

# ATRIAL SIGNAL EXTRACTION IN ATRIAL FIBRILLATION ECGS EXPLOITING SPATIAL CONSTRAINTS

P. Bonizzi<sup>1</sup>, R. Phlypo<sup>3</sup>, V. Zarzoso<sup>1</sup>, O. Meste<sup>1</sup> and A. Fred<sup>2</sup>

<sup>1</sup>I3S - UNSA/CNRS, 2000 Route des Lucioles, Les Algorithmes Euclide B, B.P. 121, 06903 Sophia Antipolis Cedex, France

<sup>2</sup>Instituto Superior Técnico - Torre Norte, Av. Rovisco Pais, 1, 1049-001, Lisbon, Portugal

<sup>3</sup>MEDISIP - IBBT, Ghent University, De Pintelaan 185, 9000 Ghent, Belgium  
bonizzi@i3s.unice.fr

## ABSTRACT

*The accuracy in the extraction of the atrial activity (AA) from electrocardiogram (ECG) signals recorded during atrial fibrillation (AF) episodes plays an important role in the analysis and characterization of atrial arrhythmias. The present contribution puts forward a new method for AA signal automatic extraction based on a blind source separation (BSS) formulation that exploits spatial information about the AA during the T-Q segments. This prior knowledge is used to optimize the spectral content of the AA signal estimated by BSS on the full ECG recording. The comparative performance of the method is evaluated on real data recorded from AF sufferers. The AA extraction quality of the proposed technique is comparable to that of previous algorithms, but is achieved at a reduced cost and without manual selection of parameters.*

## 1. INTRODUCTION

Atrial Fibrillation (AF) represents the most common sustained cardiac arrhythmia in adults. It consists of a malfunction of the atrium characterized by a modification of the normal atrial activity (AA) pattern on the electrocardiogram (ECG) signal. Epidemiologic studies have shown that its prevalence and incidence doubles with each advancing decade beyond 50 years reaching 10% in people over 80 and has direct impact on mortality and morbidity [1, 2].

The accurate extraction of the AA signal from the ECG of AF is of great interest for subsequent analysis. For instance, when the behaviour of the atrioventricular node during AF is addressed, the precision in the relative amplitude of the AF estimated signal plays a critical role. A good estimate of the AA signal is also important for an accurate analysis of the temporal evolution of the spectral content of the AA signal. This analysis is justified by the evident correlation between the spontaneous termination of the episode and the decreasing trend of the AA signal main frequency [3].

It follows that the proper analysis and characterization of AF from ECG recordings requires the cancellation of the signal components associated with ventricular activity (VA), that is, the QRS-T complex. However, this is not a simple task. Indeed, a lot of facts hinder this operation. In particular, the much lower amplitude of the AA signal compared to the ventricular one and the spectral overlapping of the two phenomena, so that linear filtering solutions in the frequency domain are unsuccessful [4].

There exist in the literature two different families of methods applied to cancel out VA in the ECG. The first involves methods that aim for a direct suppression of the QRS-T complex, e.g., using an adaptive template in conjunction

with the correct spatio-temporal alignment of every QRS-T complex [5, 6]. The second involves all the methods based on the blind source separation (BSS) approach. All the methods belonging to the first class share similar limitations such as high sensitivity to QRS morphological changes over time and inability to eliminate artifacts other than VA. Moreover, a common limitation to these methods is their inability to exploit the global spatial diversity of an ECG recording.

Starting from the key observation that AA and VA are decoupled, a new interesting perspective has been introduced recently which does not rely on direct elimination of the QRS-T complex [4]. Under this assumptions, the AA extraction problem accepts a formulation based on BSS of instantaneous linear mixtures, in which atrial and ventricular source contributions appear mixed at the electrode outputs in the ECG. First hopeful results obtained in the separation of AA sources through a BSS method gave rise to the definition of more suitable methods exploiting a priori information inside the BSS model.

The method proposed by Castells *et al.* in [7] used one complete independent component analysis (ICA) of the observed signals, followed by a second-order blind identification (SOBI). SOBI exploits the time coherence of the source signals and relies on stationary second-order statistics by performing a joint diagonalization of a set of covariance matrices. A limitation of this method is the presence of two parameters that are to be manually defined. Indeed, sources given by ICA are selected in relation with their kurtosis value, the first parameter. Only sources that satisfy a particular threshold are kept and introduced in SOBI. Moreover, also suitable correlation matrices' time lags must be manually defined. Our method, inspired to that presented by Hesse and James in [8], uses a spatial constraint as an a priori information inside the model. The spatial constraint used is based on an initial estimation of the AA source direction or spatial topography from the T-Q segments. Differently from [8], we use this spatial constraint not directly inside a suitable ICA model, but after a conventional ICA. In conjunction with a spectral concentration criterion, this topography is employed to enhance the separation of AA from VA and other artifacts in the whole recording.

