A Linear Regression based Cost Function for WSN Localization

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Abstract: **Localization with Wireless Sensor Networks (WSN) creates new opportunities for location-based consumer communication applications. There is a great need for cost functions of maximum likelihood localization algorithms that are not only accurate but also lack local minima. In this paper we present Linear Regression based Cost Function for Localization (LiReCoFuL), a new cost function based on regression tools that fulfills these requirements. With empirical test results on a real-life test bed, we show that our cost function outperforms the accuracy of a minimum mean square error cost function. Furthermore we show that LiReCoFuL is as accurate as relative location estimation error cost functions and has very few local extremes.**

1. INTRODUCTION

Researchers have already been investing a lot of effort in localization-aware applications [1]. Within the DEUS-project [2], we already implemented a next generation network and service by the use of T-mote Sky modules in an elderly surveillance localization system. In this paper, we push the limits further.

Modern widely accepted methods use statistics, like Bayesian estimators [3, 4, 5] and maximum likelihood estimation [6], to improve the accuracy of the position. In previous work [7], we presented Linear Regression based Fast Localization Algorithm (LiReFLoA). This is an automated method to optimize and calibrate the experimental data before offering them to our positioning tool. This tool is based on elimination and controlling distance circles. In this paper, we use the same selection and calibration method, present a new maximum likelihood cost function and compare it with cost functions that are more traditional, like Minimum Mean Square Error function (MMSE) [8], Relative Location Estimation (RLE) [9] and Reduced Biased Relative Location Estimation (RBRLE) [9].

This paper is organized as follows: In section 2 related work is described. The used equipment can be found in section 3. Section 4 presents LiReCoFuL. In section 5 test results are presented and LiReCoFul is compared with other cost functions. Finally, in section 6 conclusions are drawn.

2. RELATED WORK

Both the Bayesian algorithm and the Maximum LikeliHood (MLH) algorithm are widely accepted as localization tools in WSN. In this paper, we concentrate on MLH. The starting point of a MLH algorithm is a cost function. Several cost functions exist: the simplest and widely used cost function is the Minimum Mean Square Error function (MMSE) [8]:

$$(\tilde{x}, \tilde{y}) = \operatorname{argmin}_{(x,y)} \sum_{j \in \text{anchor}(i)} (\tilde{d}_{i,j} - d_{i,j})^2 \qquad (1)$$

where $d_{i,j}$ is the Euclidean distance between a point j and an anchor i. (please recall that an anchor is a node knowing its own position). Furthermore ~ denotes the estimate, i.e. $(\tilde{x}, \tilde{y})$ is the estimated (the most likely) position, and $\tilde{d}_{i,j}$ is the estimated distance between point j in the x-y plane and anchor i. Although we estimate this distance with the Received Signal Strength Indicator (RSSI)-values of the radio chip and the propagation constants, the lognormal relationship between RSSI and distance is not a prior assumption.

Thus, this cost function means that the most likely position is a point in the x-y plane where the sum of squared position errors between estimated and Euclidean distances to the anchors is minimal.

Equation (1) does not take into account that the underlying physics dictates the relationship between the RSSI and the distance to be semi-logarithmic [10]. Therefore Patwari et al.

start with this assumption and propose the Relative Location Estimation (RLE) cost function [9]:

$$(\tilde{x}, \tilde{y}) = \text{argmin}_{(x,y)} \sum_{j \,\epsilon\, \text{anchor}(i)} \ln^2(\frac{\tilde{d}^2_{i,j}}{d^2_{i,j}}) \qquad (2)$$

where ln stands for the natural logarithmic function. This cost function implies that the most likely position is a point in the x-y plane where the sum of squared logarithms of the squared quotient of the Euclidean and estimated distance is minimal.

