EEG inverse problem solution with minimal influence of the conductivity

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***Abstract*—In this paper, we propose a novel method that improves the accuracy of the estimation of neural electrical dipoles when solving the EEG inverse problem. A spherical head model is used where we limit the influence of the unknown conductivity brain-skull ratio on the inverse problem. We redefine the cost function that is used in the EEG problem where only useful information is used as input in the inverse problem. In contrast to previous approaches, weighting factors are used where the electrodes are strategically chosen so to reduce the error made on EEG dipole source localization. The proposed method enhances the source localization accuracy from approximately 9mm to 1mm for dipoles near the edge and from 2.1mm to 0.4mm for dipoles near the center of the brain.**

I. Introduction

Electroencephalography (EEG) is a medical imaging technique that neurologists use to investigate neurological disorders. Using metal electrodes, brain activity can be recorded non-invasively. EEG source analysis is particularly useful in the diagnosis of neurological disorders like epilepsy. Indeed, the determination of the origin of specific EEG waveforms helps neurologists to pinpoint the origin of the epilepsy and to evaluate the patient for resective surgery. However, when coupling the non-invasive EEG measurements to a numerical method, inaccuracies in the neural source localization are introduced. Indeed, the accuracy of EEG source analysis is mainly determined by the noise in the measurement and the accuracy of the numerical head model parameters. Also, the source modelling of the brain activity introduces an error. Since the brain electrical activity of patients suffering from epilepsy are characterized by a limited number of electrical dipoles [1], we do not investigate the influence of the used source model. The head model on the other hand has a large impact on the solution of the EEG inverse problem where important errors are introduced by the uncertainties of the values of the electrical conductivity of the brain and the skull. The quantitative values of the electrical conductivity of the brain and the skull remain a very important parameter that attract a lot of debates in EEG source analysis field, see e.g. [2], [3]. In numerical methods, the brain to skull ratio of the conductivity is the important parameter and may vary between $1/9$ to $1/60$. This paper presents a novel numerical scheme, the so-called Reduced Conductivity Dependence (RCD) method, that minimizes the influence of the conductivity on the localization errors. This method introduces a selection procedure of the EEG electrodes that are minimally influenced by the conductivity values. We validate the method onto a widely-used approximation of the head: the semi-analytical spherical head model. Comparisons are made with traditional least-squares minimization methods. For simplicity of analysis, we impose that the neural activity is represented by a single electrical dipole.

II. EEG source analysis

A. Forward problem

The forward problem starts from a given electrical dipole and calculates the potentials at the electrodes. For this, the brain to skull ratio of the conductivity X needs to be provided. The spherical head model is a widely-used approximation of the head where the head is represented by three spheres: the inner sphere represents the brain, the intermediate layer represents the skull and the outer layer represents the scalp. The forward problem needs to solve the Poisson's equation:

$$\nabla \cdot (\sigma(\mathbf{r}) \nabla \mathrm{V}(\mathbf{r})) = \mathbf{d}\, \delta(\mathbf{r} - \mathbf{r}_d) \tag{1}$$

with $\sigma(\mathbf{r})$ the place dependent conductivity determined by X, $\mathrm{V}(\mathbf{r})$ the place dependent potential, $\mathbf{d}$ the dipole orientation vector (with intensity $I = \|\mathbf{d}\|$) and $\mathbf{r}_d$ the dipole location vector. $\delta(.)$ is the three-dimensional delta Dirac function. An analytical expression for the potential values can be calculated using [4]. In this study, a standard configuration of $m = 27$ electrodes is used to compute the potential distribution of the forward model. For given $\mathbf{r}_d$ and dipole orientation $\mathbf{d}$, the electrical potential values at the given electrodes can be calculated: $\mathbf{V}\mathrm{m}(\mathbf{r}_d, \mathbf{d}) \in \mathbb{R}^{m\times 1}$. The potential values are a linear function of the dipole orientation: $\mathbf{V}\mathrm{m} = \mathbf{L}(\mathbf{r}_d) \cdot \mathbf{d}$ with $\mathbf{L} \in \mathbb{R}^{m\times 3}$ the so-called lead field matrix.