## 2. METHODS

### 2.1 Data and Preprocessing

A dataset composed of 22 recordings (all presenting AF) was employed to analyze the proposed idea. All signals were recorded and digitized at a sampling rate of 1KHz. Among the segments employed in this analysis 20 were recorded us-

ing a standard 12-lead system while 2 were recorded using a 9-lead system. Pre-processing was done by applying a zero-phase high pass filter with a cut off frequency of 0.5Hz to remove physiologically irrelevant low frequency signal variations (<1Hz) [9], while a notch filter was implemented to suppress power line noise at 50Hz, applying it in a forward-backward way to eliminate any phase jump [10].

## 2.2 Blind Source Separation

The BSS consists of recovering a set of source signals from the observation of linear mixtures of the sources. The term blind underlines that little is known about the source signals or the mixing structure, the only hypothesis being the sources' mutual independence [11, 12]. Under this hypothesis, BSS can be carried out by ICA, a technique used to transform multisensor signals into statistically independent components [11]. Mathematically, given  $N$  observations of  $n$  time series  $\mathbf{y}(t) \in \mathfrak{R}^n$ , the observed signals, it is possible to write them as a linear combination  $\mathbf{M} \in \mathfrak{R}^{n \times m}$  of the original sources  $\mathbf{s}(t) \in \mathfrak{R}^m$  ( $m \leq n$ ). BSS searches for this linear combination and the corresponding sources given the observations. In the noiseless case, the BSS model for an instantaneous linear mixtures is:

$$\mathbf{y}(t) = \mathbf{M}\mathbf{s}(t) \quad (1)$$

where the  $i$ th column of  $\mathbf{M}$  represents the spatial topography that links the  $i$ th source with the observed signals. ICA aims to estimate the sources  $\hat{\mathbf{s}}(t)$  and the separating matrix  $\hat{\mathbf{W}}$ :

$$\hat{\mathbf{s}}(t) = \hat{\mathbf{W}}\mathbf{y}(t) \quad (2)$$

with  $\hat{\mathbf{W}} \approx \mathbf{M}^\sharp$ , and where the  $\sharp$  operator stands for pseudo-inverse of the matrix.

Spatial whitening involves a linear transformation of the mean corrected observed signals  $\mathbf{y}(t)$ , which produces a set of uncorrelated waveforms with unit variance  $\mathbf{z}(t)$ :

$$\mathbf{z}(t) = \mathbf{V}\mathbf{y}(t) = \mathbf{V}\mathbf{M}\mathbf{s}(t) = \mathbf{H}\mathbf{s}(t) \quad (3)$$

The whitening matrix  $\mathbf{V}$  can be obtained from the singular value decomposition (SVD) of the observation matrix  $\mathbf{y}(t) = \mathbf{U}\mathbf{S}\mathbf{R}^T$ , and  $\mathbf{V} = \sqrt{\mathbf{N}}\mathbf{S}^{-1}\mathbf{U}^T$ . Since whitening identifies the independent components up to a rotation, the mixing matrix  $\mathbf{H} = \mathbf{V}\mathbf{M}$  for whitened data is orthonormal, i.e.  $\mathbf{H}^{-1} = \mathbf{H}^T$  with unit norm columns. Therefore, sources estimated from whitened data  $\hat{\mathbf{s}}(t) = \hat{\mathbf{H}}^T\hat{\mathbf{z}}(t)$  (with  $\hat{\mathbf{H}} \approx \mathbf{H}$  and  $\hat{\mathbf{z}}(t) = \hat{\mathbf{V}}\mathbf{y}(t)$ ) do not involve matrix inversion. This gives the possibility to apply the transpose of matrix  $\hat{\mathbf{H}}$  on  $\hat{\mathbf{z}}(t)$  directly, without further computations.

