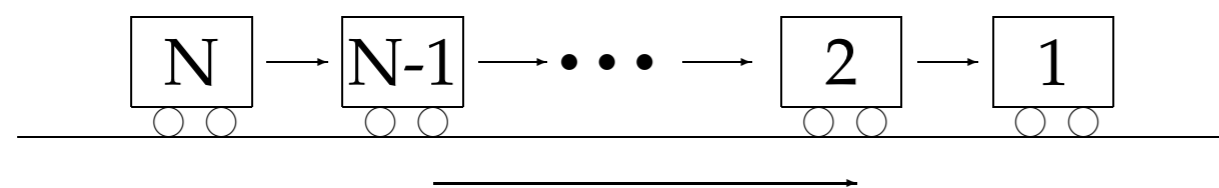
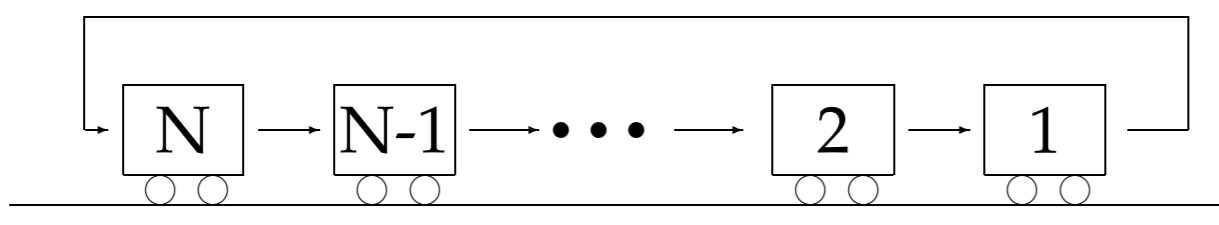


## Interconnection topology

- A classical control structure: the leading vehicle drives freely - the other vehicles follow (**supervisory-type control**)



- Our proposal: the behavior of the platoon is a result of cooperation between the vehicles (**self-organization**)



## System dynamics

Each vehicle is described by:

$$\ddot{x} + p\dot{x} = u,$$

- $x$ : position of the vehicle,
- $u$ : input,
- $p$ : drag coefficient per unit mass.

The coupling between consecutive vehicles is described by:

$$u_i = K(x_{i-1} - x_i - L_i), \quad i \in \mathcal{N} \triangleq \{1, \dots, N\},$$

- $K > 0$ : the uniform coupling strength,
- $L_i$ : chosen set points, where  $L_1 \leq 0$ ,  $L_i \geq 0$ ,  $i \in \mathcal{N} \setminus \{1\}$ .

## Existence of solutions

Equilibrium solutions of the system:

$$x_i(t) = \alpha t + \beta_i, \quad \forall i \in \mathcal{N},$$

with

$$\alpha = \frac{-K}{Np} \sum_{j=1}^N L_j \text{ and } \beta_i - \beta_{i-1} = \left( \frac{1}{N} \sum_{j=1}^N L_j \right) - L_i, \quad i \in \mathcal{N},$$

