Partial Element Equivalent Circuit Models in the Solution of the Electric Field Integral Equation

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Abstract — **3-D electromagnetic methods are fundamental design tools for complex high-speed systems. Among the integral equation-based techniques, the Partial Element Equivalent Circuit (PEEC) method has received a special attention in interconnect modeling, where mixed electromagnetic/circuit problems need to be solved. Retardation effects and the resulting delays must be taken into account and included in the modeling, when signal waveform rise times decrease and the corresponding frequency content increases or the geometric dimensions become electrically long. In this case, the enforcement of the Kirchhoff laws to PEEC delayed models leads to a set of delayed differential equations in a neutral form. The aim of this contribution is to present an overview of the PEEC method with special focus on the analysis of electrically long structures that require taking delays into account.**

1 Introduction

The increasing demand for performance of integrated circuits (ICs) pushes operation to higher signal bandwidths and accurate modeling of previously neglected second order effects, such as crosstalk, reflection, delay and coupling, becomes increasingly important during circuit and system simulations [1, 2]. Electromagnetic (EM) 3-D methods [3, 4, 5] have become increasingly indispensable analysis and design tools for a variety of complex high-speed systems. Among all EM methods, the Partial Element Equivalent Circuit (PEEC) method [4] has been found particularly useful for modeling PCBs and interconnects. The PEEC method uses a circuit interpretation of the Electric Field Integral Equation (EFIE) [6], thus being especially suitable to problems involving both electromagnetic fields and circuits [2, 4, 7]. Nonlinear circuit devices such as drivers and receivers are usually connected to PEEC equivalent circuits using a time domain circuit simulator (e.g. SPICE [8]). The quasi-static PEEC formulation [4], which approximates the full-wave PEEC approach [9],

yields an equivalent RLC circuit by neglecting the time delays between the elements in the full-wave PEEC formulation. Systems of ordinary differential equations (ODE) with constant coefficients in the time-domain and complex algebraic systems of equations with frequency-independent matrices in the frequency domain are obtained. The quasi-static PEEC formulation is reasonable for problems with electrically small geometries. When signal waveform rise times decrease and the corresponding frequency content increases or the geometric dimensions become electrically large, time delays must be taken into account and included in the modeling.

A PEEC formulation which includes delay elements, called τPEEC method [10], becomes necessary and leads to systems of neutral delayed differential equations (NDDE) [9] with constant coefficients and constant delay times in the time domain and to complex algebraic systems of equations with frequency-dependent matrices in the frequency domain. Simply using quasi-static PEEC models can result in significant errors and artifacts in the modeling [11].

This paper presents an overview of the PEEC method in the solution of the EFIE with special focus on the analysis of electrically long structures that require taking delays into account.

2 Delayed PEEC formulation

The PEEC method [4] stems from the integral equation form of Maxwell's equations.

The main difference of the PEEC method with other integral equation based techniques such as the Method of Moments [3] resides in the fact that it provides a circuit interpretation of the EFIE [6] in terms of partial elements, namely resistances, partial inductances and coefficients of potential. Thus, the resulting equivalent circuit can be studied by means of SPICE-like circuit solvers [8] in both time and frequency domain.

The PEEC method is based on the EFIE [6] of the form

$$\boldsymbol{E}^i = \frac{\boldsymbol{J}(\boldsymbol{r},t)}{\sigma} + \frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t} + \nabla\phi(\boldsymbol{r},t) \qquad (1)$$

where $\boldsymbol{E}^i$ is an incident electric field, $\boldsymbol{J}$ is a current

