

Predictive Control Strategies  
Applied to the Management of a Supply Chain

Voorspellende regelstrategieën  
toegepast op het beheer van een bevoorradingsketen

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# Predictive Control Strategies Applied to the Management of a Supply Chain

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# Nederlandse Samenvatting

De laatste drie decennia worden gekenmerkt door een verschuiving in de manier waarop de industriële goederen worden geproduceerd en gedistribueerd. De nationale bedrijven worden steeds geïntegreerd in grote internationale waardeketens. Deze ontwikkeling heeft geleid tot een belangrijke paradigma verschuiving in het modern bedrijfsbeheer, d.w.z. individuele bedrijven opereren niet langer als compleet onafhankelijke entiteiten maar eerder als schakels in een bevoorradingsketen. Praktisch, alle hedendaagse bedrijven maken gebruik van bevoorradingsketens (en: Supply Chain - SC) om hun flexibiliteit, betrouwbaarheid en responsiviteit ten opzichte van veranderlijke marktvereisten te verbeteren. Het beheer van meervoudige relaties in de bevoorradingsketen zoals de organisatorische en functionele aspecten is gebaseerd op SCM (en: Supply Chain Management). Het basisidee achter SCM is de integratie van de organisatorische eenheden (zoals leveranciers, fabrikanten, verdelers, kleinhandelaars, enz.) en de coördinatie van materiaal-, informatie-, en financiële stromen met als doel het concurrentievermogen van de bevoorradingsketen te verhogen. Het beheer van bevoorradingsketens houdt in het ontwerp, planning, regeling en bewaking van de activiteiten in de keten met als doelstellingen: voldoen aan de vraag van de klant, operationele kosten minimaliseren, een competitieve infrastructuur uitbouwen en de globale prestaties verbeteren.

SCM levert vele uitdagende onderzoeksproblemen op. Een essentieel achterliggende gedachte bij SCM is dat er bepaalde beheercomponenten bestaan die gemeenschappelijk zijn voor alle bedrijfsprocessen. Het beheer van deze componenten is belangrijk aangezien ze bepalen hoe de bevoorradingsketen beheerd en gestructureerd wordt. Eerder dan elk aspect van SCM te behandelen, bespreekt dit proefschrift de integratie van de bedrijfsprocessen in de bevoorradingsketen die gerelateerd zijn aan de voorraadbeheer en de opvolging van orders. Dit proefschrift heeft als doel het ontwikkelen van een voorraadbeheerstrategie die gebaseerd is op regeltechnische principes, hiervoor wordt de gelijkenis tussen netwerken van bevoorradingsketens enerzijds en dynamische systemen anderzijds benut. De focus van een voorraadbeheerstrategie is om de voorraad in elke schakel van de keten op het gewenste niveau te handhaven door producten te bestellen bij de toeleveranciers naargelang de vraag van de klant. De afwaartse productstroom in de bevoorradingsketen hangt af van de vraag van de klant, van de stroomopwaartse bestellingsinformatie en van het beleid van iedere lokale manager om bestellingen te plaatsen en de voorraad aan te vullen. Daardoor kan de bestellingstrategie beschouwd worden als een regelstrategie.

De strategie die in het voorraadbeheer wordt gebruikt, heeft een significant effect op de variabiliteit van de bestelde hoeveelheden en op de voorraadmiveaus in de verschillende knopen van de bevoorradingsketen. Een ongewenst effect, dat door de SCM literatuur erkend is als één van de belangrijkste belemmeringen bij de optimali-

satie van de bevoorradingsketen, is het ‘*bullwhip*’ effect dat tot overmatige fluctuaties van het voorraad- en bestellingsniveauleidt. Dit proefschrift bespreekt danook een methode om dit ‘*bullwhip*’ effect te kwantificeren en te verminderen wanneer een voorspellende regelstrategie (en: Model Predictive Control – MPC ) wordt gebruikt als bestelproces.

Binnen het brede spectrum van de systeemtheorie en de regeltechniek is dit proefschrift vooral geïnteresseerd in de integratie van de MPC principes in het voorraadbeheer. MPC is een veelbelovende regelmethode die aan de behoeften voor het regelen en coördineren van een grootschalig SC netwerk kan voldoen. Ondanks de snelle vooruitgang gedurende de laatste twee decennia, is er nog een groot onontgonnen onderzoeksgebied binnen SCM. Gedurende deze laatste twee decennia werd slechts sporadisch een zeer beperkt aantal onderzoeksrapporten gepubliceerd waarbij MPC wordt toegepast op SCM. Het doel van dit proefschrift is om de kloof in de huidige literatuur te helpen dichten.

Hoofdstuk 1 beschrijft de onderzoeksachtergrond met betrekking tot bevoorradingsketens en waarom MPC een natuurlijke oplossing is om met problemen omtrent voorraadbeheer aan te pakken. Een *state-of-art* op het gebied van MPC regeling voor bevoorradingsketens wordt gegeven en de doelstellingen en originele bijdragen van dit werk worden opgelijst.

Hoofdstuk 2 behandelt de SC topologie op basisniveau en stelt een systeembenadering voor om een benchmark SC te modelleren binnen een conceptueel kader. Zo wordt het overbodig om de verschillende types SC netwerken nog op te sommen en te modelleren met de vereiste nauwkeurigheid en detail. De motivatie voor het kiezen van deze *benchmark* bevoorradingsketen wordt besproken. Het hoofdstuk gaat verder met de bespreking van de structuur, veronderstellingen en variabelen van de bevoorradingsketen vanuit het standpunt van procestechniek teneinde de eropvolgende dynamische analyse en modellering te vergemakkelijken. Deze bestaat uit de ontwikkeling van wiskundige modellen die de typische kenmerken van SC's beschrijven, en de geschakelde systeembenadering om de echte SC dynamiek te beschrijven. Het dynamisch gedrag is volledig beschreven als het geschakelde systeem en wordt uitgebreid met de bestelstrategie, die - zoals eerder vermeld - beschouwd kan worden als een regelstrategie. Bijgevolg worden verschillende types bestelbeleid geïntroduceerd voor een vergelijkende studie waaronder de EOQ (en: Economic Order Quantity) strategie, de aangepaste FO (en: Fractional Order-Up-To) strategie, een PID (en: Proportional-Integral-Derivative) en een MPC gebaseerde regelstrategie.

De prestaties van de bevoorradingsketen worden geëvalueerd aan de hand van een aantal criteria die in Hoofdstuk 3 worden gepresenteerd (waaronder het belangrijke ‘*bullwhip*’ effect evenals kost-gebaseerde maatstaven). Verder ligt de nadruk op deze laatste genoemde criteria omdat 1) deze kwantitatief zijn en dus objectief en direct bruikbaar zijn voor evaluatie en vergelijking; 2) een veelzijdige index vormen en dus een maatstaf voor zowel de efficiëntie als de effectiviteit van de SC. Naast de meest gebruikte maatstaf (de variantieversterking) worden ook enkele aanvullende indicatoren besproken om dit fenomeen volledig te beschrijven. In de

huidige literatuur, wordt het ‘*bullwhip*’ effect meestal gekwantificeerd gebruikmakend van statistische methodes. In dit proefschrift wordt echter een gelijkwaardige uitdrukking als ‘*bullwhip*’ maatstaf afgeleid via een regeltechnische aanpak, door discrete Fourier transformatie en (inverse)  $z$ -transformatie toe te passen als de vraag een stationair stochastisch signaal is.