B. Traditional solution of EEG inverse problem

The aim of the EEG inverse problem is to start from measured EEG potentials $\mathbf{V}\mathrm{meas} \in \mathbb{R}^{m\times 1}$ and to recover the neural dipole location $\mathbf{r}_d^*$ and orientation $\mathbf{d}^*$. This is carried out by minimizing a cost function, the so-called relative residual energy (RRE):

$$\{\mathbf{r}_d^*, \mathbf{d}^*\} = \arg\min_{\mathbf{r}_d, \mathbf{d}} \mathrm{RRE}(\mathbf{r}_d, \mathbf{d}) \tag{2}$$

with

$$\mathrm{RRE}(\mathbf{r}_d, \mathbf{d}) = \frac{\|\mathbf{V}\mathrm{meas} - \mathbf{V}\mathrm{m}(\mathbf{r}_d, \mathbf{d})\|}{\|\mathbf{V}\mathrm{meas}\|} \tag{3}$$

where $||.||$ is the L_2 norm. The number of parameters in this least-squares cost function can be reduced by considering the optimal dipole components: $\mathbf{d}_{\text{opt}} = \mathbf{L}^{\dagger} \cdot \mathbf{V}_{\text{meas}}$ with $\mathbf{L}^{\dagger}$ the Moore-Penrose pseudo inverse of the lead field matrix. Equation (3) becomes then, see e.g. [5]:

$$\text{RRE}(\mathbf{r}_d) = \frac{\|\mathbf{V}_{\text{meas}} - \mathbf{L}(\mathbf{r}_d)\mathbf{L}(\mathbf{r}_d)^{\dagger}\mathbf{V}_{\text{meas}}\|}{\|\mathbf{V}_{\text{meas}}\|}. \tag{4}$$

The widely Nelder-Mead simplex method is used here to find the global minimum of the Relative Residual Energy (RRE).

III. Reduced Conductivity Dependence (RCD) Method

A. Description of the method

The RCD method proposes an alternative cost function that needs to be minimized for EEG source analysis. The main idea lies in the selection of electrodes that provide useful information in the sense that the electrodes which are selected, are minimally affected by the unknown conductivity in the forward model. Indeed, depending on the location of the electrical dipole and its orientation, some potentials are highly affected by X and others are not. The selection procedure needs to be performed in each iteration k of the minimization scheme, in this case the Nelder-Mead simplex method. In the following, we explain the basic steps taken by the RCD method.

Step 1: Start value $\mathbf{r}_d^{(0)}$ is evaluated in the forward model, yielding the lead field matrix $\mathbf{L}(\mathbf{r}_d^{(0)})$, and simulated potential values $\mathbf{V}_{\text{m}}(\mathbf{r}_d^{(0)}) = \mathbf{L}(\mathbf{r}_d^{(0)})\mathbf{L}(\mathbf{r}_d^{(0)})^{\dagger}\mathbf{V}_{\text{meas}}$. We initialize $k = 0$.

Step 2: Calculate the sensitivity $\mathbf{W}$ and the normalized sensitivity $\mathbf{w}$ of the simulated electrode positions to the conductivity for a certain conductivity X_0:

$$\mathbf{W} = \frac{\partial \mathbf{V}_{\text{m}}(\mathbf{r}_d^{(k)})}{\partial X}|_{X=X_0}, \quad \mathbf{w} = \frac{|\mathbf{W}|}{\|\mathbf{W}\|} \tag{5}$$

In the case of the spherical head model, $\mathbf{W}$ and $\mathbf{w}$ can be calculated analytically. When considering more complex realistic head models, this can be calculated by finite differentiation.

Step 3: Selection of least sensitive electrodes, based on (5). Largest values are not considered in the EEG inverse problem, since their potential values are affected by the conductivity. A new set of potential values are obtained: $\mathbf{S}_{\text{m}} \in \mathbb{R}^{N\times 1}$ and the corresponding set of measured EEG potentials are considered $\mathbf{S}_{\text{meas}} \in \mathbb{R}^{N\times 1}$. N is the number of selected potentials.