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density, $\boldsymbol{A}$ is the vector magnetic potential, and ϕ is the scalar electric potential at observation point $\boldsymbol{r}$. The standard Galerkin-type approach [3, 12] is applied by expanding current and charge densities using pulse basis functions. Scalar and vector potentials, $\phi(\boldsymbol{r}, t)$ and $\boldsymbol{A}(\boldsymbol{r}, t)$ respectively, are analogously expanded. By defining a specific inner product, as weighted volume integral over an elementary volume, (1) can be interpreted as Kirchhoff Voltage Law (KVL). The discretization process leads to topological entities such as nodes and branches, thus making the continuous electromagnetic problem solvable as an equivalent circuit [4]. In addition, the continuity equation is to be enforced

$$\nabla \cdot \boldsymbol{J}(\boldsymbol{r}, j\omega) + j\omega\rho(\boldsymbol{r}, j\omega) = 0 \qquad (2)$$

which can be easily converted in an equivalent circuit form leading to the Kirchhoff Current Law (KCL). The charge density is represented by $\rho(\boldsymbol{r}, j\omega)$. The resulting equivalent circuit can be studied by means of SPICE-like circuit solvers [8] in both time- and frequency-domain.

Following the standard approach [4], volumes and surfaces are discretized into elementary regions, hexahedra and patches respectively [10]. The current and charge densities are expanded into a series of basis functions which are usually assumed as pulse basis functions. The choice of pulse basis functions implies to assume constant current and charge densities over the elementary volume (inductive) and surface (capacitive) cells, respectively.

Conductors are modeled by their ohmic resistance, dielectrics by the excess capacitance [13]. Hence, magnetic and electric field coupling are modeled by partial inductances and coefficients of potential, respectively.

The magnetic field coupling between two inductive volume cells α and β is modeled by the partial inductance

$$Lp_{\alpha\beta} = \frac{\mu}{4\pi} \frac{1}{a_\alpha a_\beta} \int_{u_\alpha} \int_{u_\beta} \frac{1}{R_{\alpha\beta}} du_\alpha du_\beta \qquad (3)$$

where $R_{\alpha\beta}$ is the distance between any two points in volumes u_α and u_β with a_α and a_β their cross sections. The electric field coupling between two capacitive surface cells γ and δ is modeled by the coefficient of potential

$$P_{\gamma\delta} = \frac{1}{4\pi\varepsilon} \frac{1}{S_\gamma S_\delta} \int_{S_\gamma} \int_{S_\delta} \frac{1}{R_{\gamma\delta}} dS_\gamma dS_\delta \qquad (4)$$

where $R_{\gamma\delta}$ is the distance between any two points on surfaces γ and δ, while S_γ and S_δ denote the area of their respective surfaces.

Due to the finite value of the speed of light, partial inductances and coefficients of potentials relate causes and effects delayed in time:

$$v_{L,\alpha}(t) = Lp_{\alpha\beta} \frac{di_\beta(t - \tau_{\alpha\beta})}{dt} \qquad (5)$$
$$v_{P,\gamma}(t) = P_{\gamma\delta} q_\delta(t - \tau_{\gamma\delta}) \qquad (6)$$

where $\tau_{\alpha\beta} = R_{\alpha\beta}/c_0$ and $\tau_{\gamma\delta} = R_{\gamma\delta}/c_0$ are the center-to-center delays between the corresponding basis-function domains, and c_0 denotes the free-space speed of light. Hence, partial inductance and coefficient of potential matrices act as a delay operator for time derivatives of currents and charges, respectively

$$\mathbf{v}_L(t) = \mathbf{L}_p \frac{d\mathbf{i}(t - \boldsymbol{\tau}_L)}{dt} = \widetilde{\mathbf{L}}_p(\boldsymbol{\tau}_L) \frac{d\mathbf{i}(t)}{dt} \quad (7a)$$
$$\mathbf{v}(t) = \mathbf{P}\mathbf{q}(t - \boldsymbol{\tau}_C) = \widetilde{\mathbf{P}}(\boldsymbol{\tau}_C) \mathbf{q}(t) \qquad (7b)$$

where $\boldsymbol{\tau}_L$ and $\boldsymbol{\tau}_C$ denote the center-to-center delay matrices for the magnetic and electric field couplings, respectively.

