

A NOVEL APPLICATION OF RELAY FEEDBACK FOR PID AUTO-TUNING

Ioan Nascu⁽¹⁾, Robin De Keyser⁽²⁾

⁽¹⁾ *Technical University of Cluj Napoca, Department of Automation
Baritiu 26, Cluj Napoca, 3400, Romania
E-mail: Ioan.Nascu@aut.utcluj.ro*

⁽²⁾ *Ghent University, EeSA-department of Electrical Energy, Systems & Automation
Technologiepark 913, 9052 Gent, Belgium
E-mail: rdk@autoctrl.rug.ac.be*

Abstract: This paper presents a novel application of the widely used relay-feedback PID-autotuner for stable SISO processes. The proposed method consists of two steps: process identification and controller design. First, a non-iterative procedure is suggested for identification of two points on the process Nyquist curve. A second-order-plus-dead-time model is obtained and then used for PID controller design based on the internal model principle (IMC). For the identification of the two points on the Nyquist curve a pure relay in the feedback loop (as used in standard auto-tuning) and a relay which operates on the integral of the error are used. *Copyright © 2003 IFAC*

Keywords: auto-tuning, PID controller, relay experiment, Nyquist plot .

1. INTRODUCTION

Despite the development of more advanced control strategies, the majority of controllers used in industrial instrumentation are still of the PID type. Their popularity is easy to understand: they have a simple structure, their principle is well understood by instrumentation engineers and their control capabilities have proven to be adequate for most control loops. Moreover, due to process uncertainties, a more sophisticated control scheme is not necessarily more efficient in real-life applications than a well-tuned PID controller. However, it is common that PID controllers are often poorly tuned in practice because the choice of controller parameters requires professional knowledge by the user. To simplify this task and to reduce the time required for it, PID controllers can incorporate *auto-tuning* capabilities, i.e. they are equipped with a mechanism capable of computing the ‘correct’ parameters automatically when the regulator is connected to the field (Aström, *et al.*, 1984; Aström, *et al.*, 1992; Aström, *et al.*, 1995; Leva, *et al.*, 2002). Auto-tuning is a very desirable feature and almost

every industrial PID controller provides it nowadays. These features provide easy-to-use controller tuning and have proven to be well accepted among process engineers.

For the automatic tuning of the PID controllers, several methods have been proposed. Some of these methods are based on identification of one point of the process frequency response, while the others are based on the knowledge of some characteristic parameters of the open-loop process step response. The identification of a point of the process frequency response can be performed either using a proportional regulator, which brings the closed-loop system to the stability boundary, or by a relay forcing the process output to oscillate. Åström and Hägglund (Åström, *et al.*, 1984) report an important and interesting approach. Their method is based on the Ziegler-Nichols frequency domain design formula. A relay connected in a feedback loop with the process is used in order to determine the critical point.

This paper describes the development of an auto-tuning method based on the identification of *two*

points on the process Nyquist curve: the intersection with the real negative axis and that with the imaginary negative axis. The use of two relay feedback experiments provides information to determine a four parameters second-order-plus-dead-time (SOPDT) model.

2. THE EXPERIMENTAL SET-UP

A standard PID control system with single input single output, as shown in figure 1, is considered.

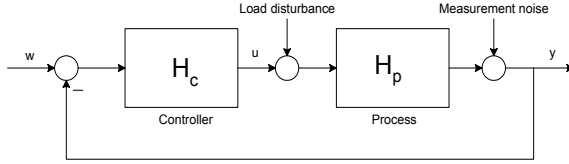


Figure 1. Control system structure

The process H_p is assumed to be linear, stable and proper. The PID controller has a non-interacting structure cascaded with a first order filter:

$$H_c(s) = K_c \left(1 + \frac{1}{sT_i} + sT_d \right) \frac{1}{T_f s + 1} \quad (1)$$