Step 4: Calculation of RCD cost function:

$$\text{RCD}(\mathbf{r}_d^{(k)}) = \frac{\|\mathbf{S}_{\text{meas}} - \mathbf{S}_{\text{m}}(\mathbf{r}_d^{(k)})\|}{\|\mathbf{S}_{\text{meas}}\|} \tag{6}$$

Step 5: Based on (6), the next iterate $\mathbf{r}_d^{(k+1)}$ can be calculated. If the termination criteria of the minimization procedure are met, i.e. $\text{RCD}(\mathbf{r}_d^{(k)})$ reaches tolerance, then stop the algorithm. Otherwise, go to step 2.

B. Results and discussion

The efficiency of the RCD method is illustrated by performing Monte Carlo simulations. Starting from known dipole locations $\widetilde{\mathbf{r}}$ and a given conductivity X_0, we compute EEG potentials. Gaussian noise is added to these potentials in order to simulate real measured EEG potentials. Dipole locations $\widehat{\mathbf{r}}$ are then estimated using the traditional method, i.e. solution of (2), and estimated using the RCD method, explained in III.A. The accuracy of both methods is determined by the error $E = \|\widetilde{\mathbf{r}} - \widehat{\mathbf{r}}\|$ for different values X and noise levels. The white zero mean Gaussian noise with standard deviation Σ has a noise level defined as $n = \Sigma/\mathbf{V}_{RMS}$ with $\mathbf{V}_{RMS}$ the root mean square of the potentials $\mathbf{V}$. Fig. 1 illustrates the decrease in localization error due to the use of the RCD method (with varying number of selected potentials N=4, 6, 8, 10 in step 3 of III.A.) compared to the traditional method (N=27). A reduction of the error is introduced due to the use of the RCD method.

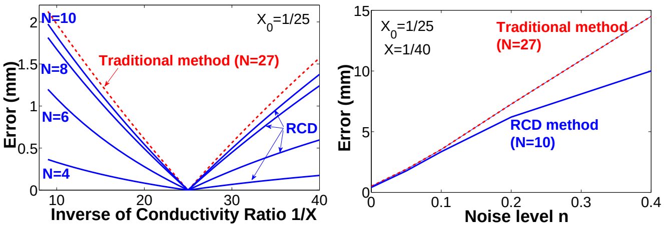

Fig. 1. Plot of Conductivities Vs Error (left) and for several noise levels (right) at X=1/40, for dipoles located near the center of the head.

For dipoles located near the center of the head, the localization error can be reduced from 2 mm to 0.4mm. For dipoles located near the edge of the head, dipole errors (with $X_0 = 25$ and $X = 40$) are approximately 9mm using the traditional method and are reduced to 1mm. The efficiency of the RCD method is reduced when noise is included, see Fig. 1, but remains accurate.

IV. Conclusion

This paper proposes a method that decreases the error introduced by the uncertainties of the conductivity. The results show that the EEG inverse problem can be solved with considerably improved quality, as compared to the traditional inverse solutions.

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References

[1] J. de Munck, B. Van Dijk, and H. Spekreijse, "Mathematical dipoles are adequate to describe realistic generators of human brain activity," *IEEE Transactions on Biomedical Engineering*, vol. 35, pp. 960-965, 1988.

[2] L.A. Geddes, and L.E. Baker, "The specific resistance of biological material-a compendium of data for the biomedical engineer and physiologist," *Medical and Biological Engineering*, vol. 5, pp. 271-293, 1967.

[3] D. Guttiérrez, A. Nehorai, and C.H. Muravchik, "Estimating brain conductivities and dipole source signals with eeg arrays," *IEEE Transactions on Biomedical Engineering*, vol. 51, no. 12, pp. 2113-2122, 2004.

[4] Y. Salu, L. Cohen, D. Rose, S. Sato, C. Kufta, and M. Hallett, "An improved method for localizing electric brain dipoles," *IEEE Transactions on Biomedical Engineering*, vol. 37, pp. 699-705, 1990.

[5] J. Mosher, R. Leahy, "Source localization using recursively applied and projected (RAP) MUSIC," *IEEE Transactions on Signal Processing*, vol. 47, pp. 332-340, 1999.