# Full-wave 3D Simulations Using the Broadband NSPWMLFMA

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## Abstract

Full-wave simulation of complex structures and devices, based on the Maxwell equations, is gaining importance now that technology has reached a point where many of the quasi-static or high-frequency approximations are no longer sufficiently accurate. A major drawback of the so-called exact methods is their greed for resources, namely CPU-time and computer memory. Industry puts a limit to both, inspiring the quest for ever more powerful methods. The Fast Multipole Method (FMM) breathed new life in the well-renowned Method of Moments (MoM). The original formulation of the FMM suffered from breakdown at low frequencies. A different formulation, efficient only at low frequencies, was also developed but is not very efficient. This paper presents the use of a novel method stable at all frequencies, the Nondirective Stable Plane Wave Multilevel Fast Multipole Algorithm (NSPWMLFMA) [1], in a 3D vectorial Boundary Integral Equation MoM solver, thus enabling efficient simulations at all frequencies.

## 1. Introduction

The Method of Moments (MoM) is a popular method for full-wave simulation. The restriction to piecewise homogeneous media allows for a formulation of the problem using boundary integral equations (BIEs) in terms of equivalent surface current densities. Expansion of these current densities in basis functions and testing of the BIEs with test functions will lead to a linear set of equations, that can be solved for the unknown coefficients of the expansion. A major advantage is that only the interfaces must be discretised, instead of volumes, as is the case for many other methods such as Finite Elements or FDTD. However, because a dense matrix results, memory complexity varies as $O(N^2)$, with N the number of unknowns. Additionally, iterative solution of the linear system requires $O(N^2)$ calculations for each iteration. The next section will briefly deal with the choices that can be made within the framework of the MoM, but in general the reader is assumed to be familiar with the method. For scattering simulations at large objects the computational complexity of the MoM becomes unacceptable and faster methods are required. The FMM starts from the MoM but treats the distant interactions in a much more efficient manner, by collecting basis and test functions into localised groups and a plane wave decomposition of the Green function. This concept can be generalised to the MLFMA, which introduces more levels by assembling the groups into larger groups, much like the organisation of an efficient telephone network. We will re-introduce some of the concepts of the MLFMA that are relevant to this paper, without going into detail. It is noteworthy that the MLFMA has an O(NlogN) complexity, both in terms of memory and number of calculations, which means almost linear behaviour for very large systems. Testimony to this great achievement is the increasing popularity of the method. However, the original formulation of the MLFMA suffers from a low-frequency (LF) breakdown, limiting its applicability to high frequency simulations. A different formulation of the MLFMA, stable at low frequencies, was also developed but is rather inefficient. A number of algorithms have been proposed that are truly broadband, enabling the simulation of scattering at large objects that nevertheless contain considerable sub-wavelength detail. We have proposed a novel method, the NSPWMLFMA, which essentially alleviates the LF-breakdown by shifting the k-space integration into the complex plane. We summarise the method, formulated for the scalar case in [1]. We have fully integrated the NSPWMLFMA in our existing 3D parallel solver, named Cassandra. It features the broadband method in a vectorial form. Note that the solver is open source and free for download by the entire community [5]. In the final section a demonstration of the broadband capabilities will be given through a number of examples of some practical interest.

## 2. Simulation Method

The MLFMA is used to accelerate the matrix vector products in the iterative solution of the MoM. It is therefore desirable to start with some short comments regarding the choices made within the framework of the MoM. We restrict ourselves to piecewise homogeneous objects, allowing a formulation in terms of equivalent surface current densities. The current densities are discretised in the well-known Rao-Wilton-Glisson (RWG) basis functions, non-zero over two neighbouring triangles. The coefficients of the expansion can be determined by solving a BIE. Depending on the type of interface and the frequency different types of BIEs are available. Essentially they are all linear combinations of two basic types, the Electric Field Integral Equation (EFIE) and the Magnetic Field Integral Equation (MFIE), imposing the boundary conditions for, respectively, the tangential electric and magnetic field at the interface. Much research [2] has been devoted to the optimal choice of BIEs. We have also chosen to increase the performance of our MoM through fully allowing junctions between objects, which leads to lines along which three or more objects touch. After selecting the appropriate BIEs, they are tested by the same RWGs, leading to a Galerkin formulation. This results in a linear system, with N unknowns (the number of basis functions), which is solved with an iterative algorithm. Our principal choice is the TFQMR, block-diagonally preconditioned. As noted before, the MoM leads to $O(N^2)$ complexity for memory and $O(PN^2)$ for CPU-time, with P the number of iterations, which is hoped to be sub-linearly dependent on the number of unknowns.

