

Frequency- and Time-Domain Stochastic Analysis of Lossy and Dispersive Interconnects in a SPICE-Like Environment

Paolo Manfredi*, Dries Vande Ginste[†], Daniël De Zutter[†] and Flavio G. Canavero*

*EMC Group, Dipartimento di Elettronica e Telecomunicazioni
Politecnico di Torino, 10129 Torino, Italy (E-mail: paolo.manfredi@polito.it)

[†]Electromagnetics Group, Department of Information Technology
Ghent University, 9000 Gent, Belgium (E-mail: vdginste@intec.UGent.be)

Abstract—This paper presents an improvement of the state-of-the-art polynomial chaos (PC) modeling of high-speed interconnects with parameter uncertainties via SPICE-like tools. While the previous model, due to its mathematical formulation, was limited to lossless lines, the introduction of modified classes of polynomials yields a formulation that allows to account for losses and dispersion as well. Thanks to this, the new implementation can also take full advantage of the combination of the PC technique with macromodels that accurately describe the interconnect properties. An application example, i.e. the stochastic analysis of an on-chip line, validates and demonstrates the improved method.

Index Terms—Circuit modeling, circuit simulation, polynomial chaos, stochastic analysis, tolerance analysis, transmission lines, uncertainty.

I. INTRODUCTION

Due to the growing demand for high computing power and large scale integration, on-chip interconnects are packed at very high densities and the assessment of signal integrity during the early design phase is becoming of major concern. On top of that, the reduced device size is making manufacturing tolerances hard to control, thus also representing a challenge for accurate estimations. A statistical analysis is often preferred, rather than trying to predict deterministic values.

However, an accurate prediction of on-chip interconnect properties with respect to physical parameters is too time-consuming for a brute-force Monte Carlo (MC) analysis. The combination with a macromodeling approach made the MC analysis feasible for a single line [1], but generalization to multiconductor lines is not straightforward.

To overcome the limitations of the MC method, a novel methodology based on the so-called polynomial chaos (PC) technique [2] was proposed in [3]. The PC approach applies to multiconductor lines in frequency domain and is based on the expansion of the random variables and governing equations into series of polynomials. It allowed to efficiently take inherent variations of the interconnect's cross-section into account.

In order to allow greater design flexibility and to provide accurate time-domain results, an integration of PC into com-

mercial SPICE-like software was proposed in [4]. However, owing to its mathematical formulation, the resulting system was not symmetric and the implementation had to rely on Branin's equivalent for multiconductor lines [5], being therefore limited to lossless, dispersion-free lines.

In [6], combination of PC and macromodeling yielded a powerful tool that provides an efficient stochastic analysis together with a very accurate description of the interconnect properties, also accounting for losses and dispersion. Nevertheless, due to Branin's model limitations, it was not possible to take full advantage of such macromodels in a SPICE-like environment.

In this paper, we present a modified formulation of PC yielding symmetric models that can be more efficiently implemented in SPICE-like environments and also take losses and dispersion into account. The new implementation is based on a proper re-definition of the polynomial bases used for the PC expansion. The stochastic simulation of an on-chip line with geometrical uncertainties concludes the paper, validating the proposed methodology.

II. POLYNOMIAL CHAOS OVERVIEW

This section briefly recalls the application of the PC technique to distributed interconnects. For the sake of brevity, we limit ourselves to the case of single lines with one Gaussian random parameter. In [3], [6] and references therein, readers can find further details about the advocated approach, as well as for the extension to multiconductor lines and multiple random variables.

The behavior of an interconnect depends on its geometrical and material properties. When these are random, the response becomes random itself. This inherent stochasticity can be highlighted by making the above dependence explicit in the governing Telegrapher's equations:

$$\begin{cases} \frac{d}{dz}V(z, s, \xi) = -Z(s, \xi)I(z, s, \xi) \\ \frac{d}{dz}I(z, s, \xi) = -Y(s, \xi)V(z, s, \xi), \end{cases} \quad (1)$$

where ξ is a generic random variable affecting the interconnect properties, z is the direction of propagation, s is the Laplace variable, Z and Y are the per-unit-length (p.u.l.) impedance and admittance parameters, and V and I are the voltage and current along the line, respectively. It should be noted that the introduction of ξ does not affect the uniformity of the line.