For the purpose of PID-controller design, higher order models are of limited utility even if the process dynamics are theoretically of high order. From the control point of view, complex process models lead to complex controllers. Several papers on PID control are based on the idea of using second order with time delay models. In this paper it is assumed that the process dynamics can be described with reasonable accuracy by a second-order-plus-dead-time (SOPDT) model as:

$$H_p(s) = \frac{k_p e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}, \quad \theta \geq 0 \text{ and } \tau, \zeta > 0 \quad (2)$$

In order to tune the PID controller we will approximate $e^{-\theta s}$ by $(1-\theta s)$ and using the well-known IMC-PID design method we then obtain:

$$K_c = \frac{2\zeta\tau}{k_p(2T_c + \theta)} \quad (3)$$

$$T_i = 2\zeta\tau, T_d = \frac{\tau^2}{2\zeta\tau}, T_f = \frac{T_c^2}{2T_c + \theta}$$

thus resulting in the closed-loop transfer function:

$$H_0(s) = \frac{H_p(s) \cdot H_c(s)}{1 + H_p(s) \cdot H_c(s)} = \frac{e^{-\theta s}}{(T_c s + 1)^2} \quad (4)$$

The parameter T_c is the closed-loop time constant and is a design parameter which can be selected by the user in order to tune the controller 'aggressiveness'.

The parameters of the process transfer function (2) are assumed to be unknown and - in order to identify them - 2 points of the frequency response have to be estimated using two relay feedback experiments: the first experiment uses a relay with hysteresis or an integrating relay (= a relay without hysteresis which has as input the integral of the error) and the second experiment uses a pure relay (without hysteresis). After performing these two experiments the values of the process frequency response at two different frequencies ω_1 and ω_2 are obtained:

$$H_p(j\omega_1) = a_1 + jb_1 \text{ and } H_p(j\omega_2) = a_2 + jb_2 \quad (5)$$

The procedure how to perform these experiments and how to obtain the values for $(a_i, b_i; i=1,2)$ is further explained in section 3.

3. THE TUNING METHOD

Note from (2) that $H_p(j\omega)$ can be written as follows:

$$H_p(j\omega) = H'_p(j\omega)H''_p(j\omega), \quad (6)$$

with

$$H'_p(j\omega) = \frac{k_p}{1 - \tau^2 \omega^2 + 2\zeta\tau j\omega}, \quad H''_p(j\omega) = (1 - \theta j\omega)$$

If we define - for a certain point on the Nyquist plot - the magnitude, the phase angle and the real and imaginary components of $H_p(j\omega)$, $H'_p(j\omega)$ and $H''_p(j\omega)$ to be M, α, a, b , respectively M', α', a', b' and M'', α'', a'', b'' , then (Ogata, 1995):

$$M^2 = a^2 + b^2 = M'^2 M''^2 = (a'^2 + b'^2)(a''^2 + b''^2) \quad (7)$$

$$\alpha = \alpha' + \alpha''$$

$$\begin{aligned} \operatorname{tg}(\alpha') &= \frac{b'}{a'} = \operatorname{tg}(\alpha - \alpha'') = \frac{\operatorname{tg}(\alpha) - \operatorname{tg}(\alpha'')}{1 + \operatorname{tg}(\alpha)\operatorname{tg}(\alpha'')} \\ &= \frac{\frac{b}{a} - \frac{b''}{a''}}{1 + \frac{b}{a} \frac{b''}{a''}} = \frac{a''b - ab''}{aa'' + bb''} \end{aligned} \quad (8)$$

$$b' = a' \frac{a''b - ab''}{aa'' + bb''} \quad a' = b' \frac{aa'' + bb''}{a''b - ab''} \quad (9)$$