Hoofdstuk 4 presenteert een gedetailleerde formulering van MPC voor het voorraadbeheer probleem met als doel het beperken van het ‘*bullwhip*’ effect. Om de kwantificering te vergemakkelijken worden de resultaten van Hoofdstuk 3 toegepast. De MPC regelaar is opgebouwd zodat hij gelijkwaardig is aan EPSAC (en: Extended Prediction Self-Adaptive Control), wat leidt tot een gesloten-vorm oplossing bij afwezigheid van regelbegrenzungen. Daardoor kan een overdrachtsfunctie voor de MPC-gebaseerde bestelstrategie bepaald worden. De voorgestelde kwantificering-methode levert een analytische uitdrukking op voor de ‘*bullwhip*’ maatstaf.

Het ontwerp van een passend beleid voor voorraadbeheer speelt een essentiële rol in het temperen van voorraad instabiliteit en het ‘*bullwhip*’ effect. Verschillende beperkingen worden vaak aangetroffen in werkelijke operaties doordat managers verplicht zijn om rekening te houden met de fysieke restricties. Om dit probleem aan te pakken, behandelen Hoofdstukken 5 en 6 in parallel het voorraadbeheer voor het benchmark SC systeem door gebruik te maken van een EPSAC-gebaseerd voorraadaanvulling beleid. Hoofdstuk 5 schetst de netwerkregelstructuren die uit praktische overwegingen in SC’s volgen, zoals topologische structuren, systeem dynamica, regeldoelstellingen en regelmodellen. De specifieke implementaties van MPC regeling voor SC worden voorgesteld als gedecentraliseerde, gecentraliseerde en gedistribueerde regeling. Dit hoofdstuk behandelt de MPC strategieën voor het gedecentraliseerde en gecentraliseerde voorraadaanvulling beleid. De pragmatische MPC oplossingen worden besproken met het inacht nemen van praktische operationele beperkingen. De gedecentraliseerde MPC strategie is gebaseerd op het EPSAC algoritme met begrenzingen en een SISO (en: Single Input Single Output) model voor de voorspelling van de voorraadpositie en een prestatie-index voor de optimalisatie waarbij de economische factoren gecombineerd worden met de dynamische prestaties. In het kader van gecentraliseerde MPC, optimaliseert één enkele regelaar het globaal proces, rekening houdend met een MIMO (en: Multiple Input Multiple Output) model dat de voorraadpositie te voorspelt.

Een originele strategie voor het voorraadbeheer die op gedistribueerde MPC beginselen gebaseerd is, wordt in Hoofdstuk 6 besproken. Het globale SC netwerk wordt in een reeks van op elkaar inwerkende subsystemen ontleed en elk van hen wordt door een regelaar bestuurd die de lokale beslissingen bepaalt. De lokale regelaar implementeert een iteratief EPSAC algoritme om de gehele SC het Nash evenwicht te laten bereiken. Hij behoudt de topologische flexibiliteit van de gedecentraliseerde regeling maar verbetert de prestaties.

De voorgestelde voorraadbeheerstrategie in de Hoofdstukken 5 en 6 toont aan een duidelijke verlaging van de operationele kosten en een aanzienlijke toename van klant tevredenheid in vergelijking met de resultaten van de conventioneel voorraadbeheer. Deze strategie kan een geschikt alternatief zijn voor de industriële prak-

### *Nederlandse Samenvatting*

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tijden van de bevoorradingketen.

De belangrijkste bijdragen van dit werk zijn opgenomen in Hoofdstuk 7, en worden samen met een aantal toekomstige onderzoeksrichtingen besproken.

## English Summary

The past three decades have witnessed a transition of industrial goods production from local or national level to global outreach that serves international markets. This development has led to a significant paradigm shift of modern business management, i.e. individual businesses no longer participate as solely independent entities, but rather as supply chain members. Almost all of today's enterprises use supply chains (SC) to improve their flexibility, reliability and responsiveness to the changing market requirements. The management of multiple relationships across the supply chains such as organizational and functional aspects, the flow of materials, information and finances relies on SCM (Supply Chain Management). The basic idea behind SCM is to integrate organizational units (suppliers, manufacturers, distributors and retailers etc.) and coordinate flows of material & finance, and information in order to increase the competitiveness of the supply chain. All movements in supply chain management involves designing, monitoring, planning and controlling the supply chain activities aiming to satisfy the customers' demand, to create profit, to build a competitive infrastructure and to improve global performance.

SCM poses many challenging research problems worth of investigation. An essential underlying premise of SCM is that there are certain management components that are common across all business processes. The management of these common components is important since they determine how the supply chain is managed and structured. Rather than dealing with every SCM component, this thesis discusses the issue of integrating the business processes in the supply chain related to inventory planning and order fulfillment. This thesis advocates developing inventory management policies based on control oriented formulations on account of the resemblance between supply chain networks and the dynamic systems. The focus of the inventory management policies is to maintain the inventory to target level at each echelon by ordering products from the upstream suppliers in order to satisfy the customers' demand. The downstream flow rates of the products within supply chain network depend on the customer demands, the upstream flow of ordering information, and the policies that every echelon manager uses to place orders and to replenish its inventory. In such a case, the ordering policy can be viewed as a control strategy. This particular issue is thus called the inventory control problem.

The type of inventory policy has a significant effect on the variability of ordering quantities and inventory levels at various echelons of a supply chain. An undesired observation known as the bullwhip effect leads to excessive oscillations of the inventory and order levels, which has been recognized in SCM literature as one of the chief barriers in improving supply chain performance. Therefore, this thesis also presents how to quantify and mitigate the bullwhip effect when the Model Predictive Control (MPC) strategy is introduced into the ordering policy.

Within the broad spectrum of system theory and control engineering, this thesis is

particularly interested in the integration of MPC principles to the inventory management. MPC is a promising control methodology that can meet the needs for controlling and coordinating a large-scale SC network. Despite of rapid advances in the past two decades, there is still a large unexplored research area in SCM. Especially because a very limited number of reports on MPC applications to SCM showed up sporadically during the last two decades, employing MPC in SC control is becoming timely and crucial. The purpose of this thesis is to fill the gap in the current literature work, summarized hereafter.

Chapter 1 describes the research background regarding the supply chains and why MPC became a natural solution to deal with the inventory control problems. A state-of-art in the field of MPC control for supply chains is given and the objectives and original contributions of this work are stated.

Instead of exhaustively enumerating and modeling all types of SC networks with the required rigor and detail, Chapter 2 makes a baseline treatment to the SC topology and suggests a systems approach to modeling a benchmark SC as conceptual framework. The motivations of choosing this benchmark supply chain are discussed. The chapter continues to characterize the structure, assumptions and variables of the supply chain from point of view of process system engineering to facilitate the subsequent dynamics analysis and modeling, which consists of the development of mathematical expressions to capture typical SC's behavior and the switched system approach to describe the real-world SC dynamics. The complete description of dynamics is realized by the switched system when it is complemented with the ordering policies. As mentioned previously, the ordering policy is also regarded as control strategy. Consequently, different types of ordering policy are introduced for comparison purpose including classical EOQ (Economic Order Quantity) policy, FO (Fractional Order-Up-To) policy, PID-based (Proportional-Integral-Derivative) and MPC ordering policies. These results provide the basis for the next steps of inventory control task.

The metrics for evaluating supply chain performance are presented in Chapter 3 including significant bullwhip effect measures and cost-based measures. The former are especially stressed because 1) being quantitative it is objective and direct for evaluation and comparison, 2) being multi-faceted index it is a measure that can assess both efficiency and effectiveness aspects of SC performance. In addition to the most commonly used measure, i.e. the variance amplification ratio, some complementary indicators are also presented to fully describe this phenomenon. Rather than quantifying the bullwhip effect using statistical methods as previous literature did, we derive equivalently the expression of bullwhip metric via control-theoretic approach by applying discrete Fourier transform and (inverse)  $z$ -transform when the demand is stationary stochastic signal.

