Low-Frequency Regularization of the Mixed-Discretized Calderon CFIE

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Among the many integral equations for analyzing scattering and radiation by perfect electrically conducting objects, the Electric and Magnetic Field Integral Equations (EFIE and MFIE) stand out: they not only are the most often used but also serve as building blocks for many other equations. Unfortunately, the EFIE and MFIE have several deficiencies. The discretized EFIE is ill-conditioned when the frequency approaches zero or the discretization density tends to infinity. The equation's high condition number in these regimes results in elevated iteration counts when solved iteratively. These low-frequency and dense-discretization breakdown phenomena render application of the EFIE problematic in many real-world scenarios. The MFIE, while well-conditioned in the dense-discretization limit, is known to produce less accurate solutions than the EFIE. Recent studies have traced this phenomenon to the unfortunate choice of basis and testing functions in traditional MFIE discretizations. A solution to this problem that leverages a mixed discretization, i.e. a judicious choice of different source and testing spaces that respect the operator mapping properties of the continuous MFIE operator, recently has been proposed.

When the scatterer under consideration is closed, both the EFIE and MFIE suffer from interior resonances, i.e. null-spaces for wavenumbers that corresponds to a resonance of the interior problem. The presence of these resonances often negatively impacts the accuracy and solution time of both the EFIE and MFIE. This problem can be remedied by using a Combined Field Integral Equation (CFIE), viz. a linear combination of the EFIE and MFIE that is provably resonance-free. Unfortunately, the CFIE inherits the ill-conditioning properties of the EFIE and the inaccuracies of the MFIE. To solve the ill-conditioning of the CFIE, one may adopt a Calderon-preconditioned CFIE (CP-CFIE), i.e. a linear combination of a modified Calderon-preconditioned EFIE and the MFIE. However, when using standard discretization strategies, this equation still inherits the inaccuracies of the MFIE. Here, we propose to remedy this problem by using a mixed discretization. First, a rigorous analysis of a mixed-discretized CP-CFIE will be presented. Specifically, the equation's resonance-free nature will be established and its convergence properties for low frequencies rigorously analyzed. The necessity of using a quasi-Helmholtz decomposition will be exposed and a regularizer for the parasitic h-refinement ill-conditioning of the Helmholtz decomposed equation presented. Numerical results will show the effectiveness of the proposed scheme and its applicability to the analysis of real-world problems.