442$ μm, respectively. Figure 4 shows the good correlation between FEA results and analytical solution [17] for the frictional shear stress distribution at the contact interface.
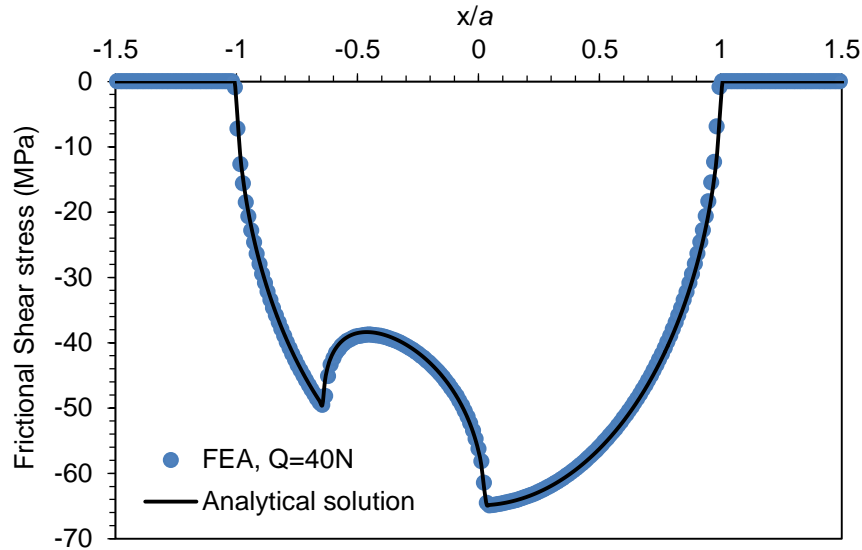

Figure 4. Validation of FEA model of fretting fatigue with analytical solution

Figure 5 illustrates the relation between the calculated SIFs at the crack tip using FEM, XFEM and analytical solution [18] for of DENT specimen. Moreover, Figure 5 shows good correlation between numerical results and analytical solutions for each crack length.
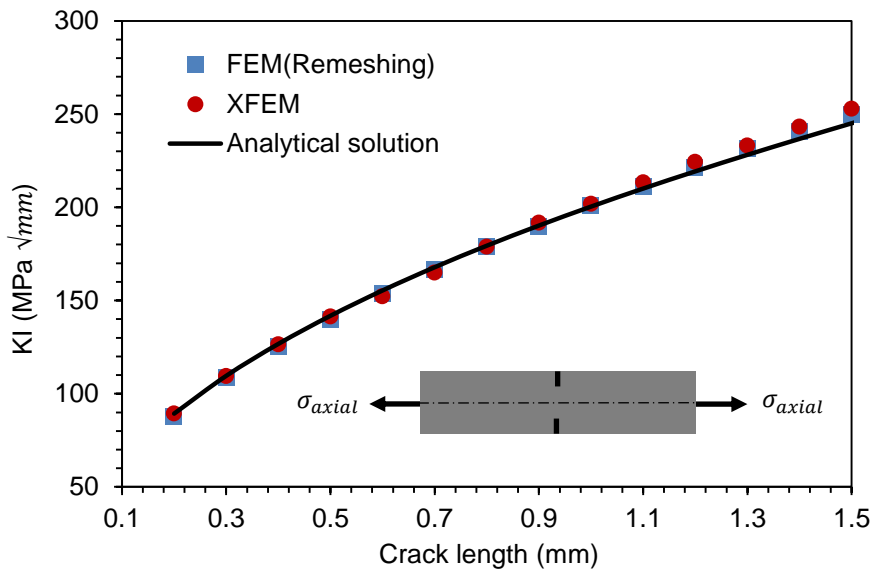

Figure 5. Distribution of SIFs VS crack length

Figure 6 depicts the variation of mode I SIFs versus different crack lengths for fretting fatigue problem using FEM, XFEM and different analytical solutions, namely DENT specimen and weight function approach for both SENT and DENT specimens. As it can be seen from the figure, the results from FEM and XFEM for

calculated SIFs nicely correlate to each other. However, the analytical values of SIFs estimate lower value at earlier stage of crack propaction to compare with FEM and XFEM crack propagation results. This is because of the effect of crack contact interaction on crack propagation which is not taken into account in analytical solutions. Nevertheless, far from the contact zone analytical solutions, FEM and XFEM results converge approximately to same values. The reason for this convergence is related to dominate behaviour of axial stress far from the contact zone.
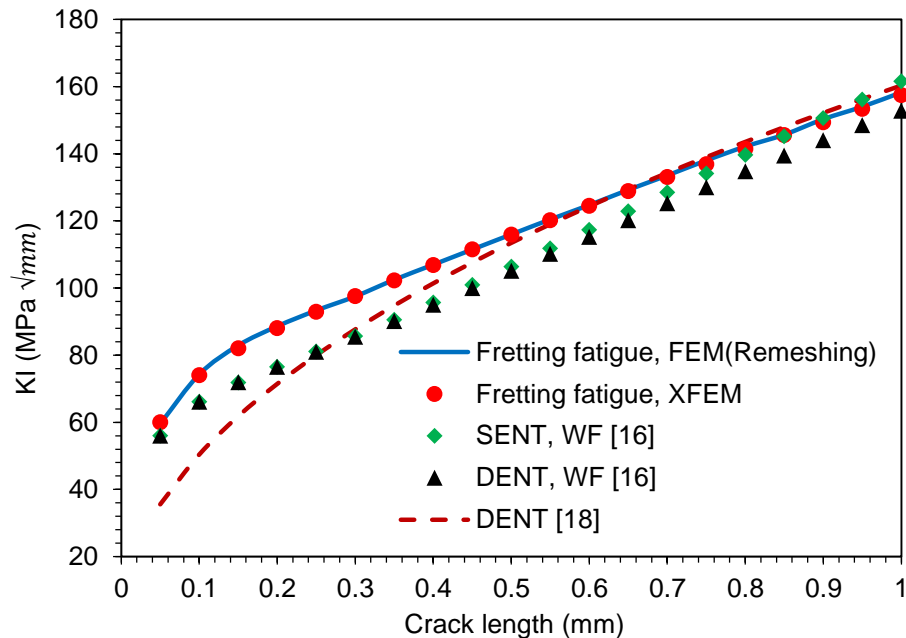

Figure 6. Calculated SIFs for different crack lengths

7   CONCLUSION

In this study, a modified fretting fatigue contact model is used to compare conventional Finite Element Method (FEM) and eXtended Finite Element Method (XFEM) techniques for 2D fretting fatigue crack propagation model. For this purpose, Python programming language along with ABAQUS software were used to implement FEM and XFEM to fretting fatigue crack propagation. The accuracy of models was checked by comparing numerical results with analytical solutions for crack propagation of DENT specimen. Then, the validated codes were applied to modified fretting fatigue model to compare the extracted SIFs at crack tips in each steps of crack propagation. The results showed good correlation between FEM models using re-meshing technique and XFEM technique using single mesh approach. Moreover, by comparing the numerical results with analytical solutions, it can be seen that the analytical values of SIFs are less than numerical values at earlier stage of crack propagation and converge far from the contact region. As a conclusion, implementing XFEM method to fretting fatigue crack propagation was found to be really accurate and robust. This method overcomes the problem of modleing singularity and re-meshing techniques of FEM techniques. As a future work, it will be implemented to model 3-D crack propagation of fretting fatigue problems.

8   ACKNOWLEDGMENTS

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