Time–Domain Parametric Sensitivity Analysis of Multiconductor Transmission Lines

D. Spina*, F. Ferranti*, G. Antonini[+], T. Dhaene*, L. Knockaert*

*Dept. of Information Technology, Internet Based Communication Networks and Services (IBCN)
Ghent University – IBBT, Gaston Crommenlaan 8 Bus 201, 9000 Ghent, Belgium
Email: domenico.spina@intec.ugent.be; francesco.ferranti@intec.ugent.be;
tom.dhaene@intec.ugent.be; luc.knockaert@intec.ugent.be

[+]UAq EMC Laboratory, Dipartimento di Ingegneria Elettrica e dell'Informazione,
Università degli Studi dell'Aquila, Via G. Gronchi 18, 67100, L'Aquila, Italy
Email: giulio.antonini@univaq.it

Abstract

We present a new parametric macromodeling technique for lossy and dispersive multiconductor transmission lines (MTLs). This technique can handle design parameters, such as substrate or geometrical layout features, and provide time–domain sensitivity information for voltage and currents at the ports of the lines. It is based on a recently introduced spectral approach for the analysis of lossy and dispersive MTLs [1], [2] and it is suited to generate state–space models and synthesize equivalent circuits, which can be easily embedded into conventional SPICE–like solvers. Parametric macromodels which provide sensitivity information are well suited for design space exploration, design optimization and crosstalk analysis. A numerical example validates the proposed approach in both frequency and time domain.

Introduction

The recent advances in fabrication methods and the rapid increase in operating speeds, density, and complexity of modern integrated circuits has made signal integrity a challenging task for high–frequency circuit designers. Consequently, high–speed interconnect modeling has become of paramount importance to properly capture physical effects such as reflection, crosstalk and propagation delays. The increased circuit complexity requires that designers make the proper trade–offs between conflicting design requirements using optimization techniques, in order to obtain the best possible performance. To achieve this goal, efficient and accurate sensitivity information with respect to interconnect parameters can be used to boost the efficiency of powerful optimizers that employ gradient–based techniques.

In [3] a parametric macromodeling technique for lossy and dispersive MTLs is presented: it provides time–domain information for voltage and current at the ports of the lines, starting from the knowledge of the MTL per–unit–length (p.u.l.) parameters. In the present paper the cited method is extended to perform the parametric sensitivity analysis of MTLs with respect to either geometric or physical parameters directly in the time–domain. the proposed technique it is based on the spectral decomposition proposed in [1], [2] and on the numerical interpolation of the p.u.l. parameters.

Time–domain parametric sensitivity analysis

In the sequel capital boldfaced fonts such as $\boldsymbol{A}$ will be used to represent matrices, lowercase $\boldsymbol{x}(t)$ will be used for vectorial functions in the time domain and the *hat* notation $\widehat{\boldsymbol{A}}$ will indicate the partial derivative of the matrix $\boldsymbol{A}$ with respect to the parameter g.

Let us consider a homogeneous multiconductor transmission line (MTL) of length d, with $N-1$ conductors and p.u.l. impedance and admittance $\boldsymbol{Z}_{pul}(s,g)$ and $\boldsymbol{Y}_{pul}(s,g)$, respectively [4]. Assuming the quasi–transverse electromagnetic (TEM) hypothesis, the propagation of voltages and currents along the line is described by the Telegrapher's equations [4]. Using the dyadic Green's function method proposed in [1], the voltage along the MTL with respect to the parameter g, which can be either physical or geometrical, can be evaluated as

$$\boldsymbol{V}(z,s,g) = \boldsymbol{G}(z,0,s,g)\left(-\boldsymbol{Z}_{pul}(s,g)\right)\boldsymbol{I}_0(s,g) \\ + \boldsymbol{G}(z,d,s,g)\left(-\boldsymbol{Z}_{pul}(s,g)\right)\boldsymbol{I}_d(s,g) \quad (1)$$

where $\boldsymbol{I}_0(s,g)$ and $\boldsymbol{I}_d(s,g)$ are the MTL port currents at the input and output ports respectively and z is the propagation distance. The dyadic Green's function for uniform MTLs can be written in a spectral form [1] as

$$\boldsymbol{G}(z,z',s,g) = -\sum_0^\infty \boldsymbol{\Psi}_n(s,g)^{-1}A_n^2\varphi_n(z)\varphi_n(z') \quad (2a)$$