For the estimate of the a priori information that is used in the proposed method, a further model based only on the temporal segments in the observations free from any VA is needed. This model is obtained in the following way. Firstly, the set of ECG recordings under analysis (e.g. Fig. 1(a)) is taken and, after the QRS-T complexes detection, only the T-Q segments are isolated, so that  $\mathbf{y}_{AA}(t) = \{\mathbf{y}(t_i) \mid t_i \notin \text{QRS-T}\}$ . This new set of signals contains only AA and possible noise, but it is quite reasonable to suppose it free from any VA, confined in the QRS-T segment. Secondly, the BSS model for this new set is generated in two different ways, that is, applying either ICA or principal component analysis (PCA) (e.g., through SVD):

$$\mathbf{y}_{AA}(t) = \mathbf{M}_{AA}\mathbf{s}_{AA}(t) \quad (4)$$

$$\mathbf{y}_{AA}(t) = \mathbf{B}_{AA}\mathbf{z}_{AA}(t) \quad (5)$$

where  $\mathbf{B}_{AA}^\sharp$  is the whitening matrix and  $\mathbf{z}_{AA}(t)$  the set of decorrelated sources. In this way, two sets of independent (4) or simply decorrelated (5) sources respectively, formed by the components present in T-Q segments only, are obtained.

## 2.3 ICA and Spatial Constraint

In many BSS problems exploiting independence, one may only have particular interest in a component or a set of desired sources, and automatically discard the remainder of uninteresting signals or noise. To this end, ICA methods exploiting some a priori information as a referential constraint inside the problem have been presented in the literature. Both signal extraction and noise rejection essentially involve the estimation of a target source, in a more precise way than conventional ICA. Therefore, the achievement of a suitable constraint becomes a crucial task.

The observation that AA and VA are decoupled underlines the idea that their electrical vectors inside the heart should be different, and so their topographies. This naturally draws our attention to their spatial differences, rather than on their temporal ones. The importance of exploiting spatial diversity of an ECG recording is then clear. Therefore, a particular AA spatial constraint, as the AA spatial topography, can be used as a tool to get rid of the VA present in the ECG.

A spatial constraint can be defined either as an abstract prior knowledge (e.g., all the constraints defined on the mixing matrix structure, as orthogonality, orthonormality etc.) or in a more specific way. We use a specific spatial constraint, for each particular subject under analysis: the estimation of the AA spatial topography  $\hat{\mathbf{m}}_{AA}$ . As said before, this spatial constraint can be generated applying either ICA (4) or PCA (5) to the set  $\mathbf{y}_{AA}(t)$ . When the spatial constraint is constructed using ICA, as in the model described in (4), the second step is to search for the best AA source that describes the AF,  $\hat{s}_{AA}$ , inside the set of the estimated output sources  $\hat{\mathbf{s}}_{AA}(t)$ . The criterion used for selecting the best AA source is Spectral Concentration (SC) of the AA around its main peak, computed according to the following expression [7]:

$$SC = \frac{\int_{0.82f_c}^{1.17f_c} P_{AA}(f) df}{\int_0^{f_s/2} P_{AA}(f) df} \quad (6)$$

The above equation is a measure for the compactness of the spectrum around the central frequency  $f_c$ , that is the modal frequency in the 3-12Hz interval.  $P_{AA}$  is the power spectrum of the AA signal,  $f_s/2$  is the half of the sampling frequency [7]. The column of the estimated mixing matrix  $\hat{\mathbf{M}}_{AA}$  associated to the selected source is the topography of interest and defines the spatial constraint  $\hat{\mathbf{m}}_{AA}$ .

Alternatively, when the spatial constraint is constructed using PCA, as in the model described in (5), we can take as reference topography  $\hat{\mathbf{m}}_{AA}$  the first column of the estimated matrix  $\hat{\mathbf{B}}_{AA}$ , that is the column associated with the decorrelated source with the highest energy. This is because AA is expected to be the component contributing to  $\mathbf{y}_{AA}(t)$  with the highest variance. Moreover, it is possible to distinguish between spatial constraints of different severity, namely *hard* and *soft* spatial constraints, according to the uncertainty about their constraint topographies.