To obtain statistical information, MC sampling of (1) is possible, but it can become rather time-consuming even for simple structures, due to the large number of samples that it required to converge. According to the standard PC approach [3], the stochastic p.u.l. parameters are expanded into series of proper *orthogonal* polynomials $\{\phi_k\}$:

$$Z(s, \xi) \approx \sum_{k=0}^P Z_k(s) \phi_k(\xi), \quad Y(s, \xi) \approx \sum_{k=0}^P Y_k(s) \phi_k(\xi), \quad (2)$$

where Z_k and Y_k are coefficients that can be obtained with a projection procedure. When dealing with Gaussian random variables, $\{\phi_k\}$ is represented by the probabilists' Hermite polynomials, the first four being $\phi_0 = 1$, $\phi_1 = \xi$, $\phi_2 = \xi^2 - 1$, $\phi_3 = \xi^3 - 3\xi$. It can be proven that these polynomials are orthogonal with respect to the following inner product:

$$\langle f(\xi), g(\xi) \rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(\xi)g(\xi)e^{-\frac{1}{2}\xi^2} d\xi. \quad (3)$$

The introduction of similar expansions for the unknown voltage and current variables, and subsequent Galerkin weighting, allow to rewrite the stochastic problem (1) as an augmented but deterministic one

$$\begin{cases} \frac{d}{dz} \tilde{\mathbf{V}}(z, s) = -\tilde{\mathbf{Z}}(s) \tilde{\mathbf{I}}(z, s) \\ \frac{d}{dz} \tilde{\mathbf{I}}(z, s) = -\tilde{\mathbf{Y}}(s) \tilde{\mathbf{V}}(z, s), \end{cases} \quad (4)$$

which is $P+1$ times larger and analogous to a multiconductor problem where the new unknowns are the coefficients for the voltage and current variables, collected into vectors $\tilde{\mathbf{V}}$ and $\tilde{\mathbf{I}}$. The entries of the new p.u.l. matrices are given by

$$\tilde{\mathbf{Z}}_{ij} = \sum_{k=0}^P Z_k \alpha_{kji}, \quad \tilde{\mathbf{Y}}_{ij} = \sum_{k=0}^P Y_k \alpha_{kji}, \quad (5)$$

where

$$\alpha_{kji} = \frac{\langle \phi_k(\xi) \phi_j(\xi), \phi_i(\xi) \rangle}{\langle \phi_i(\xi), \phi_i(\xi) \rangle}. \quad (6)$$

Specifically, in the case of Hermite polynomials, analytical expressions exist for both the numerator and the denominator of (6), i.e.,

$$\langle \phi_k(\xi) \phi_j(\xi), \phi_i(\xi) \rangle = \frac{k!j!i!}{(m-k)!(m-j)!(m-i)!}, \quad (7)$$

where $2m = k + j + i$, and

$$\langle \phi_j(\xi), \phi_i(\xi) \rangle = i! \delta_{ij}, \quad (8)$$

where δ_{ij} denotes the Kronecker's delta. It should be noted that (7) holds when the arguments of the involved factorials are integer and non-negative, being zero otherwise.

Hence, the construction of the augmented system (4) can be automated and, given a model for random parameter variations, the stochastic analysis can be performed through its solution and the subsequent extraction of statistical information from the PC expansions by means of analytical formulae or numerical techniques. This solution generally turns out to be much faster than computing a large number of MC samples.

III. SPICE IMPLEMENTATION

The numerical solution of lossy and dispersive multiconductor transmission lines like (4) can be achieved in SPICE-like solvers by means of the W-element, available in the most advanced tools such as HSPICE or Agilent's Advanced Design System (ADS). However, the input p.u.l. matrices are inherently assumed to be symmetric, which is obviously always the case for physical lines (when reciprocal materials are used).

Yet, according to the expression of coefficients (6), the augmented p.u.l. matrices (5) turn out not to be symmetric. Therefore, direct implementation of the augmented PC problem (4) cannot be achieved. In order to partially overcome this limitation, in [4] a SPICE model for the augmented problem was created by means of the Branin's equivalent for multiconductor transmission lines [5]. However, this approach is based on the decomposition of the multiconductor structure into lossless uncoupled single lines, and therefore it is not suitable to manage lines with lossy and frequency-dependent p.u.l. parameters. Furthermore, it requires the introduction of a subcircuit with additional auxiliary nodes, thus also reducing the efficiency.