Chapter 4 details a MPC formulation of inventory control problem to mitigate the bullwhip effect aiming to facilitate the quantification by directly employing the previous chapter's results. The MPC controller is constructed as equivalence to the Extended Prediction Self-Adaptive Control (EPSAC), leading to a closed-form solution in the unconstrained case hence the transfer function for MPC-based ordering policy



can be determined. The proposed quantification method yields an analytical expression of the bullwhip metric.

The design of an appropriate inventory management policy plays an essential role in tempering inventory instability and bullwhip effect. Several constraints are commonly encountered in actual operations so managers are required to take these physical restrictions into account. To address this issue, Chapters 5-6 proceed in parallel to treat the inventory control for the benchmark SC system by utilizing EPSAC based replenishment policies. Chapter 5 outlines the network control structures due to practical considerations in SCs such as topological structures, system dynamics, control objectives and control models. The specific implementations of MPC scheme to SC are depicted as decentralized, centralized and distributed control, which assigns the ensuing tasks of inventory management policies. This chapter deals with the MPC schemes for decentralized and centralized replenishment policies. The pragmatic MPC solutions with consideration of practical operational constraints are discussed. The decentralized MPC strategy is based on constrained EPSAC algorithm relying on a SISO (Single Input Single Output) model for inventory position prediction and a performance index for optimization that combines both economic factors and dynamic performance characteristics. In the centralized MPC replenishment framework, a single controller relying on a MIMO (Multiple Input Multiple Output) model to predict inventory position optimizes globally and finds the optimal ordering policies for each node. An original decision framework based on distributed MPC principles for the inventory control is presented in Chapter 6, where the whole SC network is decomposed into a series of interacting subsystems and each of them is controlled by an agent who determines the local ordering decisions. The local agent implements an iterative EPSAC algorithm in order to reach the whole SC to Nash Equilibrium. It maintains the topological flexibility of decentralized control but improves the performance. To conclude, the proposed replenishment policies in these chapters may be appropriate for industrial practice because they can bring an obvious reduction on operating cost and a significant increase of customer satisfaction level compared with that of the conventional replenishment policies.

The main contributions of this work are concluded in Chapter 7, along with some further research directions.



# List of Acronyms

## A

ABO	Average Backorder
AEI	Average Excess Inventory
AMCS	Average Measure of Customer Satisfaction
APIOBPCS	Automatic Pipeline Inventory and Order Based Production Control System
APVIOBPCS	Automatic Pipeline Variable Inventory and Order Based Production Control System
ARIMA	Auto-Regressive Integrated Moving Average
ARMA	Auto-Regressive Moving Average

## C

CV	Coefficient of Variation
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## D

DC	Direct Current (static gain)
DMPC	Distributed Model Predictive Control

## E

EOQ	Economic Order Quantity
EPSAC	Extended Predictive Self-Adaptive Control

## F

FIR	Finite Impulse Response
FO	Fractional Order-Up-To
FRF	Frequency Response Function

## I

iIL	initial Inventory Level
iWIP	initial Work-In-Progress
IMC	Internal Model Control
IOBPCS	Inventory and Order Based Production Control System
IP	Inventory Position
ISHS	Infinite Supply and High Stock
ISLS	Infinite Supply and Low Stock

## L

LAN	Local Area Network
LS	Limited Supply

### *List of Acronyms*

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LTI	Linear and Time-Invariant
<b>M</b>	
MILP	Mixed Integer Linear Programming
MIMO	Multiple Input Multiple Output
MLD	Mixed Logical Dynamical
MPC	Model Predictive Control
MSE	Mean Squared Error
<b>O</b>	
OUT	Order-Up-To
<b>P</b>	
PID	Proportional-Integral-Derivative
PTT	Production/Transportation Time
<b>Q</b>	
QP	Quadratic Programming
<b>R</b>	
RHS	Right Hand Side
<b>S</b>	
SC	Supply Chain
SCM	Supply Chain Management
SISO	Single Input Single Output
SKU	Stock Keeping Units
<b>T</b>	
TF	Transfer Function
<b>V</b>	
VAR	Variance Amplification Ratio
VIOBPCS	Variable Inventory and Order Based Production Control System
<b>W</b>	
WIP	Work-In-Progress

# 1

## Introduction

### 1.1 Research Background

Over the last decades, modern business has witnessed a transition of industrial goods production from local or national level to global outreach that serves international markets. This development has put a substantial stress on the supply chain of today's enterprises since it is widely used by companies to improve their flexibility, reliability and responsiveness to the changing market requirements. A supply chain system is an integrated network of highly interconnected facilities and distribution channels that function together to:

- (1) acquire raw materials;
- (2) transform the raw materials into intermediate and final products;
- (3) add values to these products and distribute them to retailers or customers;
- (4) promote information exchange among various business entities [1].

The supply chain (SC) is typically characterized by a set of interrelated structures such as organizational, functional, informational and financial. Decisions in all the structures are interrelated too. Moreover, their structures are subject to changes. To reduce unnecessary complexity in analysis, this thesis highlights the counter-current flows of finance & materials, and information, where the finance accompanying the products flows from the raw material suppliers through the manufacturing and distribution facilities to the end customers, while information propagates from the end customers upstream to the suppliers in the forms of demand and order.

Since 1982 when the term supply chain management (SCM) was first introduced [2], it has been investigated in various aspects. Around 1990s, academics first defined SCM from a theoretical standpoint [3] to clarify the difference to more traditional ways of managing the flow of materials and the associated flow of in-

## 1.1 Research Background

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formation. Generally speaking, academia is leading rather than following business practice regarding SCM. SCM has become an independent scientific discipline and has been as one of the key management functions in enterprises [4, 5]. However, there is still no consistent definition of the SCM till now. The Council of Supply Chain Management Professionals [6] gives the official definition of supply chain management as follows:

*“Supply chain management encompasses the planning and management of all activities involved in sourcing, procurement, conversion, and all logistics management activities. Importantly, it also includes the coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies. Supply chain management is an integrating function with primary responsibility for linking major business functions and business processes within and across companies into a cohesive and high-performing business model. It includes all the logistics management noted above, as well as manufacturing operations, and it drives coordination of processes and activities with and across marketing, sales, product design, finance and information technology.”*

There are other similar definitions commonly accepted by academics, for example, [7, 8] consider SCM as

*“a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements.”*

In the definition of [9], SCM is

*“the task of integrating organizational units along a supply chain and coordinating material, information and financial flows in order to fulfill (ultimate) customer demands with the aim of improving the competitiveness of a supply chain as a whole.”*

From the above definitions, the overall goal of SCM is to integrate organizational units and coordinate flows of material & finance, and information so that the competitiveness of the supply chain is improved. Thus SCM deals *a priori* with the materials transfer and information processing so that SC can respond to uncertainties in customer demand without generating excessive inventories [10]. Such integration of material and information flows at various echelons of the SC becomes increasingly important, as this collaborated perspective changes traditional channel arrangements from loosely linked independent business units toward a managerial coordination. The outlook for global supply chains is under constant review since

the global economic climate has made the financial consequences of SCM even more important than before.

Supply chain management involves various decisions at three hierarchical levels with respect to the time scales (length of planning horizon) and their impacts (the importance of the decisions to be made): long-term strategic planning, mid-term decisions and short-term execution [11, 12]. The last two are called operational planning level. Some authors term the second level tactical, but as this notion has several contradictory meanings in the literature, it is not used in this thesis.