## 2.4 Hard constraints

If the degree of certainty about a particular spatial constraint topography  $\hat{\mathbf{m}}_{AA}$  is quite high, it is possible to use it as a *hard* constraint. Indeed, in this case  $\hat{\mathbf{m}}_{AA}$  can be used to define the weight vector of a spatial filter applied to the whitened set  $\mathbf{z}(t)$  of observed signals. The spatial filter applies the AA signal topography on the prewhitened waveforms, for VA removal, as follows:

$$\hat{s}_{AA}(t) = \hat{\mathbf{h}}_{AA}^T \mathbf{z}(t) \quad (7)$$

where  $\hat{s}_{AA}(t)$  is the output of the filter, that is, the estimated AF signal, and  $\hat{\mathbf{h}}_{AA}$  is obtained by transformation of  $\hat{\mathbf{m}}_{AA}$  as  $\hat{\mathbf{h}}_{AA} = \mathbf{V}\hat{\mathbf{m}}_{AA}$ . This transformation allows the projection of  $\hat{\mathbf{m}}_{AA}$  on the whitened signal subspace insuring the appropriate use of the spatial constraint on the full recording.

## 2.5 Soft constraints

If the degree of uncertainty about the spatial constraint topography  $\hat{\mathbf{m}}_{AA}$  is not negligible, it is better to introduce a *soft* constraint. With respect to other methods (e.g., [8]), we use this kind of a priori information on AF not directly inside a suitable ICA algorithm, but after a conventional ICA. First, AA spatial topography  $\hat{\mathbf{m}}_{AA}$  is obtained, as explained in Section 2.3, and a conventional ICA is applied to the observed signals  $\mathbf{y}(t)$ , obtaining the set of independent sources  $\hat{\mathbf{s}}(t)$  related to them, according to the model introduced in (1)-(3). Secondly, the best source that describes the AF, inside the set of output sources  $\hat{\mathbf{s}}(t)$  is searched. The criterion used for selecting the best AF source is the SC of the AA source around its main peak [7]. We denote  $\mathbf{m}$  the column of the mixing matrix associated with the selected source. Once we have obtained both the topography of the reference  $\hat{\mathbf{m}}_{AA}$  and that of the source of interest  $\hat{\mathbf{m}}$ , we search for the topography  $\hat{\mathbf{h}}'_{\text{opt}}$  maximizing the SC in the plane defined by the two whitened vectors  $\hat{\mathbf{h}}_{AA}$  and  $\hat{\mathbf{h}} = \mathbf{V}\hat{\mathbf{m}}$ . An orthonormal basis of that plane can be defined as:

$$\mathbf{e}_1 = \hat{\mathbf{h}}_{AA} \quad (8)$$

$$\mathbf{e}_2 = \frac{\hat{\mathbf{h}} - \text{proj}_{\hat{\mathbf{h}}_{AA}} \hat{\mathbf{h}}}{\|\hat{\mathbf{h}} - \text{proj}_{\hat{\mathbf{h}}_{AA}} \hat{\mathbf{h}}\|} \quad (9)$$

where notation  $\text{proj}_c \mathbf{d}$  stands for the projection of vector  $\mathbf{d}$  on vector  $\mathbf{c}$ . Accordingly,

$$\hat{\mathbf{h}}'_\alpha = \mathbf{e}_1 \cos(\alpha) + \mathbf{e}_2 \sin(\alpha) \quad (10)$$

$$\hat{s}'_\alpha(t) = \hat{\mathbf{h}}'^T_\alpha \hat{\mathbf{z}}(t) \quad (11)$$

where  $\hat{\mathbf{h}}'_\alpha$  and  $\hat{s}'_\alpha(t)$  represent respectively the generic spatial topography and the generic source, defined in the aforementioned plane, to be optimized. The source estimate associated with the largest SC value,  $\hat{s}'_{\text{opt}}(t) = \hat{s}'_{\alpha_{\text{opt}}}(t)$ , is taken as the best estimation of the AA source  $\hat{s}_{AA}(t)$ , its corresponding topography being  $\hat{\mathbf{h}}'_{\text{opt}} = \hat{\mathbf{h}}'_{\alpha_{\text{opt}}}$ , where

$$\alpha_{\text{opt}} = \arg \max_{\alpha} \text{SC}(\hat{s}'_\alpha(t)) \quad (12)$$

The above SC optimization can be carried out algebraically at very little computational cost.