From (9) - and afterwards (7) - it can be written:

$$\begin{aligned} (a'^2 + b'^2) &= a'^2 \left[1 + \left(\frac{a''b - ab''}{aa'' + bb''} \right)^2 \right] = \\ &= a'^2 \frac{(a''^2 + b''^2)(a^2 + b^2)}{(aa'' + bb'')^2} = a'^2 \frac{(a''^2 + b''^2)^2 (a'^2 + b'^2)}{(aa'' + bb'')^2} \end{aligned}$$

thus leading to:

$$aa'' + bb'' = a'(a''^2 + b''^2) = a'M''^2$$

and using (9) again also to:

$$a''b - ab'' = b'M''^2$$

Thus for the two points obtained from the relay feedback experiments we have:

$$a_1 a_1'' + b_1 b_1'' = a_1' M_1''^2 \quad (10a)$$

$$a_1'' b_1 - a_1 b_1'' = b_1' M_1''^2$$

$$a_2 a_2'' + b_2 b_2'' = a_2' M_2''^2 \quad (10b)$$

$$a_2'' b_2 - a_2 b_2'' = b_2' M_2''^2$$

The parameters a_i and b_i ($i=1, 2$) can be obtained using the two relay feedback experiments. In order to identify a point on the process Nyquist curve, a relay connected in a feedback loop with the process is used, forcing the process output to oscillate. Therefore assume that d_i is the relay amplitude and ε_i is the relay hysteresis width. For the given values d_i and ε_i we will obtain oscillation with amplitude h_i and period T_i in the process output.

In the first experiment there are two possibilities: using a relay with hysteresis, a point in the third quadrant on the process Nyquist curve, $P_h(a_1, j b_1)$, is identified (Aström, *et al.*, 1995):

$$\omega_1 = \frac{2\pi}{T_1}, \quad a_1 = \frac{-\pi\sqrt{h_1^2 - \varepsilon_1^2}}{4d_1}, \quad (11a)$$

$$b_1 = -\frac{\pi\varepsilon_1}{4d_1}, \quad M_1''^2 = a_1^2 + b_1^2$$

or, using an integrating relay ($\varepsilon_i=0$) the point given by the intersection of the process Nyquist curve and the negative imaginary axis, $P_i(a_1, j b_1)$, is identified:

$$\omega_1 = \frac{2\pi}{T_1}, \quad a_1 = 0, \quad b_1 = -\frac{\pi h_1}{4d_1}, \quad M_1''^2 = b_1^2 \quad (11b)$$

In the second experiment, using a pure relay ($\varepsilon_2=0$), the point given by the intersection of the process Nyquist curve and the negative real axis, $P_p(a_2, j b_2)$, is identified:

$$\omega_2 = \frac{2\pi}{T_2}, \quad a_2 = \frac{-\pi h_2}{4d_2}, \quad b_2 = 0, \quad M_2''^2 = a_2^2 \quad (12)$$

The real and imaginary components of $H_p'(j\omega_i)$ and $H_p''(j\omega_i)$ can be written as:

$$a_i' = \text{Re}(H_p'(j\omega_i)) = \frac{k_p(1 - \tau^2\omega_i^2)}{(1 - \tau^2\omega_i^2)^2 + (2\zeta\tau\omega_i)^2} = \frac{(1 - \tau^2\omega_i^2) M_i''^2}{k_p}$$

$$b_i' = \text{Im}(H_p'(j\omega_i)) = \frac{k_p(-2\zeta\tau\omega_i)}{(1 - \tau^2\omega_i^2)^2 + (2\zeta\tau\omega_i)^2} = \frac{-2\zeta\tau\omega_i M_i''^2}{k_p}$$

$$a_i'' = \text{Re}(H_p''(j\omega_i)) = 1 \quad b_i'' = \text{Im}(H_p''(j\omega_i)) = -\theta\omega_i \quad (13)$$

Inserting in (10) the relations (13) for a_i', b_i', a_i'', b_i'' ; $i=1, 2$ gives 4 equations with 4 unknown parameters: k_p, θ, τ^2 and $2\zeta\tau$:

$$a_1 - b_1\theta\omega_1 = \frac{(1 - \tau^2\omega_1^2) M_1''^2}{k_p} \quad (14.a)$$

$$b_1 + a_1\theta\omega_1 = \frac{-2\zeta\tau\omega_1 M_1''^2}{k_p} \quad (14.b)$$

$$a_2 - b_2\theta\omega_2 = \frac{(1 - \tau^2\omega_2^2) M_2''^2}{k_p} \quad (14.c)$$

$$b_2 + a_2\theta\omega_2 = \frac{-2\zeta\tau\omega_2 M_2''^2}{k_p} \quad (14.d)$$