Generalized Kirchoff's laws, for conductors, can be rewritten as

$$\frac{d\mathbf{q}(t)}{dt} - \mathbf{A}^T \mathbf{i}(t) + \mathbf{i}_e(t) = 0 \qquad (8a)$$

$$-\mathbf{A}\mathbf{v}(t) - \widetilde{\mathbf{L}}_p(\boldsymbol{\tau}_L) \frac{d\mathbf{i}(t)}{dt} - \mathbf{R}\mathbf{i}(t) = \mathbf{0} \quad (8b)$$

where $\mathbf{A}$ is the connectivity matrix, $\mathbf{v}(t)$ denotes the node potentials to infinity, $\mathbf{i}(t)$ and $\mathbf{i}_e(t)$ represent the currents flowing in volume cells and the external currents, respectively.

The previous equation (8b) has to be modified when dielectrics are considered, since the resistance $\mathbf{R}$ is substituted by the excess capacitance which relates the polarization charge and the corresponding voltage drop as $\mathbf{v}_d(t) = \mathbf{C}_d^{-1}\mathbf{q}_d(t)$ [13]. Hence, for dielectric elementary cells, (8) become

$$\frac{d\mathbf{q}(t)}{dt} - \mathbf{A}^T \mathbf{i}(t) + \mathbf{i}_e(t) = 0 \qquad (9a)$$

$$-\mathbf{A}\mathbf{v}(t) - \widetilde{\mathbf{L}}_p(\boldsymbol{\tau}_L) \frac{d\mathbf{i}(t)}{dt} - \mathbf{v}_d(t) = \mathbf{0} \quad (9b)$$

$$\mathbf{i}(t) = \mathbf{C}_d \frac{d\mathbf{v}_d(t)}{dt} \qquad (9c)$$

A selection matrix $\mathbf{K}$ is introduced to define the port voltages by selecting node potentials. The same matrix is used to obtain the external currents $\mathbf{i}_e(t)$ by the currents $\mathbf{i}_s(t)$ which are of opposite sign with respect to the port currents $\mathbf{i}_p(t)$

$$\mathbf{v}_p(t) = \mathbf{K}\mathbf{v}(t) \qquad (10a)$$
$$\mathbf{i}_e(t) = \mathbf{K}^T \mathbf{i}_s(t) \qquad (10b)$$

An example of τPEEC circuit for a conductor elementary cell is illustrated, in the Laplace domain, in Fig. 1 where the current controlled voltage sources $sL_{p,ij}I_j$ and the charge controlled current sources sQ_i model the magnetic and electric field couplings, respectively.
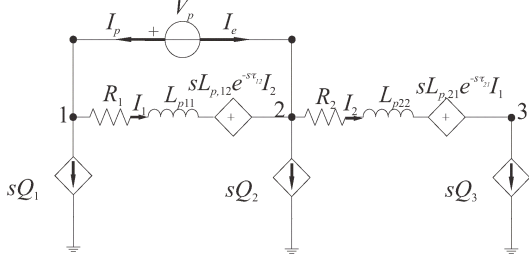

Figure 1: Illustration of τPEEC circuit electrical quantities for a conductor elementary cell.

3 Descriptor representation of τPEEC circuits

We assume that the system under analysis consists of conductors and dielectrics. Let us denote with n_n the number of nodes and n_i the number of branches where currents flow. Among the latter, we denote with n_c and n_d the number of branches of conductors and dielectrics, respectively. Furthermore, let us assume to be interested in generating an admittance representation having n_p output currents $\mathbf{i}_p(t)$ under voltage excitation $\mathbf{v}_p(t)$. Since dielectrics require the excess capacitance to model the polarization charge [13], additional n_d unknowns are needed in addition to currents. Hence, if the MNA approach [14] is used, the global number of unknowns is $n_u = n_i + n_d + n_n + n_p$. In a matrix form, (8)-(10), taking (7) into account, read