	$\mu_{SC} \pm \sigma_{SC} (\%)$	$\mu_k \pm \sigma_k (n.u.)$	$\mu_{f_c} \pm \sigma_{f_c} (Hz)$
COM2	52.00 ± 14.69	-0.0951 ± 0.5587	5.5154 ± 1.29
SCICA <sup>hard</sup> <sub>ICA</sub>	46.56 ± 18.76	0.0519 ± 0.589	5.421 ± 1.2678
SCICA <sup>hard</sup> <sub>PCA</sub>	36.09 ± 22.618	1.0329 ± 1.8245	4.6442 ± 1.2348
SCICA <sup>soft</sup> <sub>ICA</sub>	58.39 ± 10.57	-0.2085 ± 0.4403	5.5154 ± 1.2656
SCICA <sup>soft</sup> <sub>PCA</sub>	58.01 ± 12.14	-0.1717 ± 0.526	5.5209 ± 1.2736
SOBI	60.82 ± 9.21	-0.1391 ± 0.4967	5.3711 ± 1.3255
ST-Canc	57.01 ± 11.98	0.5511 ± 2.8898	5.4321 ± 1.2159

Table 1: Mean performance estimates of Spectral Concentration (SC), kurtosis (k) and characteristic frequency ( $f_c$ ) for the different methods under analysis.

## 3. RESULTS

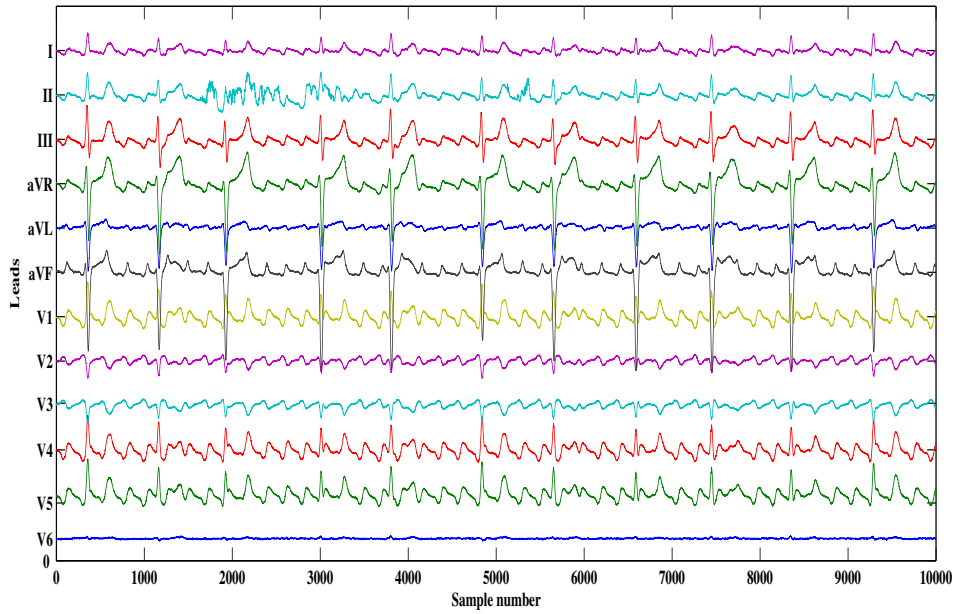
Since methods that exploit spatial constraints, even if in different ways from how presented here, already exist in the literature, the proposed method is named Spatial Constrained ICA (SCICA), choosing the same name of that proposed by Hesse and James [8]. This method for the automatic extraction of the AF from a set of observed ECG signals was applied to a dataset of 22 patients. Its performance is evaluated both for the PCA- and the ICA-defined constraints, and they are compared to those of some classical ones, among which, a conventional ICA (COM2) [11], a spatio-temporal cancellation approach (ST-Canc) [6] and a spatio-temporal BSS approach (SOBI) [7]. For completeness, the performance of the presented method, using either *hard* or *soft* constraints, was evaluated in terms of SC of the AA estimated source around its main peak, characteristic AF frequency value at the peak, and excess kurtosis of the estimated source. Mean value  $\mu$  and standard deviation  $\sigma$  of each index are presented for each method.

Results are reported in Table 1, while an example of final estimation of the AF source  $\hat{s}_{AA}(t)$  for different methods is shown in Fig. 1(b), when *soft* constraints are used for SCICA. Finally, Fig. 2 shows the box and whisker plot of the SC parameter only, for SCICA with *soft* constraints (both ICA and PCA defined), and for the other methods.