Since this cost function is biased (this means that the mean of the estimated position does not equal the Euclidean distance), the same authors suggest a better cost function with reduced bias (RBRLE)

$$(\tilde{x}, \tilde{y}) = \text{argmin}_{(x,y)} \sum_{j \,\epsilon\, \text{anchor}(i)} \ln^2(\frac{\tilde{d}^2_{i,j}}{C^2 d^2_{i,j}}) \qquad (3)$$

where C is calculated with the propagation parameters and the standard deviation on the RSSI. This standard deviation is estimated with the Cramer-Rao lower bound (CRLB). The authors of [9] notice that $C \approx 1.2$ for typical channels. Therefore, we use this value in this paper.

In section 4 we will suggest a new cost function, based on linear regression and probabilities around a point on the regression line. In the next section, our test environment is described.

3. USED EQUIPMENT

The Interdisciplinary Institute for Broadband Technology (IBBT) iLab.t Wireless Lab or W-iLab.t test bed is used in our experiment. More about this test bed can be found in [11]. Only the second floor is used in this paper. On figure 1, this floor is shown with the position of the 51 active nodes. The floor is rectangular shaped, but in the center of the floor, there are also outside walls, almost cutting the floor in two smaller rectangles. In this paper, we use not only the same selection method of best anchors but also the same calibration method as in our previous work [7, 12]. We swap the RSSI- and (logarithmic) distance axes, perform a linear regression and use regression properties to obtain the "well-behaving" and calibrated anchors. These anchors are marked with a red circle in figure 1.

4. OUR COST FUNCTION

The RSSI-distance plane in figure 2 presents the measurements for a well-behaving anchor with the regression line. This line reduces the mean squared errors, thus the measurements are close to this regression line. A point further away from this line will therefore result in a lower probability of occurrence. The distance probability distribution is shown for three different values of the RSSI in the third dimension of this plot, according to the basics of the linear regression technique [13]. This is also valid for other RSSI-values and thus a kind of "tunnel" is formed around this regression line. An assumption of linear regression theory is that the y-coordinate values are normally distributed with the same standard deviation. Therefore the width of the tunnel remains constant for a specific regression line. Having defined an error on distance [7] we were the first to assume a normal distribution on the (logarithmic) distance. Many other authors, including [9], assumed a normal distribution on the

