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Reply to Comment on "On the origin dependence of multipole moments in electromagnetism" by P De Visschere [J.Phys. D: Appl. Phys. 39 (2006) 4278–4283]

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In their Comment [1] on our paper [2], Raab and de Lange disregard the main critiscism we formulated against their work [3], which started from the premise that the "direct multipole theory" is unphysical and must somehow be remedied. We showed that there is no problem with "direct multipole theory". We dismissed the remedy (the transformation theory) proposed by Raab and de Lange as unphysical since it modifies the boundary conditions, which we considered unacceptable at the time. However we now realize that one might modify the boundary conditions provided the change merely induces a phase shift. Whether the transformation proposed by Raab and de Lange fulfils this condition is not obvious. In what follows we give a more detailed reply to the numbered comments by Raab and de Lange.

1. We [2] did not "advocate" the use of origin dependent material constants. We merely tried to make clear that the "direct application of multipole theory" which leads to origin dependent polarizabilities and material properties, when applied correctly is not "unphysical". "Correctly" means that one should always take into account the two changes caused by a shift of origin: (i) the multipoles themselves do change and (ii) these multipoles must be positioned at new locations. Whereas Raab and de Lange seem to agree that origin-dependence is harmless as far as source densities are concerned, hence the restriction in comment 3, they are not prepared to accept the consequences of our calculations at a higher level, notwithstanding that doing so explains a number of "surprising" observations made in [3]: (i) the wave equation is origin-independent despite the use of origin dependent polarizabilities and material properties. (ii) The

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origin-dependent material properties seem to have a deleterious effect on the reflected intensity only and not on the transmitted intensity.
Raab and de Lange claim that "material constants are origin-independent macroscopic observables". We say that only measured quantities are required to be origin-independent. Material properties are part of a macroscopic model, which also includes the position of the (macroscopic) boundary. Only their combined effect should be origin-independent.

2. Apparently Raab and de Lange did not get the point we tried to make: we do not agree that the reflected intensities are origin-dependent as found in [3]. In [2] we argued that the correct origin-dependence of the amplitude reflection coefficients boils down to a mere phase shift and this has no effect on the reflected intensity, as it should. We came to this conclusion by analyzing the origin dependence of the boundary conditions and we also referred to a number of special cases for which we did obtain a direct proof, which was not included. In the meantime we proved this result (still for a non-magnetic material and up to electric quadrupole/magnetic dipole order), but without any restrictions on the symmetry. Due to space constraints this proof can unfortunately not be included in this reply. It should be obvious that retaining terms in these calculations of higher than quadrupole order is inconsistent and these terms must be neglected. Otherwise one must include higher than quadrupole terms in the boundary conditions. We believe this is the reason Raab and de Lange did find an effect on the reflected intensity and not for the transmitted intensity when following the "direct multipole theory". Our proof also shows that the effects on reflection and transmission coefficients are completely similar and compatible. For the sake of completeness we repeat that this phase shift is compensated by the 2nd effect of shifting the origin: the relocation of the multipoles which for this case means a shift of the macroscopic boundary and the overall effect of the origin shift is therefore nil, as for the microscopic and macroscopic source densities.

3. It was not our intention to review or comment on the papers referred to by Raab and de Lange under comment 3. Summarizing we could say that we showed that the procedure (I) referred to is not really needed. But this does not mean that the procedure (I) could not be correct or useful. Our study was triggered by the conclusion in [3] that the "direct multipole theory" leads to unphysical results, with which we don't agree.

4. Comment 4 continues the argument under comment 3 and refers to the "uniqueness problem". For the latter see our answer to comment 7.

5. We are not sure which of our claims precisely are referred to but we believe this comment restates that only origin-independent material properties are acceptable. See our answer to comment 1 & 3 why we think this is too restrictive. To further clarify our viewpoint, consider an experimenter who measures the reflected/transmitted intensities only. Then

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since these properties are origin-independent, this experimenter will not be able to determine unique values for the macroscopic multipole related material properties. One should not blame the multipole model for this shortcoming (and change the model in such a way that it contains only origin-independent quantities) since this outcome is a limitation of the measurement. A more ambitious experimeter might measure not only the intensities of the waves but also their exact phase. This experimenter is now faced with an additional problem: he must fix the position of the (fictitious) macroscopic boundary (and this can only be done wrt the true microscopic positions of the constituent atoms). Without any detailed apriori knowledge about the further properties of these constituent atoms, he might just choose an arbitrary position and since the position of the macroscopic boundary must follow the origin used for calculating the multipole moments, this origin has thus been fixed implicitly and this experimenter should now, with the additional phase measurements in his hand, in principle be able to derive unique values for the macroscopic material properties. Once these have been obtained the experimenter is free to move the origin again but always together with the position of the macroscopic boundary, perhaps because for one particular choice of the origin some multipole moments might vanish and this leads to a simpler description of the material.

6. We believe we should have made a distinction between the intended purpose of the transformation and it's actual effect. Although this is not clear in their comment 6, we thought the purpose of the transformation theory was quite clearly to "present an alternative to the unphysical standard formulation of multipole theory for macroscopic media" [3](page vi). But since we showed that the "standard formulation" is perfectly physical, such a transformation if found must necessarily be unphysical.
   Raab and de Lange arrive at their transformation by modifying the boundary conditions so that these become formally origin independent and they claim (posing additional constraints) that this transformation is unique, which we dispute (see comment 7). We now look at the problem from our perspective: since the effect of an origin shift on the material properties results merely in a phase shift in the transmission/reflection coefficients, which must be compensated by a shift of the position of the macroscopic boundary, we could try to get rid of this remaining origin dependence, by modifying the boundary conditions so that they become formally origin independent. Possibly this can be accomplished with a transformation as considered by Raab and de Lange, but such a transformation should have no effect on the reflected/transmitted intensities, which we found already to be origin independent and therefore it's effect could be no more than a phase shift. Whether the transformation arrived at by Raab and de Lange fulfils this condition is not obvious to us.

7. Apparently the validity of the transformation found by Raab and de Lange (R&dL) rests heavily on it's uniqueness, but we believe that e.g. the con-

clusion (5.32) in [4] is not justified. The reasoning followed by R&dL is as follows. R&dL apply two different transformations: (1) a Faraday transformation depending on parameters γ_i which does not modify the fields $\overline{D}$ and $\overline{H}$ and (2) a gauge transformation depending on parameters β_i which modifies the fields $\overline{D}$ and $\overline{H}$. Next R&dL find that in order to obtain origin-independent material properties some of the $\beta_i's$ must be equal to some of the $\gamma_i's$. They then explain that "... by construction, the Faraday transformation cannot change $\overline{H}$ ..." and conclude that those parameters must be zero. What they are probably referring to is that through the required equality $\beta_i = \gamma_i$ the Faraday transformation (indirectly) modifies $\overline{D}$ and $\overline{H}$ after all, but that's no argument to zero those coefficients since the Faraday transformation has no influence on $\overline{D}$ and $\overline{H}$ *whatever* the values given to the parameters γ_i. Thus R&dL arrive at their unique transformation only when imposing the additional condition that the Faraday transformation is not allowed to change $\overline{D}$ and $\overline{H}$ in this indirect way. It's not obvious which physical requirement could justify such a condition and we believe a set of formally valid transformations has been excluded arbitrarely.

References

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[4] Lange O and Raab R 2003 *P R Soc A* **459** 1325–1341