Although in many practical situations losses can be neglected, this is not the case for on-chip lines, where they play a fundamental role due to presence of semiconductors and to the very tiny conductor cross-section. To improve the design accuracy, a better solution is then imperative.

IV. INTRODUCTION OF A NEW POLYNOMIAL BASIS

A closer investigation of the classical mathematical framework of PC theory [2] reveals that the choice of *orthogonal* polynomials for the expansions breaks the symmetry. Indeed, the asymmetry in (6) is due to the denominator (8), corresponding to the non-unitary norm of the polynomials $\{\phi_k\}$.

However, it can be shown that the introduction of the following normalized class of Hermite polynomials

$$\phi'_k(\xi) = \frac{\phi_k(\xi)}{\sqrt{\langle \phi_k(\xi), \phi_k(\xi) \rangle}} = \frac{\phi_k(\xi)}{\sqrt{k!}} \quad (9)$$

yields symmetric matrices. In fact, the corresponding coefficients α'_{kji} to be used in (5) now have a unitary denominator. As a result, the first four *orthonormal* Hermite polynomials are $\phi'_0 = 1$, $\phi'_1 = \xi$, $\phi'_2 = \frac{1}{\sqrt{2}}\xi^2 - \frac{1}{\sqrt{2}}$, $\phi'_3 = \frac{1}{\sqrt{6}}\xi^3 - \frac{3}{\sqrt{6}}\xi$. For example, the corresponding augmented impedance matrix $\tilde{\mathbf{Z}}$ for a second-order expansion ($P = 2$) then becomes

$$\tilde{\mathbf{Z}} = \begin{bmatrix} \mathbf{Z}_0 & \mathbf{Z}_1 & \mathbf{Z}_2 \\ \mathbf{Z}_1 & \mathbf{Z}_0 + \sqrt{2}\mathbf{Z}_2 & \sqrt{2}\mathbf{Z}_1 \\ \mathbf{Z}_2 & \sqrt{2}\mathbf{Z}_1 & \mathbf{Z}_0 + 2\sqrt{2}\mathbf{Z}_2 \end{bmatrix}, \quad (10)$$

which is indeed symmetric, and a similar relation holds for the admittance matrix $\tilde{\mathbf{Y}}$.

Clearly, the normalization process does not affect the accuracy and the convergence properties of the PC expansions, as the new polynomials are still orthogonal with respect to (3).

V. VALIDATION AND NUMERICAL RESULTS

The new augmented p.u.l. matrices being symmetric, they can be supplied as tabulated frequency-dependent input data for a W-element. The augmented problem (4) can be solved with a circuit solution after a proper augmentation of the line terminations. For example, for linear, deterministic RLGC loads, this simply amounts to replicating the elements onto the new line terminations. No replication is required for voltage and current sources instead [3], [6].

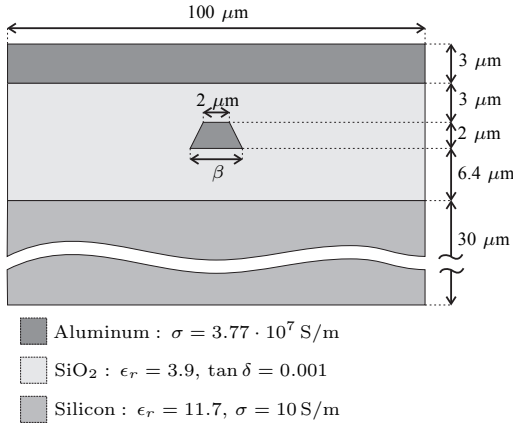


Fig. 1. Cross-section of the IEM on-chip line (not on scale).

To provide a validation example, the proposed methodology is applied to the analysis of the inverted embedded microstrip (IEM) line shown in Fig. 1. This kind of structure is becoming increasingly used as on-chip interconnection. Nonetheless, due to the presence of semiconductors, the behavior of its p.u.l. parameters is hard to predict analytically. An accurate, but slow, 2-D electromagnetic solver is then required. Moreover, owing to its tiny dimensions, the manufacturing process (e.g., etching) introduces random and significant irregularities in the conductor shape, that can be modeled as a trapezoid with uncertain base width β , as illustrated in Fig. 1.