Using in the first experiment a relay with hysteresis and in the second experiment a pure relay (the 'HR' case) then the first identified point on the process Nyquist curve is in the third quadrant and the second one is the intersection with the negative real axis (thus $b_2=0$). From (14.b) and (14.d) results:

$$\theta = \frac{a_2 b_1}{\omega_1 (M_1''^2 - a_2 a_1)} \quad (15.a)$$

From (14.c) and (14.a) results:

$$k_p = \frac{M_1''^2 M_2''^2 (\omega_2^2 - \omega_1^2)}{\omega_2^2 M_2''^2 (a_1 - b_1\theta\omega_1) - a_2 \omega_1^2 M_1''^2} \quad (15.b)$$

Now, (14.a) and (14.b) give:

$$\tau^2 = \frac{1}{\omega_1^2} + \frac{k_p (b_1\theta\omega_1 - a_1)}{\omega_1^2 M_1''^2} \quad (15.c)$$

$$2\zeta\tau = -\frac{k_p (b_1 + a_1\theta\omega_1)}{\omega_1 M_1''^2} \quad (15.d)$$

These values can then be used in (3) to calculate the PID-parameters.

Using in the first experiment an integrating relay and in the second experiment a pure relay (the 'IR' case) then the first identified point on the process Nyquist curve is the intersection with the negative imaginary axis (thus $a_1=0$) and the second one is the intersection with the negative real axis (thus $b_2=0$). In this case we obtain the following parameters which can then be used in (3) to tune the PID:

$$\theta = \frac{a_2}{\omega_1 b_1}$$

$$k_p = \frac{a_2 b_1^2 (\omega_1^2 - \omega_2^2)}{b_1 \omega_1 (a_2 \theta \omega_2^2 + b_1 \omega_1)} \quad (16)$$

$$\tau^2 = \frac{1}{\omega_1^2} \left(1 + \frac{k_p a_2}{b_1^2} \right)$$

$$2\zeta\tau = -\frac{k_p}{\omega_1 b_1}$$

On the other hand, to perform relay feedback experiments, the process is first brought to steady-state conditions in manual control or with any preliminary tuned PI controller. Measuring steady-state values u_0 and y_0 of the controller and process output, giving then a small perturbation to the control variable and measuring its effect on the process output, the process gain k_p can be easily determined instead of using (15.b) or (16.b). For processes with small nonlinearities better results are then obtained. For processes of first or second order without time delay, the value of θ in the model (2) will be zero. Strictly applying theory, these processes cannot be forced to oscillate by a pure relay. A relay without hysteresis can be used only if the process Nyquist curve crosses the negative real axis. In fact, considering that in any digital controller implementation the sampling process itself introduces a phase lag and that in real situations the process output is filtered, it can be assumed that all the processes in practical cases will oscillate when a relay controller is connected. Unfortunately, in absence of hysteresis, the relay experiment gives small values for the amplitude of oscillation, difficult to measure in the presence of measurement noise. Thus, in case of small amplitude oscillations, in the pure relay experiment a_2 and θ are fixed to zero, k_p is computed from steady-state conditions and τ^2 and $2\zeta\tau$ are computed using measurements only from the first experiment (a_1, b_1, ω_1).

If there are no others restrictions, the amplitude of the relay will be set to $0.1u_0$ and the hysteresis to $0.01y_0$. Better results can be obtained using IR method.