$$\underbrace{\left[\begin{array}{cccc} \mathbf{I}_{n_n,n_n} & \mathbf{0}_{n_n,n_i} & \mathbf{0}_{n_n,n_d} & \mathbf{0}_{n_n,n_p} \\ \mathbf{0}_{n_i,n_n} & \widetilde{\mathbf{L}}_p(\boldsymbol{\tau}_L) & \mathbf{0}_{n_i,n_d} & \mathbf{0}_{n_i,n_p} \\ \mathbf{0}_{n_d,n_n} & \mathbf{0}_{n_d,n_i} & \mathbf{C}_d & \mathbf{0}_{n_d,n_p} \\ \mathbf{0}_{n_p,n_n} & \mathbf{0}_{n_p,n_i} & \mathbf{0}_{n_p,n_d} & \mathbf{0}_{n_p,n_p} \end{array}\right]}_{\mathbf{C}} \cdot \frac{d}{dt}\mathbf{x}(t) =$$

$$-\underbrace{\left[\begin{array}{cccc} \mathbf{0}_{n_n,n_n} & -\mathbf{A}^T & \mathbf{0}_{n_n,n_d} & \mathbf{K}^T \\ \mathbf{A}\widetilde{\mathbf{P}}(\boldsymbol{\tau}_C) & \mathbf{R} & \boldsymbol{\Phi} & \mathbf{0}_{n_i,n_p} \\ \mathbf{0}_{n_d,n_n} & -\boldsymbol{\Phi}^T & \mathbf{0}_{n_d,n_d} & \mathbf{0}_{n_d,n_p} \\ -\mathbf{K}\widetilde{\mathbf{P}}(\boldsymbol{\tau}_C) & \mathbf{0}_{n_p,n_i} & \mathbf{0}_{n_p,n_d} & \mathbf{0}_{n_p,n_p} \end{array}\right]}_{\mathbf{G}} \cdot \mathbf{x}(t) +$$

$$\underbrace{\left[\begin{array}{c} \mathbf{0}_{n_n+n_i+n_d,n_p} \\ -\mathbf{I}_{n_p,n_p} \end{array}\right]}_{\mathbf{B}} \cdot \underbrace{\left[\ \mathbf{v}_p(t)\ \right]}_{\mathbf{u}(t)} \quad (11)$$

where $\mathbf{x}(t) = [\mathbf{q}(t)\ \ \mathbf{i}(t)\ \ \mathbf{v}_d(t)\ \ \mathbf{i}_s(t)]^T \in \Re^{n_u \times 1}$ and $\mathbf{I}_{n_p,n_p}$ is the identity matrix of dimensions equal to the number of ports. Matrix $\boldsymbol{\Phi}$ is

$$\boldsymbol{\Phi} = \left[\begin{array}{c} \mathbf{0}_{n_c,n_d} \\ \mathbf{I}_{n_d,n_d} \end{array}\right] \quad (12)$$

If the delay operator $\widetilde{\mathbf{P}}(\boldsymbol{\tau}_C)$ is applied to the first equation in (11), the system can be recast as

$$\underbrace{\left[\begin{array}{cccc} \widetilde{\mathbf{P}}(\boldsymbol{\tau}_C) & \mathbf{0}_{n_n,n_i} & \mathbf{0}_{n_n,n_d} & \mathbf{0}_{n_n,n_p} \\ \mathbf{0}_{n_i,n_n} & \widetilde{\mathbf{L}}_p(\boldsymbol{\tau}_L) & \mathbf{0}_{n_i,n_d} & \mathbf{0}_{n_i,n_p} \\ \mathbf{0}_{n_d,n_n} & \mathbf{0}_{n_d,n_i} & \mathbf{C}_d & \mathbf{0}_{n_d,n_p} \\ \mathbf{0}_{n_p,n_n} & \mathbf{0}_{n_p,n_i} & \mathbf{0}_{n_p,n_d} & \mathbf{0}_{n_p,n_p} \end{array}\right]}_{\mathbf{C}} \cdot \frac{d}{dt}\mathbf{x}(t) =$$