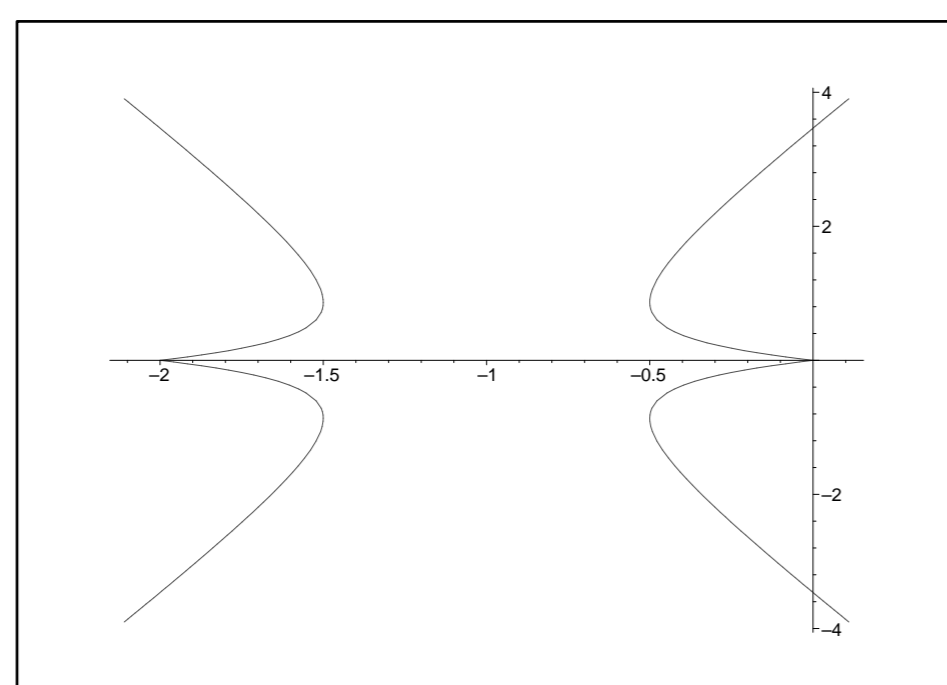
## Stability result

**Theorem 1** If and only if

$$K < \frac{p^2}{2 \cos^2(\pi/N)},$$

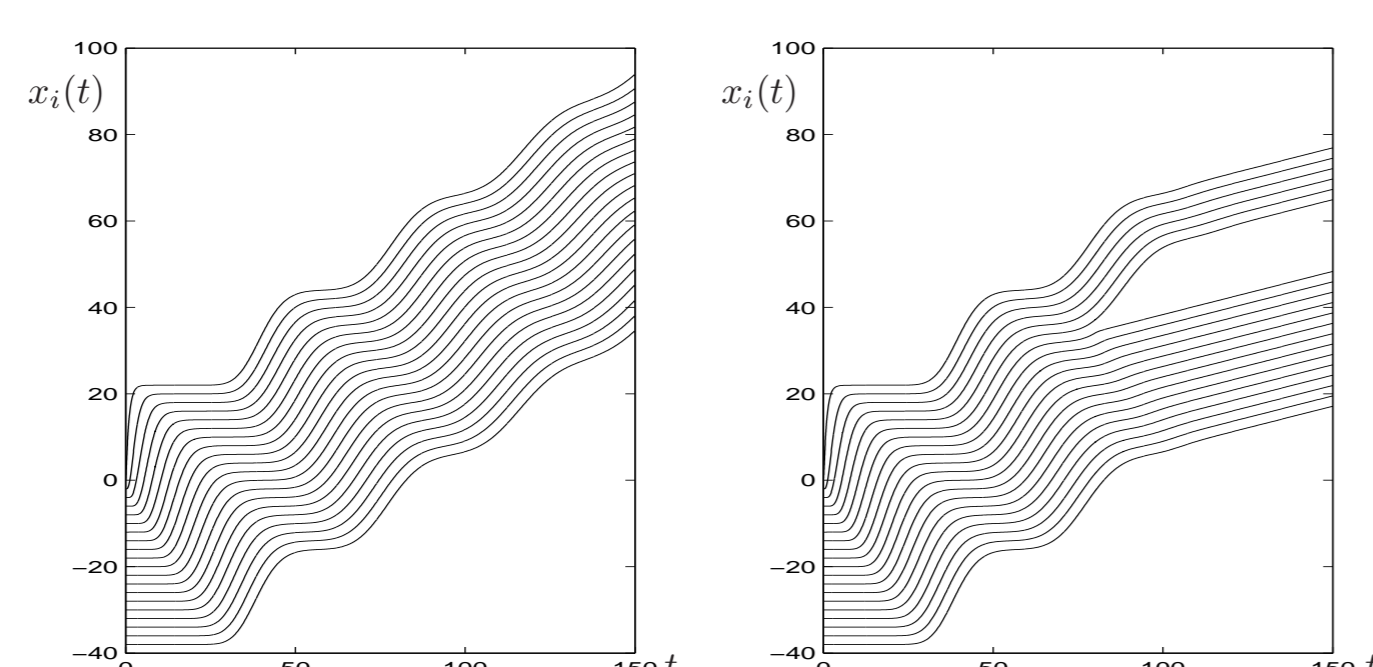
the above solution is **asymptotically stable**.