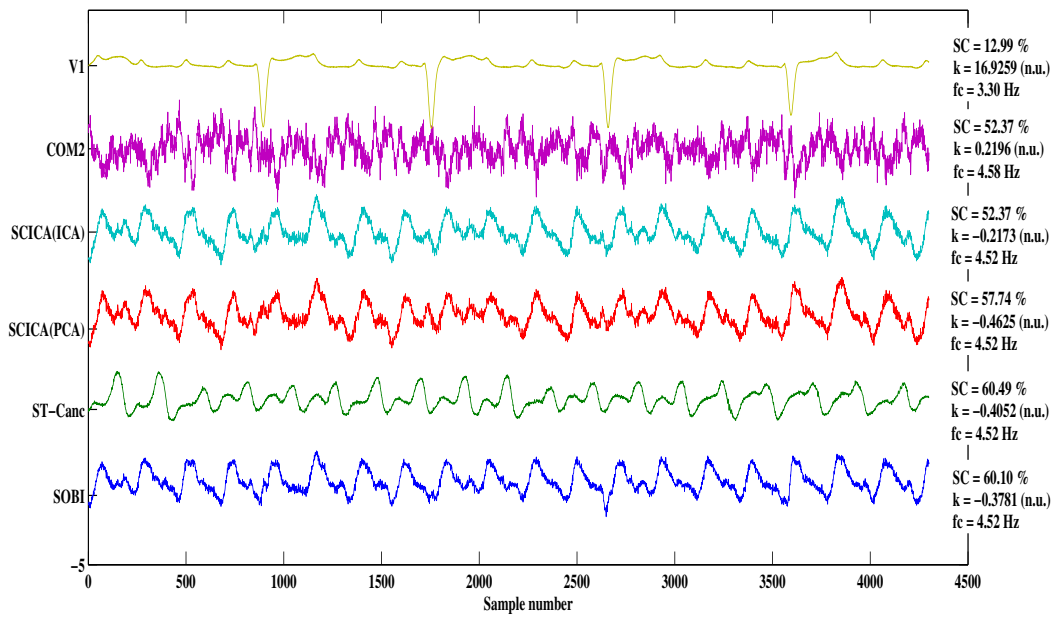
From the values of the performance indexes (Table 1), we note that SCICA shows good performances when *soft* constraints are used, obtained either using ICA or PCA, as in Section 2.3, according to the SC values found for the different methods. Whereas, performance is not so good when *hard* constraints are employed. Indeed, the variance of the AA set of sources estimated using *hard* constraints is quite high, attesting the strong correlation between the quality of the estimated source and the appropriateness of the chosen constraint.

## 4. DISCUSSION

This work has pointed out two important issues. Firstly, the importance of using a suitable a priori information in combination with the ICA algorithm when the extraction of the AF signal is addressed. Secondly, the use of SC as a preferential parameter in the search for the optimal AF signal estimate. The idea that AA and VA electrical vectors are spatially different supports the idea of using AF spatial topography esti-



(a)



(b)

Figure 1: (a) Example of a 12-leads ECG recording. Signals in the figure are 10s long; leads, specific ECG leads. (b) V1 ECG lead and AA signals estimated using different methods. For SCICA only *soft* constraints are used. Values of Spectral Concentration(SC), kurtosis (k) and characteristic frequency ( $f_c$ ) are presented for each signal. A 4.5s segment is represented for the bottom figure.

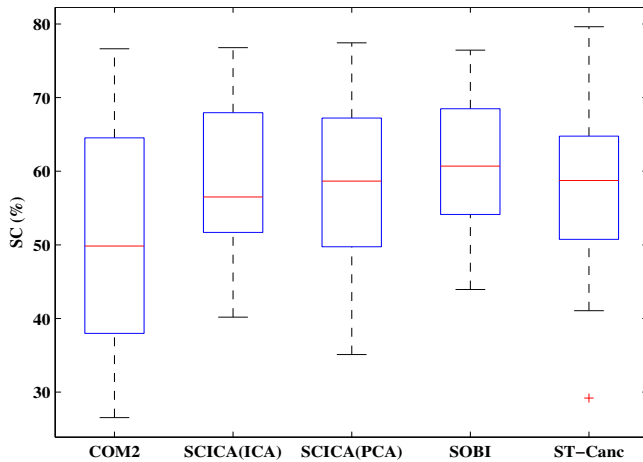


Figure 2: Box and whisker plot of the Spectral Concentration (SC) values for different methods. The box has lines at the lower quartile, median, and upper quartile values. The whiskers are lines extending from each end of the box to show the extent of the rest of the data. Outliers are data with values beyond the ends of the whiskers, and are represented as crosses; (%) percentage.

mate as spatial constraint.