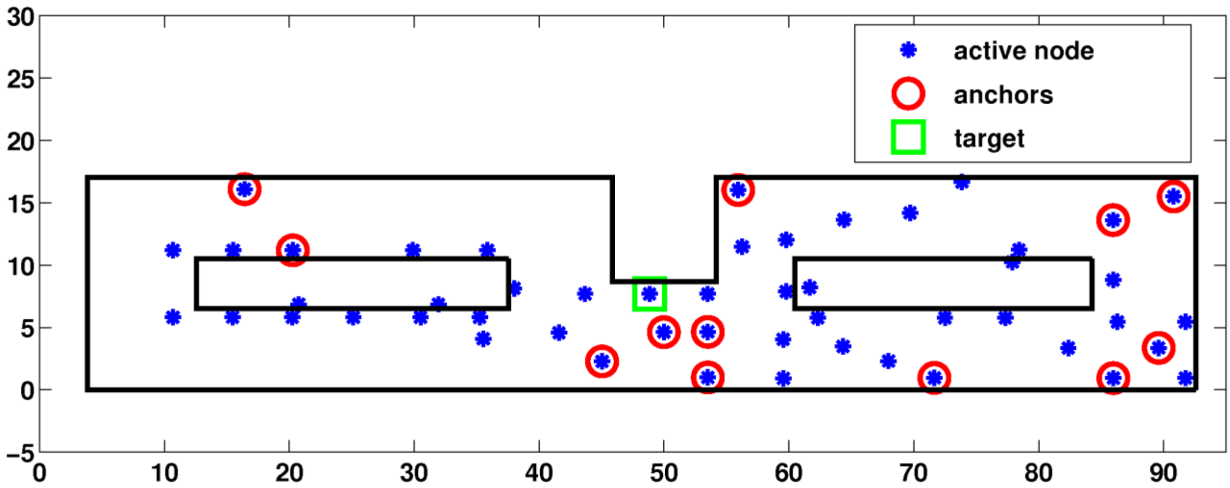

Figure 1: Position of nodes on the second floor of the IBBT

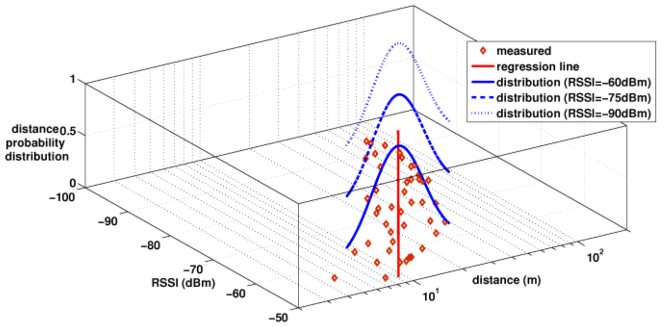

Figure 2: Linear regression and distance probability distribution

RSSI. Since the variables are linearly correlated, both assumptions are equivalent. Our approach however is more direct, since it outputs distances rather than RSSI's.
Consequently, (4) is a normal distribution with zero means and the (unknown) standard deviation. Dividing (4) by this standard deviation results in a standard normal distribution.

$$\log_{10}\left(\frac{\tilde{d}_{i,j}}{d_{i,j}}\right) \sim N(0, \sigma_i) \qquad (4)$$

For each anchor, the exact standard deviation was estimated by the measurements using the regression technique. Formula (4) divided by the standard (logarithmic distance) error (or half the error on distance [7]) converts the standard normal distribution to a t-distribution [13].
The most likely location is now found by maximizing our cost function:

$$(\tilde{x}, \tilde{y}) = \mathrm{argmax}_{(x,y)} \prod_{j\,\epsilon\,\mathrm{anchor(i)}} \mathrm{tpdf}\left[\frac{\log_{10}\left(\frac{\tilde{d}_{i,j}}{d_{i,j}}\right)}{SE_{\mathrm{anchor(i)}}}, n(i)\right] \qquad (5)$$

where tpdf(t,n) denotes the Student's T probability distribution function with n degrees of freedom at the t-value of t [13]. The anchor dependent degrees of freedom n(i) can also be obtained by linear regression: for each sending anchor, it equals the number of receivers where the RSSI is above the noise floor minus two. Indeed two degrees of freedom are lost: one for calculating the mean and one for calculating the standard deviation [13].
When the Euclidean distance of a point in the x-y plane to a particular target equals the estimated distance, the t-value is zero and the t-distribution peaks. This is the case for all anchors. Assuming that the anchors are independent, the overall probability is found by multiplying the probabilities of the individual anchors. Therefore multiplication needs to be done for all points that are anchors and the cost function needs to be maximized.

Mostly the conjugate gradient algorithm is used to find the extremes of the cost functions (1-3), (5) [14]. A drawback of this method is that it does not always converge to the wanted extreme of the function, or that it converges to a local extreme [15]. Some authors [16] therefore use this algorithm in combination with another coarse positioning algorithm. In this paper we put a grid on our building and calculate the cost function for each grid point. This algorithm is "safer" since it always finds the true extreme and allows easy visualization.

5. RESULTS

In this section, the test results of the different cost functions are compared. The first subsection starts with the comparison of the plots of the cost functions and a second follows with a cumulative distribution plot of the position error.

5.1 Graphical comparison of the cost functions.

Figure 3 plots the cost functions (1), (2), (3) and (5) on a 0.25 m grid (for the same central target) respectively. In figure 1 this target is marked with a green square. This target is chosen randomly. Other targets have similar graphs.

For RBRLE, a C-value of 1.2 is chosen. Please recall from section 2, that MMSE, RLE and RBRLE need to be minimized.