The original formulation of the MLFMA [3] is based on a plane wave decomposition of the Green function, allowing diagonal interactions. Assembling the basis (and test) functions into groups then leads to the efficient process of aggregation, translation and disaggregation. In the high frequency regime, the number of plane waves or the number of k-space samples required to accurately represent a radiation pattern increases with increasing group size. Hence, when taking a multilevel approach, a method for interpolation and anterpolation between levels is necessary. While local Lagrangian interpolation certainly has its attractions, we nevertheless opt for the more error-controllable FFT-interpolation, which requires uniform sampling in both angular coordinates. The infamous LF-breakdown of MLFMA finds its origin in the behaviour of the Hankel function for large order, leading to unavoidable round-off errors that unavoidably lead to unacceptable loss of accuracy. Current LF-methods (see [1] and the references therein) can be subdivided in two classes, the multipole expansion based methods (using non-diagonal translations) and the spectral methods (requiring six radiation patterns). Both methods, however, have their drawbacks and it was worthwhile to search for a method to tame the LF-breakdown while maintaining the high frequency MLFMA formulation. A first attempt to do so was shown in [4], but is less attractive due to a limited achievable accuracy. The key idea introduced there was, however, used in the NSPWMLFMA, which is described in detail in [1] and will only be summarised here. By shifting the $\theta$-integration in k-space over a distance $\chi$ into the complex plane a normalisation factor is added to the terms in the translation operator, avoiding the exponential of the higher order terms. Unfortunately, straightforward application of this principle seems valid only for translations in the z-direction. It is possible, obviously, to rotate every translation to a z-directed translation, but this would require a different set of sample points for every direction. However, from the entire set of k-vectors a limited set can be selected through QR-factorisation. In [1] it is demonstrated that a $\chi$ and set of k-vectors can be found for every frequency. Note that this set depends on the group size, which means that interpolation between levels is always required, even though the number of sample points may be the same. This interpolation is achieved through a dense matrix. Because the QR algorithm is relatively expensive in terms of L, it is advisable to set $\chi$ to zero when the group size becomes sufficiently large at higher levels in the tree and use the uniformly sampled method with the FFT-interpolation. This transition is seamless. As demonstrated in [1], the method is highly error-controllable and is capable of achieving very high accuracy. It is also shown that it is stable down to the static case.

We have implemented these algorithms under the name “Cassandra”, as part of the OpenFMM project [5]. It can be used for scattering simulations at homogeneous dielectric and perfectly electric conducting objects. It also makes use of an asynchronous parallelisation scheme [5], allowing for fast solution in a GRID computing environment. However, while [1] explains the NSPWMLFMA in the scalar case, here we have extended this to the vectorial case. In every k-space direction the radiation pattern is expressed for two orthogonal components. Note that, because $\theta$ is complex, these directions do not lie in real space. Interpolation is done through a dense matrix, based on an expansion of the radiation pattern in spherical harmonic functions.

## 3. Examples

The great advantage of broadband simulation is the possibility to include sub-wavelength detail in geometries that are several wavelengths in size. We start by demonstrating that we can indeed choose the group size arbitrarily small. It is, admittedly, largely an academic example but it shows the effectiveness of the algorithm. We consider plane wave scattering at a dielectric ($\varepsilon=2$) sphere of 2.25 internal wavelengths in diameter. We discretise the geometry in very small triangles, about $0.015\lambda$. This leads to approximately 200.000 unknowns. We choose the box-size to be only slightly larger than the size of the RWGs, leading to the most efficient use of the MLFMA. The bistatic RCS is shown in Figure 1, in comparison with the closed-form Mie solution. The agreement is extremely good, demonstrating the stability of the NSPWMLFMA. Note that this also shows that the Combined Transversal Field (CTF) BIE [2], which is an LF-unstable integral equation, is sufficiently well-conditioned for this discretisation size. The alternative, the nMüller BIE [2], is LF-stable but less accurate.

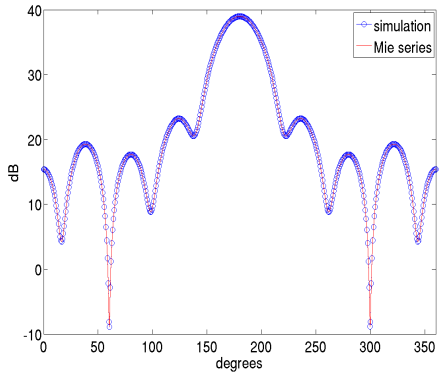


Figure 1: Comparison with analytical solution for RCS of a $2.25\lambda$ sphere discretised with very small triangles