$$\boldsymbol{\Psi}_n(s,g) = \boldsymbol{\gamma}^2(s,g) + \left(\frac{n\pi}{d}\right)^2\boldsymbol{U} \quad (2b)$$

$$\boldsymbol{\gamma}^2(s,g) = \boldsymbol{Z}_{pul}(s,g)\boldsymbol{Y}_{pul}(s,g) \quad (2c)$$

$$\varphi_n(z) = \cos\left(\frac{n\pi z}{d}\right) \quad (2d)$$

where $A_n = \sqrt{\frac{1}{d}}$ if $n=0$, otherwise $A_n = \sqrt{\frac{2}{d}}$ and $\boldsymbol{U}$ is the identity matrix. Equation (1) can be written in a more compact form

$$\boldsymbol{V}(z,s,g) = \boldsymbol{Z}(z,s,g)\left[\begin{array}{c} \boldsymbol{I}_0(s,g) \\ \boldsymbol{I}_d(s,g) \end{array}\right] = \boldsymbol{Z}(z,s,g)\boldsymbol{I}(s,g) \quad (3)$$

Equation (3) introduces the impedance matrix $\boldsymbol{Z}(z,s,g)$ which can be expressed with respect to the Green's function [1] as

$$\boldsymbol{Z}(z,s,g) = \sum_{n=0}^\infty \boldsymbol{\Psi}_n(s,g)^{-1}A_n^2(-\boldsymbol{Z}_{pul}(s,g))\varphi_n(z)\boldsymbol{U}_n \quad (4)$$

The symbol $\boldsymbol{U}_n$ represent the matrix

$$\boldsymbol{U}_n = \left[\begin{array}{cc} 1 & (-1)^n \\ (-1)^n & 1 \end{array}\right]$$

Equation (3) leads to an analytical expression for the voltage sensitivity with respect to parameter g at the MTL ports

$$\widehat{\boldsymbol{V}}(z,s,g) = \widehat{\boldsymbol{Z}}(z,s,g)\boldsymbol{I}(s,g) + \boldsymbol{Z}(z,s,g)\widehat{\boldsymbol{I}}(s,g) \quad (5)$$

The matrix $\widehat{\boldsymbol{Z}}(z,s,g)$ can be written as [2]

$$\begin{aligned}
\widehat{\boldsymbol{Z}}(z,s,g) = & \sum_{n=0}^{\infty} \boldsymbol{\Psi}_n(s,g)^{-1} A_n^2 \widehat{\boldsymbol{\gamma}^2}(s,g)\boldsymbol{\Psi}_n(s,g)^{-1} \\
& \quad (-\boldsymbol{Z}_{pul}(s,g))\varphi_n(z)\boldsymbol{U}_n \quad (6) \\
& + \sum_{n=0}^{\infty} \boldsymbol{\Psi}_n(s,g)^{-1} A_n^2 (-\widehat{\boldsymbol{Z}}_{pul}(s,g))\varphi_n(z)\boldsymbol{U}_n
\end{aligned}$$

where the constant propagation sensitivity is

$$\widehat{\boldsymbol{\gamma}^2}(s,g) = \widehat{\boldsymbol{Z}}_{pul}(s,g)\boldsymbol{Y}_{pul}(s,g) + \boldsymbol{Z}_{pul}(s,g)\widehat{\boldsymbol{Y}}_{pul}(s,g) \quad (7)$$

Note that only the matrices $\boldsymbol{Z}_{pul}(s,g)$, $\boldsymbol{Y}_{pul}(s,g)$ and the corresponding derivatives with respect to parameters g (i.e., the $\widehat{\boldsymbol{Z}}_{pul}(s,g)$ and $\widehat{\boldsymbol{Y}}_{pul}(s,g)$ matrices) are required in order to obtain equations (4) and (6).

In [3] only equation (4) must be solved: to calculate a macromodel for the impedance matrix $\boldsymbol{Z}(s,g)$ the p.u.l. parameters must be calculated in the entire design space. To this aim the Multivariate Orthonormal Vector Fitting technique [5] is used to build a rational parametric macromodel of $\boldsymbol{Z}_{pul}(s,g)$ and $\boldsymbol{Y}_{pul}(s,g)$, leading to a time–domain model for the impedance matrix for each desired value of g [3]. In our case also the $\widehat{\boldsymbol{Z}}_{pul}(s,g)$ and $\widehat{\boldsymbol{Y}}_{pul}(s,g)$ matrices must be calculated to solve equation (6), suggesting a different approach to tackle the problem. The $\boldsymbol{Z}_{pul}(s,g)$ and $\boldsymbol{Y}_{pul}(s,g)$ are usually smooth functions with respect to both frequency and the geometrical parameter: different interpolation techniques can be used to express these matrices in the entire design space, starting from initial data which can be based on measurements or on electromagnetic simulations of the MTL. Once the p.u.l. parameters are well approximated in the entire design space, the corresponding sensitivities (i.e., the $\widehat{\boldsymbol{Z}}_{pul}(s,g)$ and $\widehat{\boldsymbol{Y}}_{pul}(s,g)$ matrices) can be evaluated with numerical techniques.