- The *strategic planning* considers long-term decision making for the supply chain, which determines the business goals of the supply chain and prepares the resources to achieve this objective, such as the SC network design, facilities locations, etc. Decisions at this level have a significant impact on the supply chain over years into the future.
- The *mid-term management* deals with tactical decisions about how to do to ensure the effective and efficient utilization of the resources from the strategic level decisions. The typical decisions are updated from once a few weeks to once a few years, include production and distribution planning, inventory policies, etc.
- On a short time range, *operational decisions* with high details are made to implement the operations and tasks in order to fulfill the objective at the mid-term level. The operational level decisions involve a variety of decisions which are related to optimally operating the supply chain, such as inventory management, shipping policies and ordering policies design.

The SCM problems addressed in this thesis mainly cover the decision makings in operational level. In the major part of the literature SCM employed only heuristics or mathematical programming techniques to control the simplified representations of the real SC systems [13]. It is becoming increasingly difficult for the companies, with only heuristic decision-making tools, to compete on a global level. In many corporations, management has reached the conclusion that optimizing the operations cannot be achieved without implementing a systematic approach to the business. As indicated by the thorough literature reviews [14, 15], increasingly more methods and techniques from control engineering are now utilized in designing SCM strategies to accomplish various goals. The methods that have been employed to study SC are interdisciplinary by nature. From the control point of view, they are developed from model-free controllers to model-based controllers and from heuristic controllers to optimization-based controllers. This thesis emphasizes applying control system theory to the modeling and analysis of the decision-making problems for SC. More specifically, it focuses on how the decision policies inspired from process control can be effectively applied for SCM problems in the presence of uncertainties, as is typical in industrial practice.

## 1.1 Research Background

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The SCs are viewed as complex dynamic systems subject to various disturbance-based random/deterministic disruptions so the controllers are designed to cope with these changes, the objectives of which are to react to deviations from the goal in system's behaviour. In order to be able to control a SC system, one must specify the goal or the required system's performance, and a set of rules that determines how the control system should react given the deviations of its performance from the goal. There are some advantages of applying control theoretic approaches to meet this requirement. Above all, well-designed control formulation has an intrinsic mechanism to reject disturbances. Some sources of uncertainties are very difficult to influence or alter by nature in SCs, e.g. customer demand fluctuation (although the suitable pricing strategy or advertising incentive may result in customer demand change). Fortunately, the control engineering methods can provide good decision support in adapting and hence minimizing the impact of these uncertainties. Moreover, the control of SC system relies on a formal model, which renders a quantitative analysis of control strategy and leads to a clear understanding of the behaviour for system elements. In addition, control theory can serve as a unified methodical basis for solving various SC planning and control problems. Therefore, control theory knowledge is a natural choice to handle SC dynamics and in fact it is regarded as the next crucial step for exploring SCM [16]. In recent years, the works on SCM have been broadened to cover the whole SC dynamics, planning and control. Under these settings, control theory is becoming of a greater interest to SC researchers and practitioners [17-20].

Within the broad spectrum of system theory and control engineering, this thesis is particularly interested in the application of model predictive control to the SCM due to the wide acceptance of its ability to deal with constraints and nonlinearity of multivariable dynamic systems. MPC is a viable approach to cope with some SC planning and control problems on account of the resemblance of SC networks to dynamic control systems, but some SCM domain-specific modifications are required in many aspects. Recent relevant studies have shown that MPC provide an attractive solution to SCM as it increasingly becomes a popular technique in adaptive planning and formulating decision policies. Several extensive areas of MPC applications are concerned with inventory control, uncertainty analysis, adaptive planning at operational level and so on. It is worth mentioning that MPC can deal with problems comprising different decision levels as long as an integrated predictive control is incorporated in the control algorithm. One thing to note is that in technical control systems, the controllers are devices that adjust the systems' dynamics, which lead to automatic control. Very differently, in SC systems the controllers are devised to provide the support for the human beings, i.e. the real decision-makers). Hence, the applicability of MPC to human-driven SCM demands additional attention from the SC practitioners [21].



## 1.2 State of the Art

The introduction of control engineering methods to management science is hardly new. The Nobel Prize winner Simon [22] seems to have been the first to apply the *Laplace* transform to an analysis of inventory and order-based production scheduling system. The idea was extended to discrete time models [23] with a production and inventory control framework based on  $z$ -transform. In the work [24], the block diagram framework of IOBPCS was proposed. This framework can be modified to represent a family of models for a single node including extensions to discrete-time systems (e.g. VIOBPCS [25], APIOBPCS [26], and APVIOBPCS [27]). The researchers [28] have investigated the bullwhip effect (BW) caused by OUT policy in APVIOBPCS framework using  $z$ -transform and frequency response plot and then extended these results to the case of a centralized supply chain. The serial works [17, 29] have developed an integrated continuous-time approach to modeling a supply chain. This framework is similar to the deterministic supply chain models but uses differential equations to describe their dynamics. A heuristic shipping policy is chosen. They developed a Proportional (P) controller to track inventory, backorder or a combination of both, but they did not suggest any tuning methods for the controllers. Lin et al. [30] presented a discrete time model and an approach to obtain the transfer function for each node using  $z$ -transform. This transfer function approach facilitates the closed-loop stability analysis and controller synthesis. Compared with the works of [17, 29], Lin et al. proposed P and Proportional-Integral (PI) controllers tuning criterion based on frequency domain analysis of the  $z$ -transfer functions. The response and stability conditions of a supply chain system were also studied in [31].

A systematic overview on the application of control theory to the practical SCM problems can be obtained from several excellent review papers [14, 15, 32]. Most of the classical control applications to SCM problems concern linear systems and are performed in the frequency domain, in particular *Laplace* transfer functions and  $z$ -domain transfer functions are used to model the dynamics. It is true that control theory provides a variety of tools/techniques, e.g. fuzzy control, neural network, robust control to analyze, design and control SC systems. But MPC technique is a natural choice to resolve the SCM problems because the SC process itself is suitable for MPC application. The following gives a detailed state of the art on MPC application to SCM in particular.

MPC is nowadays recognized as a standard technique for the control of industrial and process system. It makes full use of a dynamic model to predict outputs and solves a constrained optimization problem over the future outputs to find the best operational decisions. In general, the reports on the application of MPC algorithms in the SCM field are limited in terms of number but vary greatly regarding the problems that have been solved. However, in majority of these works the strategy depicted in Figure 1.1 is applied on-line following the MPC philosophy, i.e. by solving the resulting mathematical programming problem at each discrete time in-

## 1.2 State of the Art

stance, applying on the first decision and moving to a new state, where the procedure is repeated.

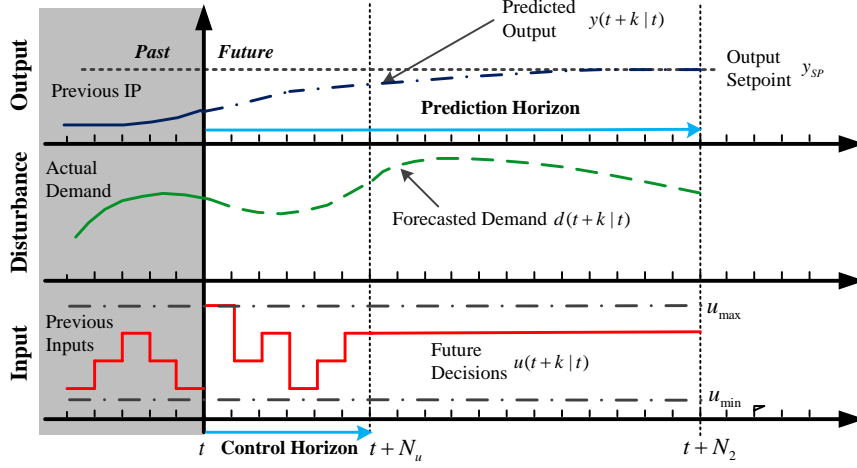


Figure 1.1: The philosophy of model predictive control

The general formulation of the MPC on-line optimization problem can be written as follows. At time instant  $t$  with the prediction over  $(t+k)$  samples, the following finite-horizon optimal control problem is solved:

$$\min_{\{u(t+k|t)\}_{k=0}^{N_u-1}} \left\{ \sum_{k=N_1}^{N_2} \left[ \|y(t+k|t) - y_{sp}\|_{Q_y}^2 \right] + \sum_{k=0}^{N_u-1} \left[ \|u(t+k|t) - u_{sp}\|_{Q_u}^2 + \|\Delta u(t+k|t)\|_{Q_{\Delta u}}^2 \right] \right\} \quad (1.1)$$

$$\text{s.t.} \quad y(t+k|t) = f(y, u, d) \quad (1.2)$$

$$\begin{aligned} u_{\min} &\leq u(t+k|t) \leq u_{\max} \\ y_{\min} &\leq y(t+k|t) \leq y_{\max} \\ \Delta u_{\min} &\leq \Delta u(t+k|t) \leq \Delta u_{\max} \end{aligned} \quad (1.3)$$

where  $y_{sp}, u_{sp}$  are the setpoints (target values) of input and output vectors,  $y(\cdot|t)$  and  $d(\cdot|t)$  are the predictions for the output and the disturbance (i.e. the customer demand) over prediction horizon from  $N_1$  to  $N_2$ ,  $f(\cdot)$  is the function of the SC model describing the dynamics of the system. The default value of  $N_1$  equals to the system time-delay and it equals to 1 when there is no system delay. The design parameter  $N_u$  is the control horizon with  $\Delta u(t+k|t) = 0, \forall k \in [N_u, N_2]$ . The notation

$\|\cdot\|_Q$  stands for the  $Q$ -weighted norm, i.e.  $\|\xi\|_Q = \sqrt{\xi^T \cdot Q \cdot \xi}$ . In this section, a general literature review is conducted from the standpoint of the application of MPC. The selection criteria of the reviewed papers are: using MPC techniques to solve SCM problems, and being quantitative on the analysis of SCM problems.

During the last two decades there have been a limited number of reports on the application of MPC to the SCM problems. It seems that MPC was first applied by Kapsiotis and Tzafestas [33] to inventory management problem for a single manufacturing site. Their subsequent work [34] introduced the concept of MPC used as a decision-making tool for generalized production planning problem. The sales as the controlled variable are coupled with the inventory balance, the advertisement effort and the production levels as the manipulated variables are optimized by an MPC problem. Bose and Pekny [35] concentrated on a decomposition framework (centralized vs. decentralized vs. distributed control) that computes the target inventory by means of a planning model in a prediction horizon and then evaluates a schedule to satisfy them. The idea is to find the target that ensures a desired service level with a planning tool, and then track them with MPC algorithm. The potential of MPC as a good decision-supporting tool for SCM did not receive the deserved attention during the first decade. The researchers' interest in investigating the applicability of MPC in many aspects of SCM problem has increased in the last decade, resulting in a growing number of papers on relevant topics.

Braun et al. [36] presented a partially decentralized MPC framework for managing inventories and meeting customer requirements for a SC originated from semiconductor industry. They showed the control performance is guaranteed under significant uncertainties caused by plant-model mismatch and forecasting error. Perea-López and co-workers [37] modeled the multi-echelon multi-product SC as a MILP framework, which was used as prediction model in MPC framework. The cost function is related to the economic performance measures i.e. maximizing the profit. Seferlis and Giannelos [38] proposed a two-layered control approach for multi-echelon multi-product SC system based on multivariable MPC principles. The prediction of output relied on a state space model and the performance index consisted of four terms to account for transportation cost and inventory cost, penalties on back orders and rate of change for decision variables. A two-loop control technique for the SC optimization problem in semiconductor manufacturing was described by [39]. The outer loop combining strategic planning and inventory planning to generate the inventory and order set points (reference trajectories), while the inner loop based on MPC can track these set points. Lin et al. [40] discussed Minimum Variance Control (it is a MPC configuration in essence) for a single supply node with two separate set points for the actual inventory level and for the WIP level. In their work, the MPC control scheme outperformed classical order-up-to, PI control and APVIOBPCS model in maintaining inventory levels at a desired level while mitigating the bullwhip effect. Mestan et al. [13] addressed an optimal operation for a hybrid (includes continuous/discrete dynamics and logic rules) supply chain system. The decision values are obtained by optimization under central-

## 1.2 State of the Art

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ized, fully decentralized, and semi-decentralized MPC configurations. In the study of Wang et al. [41] the MPC was used as a tactical planning policy in a hierarchical decision framework for the SCM problems associated with semiconductor industry. Wang and Rivera's later work [42] on the same SC systems incorporated MPC and multiple-degree-of-freedom tuning to achieve robustness and performance when stochastic demand was present. In the work of [43], an adaptive MPC strategy was investigated to identify the coefficients of FIR model by RLS method and to control inventory for production-inventory systems. Li et al. [44] adopted robust MPC for a multi-echelon supply chain under uncertainties in both the feedback process and the disturbance forecast. This method optimized an economic objective function. They concluded that the robust MPC can resolve both the model mismatch as well as disturbance uncertainty for optimization problem. The study [45] combined *min-max* optimization and MPC to solve inventory control problems for multi-echelon, multi-product distribution centre. On strategic level, the solution of *min-max* problem was obtained to select such policy parameters as safety stock and delivery cycle time. On the tactical level, the shipment decision was treated as the control input in MPC formulation. The work [46] proposed a robust decision-making tool based on MPC strategy to address uncertainty in demand and model parameters explicitly.

The latest direction toward control problems for SCM is to apply the distributed MPC scheme to the decision-making process, which is a natural choice for SC control since various parties of SC are distributed geographically or logically. Dunbar and Desa [47] studied a three-node supply network and obtained a continuous-time dynamic model. Then they applied distributed nonlinear MPC to the model, in which each node subsystem was regulated by a MPC controller and was optimized locally for its own policy in parallel with the other node subsystems. During optimization process local node subsystem was required to communicate the most recent control profile to those subsystems to which it was coupled. Maestre et al. [48] proposed a distributed MPC scheme based on cooperative game and tried to apply the algorithm to a two-node SC system. Subramanian et al. [32] presented cooperative and parallel algorithms based on distributed MPC structure. The three algorithms were then implemented to a very simple two-node SC as an illustrative example.

The above listed work differs in four aspects:

- The prediction models in Eq. (1.2) are developed specific to the diverse problems. This means not only that different decision variables in SC operations are chosen as the output and control effort but also that various types of mathematical models are employed. These prediction models vary from input-output models in either the most common discrete time [34, 36, 40] or the continuous time [47], to FIR model [43] and state-space models [41, 42, 44, 45]. For optimization purposes, the SC is also modeled within the framework of the hybrid system, e.g. MLD [13], MILP [37].

- Different performance measures are emphasized in the specific problems solved in the literature. They may choose different forms of the objective functions to the quadratic form in Eq. (1.1). Instead of pure control-theoretic point of view in formulating the objective function [36, 40-42, 44, 45, 47], some researchers integrated the economic measure into the optimization process [13, 37, 38].
- Some works considered the physical constraints on the variables (1.3) in their formulation while most of the others just ignore this practical issue.
- The tailored MPC algorithms are developed for their specific problems.

In view of these points, this thesis develops further MPC schemes within SCM context and extends them to be applicable in benchmark cases, leading to general rules of modeling and MPC control for even more complex SC.