We now proceed to a second, more involved example. We are considering the accurate simulation of monopole antennas, fed by a coaxial cable. Because our MoM supports junctions, we can elegantly model the different parts. The meshed geometry of a simple quarter wavelength monopole with a finite $2\lambda$ x $2\lambda$ ground plane, connected to an almost matched coaxial cable, is shown in Figure 2. The mesh is non-uniform, in order to accurately capture the small detail of the antenna and the cable (N = 22544). The structure is excited by placing a small radial electrical dipole somewhere at the lower port of the coaxial cable. The coaxial cable is short circuited at the bottom and filled with a dielectric material with a relative permittivity of 2.5. This procedure allows very accurate calculation of, for instance, the radiation pattern of such an antenna, shown in Figure 3. Also, because we are using a BIE, it is easy to add another antenna to the configuration at an arbitrary distance, and study the communication between both, even in the presence of other objects. We have compared the field inside the coaxial cable of the receiving monopole antenna when they are communicating through vacuum (N = 45088) and when an identified flying object (IFO), in this case the Thunderbird 2 (conveniently rescaled to dwarfish size), is hovering somewhere between them (N = 87915). The configuration is shown in Figure 4 and the results in Figure 5.

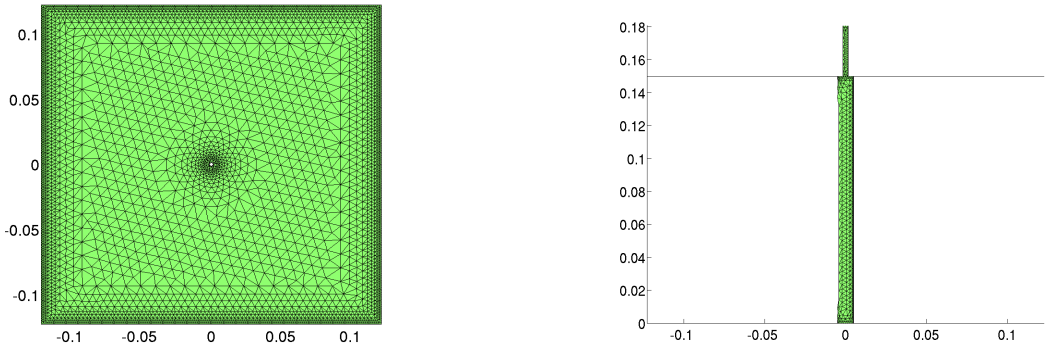


Figure 2: Mesh of the monopole antenna from top view (left) and front view (right)

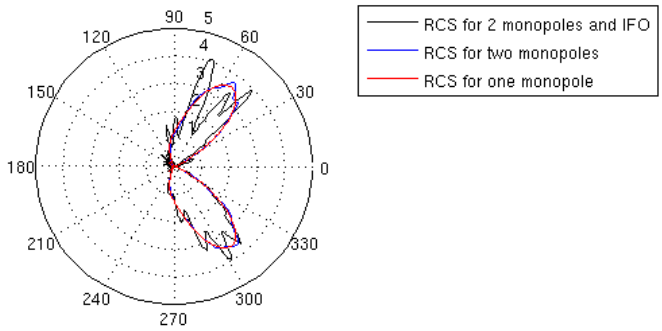


Figure 3: Radiation pattern along θ for three systems when one monopole is excited. Note the dead zone at 60° where the IFO is blocking direct radiation from the monopole.

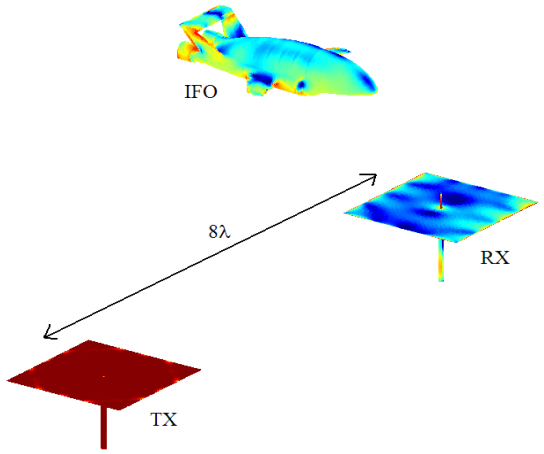


Figure 4: The induced currents on the monopoles and the IFO

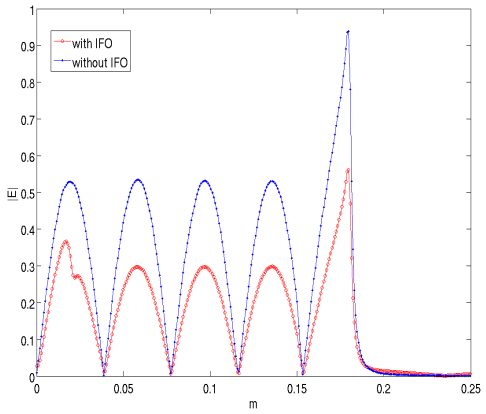


Figure 5: Field in the coaxial line at the receiving antenna

## 4. Conclusion

We have demonstrated the accuracy and capabilities of a truly broadband fast multipole method, the NSPWMLFMA, in three dimensions.

## 5. References

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