Assuming that the impedance and sensitivity matrices (4), (6) are expressed in a rational form it is easy to find the poles and residues. As a consequence, the generation of the time–domain state–space equations in Jordan form [6], [7] is straightforward. In fact, based on equation (3), it is easy to calculate the time–domain model for the voltage at the MTL ports, i.e.,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_Z\,\boldsymbol{x}(t) + \boldsymbol{B}_Z\,\boldsymbol{i}(t) \quad (8a)$$
$$\boldsymbol{v}(t) = \boldsymbol{C}_Z\,\boldsymbol{x}(t) + \boldsymbol{D}_Z\,\boldsymbol{i}(t) \quad (8b)$$

Hence, based on equation (5), the voltage sensitivity can be represented by two different sets of space–state equations

$$\dot{\boldsymbol{x}}_1(t) = \boldsymbol{A}_{\widehat{Z}}\,\boldsymbol{x}_1(t) + \boldsymbol{B}_{\widehat{Z}}\,\boldsymbol{i}(t) \quad (9a)$$
$$\widehat{\boldsymbol{v}}_1(t) = \boldsymbol{C}_{\widehat{Z}}\,\boldsymbol{x}_1(t) + \boldsymbol{D}_{\widehat{Z}}\,\boldsymbol{i}(t) \quad (9b)$$

$$\dot{\boldsymbol{x}}_2(t) = \boldsymbol{A}_Z\,\boldsymbol{x}_2(t) + \boldsymbol{B}_Z\,\widehat{\boldsymbol{i}}(t) \quad (10a)$$
$$\widehat{\boldsymbol{v}}_2(t) = \boldsymbol{C}_Z\,\boldsymbol{x}_2(t) + \boldsymbol{D}_Z\,\widehat{\boldsymbol{i}}(t) \quad (10b)$$

where $\boldsymbol{x}(t)$, $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$ are space state variables, while $\boldsymbol{i}(t)$ and $\widehat{\boldsymbol{i}}(t)$ are the inputs [7].

Clearly, the rational model for $\boldsymbol{Z}(s,g)$ and $\widehat{\boldsymbol{Z}}(s,g)$ must be calculated for multiple values of g in the design space. To attain this goal it is necessary to switch from a multivariate model to a univariate model and this can be done by fixing the values of the parameter: $g = \overline{g}$. If so, the impedance matrix and its sensitivity can be calculated using the spectral decomposition (4) and (6), once the infinite summation of modes is truncated. Only a limited number of modes it is needed to model $\boldsymbol{Z}(s,g)$ and $\widehat{\boldsymbol{Z}}(s,g)$ accurately over the frequency bandwidth of interest [3], [2]. In the present paper an adaptive criteria is proposed to choose the required number of modes for the different values of the geometrical or physical parameter and the adopted solution will be shown only for the $\widehat{\boldsymbol{Z}}$ matrix, since it is similar for the MTL impedance matrix. With $\overline{g}$ a fixed value of the parameter g and with $\widehat{\boldsymbol{Z}}_n(s,\overline{g})$ the corresponding sensitivity modal impedance, the adopted criterion is described in algorithm (1).

Input: $\boldsymbol{Z}_{pul}(s,\overline{g}), \boldsymbol{Y}_{pul}(s,\overline{g}), \widehat{\boldsymbol{Z}}_{pul}(s,\overline{g}), \widehat{\boldsymbol{Y}}_{pul}(s,\overline{g})$
Output: Number of modes N, $\widehat{\boldsymbol{Z}}_N(s,\overline{g})$