To efficiently account for such variations, a parametric frequency-dependent macromodel was built for the IEM p.u.l. parameters [1], [6]. This minimized the calls to the slow 2-D solver, providing however an accurate analytical description of the behavior of the p.u.l. parameters as a function of the frequency and of the uncertain parameter β .

In the following, these macromodels are used to describe random variations of the conductor shape and for the computation of the pertinent PC expansions (2). The nominal value of β is supposed to be $2 \mu\text{m}$, thus leading to a square cross-section. Gaussian 1σ -variations of $\pm 10\%$ with respect to this nominal value are considered to model the uncertainty introduced by the etching process. It should be noted that

positive and negative variations correspond to underetching and overetching, respectively.

For the simulation, the line is supposed to be 1-mm long and to be driven by a voltage source with a $1\text{-}\Omega$ internal resistance. The far-end extremity is terminated by a capacitive 1-pF load.

A. Time-domain analysis

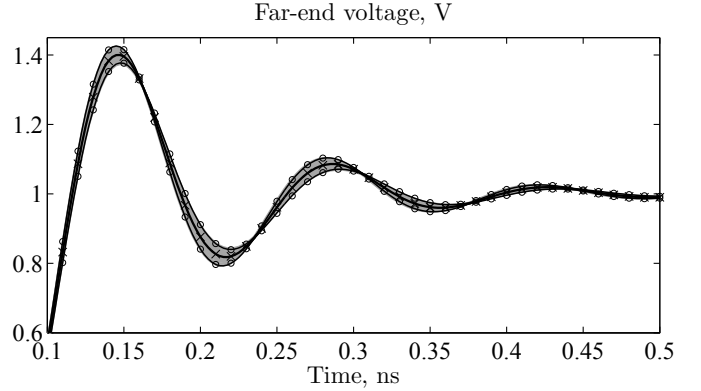


Fig. 2. Time-domain far-end voltage. Solid black lines: mean value and $\pm 3\sigma$ limits computed with MC; crosses and circles: mean value and $\pm 3\sigma$ limits obtained from the PC expansion; gray area: a sample of responses obtained by means of the MC method.

A time-domain analysis is presented first. In this case, the voltage source is a ramped step from 0 V to 1 V with a rise time of 50 ps. Fig. 2 shows the voltage waveform produced on the far-end termination. The solid black lines refer to the average value and the $\pm 3\sigma$ bounds estimated by means of 10000 MC simulations. Markers refer to the same statistical information obtained by means of the PC approach, which are in perfect agreement. Additionally, a subset of 100 samples of the response (resulting in the gray area) are reported to provide a qualitative idea of its spread. The impact of the variations of the conductor cross-section is clearly established.

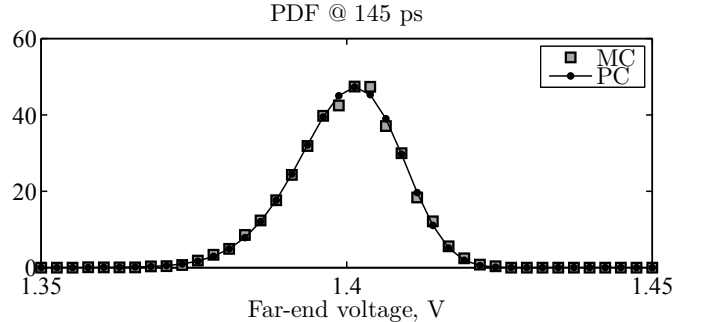


Fig. 3. Probability density function of the far-end voltage at 145 ps.

As designers might be interested in more quantitative information, such as the maximum overshoot that is likely to be expected, the computation of probability density functions (PDFs) is required. This information can be extracted from the PC expansions as well. Fig. 3 reports the comparison between

MC and PC in the computation of the PDF of the far-end voltage at 145 ps, i.e., the point at which the largest overshoot occurs (see Fig. 2). The comparison again reveals excellent agreement between the two techniques.