4. EXAMPLES

First the proposed identification method has been tested. An example of the identification with both methods (HR and IR) is shown in Figure 2. The real process transfer function is given by:

$$H_p(s) = \frac{2e^{-50s}}{(80s+1)^3} \quad (17)$$

We identified a second-order-plus-dead-time (SOPDT) model (equation 2). The estimated parameter values are presented in table 1. In this example better results are obtained using an integrating relay (IR case) instead of a relay with

hysteresis (HR case). Also, the computations are simpler in the IR case. In both cases better results are obtained determining k_p from steady-state conditions instead of using (15.b) or (16.b). In the IR case (Fig. 2.a) the real process Nyquist plot (solid line) is not too separate from the Nyquist plot of the identified model using steady-state conditions for k_p (dashdot line)

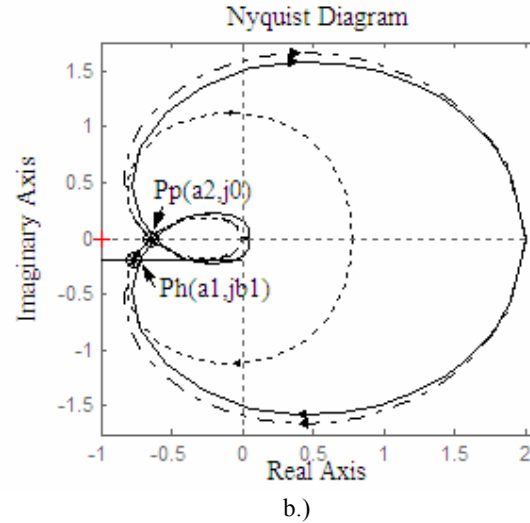
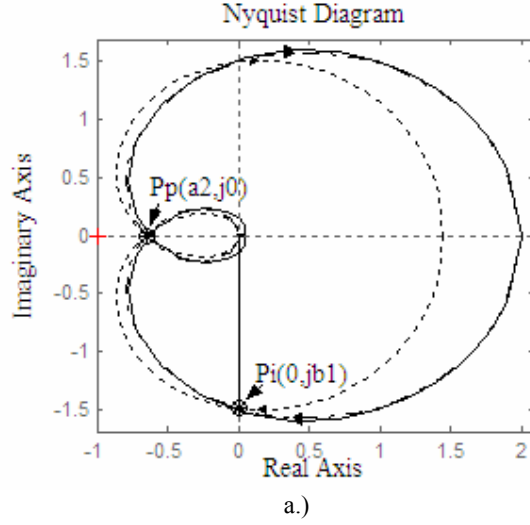


Figure 2. Nyquist plot of the process given by the transfer function (17) (solid line) and of the process model (2) identified using the two relay experiments IR/HR, with k_p computed from steady-state conditions (dashdot line) respectively from relay experiment (dotted line); a) IR case , b) HR case

Table 1

k_p determination	Method	k_p	θ	$2\zeta\tau$	τ^2
steady-state conditions	HR	2	84	257	20225
	IR	2	78	239	12878
relay-experiment measurements	HR	0.77	84	99	18012
	IR	1.45	78	173	12217

The proposed auto-tuning algorithms have been tested by simulated examples. Different aspects, such as setpoint changes and load disturbances, have been analyzed. The amplitude of the setpoint and the load disturbance is 1. The closed loop step responses using IR method are presented in the left plots and the load disturbance responses in the right plots of Figs. 3/4. The design parameter T_c can be specified by the user or can be chosen automatically as a percent of $2\zeta\tau$ or $2\zeta\tau+\theta$. Here T_c is fixed at $(2\zeta\tau+\theta)/2$ for the upper plots and respectively $(2\zeta\tau+\theta)/10$ for lower plots in Figs. 3/4.

In Fig. 3 the results are presented for processes of order 1 to 4 with no delay:

$$H_p(s) = \frac{2}{\left(\frac{240}{i}s+1\right)^i}, \quad i=1,\dots,4. \quad (18)$$

In Fig. 4 the results are presented for the same processes with time delay 50s.