$\widehat{\boldsymbol{Z}}_N(s,\overline{g}) = \widehat{\boldsymbol{Z}}_{prev}(s,\overline{g}) = \boldsymbol{0};$
$n = 0;$
$N = N_{init};$
$error = error_{chosen};$
$rms\_error = \infty$

if $rms\_error(\widehat{\boldsymbol{Z}}_N(s,\overline{g}), \widehat{\boldsymbol{Z}}_{prev}(s,\overline{g})) > error$ **then**
    while $n < N$ **do**
        $\widehat{\boldsymbol{Z}}_N(s,\overline{g}) = \widehat{\boldsymbol{Z}}_N(s,\overline{g}) + \widehat{\boldsymbol{Z}}_n(s,\overline{g});$ [2]
        $n = n + 1;$
    end
    $N = N + N_{init};$
end

Algorithm 1: *Adaptive mode selection strategy.*

The desired stable rational model for the impedance matrix and its sensitivity has been calculated using the Vector Fitting algorithm [8], [9] for each value of the geometrical parameter in the design space. The passivity of the models for $\boldsymbol{Z}(s,\overline{g})$ can be checked and enforced in a post–processing step by means of standard techniques [10], [11].

Our approach offers several advantages:

- The proposed method needs a numerical approximation only on the initial data, expressing the sensitivity of the MTL impedance matrix in a closed form.

- The entire design space can be efficiently explored based on a limited number of initial samples.

- The time–domain macromodel can be easily embedded into conventional SPICE–like solvers, since it is represented in state–space form [7].

- Voltage and current sensitivity at MTL ports are calculated directly in the time–domain, thereby avoiding the time consuming similarity transformation and the use of the In-

verse Fast Fourier Transform and allowing to incorporate non–linear terminations in a straightforward way [7].

Numerical Results

A three–conductor transmission line (length $d = 15cm$) with frequency–dependent per–unit–length parameters has been modeled [3] over the frequency range $[0 - 15]GHz$. It consists of two coplanar microstrips over a ground plane. The cross section is shown in Fig. 1.
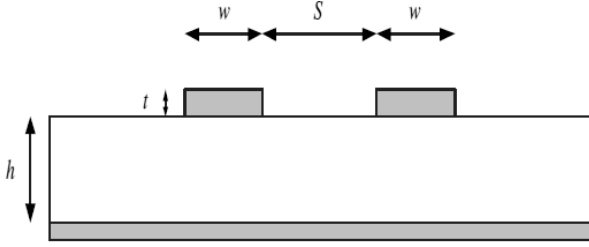

Figure 1: Cross section of the two coupled microstrips.

The conductors have width $w = 100\mu m$ and thickness $t = 50\mu m$. The spacing S between the microstrips vary over the design range $[100 - 500]\mu m$. The dielectric is $300\mu m$ thick and characterized by a dispersive and lossy permittivity which has been modeled by the wideband Debye model [12]. The frequency–dependent p.u.l. parameters are evaluated using a commercial tool [13] over a reference grid of 250×40 samples, for frequency and spacing respectively. The $\widehat{Z}_{pul}$ and $\widehat{Y}_{pul}$ matrices are interpolated by two polynomial techniques: *spline* and *pchip* [14], [15]. *pchip* finds the value of an underlying interpolant function $P(x) \in C^1$ at intermediate points under the constraint that in each subinterval $P(x)$ is the cubic Hermite interpolant, thereby preserving shape and monotonicity of the initial data. *spline* returns the polynomial form of the cubic spline interpolant $S(x) \in C^2$ calculated at the data points, resulting in a not necessarily monotonic interpolation. To validate the proposed method the voltage and current sensitivities at the MTL ports are calculated using both numerical approximations and the results are compared with the perturbative approach. The accuracy of the parametric macromodeling strategy is good and some results are shown in Figures 2 and 3.

The next step is to show that the accuracy is preserved in the time–domain as well. One line is excited by a voltage step with amplitude $1V$, rise/fall times $300ps$, width $1ns$ and initial delay $8ns$. The victim line is terminated on the near and far–end by $R_{NE} = 50\Omega$ and $R_{FE} = 50\Omega$, $C_{FE} = 1pF$, while the driven line is terminated with driver and load impedance equal to $R_S = 50\Omega$ and $R_L = 50\Omega$, $C_L = 1pF$. The port voltages and currents have been computed using the classic transmission line theory (TLT–IFFT) and are compared with the ones obtained with the state–space realization for the macromodel of the $Z(s, g)$ matrix, while the port voltage and current sensitivities are calculated using the different interpolation approaches for the p.u.l. parameters. Results shown in Figures 4 and 5 demonstrate the high accuracy in the time–domain of the proposed method.
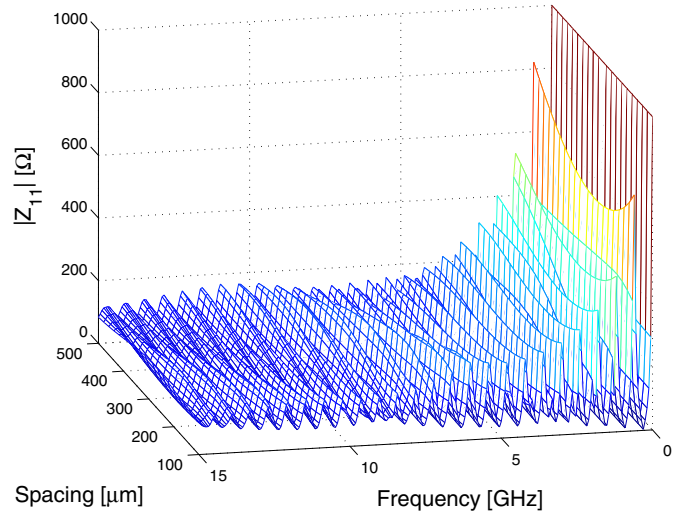