### 1.3 Motivations and Objectives

As the SCs are increasingly complex, they pose many practical challenges, such as:

- dynamic inventory control policies
- rejection of disturbances and fluctuations (e.g. bullwhip effect)
- adaptation and real-time control
- multi-objective decision-making

MPC is the promising control methodology that can meet these needs for controlling and coordinating a large-scale SC network. There are several advantages of applying MPC to SCM. Above all, MPC can easily deal with multi-input and multi-output problems with constraints. Therefore, different operational goals (e.g. inventory target tracking, customer demand fulfillment etc.) are able to be controlled and coordinated simultaneously. Since MPC approach is a model-based optimization control strategy, it can combine multiple operational objectives into one control problem. Moreover, due to the rolling horizon procedure, MPC becomes a closed-loop controller that can be adapted in real-time by the feedback information measured from the real-life SC operations. Consequently, MPC has the ability to deal with the uncertainty of the SC, which may be caused by the unpredictable disturbances, the variation over time of the parameters, and model mismatches. Another advantage is that SC engineers can easily select and replace the prediction model based on the control requirements. They also have flexibility to design an objective function that represents a suitable SC performance measure. Finally, the commonly used heuristic or mathematical programming in traditional SCM requires many “what-if” rules to be run and examined by highly skilled professionals. By comparison, the MPC approaches are conceptually different and require less detailed

### 1.3 Motivations and Objectives

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knowledge yet offer the same flexibility in information sharing & network topology (resulting in centralized, decentralized or distributed control structure), constraints (arise from SC practice) that can be handled.

Despite of rapid advances in the past decades, there is still a large unexplored research area in SCM. Especially because a very limited number of reports on MPC applications to SCM showed up sporadically during the last two decades and then employing MPC in SC control is becoming timely and crucial. The concise objectives of this thesis can be formulated as follows:

- (1) to employ the available knowledge from the literature on SCM to select the benchmark structure in order to represent the typical SC. While most SCs could be described by models based on law of conservation and quantified by several states, the necessity arises to introduce dynamics and general modeling framework for this benchmark SC.
- (2) to choose the SC performance measures and to quantify them for evaluation of the SC planning and control that are suitable in the context of MPC application.
- (3) to apply the MPC frameworks in order to obtain the replenishment rules for benchmark SC, using the models obtained in previous objective. To validate them through numerical simulation.

The accomplishment of these objectives requires several specifications:

- Instead of enumerating numerous SCs of different topologies and functions existed in literature, the selected SC must be typical to represent and preserve the dynamics and features of the general SCs. Thus, the benchmark supply chain similar to beer game setup [49] is selected.
- The SC dynamics should describe the most important and practical features of SC operations (the conservation of material and information), the equivalent mathematical models preserving management interpretation must be developed in order to use MPC framework.
- During the practical planning horizon, different elements (decision-makers, products, goals, control variables, constraints, perturbations etc.) are involved in decision-making, but not all of them at the same time. These elements appear and disappear from the decision-making when moving on through the planning period. As such, SC planning and control problems vary greatly from one SC to another at different time period. Among many aspects of SC planning and control, this thesis focuses on inventory control problem by designing the ordering policies.
- The chosen performance measures should reflect the control theoretic objectives and economic goals.

- The MPC-based replenishment frameworks must be designed to suit the informational and organizational requirements of SC, which may result in different control structures. The MPC frameworks will be developed from the standpoint of EPSAC (Extended Prediction Self-Adaptive Control) algorithm.
- Finally, these MPC-based replenishment frameworks should be tested on the MATLAB simulation platform and compared with other replenishment rules in literature.

This thesis presents a solution to each of the above objectives. Rather than resolving a specific case study, the modeling and control are generic methods which can be used not only in the benchmark SC application but also in more complex SCs subject to proper extension. The original contributions are listed in the following section.

## 1.4 Thesis Overview

The seven chapters contained in this thesis offer a detailed investigation of three main problems:

- (1) the dynamics description and mathematical modeling for the benchmark SC network;
- (2) the formulation of supply chain performance measures, especially the detailed discussion of the bullwhip effect;
- (3) the design of MPC frameworks that facilitate quantifying the bullwhip effect, and the application of MPC frameworks to decentralized, centralized and distributed replenishment rules.

Figure 1.2 shows a structural overview of the thesis and gives the opportunity to potential readers interested in a particular chapter to orient. The main content of the chapters is briefly summarized as follows:

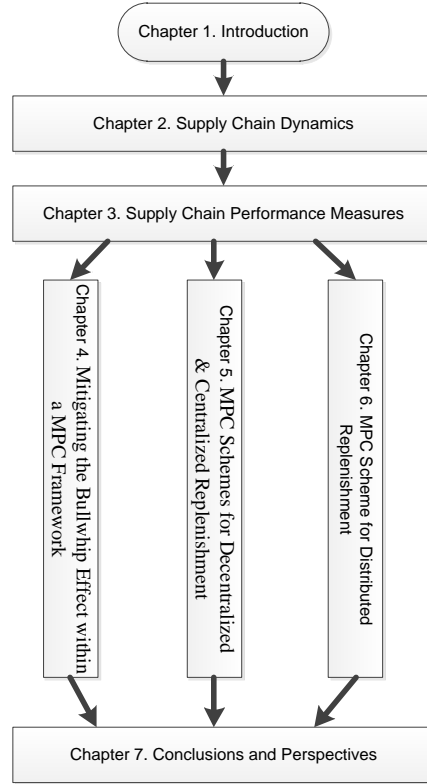
**Chapter 2** makes a detailed description of the dynamics of the benchmark supply chain system. The structure, assumptions and variables of the supply chain are characterized from the process system engineering point of view to facilitate the analysis and modeling. The switched systems are derived to represent the real dynamics. The important issues for SCM such as ordering policies and demand estimation techniques are discussed. These problems form the basis for the next step of work on inventory control and replenishment rules in the following chapters.

A number of important performance metrics to evaluate SC effectiveness and efficiency are proposed in **Chapter 3**. The bullwhip effect metric together with its complementary indicators is stressed and the quantification of bullwhip effect is focused on using the control engineering approach. The economic measures based on cost are also introduced. These metrics and measures are used to assess the per-

## 1.4 Thesis Overview

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formances of the EPSAC-based replenishment rules presented in next chapters.



*Figure 1.2: Structural overview of the thesis*

**Chapter 4** presents the Diophantine approach to MPC formulation to facilitate a closed-form expression of bullwhip effect, using the quantification methods proposed in Chapter 3.

The pragmatic solutions to inventory control of the benchmark SC system with consideration of practical constraints are discussed in **Chapter 5**. The replenishment rule is based on constrained EPSAC algorithm with decentralized and centralized control structure, determined by the real-world SC operational requirements.

**Chapter 6** focuses on an original framework for the inventory control of SC system based on DMPC principles. The whole SC network is decomposed into a series of interacting subsystems and each of them is controlled by an agent which determines the local ordering decisions. The local agent implements an iterative EPSAC algorithm in order to reach the whole SC to Nash Equilibrium. It maintains



the topological flexibility of decentralized control but improves the performance.

**Chapter 7** summarizes the main contributions of the thesis and gives recommendations for future research directions.

The following gives a list of publications and scientific outputs resulting from above chapters:

**Journal articles:**

- Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., 2014. “Decentralized and centralized model predictive control to reduce the bullwhip effect in supply chain management.” *Computers & Industrial Engineering*, 73, 21–31 (Impact Factor 2.086).
- Fu, D., Aghezzaf, E-H., De Keyser, R., 2014. “A model predictive control framework for centralised management of a supply chain dynamical system.” *Systems Science & Control Engineering: An Open Access Journal*, 21, 250–260.
- Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., 2015. “Quantifying and mitigating the bullwhip effect in a benchmark supply chain system by an extended prediction self-adaptive control ordering policy.” *Computers & Industrial Engineering*, 81, 46-57 (Impact Factor 2.086).
- Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., 2016. “A constrained EPSAC approach to inventory control for a benchmark supply chain system.” *International Journal of Production Research*, 54(1), 232-250 (Impact Factor 1.693).
- Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R.. “An efficient, bullwhip reducing, predictive control strategy for inventory management in supply chains.” submitted to *IEEE Trans. on Control Systems Technology*.