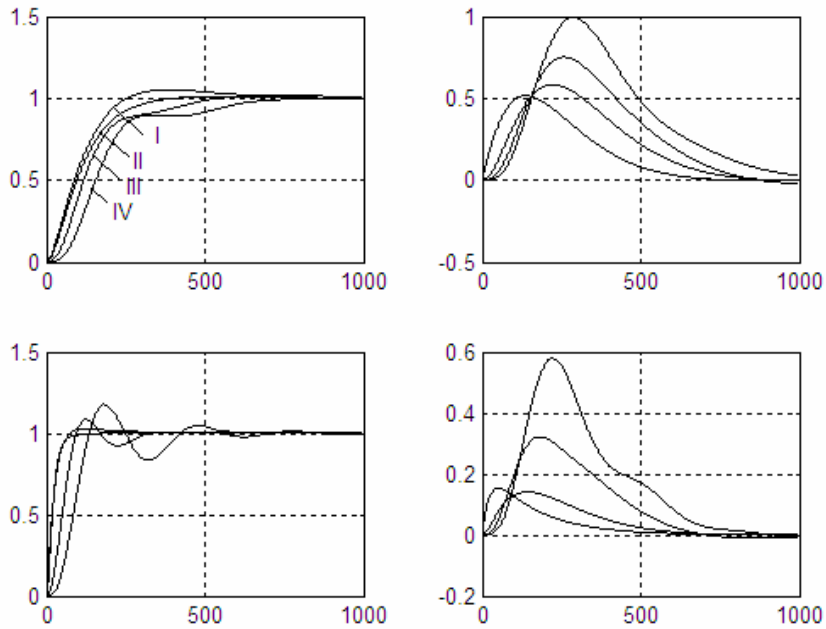


Figure 3. Responses for processes without delay

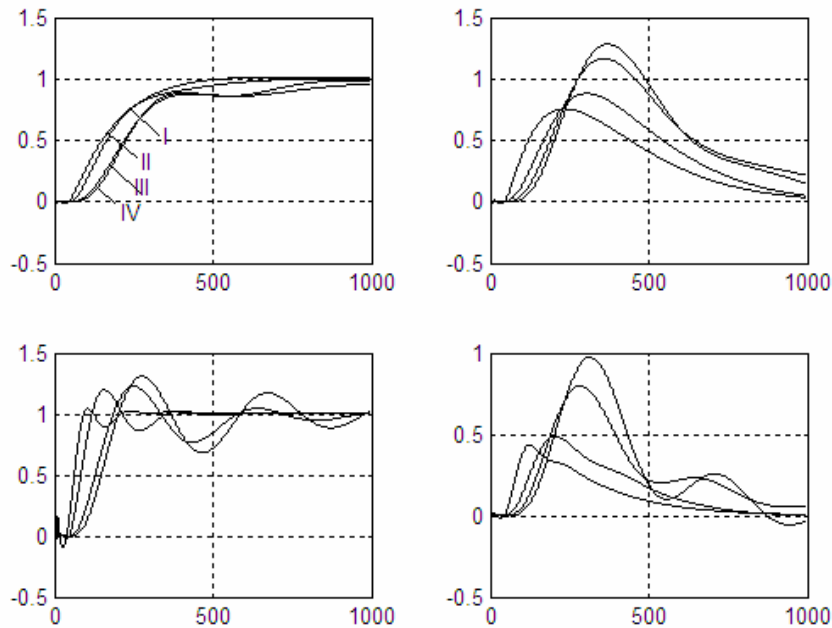


Figure 4. Responses for processes with delay

In figure 5 and 6 the performances of the IR method are compared with two other methods: Ziegler-Nichols method (ZN) and internal model control (IMC). Notice that here IMC can be considered as a kind of reference but this is not an auto-tuning method and it requires the full process model. In figure 5 a second order process given by (18) is considered. The same process with time delay 50 s is considered in figure 6. T_c is fixed at $(2\zeta\tau+\theta)/10$. The closed loop step responses are presented in the upper plots and the load disturbance responses in the lower plots.