**Figure 2:** Magnitude of the element $\boldsymbol{Z}_{11}$, at 20 different spacing values in the design space.
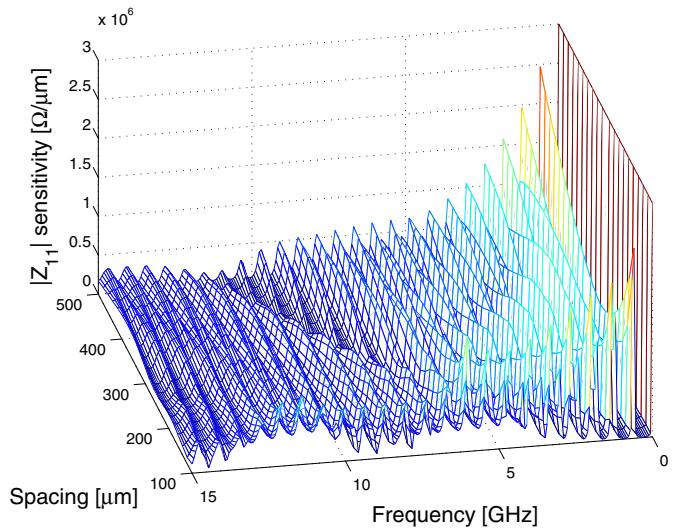
Figure 3: Magnitude of the element $\widehat{\boldsymbol{Z}}_{11}$, at 20 different spacing values in the design space.

Conclusion

In this paper, a new methodology for time–domain sensitivity analysis of multiconductor transmission lines has been presented. The dyadic Green's function of the 1–D wave propagation problem is used to compute a spectral model for the voltage sensitivity and this leads to a time–domain macromodel in state–space form. Using standard realization techniques, this macromodel can be easily embedded into conventional SPICE–like solvers. The new proposed technique is validated by comparing the MTL port currents and voltages with classic approaches (TLT–IFFT). Time–domain simulations confirm reliability and robustness of the proposed method.
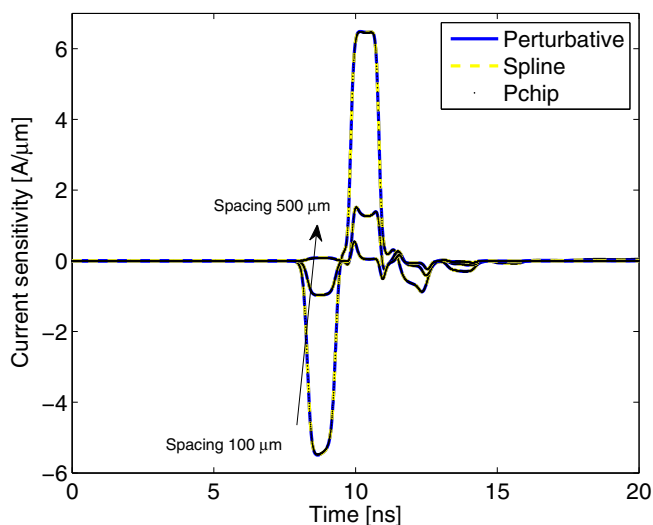
Figure 4: Current sensitivity at the input port of the driven line, for spacing values $S = [100, 253.85, 500]\ \mu m$.
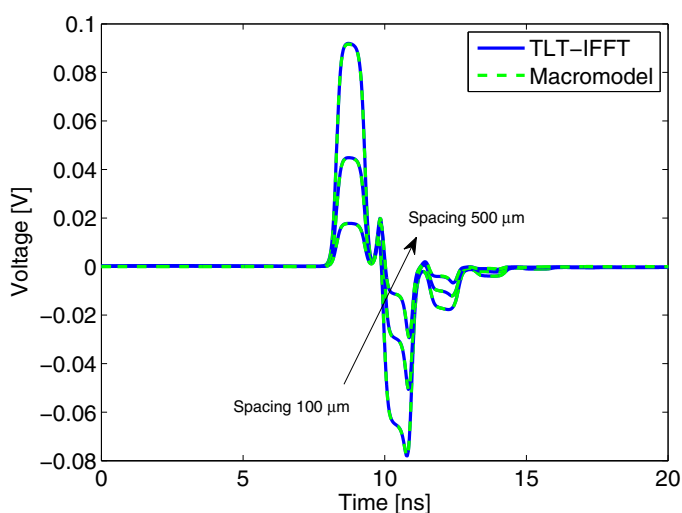
Figure 5: Voltage at the input port of the victim line, for spacing values $S = [100, 253.85, 500]\ \mu m$.