Example: Location of the eigenvalues for varying  $K$  ( $N = 3, p = 2$ ):



## Robustness

Evolution of the position for a platoon of 39 vehicles. Left: no malfunctions. Right: at  $t = 80$  s, the 12th vehicle starts malfunctioning and cannot drive faster than 0.3 m/s.



## String stability

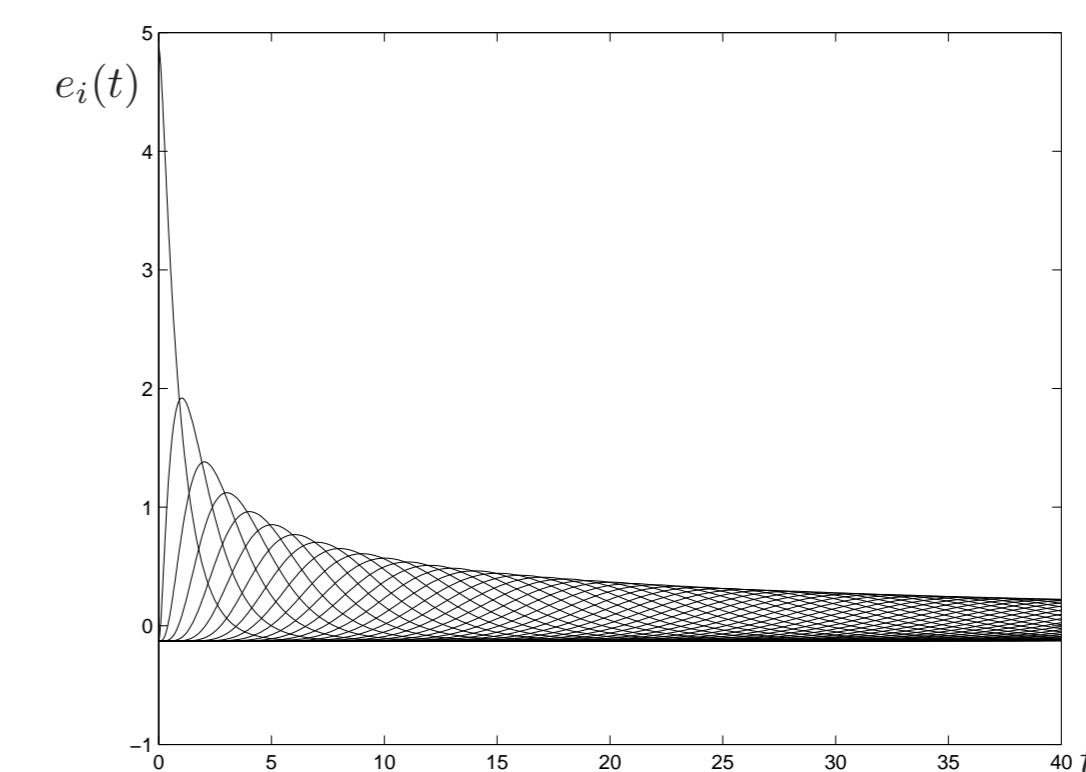
- Common problem in platoons: the slinky effect.
- Solution: Construct a controller that renders the platoon **string stable**:  
**Definition 1** Let  $e_i$  be the distance error between the  $i$ -th and  $(i-1)$ -th vehicle. The platoon is called **string stable** if

$$\|e_i(t)\|_\infty < \|e_{i-1}(t)\|_\infty, \quad \forall i > 1,$$

where  $\|e_i(t)\|_\infty$  denotes  $\sup_{t \geq 0} |e_i(t)|$ .