**Conference papers:**

- Fu, D., Dutta A, Ionescu, C., De Keyser, R., “Reducing the bullwhip effect in supply chain management by applying a model predictive control ordering policy,” in *Information Control Problems in Manufacturing*, 2012 *14th IFAC Symposium on*, pp. 481-486, 2012.
- Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., “A centralized model predictive control strategy for dynamic supply chain management,” in *Manufacturing, Modelling, Management and Control*, 2013 *7th IFAC Conference on*, pp. 1630-1635, 2013.

## 1.4 Thesis Overview

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# 2

## Supply Chain Dynamics

Market globalization, intensifying competition and an increasing emphasis on customer orientation have increased the demands for future engineers to apply fundamental problem solving skills in non-classical environments. In today's global marketplace, individual business entity no longer competes independently but rather as integral part of supply chains. Consequently the ultimate success of a company depends on its managerial ability to integrate and coordinate SC members. A large body of literature [14, 15, 19, 32] has indicated that the problem of supply chain management benefits substantially from the use of control engineering concepts.

However, there has been relatively little reported work so far. It is necessary to develop normative methods for successful SCM practice. As the work [21] rightly highlighted, the future research direction should focus on developing computational tools and optimization-based methodologies that enable SC managers and engineers to analyze, design and evaluate SCs as dynamic systems. A well-defined SC optimization model needs a detailed dynamic description. The exploratory findings reported in this chapter are research effort to develop a normative model to guide future research.

### 2.1 Introduction

One can find supply chains both in manufacturing and service industries. These SCs can differ greatly in structure and complexity from industry to industry and from individual company to company. There exist many alternative definitions of a SC due to the widespread prevalence and variety of SCs. The misconceptions of using the term supply chain are: (1) It evokes image of product or service moving along a chain from suppliers to customers through manufacturers, distributors and retailers. This is certainly part of the supply chain, but it is also important to visualize information, funds, and product flows along both directions of this chain [50]. (2) This term may also imply one-to-one relationship between nodes. In real world,

## 2.1 Introduction

a SC participant may receive material from several suppliers and then supply several dependents. Thus the term supply chain is somewhat of a misnomer because a supply chain is often not a single or simple chain but a complex network with many divergent and convergent flows [51]. Therefore, SC is typically of a more complex network structure as illustrated in Figure 2.1.

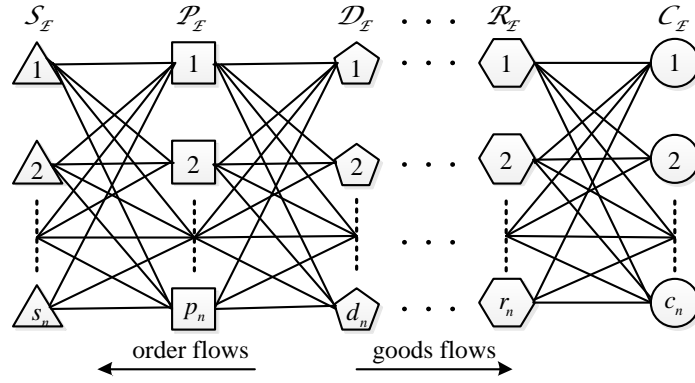


Figure 2.1: The schematic representation of a general supply chain network

### Nomenclature

#### Indices/sets

$i \in \mathcal{N}$	node/echelon index
$t$	the supply chain planning period
$\mathcal{N}$	set of echelons/nodes
$\mathcal{I}(t)$	full information set for the whole SC at time instant $t$
$\mathcal{I}_i(t)$	information subset for echelon $i$ at time instant $t$

#### Variables

$L^i$	lead-time of echelon $i$
$y_3^i$	standing order of the echelon $i$
$y_2^i$	measured inventory at echelon $i$
$y_1^i$	measured inventory position at echelon $i$
$u_1^i$	orders placed by the $i$ th echelon to

$u_2^i$	the $(i+1)$ th echelon delivery to downstream echelon $i-1$
$d_1^{R_i}$	end-customer demand
$d_1^i$	demand from echelon $i-1$
$\hat{d}_1^i$	demand estimate for echelon $i$
$\hat{\sigma}^i$	standard deviation of the demand for echelon $i$
$\hat{D}_{L^i+1}^i$	the estimate of mean demand over $L^i + 1$ periods
$\hat{\sigma}_{L^i+1}^i$	the estimate of standard deviation of the demand over $L^i + 1$ periods
$d_2^i$	received product by the $i$ th echelon
$BO^i$	backorder of the $i$ th echelon
$WIP^i$	work-in-progress of the $i$ th echelon
$w^i$	IP setpoint of subsystem $i$
$w_1^i$	the setpoint of WIP in FO policy

$w_2^i$	the inventory setpoint for FO policy	$T_m^i$	the exponential smoothing filter the number of the review periods
$e^i(t)$	error signal in PID-based ordering		within the window size for the moving average filter
<b>Parameters/Notations</b>		$\beta^i$	adjustable parameter to regulate the response to WIP discrepancy in FO policy for the $i^{th}$ node
$\delta$	the review period	$\eta^i$	adjustable parameter to regulate the response to inventory discrepancy in FO for the $i^{th}$ node
$*(z^{-1})$	polynomial $*$ in terms of the backward shift operator $z^{-1}$	$K_p^i$	the gain of PID control
$s$	threshold value of EOQ policy	$\tau_I^i$	integration time of PID control
$S$	OUT value of EOQ policy	$\tau_D^i$	differentiation time of PID control
$Q$	order quantity in $(s, Q)$ policy		
$\gamma^i$	desired service level for echelon $i$		
$\alpha^i$	exponential smoothing parameter for demand forecaster of the $i^{th}$ node		
$T_a^i$	the average age of demand data for		

The general supply chain is a network of interconnected multiple organizations, which contributes to move product or service from source nodes of supplier echelon  $S_E$  to destination nodes of customer echelon  $C_E$ . There may be multiple echelons between the initial source  $S_E$  and the final sink  $C_E$  such as production echelon  $P_E$ , distributor echelon  $D_E$  and retailer echelon  $R_E$ . The interactions among nodes of SC network can be represented by a framework of bi-directed graph  $G=(V, S)$ , where a set of vertexes  $V = \{1, \dots, m\}$  matches all the nodes and a set of directed arcs  $S \subseteq \{(i, j) \in \mathcal{V} \times \mathcal{V} | i \neq j\}$  represents couplings (interactions). An arc  $(i, j) \in S$  indicates the product transfer along downstream direction and ordering information flow along upstream direction as specified in Figure 2.1. Successful SCM requires managers to have an explicit knowledge and understanding of how the SC network structure is configured. Dimensions to consider include the length of the supply chain and the number of suppliers and customers at each echelon. More specifically, there are three primary aspects of a company's SC network structure [52]:

- *the members of the supply chain*: The SC members include all organizations/firms with which the focal company interacts directly or indirectly through its suppliers or customers, from upstream point of origin to downstream point of consumption. Including all types of members may cause the network to become too complex to be manageable. The appropriate way of solving this problem is to identify which members are critical to the success of the SC and thus should be allocated managerial attention and resources. Thus primary members and supporting members of a supply chain are distinguished [53] to provide a managerial simplification and yet this distinction captures the

## 2.1 Introduction

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essential aspects of who should be considered as key members.