References

[1] G.Antonini, "A dyadic Green's function based method for the transient analysis of lossy and dispersive multiconductor transmission lines", IEEE Trans. Microw. Theory Tech., Vol. 56, No. 4, pp. 880-895, Apr. 2008.

[2] G.Antonini, L. De Camillis, and F. Ruscitti, "A spectral approach to frequency-domain sensitivity analysis of multiconductor transmission lines", IEEE Microw. Wireless Compon. Lett., Vol. 19, No. 2, pp. 65-67, Feb. 2009.

[3] F. Ferranti, G. Antonini, T. Dhaene and L. Knockaert "Parametric Macromodeling of Lossy and Dispersive Multiconductor Transmission Lines", IEEE Trans. Adv. Packag., Vol. 33, No. 2, pp. 481-491, May 2010.

[4] C.R. Paul, Analysis of Multiconductor Transmission Lines 2nd ed. New York: Wiley, 2008.

[5] D. Deschrijver, T. Dhaene and D. De Zutter, Robust parametric macromodeling using multivariate orthonormal vector fitting, IEEE Trans. Microw. Theory Tech., Vol. 56, No. 7, pp. 16611667, Jul. 2008.

[6] C.-T. Chen, Linear System Theory and Design, ser. Electrical and Computer Engineering. London, U.K.: Oxford University Press, 1998.

[7] G. Antonini and L. de Camillis, "Time-Domain Green's Function-Based Sensitivity Analysis of Multiconductor Transmission Lines With Nonlinear Terminations", IEEE Microw. Wireless Compon. Lett., Vol. 19, No. 7, Jul. 2009.

[8] B. Gustavsen and A. Semlyen, "Rational approximation of frequency domain responses by Vector Fitting", IEEE Trans. Power Del., Vol. 14, No. 3, pp. 1052-1061, Jul. 1999.

[9] D. Deschrijver, M. Mrozowski, T. Dhaene, and D. De Zutter, "Macromodeling of Multiport Systems Using a Fast Implementation of the Vector Fitting Method", IEEE Microw. Wireless Compon. Lett., Vol. 18, No. 6, pp. 383-385, Jun. 2008.

[10] A. Semlyen and B. Gustavsen, "A half-size singularity test matrix for fast and reliable passivity assessment of rational models", IEEE Trans. Power Del., Vol. 24, No. 1, pp. 345-351, Jan. 2009.

[11] B. Gustavsen, "Fast passivity enforcement for pole-residue models by perturbation of residue matrix eigenvalues", IEEE Trans. Power Del., Vol. 23, No. 4, pp. 2278-2285, Oct. 2008.

[12] A.R. Djordjeviè, R.M. Biljiè, V.D. Likar-Smiljaniè, T.K. Sarkar, "Wideband frequency-domain characterization of FR-4 and time-domain causality", IEEE Trans. Electromagn. Compat., Vol. 43, No. 4, pp. 662-667, Nov. 2001.

[13] Simbeor, Electromagnetic Simulation Environment with 3D Full-Wave Field Solver for Multi-Layered Circuits, Simberian Inc., Seattle.

[14] C. de Boor, A Practical Guide to Splines, Springer-Verlag, 1978

[15] F. N. Fritsch and R. E. Carlson, "Monotone Piecewise Cubic Interpolation", SIAM J. Numerical Analysis, Vol. 17, pp. 238-246, 1980.