$$-\underbrace{\left[\begin{array}{cccc} \mathbf{0}_{n_n,n_n} & -\widetilde{\mathbf{P}}(\boldsymbol{\tau}_C)\mathbf{A}^T & \mathbf{0}_{n_n,n_d} & \widetilde{\mathbf{P}}(\boldsymbol{\tau}_C)\mathbf{K}^T \\ \mathbf{A}\widetilde{\mathbf{P}}(\boldsymbol{\tau}_C) & \mathbf{R} & \boldsymbol{\Phi} & \mathbf{0}_{n_i,n_p} \\ \mathbf{0}_{n_d,n_n} & -\boldsymbol{\Phi}^T & \mathbf{0}_{n_d,n_d} & \mathbf{0}_{n_d,n_p} \\ -\mathbf{K}\widetilde{\mathbf{P}}(\boldsymbol{\tau}_C) & \mathbf{0}_{n_p,n_i} & \mathbf{0}_{n_p,n_d} & \mathbf{0}_{n_p,n_p} \end{array}\right]}_{\mathbf{G}} \cdot$$

$$\cdot \mathbf{x}(t) + \underbrace{\left[\begin{array}{c} \mathbf{0}_{n_n+n_i+n_d,n_p} \\ -\mathbf{I}_{n_p,n_p} \end{array}\right]}_{\mathbf{B}} \cdot \underbrace{\left[\ \mathbf{v}_p(t)\ \right]}_{\mathbf{u}(t)} \quad (13)$$

In a more compact form, the previous equations can be rewritten as

$$\mathbf{C}(\boldsymbol{\tau})\frac{d\mathbf{x}(t)}{dt} = -\mathbf{G}(\boldsymbol{\tau})\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (14a)$$

$$\mathbf{i}_p(t) = \mathbf{L}^T\mathbf{x}(t) \quad (14b)$$

where $\boldsymbol{\tau} \in \Re^{n_\tau \times 1}$ contains all delays $\boldsymbol{\tau}_L, \boldsymbol{\tau}_C$. Since this is an n_p-port formulation, whereby the only sources are the voltage sources at the n_p-port nodes, $\mathbf{B} = \mathbf{L}$ where $\mathbf{B} \in \Re^{n_u \times n_p}$. Each delayed entry of matrices $\mathbf{C}(\boldsymbol{\tau})$ and $\mathbf{G}(\boldsymbol{\tau})$ acts as a delay operator for the corresponding entry of vector $\mathbf{x}(t)$. Hence, (14) can be re-written in the Laplace domain as:

$$s\mathbf{C}(s)\mathbf{X}(s) = -\mathbf{G}(s)\mathbf{X}(s) + \mathbf{B}\mathbf{V}_p(s) \quad (15)$$

$$\mathbf{I}_p(s) = \mathbf{B}^T\mathbf{X}(s) \quad (16)$$

$$\mathbf{C}(s) = \mathbf{C}_0 + \sum_{k=1}^{n_\tau}\mathbf{C}_k e^{-s\tau_k} \quad (17)$$

$$\mathbf{G}(s) = \mathbf{G}_0 + \sum_{k=1}^{n_\tau}\mathbf{G}_k e^{-s\tau_k} \quad (18)$$

The solution of (14) can be carried out using direct or iterative solvers. Among the direct methods, the LU decomposition has been widely used [15]. If the transient analysis is carried out for n_t time steps, the complexity of the direct solution scales

as $O(n_t n_u^3)$, where n_u is the total number of unknowns. Hence, for a large number of unknowns n_u, equations (14) cannot be solved directly and iterative solvers are needed. Assuming that an average number n_{iter} of iterations per time step is required for the iterative solver to converge to a specified residual and n_u^2 is the computational cost for the matrix-vector products involved, the iterative solution for all time steps n_t scales as $O(n_{iter} n_t n_u^2)$ [16].

4   Conclusions

We have presented an overview of the PEEC method in the solution of the EFIE with special focus on the analysis of electrically long structures that require taking delays into account. When signal waveform rise times decrease and the corresponding frequency content increases or the geometric dimensions become electrically long, time delays must be taken into account and included in the modeling.

Acknowledgments

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