For these simulations, MC samples of the p.u.l. parameters as well as the corresponding PC augmented matrices have been supplied to HSPICE as frequency-dependent tabulated data for W-elements. The MC solution took 0.23 s for each sample, thus leading to an overall simulation time of 38 min. The solution of the augmented problem required 0.28 s instead. Therefore, for this example, our new PC for general lossy, dispersive lines yields a speed-up of 8200 \times .

B. Frequency-domain analysis

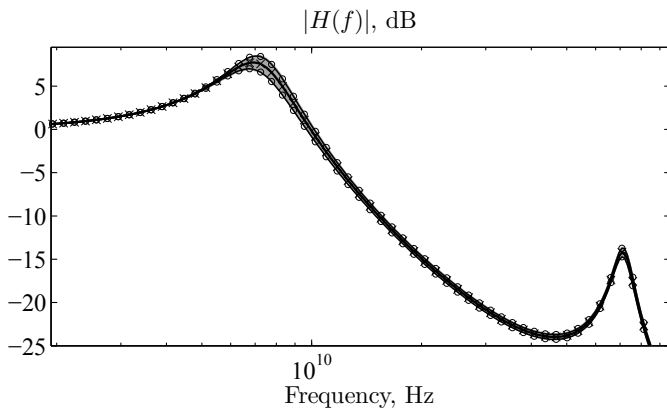


Fig. 4. Magnitude of the far-end transfer function. Solid black lines: mean value and $\pm 3\sigma$ limits computed with MC; crosses and circles: mean value and $\pm 3\sigma$ limits obtained from the PC expansion; gray area: a sample of responses obtained by means of the MC method.

The voltage source is now replaced by a sine wave of amplitude $E(f)$. Fig. 4 shows the magnitude of the far-end transfer function, defined as $H(f) = V_L(f)/E(f)$, with $V_L(f)$ the voltage at the load. As for the previous analysis, average value and $\pm 3\sigma$ limits are provided together with a reduced set of samples of the response.

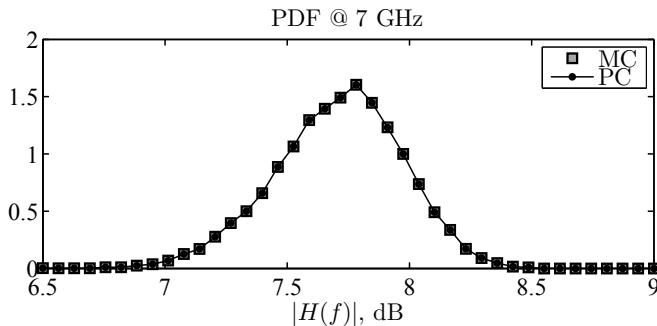


Fig. 5. Probability density function of the far-end transfer function at 7 GHz.

Fig. 5 additionally shows the PDF computed at 7 GHz, again by means of 10000 MC simulations and PC. The accuracy of

the latter is confirmed also for this second analysis, for which the simulation times are similar to those of the time-domain example.

VI. CONCLUSIONS

This paper presents an improvement of the implementation of an efficient stochastic modeling for high-speed lines in commercial SPICE-like tools. The methodology is based on the PC approach, i.e., on the expansion of random variables into polynomial bases. It amounts to creating and solving an augmented set of differential equations, similar to a multiconductor transmission-line problem. However, the state-of-the-art implementation was limited to lossless and non-dispersive lines since, according to the general framework of PC theory, the new augmented p.u.l. matrices were not symmetric.

In this paper it is shown that, thanks to a proper normalization of the polynomial bases, the symmetry of the p.u.l. matrices is preserved. This allows to solve the PC problem by means of generic multiconductor transmission-line models already available in many commercial SPICE-like solvers, which are capable of handling losses and dispersion as well. Moreover, a PC- and macromodeling-based variability analysis can now be efficiently performed in a SPICE-like environment.

The PC technique has been proven to provide accurate results with respect to traditional approaches such as the MC method, but with a remarkable reduction in computational time. Integration of such a technique into standard commercial software in industry provides designers a much larger flexibility as far as the complexity of the circuit configurations that can be analyzed is concerned.

The strength of the methodology is validated through the time- and frequency-domain analysis of an IEM on-chip line with uncertain trace cross-section, for which losses and dispersion cannot be neglected. For this example, a speed-up of about 8000 \times is obtained.

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