For this central target, the Euclidean distances to the extremities of the building are large in the cost function (1). This results in the shape of the upper left MMSE graph in figure 3.

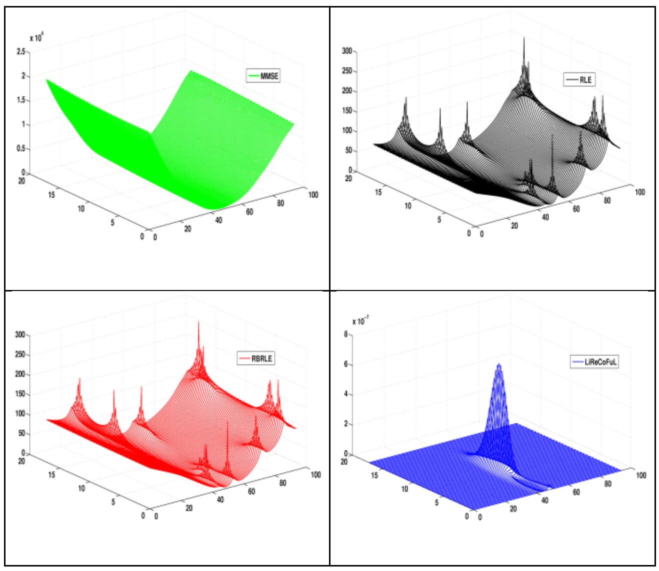

Figure 3: Comparison of the different cost functions for a central target

Near an anchor the denominator of the ln-argument of both (2) an (3) is very small. When this point is not the target, the nominator of the ln-argument is not small. This results in peaks of these cost functions at the anchor locations, forcing

the estimated position to the lower values in both the upper right-hand side RLE- and lower left RBRLE graphs of figure 3. A large value of the C-value will increase these peaks more pronouncedly.
Our cost function for the target can be found in the lower right corner of figure 3. Please recall from section 4 our cost function needs to be maximized. It has a large gradient around the maximum. It has less local maxima than other cost functions have local minima. This eases a real-time positioning algorithm based on the conjugate gradient method.

5.2 Cumulative distribution plots of the position error.

Our software now calculates the position of each of the 51 active nodes for the different algorithms and compares the results with the exact positions. In figure 4, a cumulative distribution plot (cdfplot) of the position error is given for the different cost functions. The Euclidean distance between the exact and the calculated position presents one position error point in this cdfplot.
The MMSE cost function gives the worst results. It has a median of 4.86 m. This can be explained by the fact that the

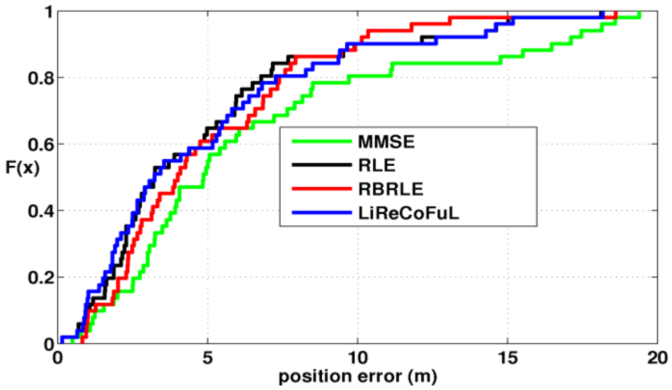

Figure 4: Cdfplot of the position error for the different cost functions on the second floor