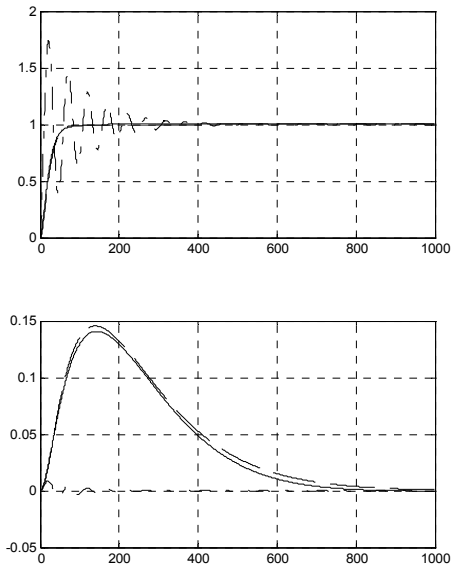


Figure 5. Comparison of the IR(solid line), IMC(dashed) and ZN (dashdot) methods, step response and load disturbance response. Second order process without delay.

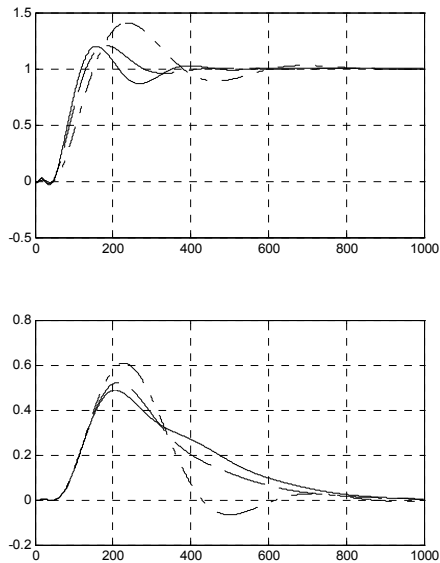


Figure 6. Comparison of the IR(solid line), IMC(dashed) and ZN (dashdot) methods, step response and load disturbance response. Second order process with delay.

5. CONCLUSION

Some relay-based algorithms for auto-tuning of PID controllers have been presented, assuming a simple process model structure and achieving the regulator tuning by identifying two points on the process frequency response and using the IMC-PID design method. This auto-tuning method yields good PID parameters for a specific class of processes (stable processes with self-regulation); it is not a general methodology for arbitrary process models.

Concerning the numerical complexity, the proposed method involves more complicated calculations than Ziegler-Nichols/Åström-Hägglund; however the required experiments are non-iterative and easy to be performed. Experiments and simulation studies have indicated that the presented auto-tuner performs well and can be easily used even by people who are not specialist in automatic control.

REFERENCES

- Aström, K.J. and T. Hägglund (1984). Automatic tuning of simple regulators with specifications on phase and amplitude margins, *Automatica*, **Vol.20**, pp.645.
- Aström, K.J., C.C. Hang, P. Persson and W.K. Ho (1992). Towards intelligent PID control. *Automatica*, **Vol.28**, pp.1.
- Aström, K.J. and T. Hägglund (1995). *PID Controllers: Theory, Design and Tuning*. Instrument Society of America, Research Triangle Park, NC, USA, 1995.
- Leva, A., C. Cox and A. Ruano (2002). Hands-on Pid autotuning: a guide to better utilisation. *IFAC Professional Brief*.
- Ogata, K. (1990). *Modern Control Engineering*. Prentice-Hall.
- Ho, W.K., C.C. Hang and L.S. Cao (1995). Tuning of PID controllers based on gain and phase margin specifications, *Automatica*, **Vol.31**, No.3, pp.497.
- Poulin, E., A. Pomerleau, A. Desbiens and D. Hodouin (1996). Development and evaluation of an auto-tuning and adaptive PID controller, *Automatica*, **Vol.32**, pp.71.
- Schei, T.S. (1994). Automatic tuning of PID controllers based on transfer function estimation. *Automatica*, **Vol.30**, pp.1983.