## String stability: simulation

- Assume the platoon is driving according to an equilibrium solution to set points  $L_i$
- Control: replace  $L_1$  by  $\tilde{L}_1$ . Keep the other set points constant. This defines the corresponding initial condition.
- Slowing down manoeuvre corresponding to  $L_1 - \tilde{L}_1 = -5$ :



## Adding integral control

The control is given by

$$u_1(t) = K(x_N(t) - x_1(t) - L_1) + q \int_0^t (x_N(\tau) - x_1(\tau) - L_1) d\tau, \quad \forall t \in \mathbb{R}^+,$$

$$u_i = K(x_{i-1} - x_i - L_i), \quad i \in \mathcal{N} \setminus \{1\},$$

Equilibrium solutions of the system:

$$x_0(t) = -\frac{\alpha p}{q},$$

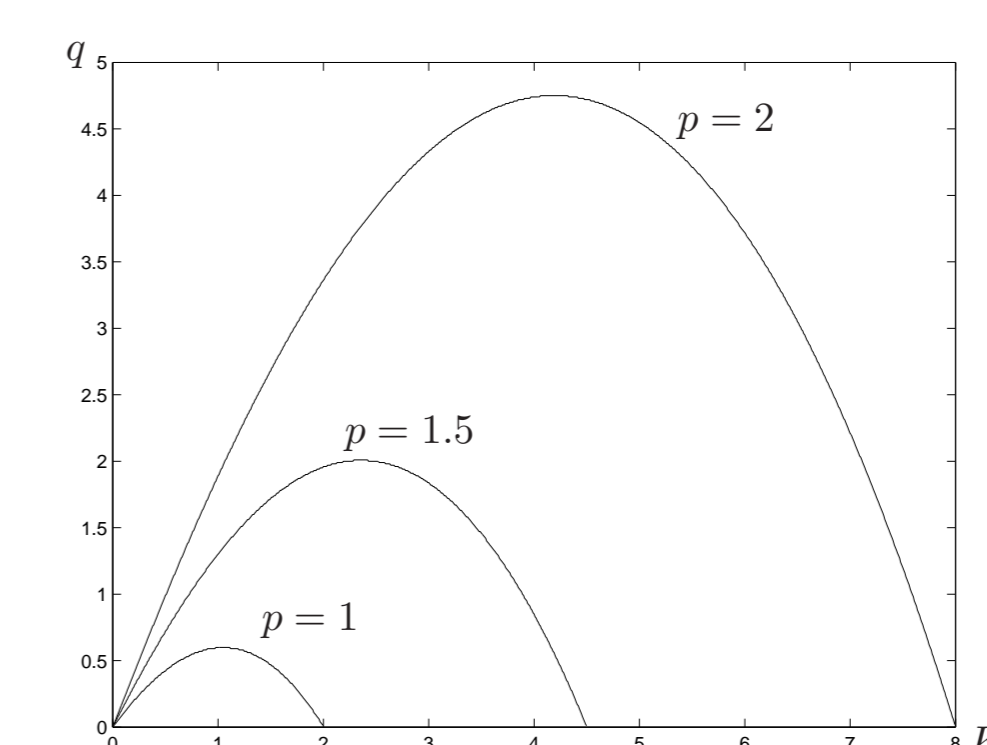
$$x_i(t) = \alpha t + \beta_i, \quad \forall i \in \mathcal{N},$$

with

$$\begin{cases} \alpha = \frac{-K}{(N-1)p} \sum_{j=1}^N L_j, \\ \beta_N - \beta_1 = L_1, \\ \beta_i - \beta_{i-1} = \left( \frac{1}{N-1} \sum_{j=1}^N L_j \right) - L_i, \quad i = 2, \dots, N. \end{cases}$$

## Stability

Stability regions for a 3-vehicle platoon at different values of the drag coefficient.



Stability region for a 7-vehicle platoon with drag coefficient  $p = 2$ .