model does not take into account the lognormal relation of the distance and the RSSI. The other medians are 3.23 m, 4.01 m and 3.23 m for the RLE, RBRLE and our cost function respectively.
It can be shown that the frequency distribution of the position error is not a normal distribution. Therefore nonparametric test are performed. A Friedman test [17] rejects the null hypothesis that the error distributions are the same for all cost functions. The p-value (defined as the probability that the test statistic is equal to or more extreme than the one observed under the null hypothesis [17]) equals 0.003 or 0.3%.
Next, 6 Wilcoxon tests [17] are done, pair wise comparing the position error of the cost functions. E.g. a first test compares the position error for MMSE and LRE (for the same target), a second MMSE and RBLRE, …. These tests confirm that RLE, RBRLE and our cost function result in lower position errors than MMSE. 1-tailed p-values are less than 0.05%, 2.9% and 0.05% respectively. The tests further fail to prove a difference between the position error of our cost function and both RLE and RBRLE.
This subsection thus shows that the position errors of LiReCoFuL are comparable with those of (RB)RLE and definitely better than those of MMSE.

5.3 Execution times.

Very fast execution times are needed for real-time localization. At the starting point of this algorithm comparison, anchors are already selected and calibrated. Therefore those execution times are not treated here. At this stage, a RSSI matrix and a distance matrix are already calculated in Matlab. The RSSI(i,j) matrix consists of averaged RSSI elements reported from receiver j with sending node i. The distance matrix contains elements with the (known) distance between receiver j and sending node i. First, the gridpoints are calculated. In our 0.25m gridded building this results in a matrix of 26000 rows and two columns (one for the longitudinal and one for the lateral coordinate). A denser grid will result in larger execution times. Now, the position errors are calculated for each algorithm. On our Dell Latitude D830 position server equipped with Matlab, the average time for calculating one of the 51 positions took 27, 45, 45 and 240 ms for the MMSE, RLE, RBRLE and LiReCoFuL algorithm respectively. Implementing the t-distribution formula [18]:

$$\mathrm{tpdf}(\mathrm{t}, \mathrm{n}) = (\mathrm{n}\Pi)^{-\frac{1}{2}} \frac{\Gamma\left(\frac{\mathrm{n}+1}{2}\right)}{\Gamma\left(\frac{\mathrm{n}}{2}\right)} (1 + \frac{\mathrm{t}^2}{\mathrm{n}})^{-(\frac{\mathrm{n}+1}{2})} \quad (6)$$

instead of using the tpdf build-in Matlab function will speed up our algorithm, when the n(i) value does not vary between anchors (n(i)=n).
Please note that (6) is differentiable. This eases the implemention of (5) in a conjugate gradient algorithm. We keep this as future work.

5.4 The cost function with different scenarios.

A similar test was done on the third floor of the IBBT building. The cdfplots can be found in figure 5. The medians of the position errors are 7.05, 4.19, 4.83 and 4.01 m for MMSE, RLE, RBRLE and LiReCoFuL respectively. This confirms the findings of Section 5.2. Furthermore, the presence of longer corridors results in higher constructive multipath fading and thus in somewhat higher medians for all algorithms.

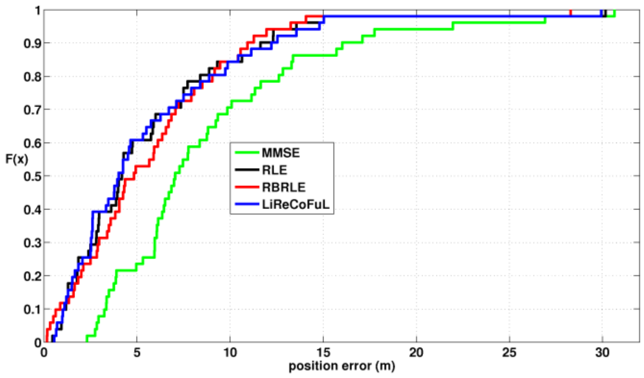

Figure 5: Cdfplot of the position errors for the different cost functions on the third floor

6. CONCLUSION

This paper presents a new cost function for localization algorithms using maximum likelihood. Our empirical tests show that the position errors are better than with a minimum mean square error cost function and equally well as with a relative location estimation cost function. The grid approach in this paper reveals that LiReCoFuL has less local maxima than the RLE and RBRLE cost functions have local minima.

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