The use of SC not simply as a performance parameter, but as an optimization criterion inside the AF signal extraction model seems to improve the AA estimation quality. This can be noted by looking at the ability of SCICA to get almost the same performance as other methods suitable for the extraction of the AF, but simply exploiting the statistical independence between AA and VA, and the optimization of SC.

An important result is the capability of getting similar performance for the proposed method when either SVD-defined or ICA-defined constraints are used in a *soft* way. This gives us the possibility to focus the attention mainly on their construction using SVD, with benefits in terms of complexity of the algorithm.

Finally, low performance values obtained when *hard* constraints are employed reveal it is inappropriate to apply the AA topography estimate directly on the observation set. To use it as *soft* constraint as a part of an optimization criterion seems to be a more appropriate option, as shown by the results.

## 5. CONCLUSION

A new fully automated method for the extraction of AA signals in ECG recordings of AF has been presented. The method is based on an initial estimation of the AA source direction or spatial topography from the T-Q segments. In conjunction with a spectral concentration criterion, this topography is employed to enhance the separation of AA from VA and other artifacts in the whole recording. Results show that the proposed methodology constitutes a cost-effective alternative to previous BSS-based methods. Indeed, a spatial reference computed from the PCA of the T-Q segments achieves a satisfactory performance while preventing the manual selection of parameters (e.g., kurtosis threshold or autocorrelation time lags).

Future works aim to exploit simultaneously reference AA topographies related not only to the AA source with the highest SC but also to other candidate AA sources in cases where more than one AA source may be present during an AF episode. A new definition of SC capable of describing the information contained in the harmonics of the characteristic frequency could allow a more efficient exploitation of this parameter.

## Acknowledgements

The authors would like to express their gratitude to Leif Sörnmo and Francisco Castells for providing the real data. The work of Bonizzi Pietro is supported by the EU by a Marie-Curie Fellowship (EST-SIGNAL program : <http://est-signal.i3s.unice.fr>) under contract No MEST-CT-2005-021175.

## REFERENCES

- [1] W. K. Kannel, R. D. Abbott, D. D. Savage, and P. M. McNamara. Epidemiologic features of chronic atrial fibrillation: the Framingham study. *N Engl J Med*, 306:1018–22, 1982.
- [2] A. D. Krahn, J. Manfreda, R. B. Tate, F. A. Mathewson, and T. E. Cuddy. The natural history of atrial fibrillation: incidence, risk factors, and prognosis in the Manitoba Follow-Up Study. *Am J Med*, 98:476–84, 1995.
- [3] Petruțiuț, S. and Sahakian, V. and Ng, J. and Swiryn, S. Analysis of the surface electrocardiogram to predict termination of atrial fibrillation: The 2004 computers in cardiology/physionet challenge. *Proc. Computers in Cardiology*, 31:105–108, 2004.
- [4] J. J. Rieta, F. Castells, C. Sánchez, V. Zarzoso, and J. Millet. Atrial activity extraction for atrial fibrillation analysis using blind source separation. *IEEE Trans on Biomed Eng*, 51, No. 7:1176–86, July 2004.
- [5] L. Sörnmo M. Stridh. Spatiotemporal QRST cancellation techniques for analysis of atrial fibrillation. *IEEE Trans. Biomed. Eng.*, 48:105–111, January 2001.
- [6] O. Meste and N. Serfaty. QRST cancellation using bayesian estimation for the auricular fibrillation analysis. In *Engineering in Medicine and Biology*, 2005.
- [7] F. Castells, J. J. Rieta, J. Millet, and V. Zarzoso. Spatiotemporal blind source separation approach to atrial activity estimation in atrial tachyarrhythmias. *IEEE Transactions on Biomedical Engineering*, 52(2):258–267, February 2005.
- [8] C. W. Hesse and C. J. James. The FastICA algorithm with spatial constraints. *IEEE Signal Process. Lett.*, 12:792–795, 2005.
- [9] Leif Sörnmo and Pablo Laguna. *Bioelectrical Signal Processing in Cardiac and Neurological Applications*. Elsevier Academic Press, 2005.
- [10] S. K. Mitra. *Digital Signal Processing: A computer-based approach*, 2nd ed. McGraw-Hill, 2001.
- [11] P. Comon. Independent component analysis: a new concept? *Signal Processing*, 36:287–314, 1994.
- [12] Hyvriinen, A. and Karhunen, J. and Oja, E. *Independent Component Analysis*. Wiley, 2001.