- *the structural dimensions of the network*: Two structural dimensions are essential when describing, analyzing and managing the supply chain. The first dimension defined as horizontal structure refers to the number of echelons. The second dimension, vertical structure, refers to the number of suppliers or customers involved within each echelon. An additional structural dimension is the company's horizontal position within SC. A company can be positioned at or near the point of origin, be at or near the point of consumption, or somewhere between these end points.
- *the different types of process links across the chain*: The key SC processes are customer service management, demand management, order fulfillment and manufacturing flow management etc. Integrating and managing all business process throughout the entire supply chain is likely inappropriate. As a consequence, the scarce resources are only allocated to the critical ones in order to achieve specific objectives.

Having these ideas in mind, the supply chain networks are known as process systems that share some commonalities. Instead of exhaustively enumerating and modeling all types of SC networks with the rigor and detail required, this thesis suggests a system approach to modeling a benchmark SC as conceptual framework. The motivations of using a benchmark SC network in Figure 2.2 are as follows:

- 1) there is no universal framework or design for supply chain networks due to the reasons mentioned above. The topic of SCM is multidisciplinary by nature and it has been subject of a growing body of literature with associated research being conceptual as well as analytical. A number of simplified SC reference models have been employed for a theoretical investigation of certain aspects of SC and for an explanation of observed phenomena [49]. However, the fundamental nature of supply chains does not change with varying numbers of echelons involved. These models share somewhat important similarities to this benchmark SC in term of network structure. The specific SC network in Figure 2.2 is suitable since it properly reflects the structural and behavioral aspects of a supply chain.
- 2) a typical supply chain may involve a variety of echelons which are customers, retailers, wholesalers/distributors manufacturers and component/raw material suppliers [50]. Every echelon need not be present in a supply chain. The appropriate structure of the supply chain depends on both the customer's needs and the roles played by the stages involved. The SC structure in this thesis has been used extensively in recent years to analyze and design various aspects of SC structure and behavior because the generic dynamic features are kept while it is easier to manipulate than the real-world SC system.

- 3) despite a nearly infinite variety of SC systems, an understanding of their most important characteristics needs to be developed from an operational point of view. A supply chain is dynamic and involves the counter-current flows of material (product), information (order) and funds (financial values accompanying the material flow in opposite direction) between different echelons. The intrinsic dynamics of the SC members are adequately retained in this benchmark structure. It also yields enhanced insight into the behavior of the real-world systems.
- 4) the thesis contributes to the SCM literature by examining the classic four-echelon supply chain schematic. Researchers since [49] have coined the expression of the ‘Forrester Supply Chain’ or ‘Forrester Model’ which essentially is a four-echelon supply chain. This baseline treatment to the SC structure can also be thought of as a representation of beer distribution game [54-57]. The Beer Game, developed at the MIT Sloan School of Management in the 1960's, is a role-playing simulation widely used to clarify the concepts of managing the supply chain. It is widely used in management schools as a tool to teach students the causal relationships between decision-making and the SC behavior.

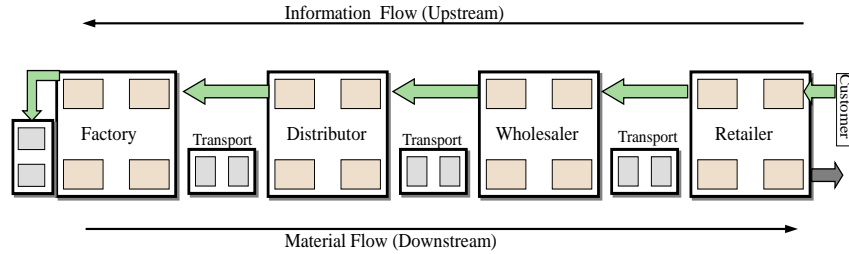


Figure 2.2: Network structure of the benchmark supply chain

To conclude, there exists a great diversity of supply chains in the real world, nevertheless the structure of supply chain under this study is generic, i.e. it has standard constituents and it captures the basic characteristics of a general SC network. The following control and decision strategies will have to exploit the further benefits of benchmark supply chain system. A dynamics analysis and modeling to facilitate the control of this SC network are to be formulated in next sections.

## 2.2 Supply Chain Dynamics and Modeling

The supply chain network is an open complex system whose constituent parts include material suppliers, production facilities, distribution channels and customers linked together via the feed-forward flow of materials and the feedback flow of information. The real logistical operation of a supply chain is inherently a discrete dynamic system, and it is more convenient to use a  $z$ -transformation technique. This section analyses the SC system dynamics from the system's interior microscopic structure, meanwhile it studies inner links between dynamic behavior and system structure.

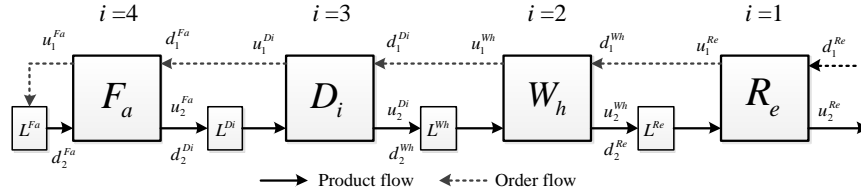


Figure 2.3: The block diagram of the benchmark production-distribution supply chain

The SC network under study is a cascade production-distribution system, which shares the similar structure to the benchmark systems used in [19, 55-60]. The system depicted in Figure 2.3 consists of three logistic echelons between the factory and the end customers: a distributor, a wholesaler, and a retailer. In practice, the SC systems with multiple nodes in each echelon can be found and there is no universal framework or topology could describe all types of SCs. However, all of them share some very important characteristics in dynamics, e.g. they all involve counter-current flow of information and material. Orders for products propagate upstream from customers to factory, and products are shipped downstream along the opposite direction. Although the real-life supply chains tend to be multi-product, multi-echelon supply networks, the benchmark is typical of the predominant real-life SC networks and it is also the simplest case to analyze the dynamics in control engineering point of view. Thus running this supply chain would still be valid, and the methods in this thesis can be easily extended to the case of multiple nodes at the expense of extra computational effort.

### 2.2.1 Supply Chain Process Description

For sake of simplicity, it is assumed that operational decisions are made at equally spaced time instants  $t \cdot \delta$ , where  $\delta$  is the review period and  $t=1, 2, \dots$ . The duration



and unit of base time periods  $\delta$ , e.g. days, weeks or months depends on the dynamic characteristics of specific supply chain network. While developing dynamic models, the components of the supply chain are named as nodes. The SC network uses arcs and vertexes to depict the connections between various nodes as shown in Figure 2.1. The set of nodes is defined as  $\mathcal{N} := \{R_e, W_h, D_i, F_a\}$ . An arbitrary node's position in the network is marked by  $i$  ( $i=1, \dots, 4$ ). Following the rules of this notation system,  $(i+1)$  indicates an immediate upstream node of  $i$ th node and  $(i-1)$  an immediate downstream node. Thereby, for this specific supply chain,  $i=1$  and  $i=4$  denote respectively the retailer ( $R_e$ ) and factory ( $F_a$ ). The terms 'node' and 'echelon' can be used interchangeably since there is only one node at each echelon.

As depicted in Figure 2.3, any arbitrary node  $i \in \mathcal{N}$  is associated with four variables and their SC denotations are defined in nomenclature. Additionally, any node is linked to three states  $[y_1^i(t), y_2^i(t), y_3^i(t)]$ . More specifically, the inventory  $y_2^i(t)$  is the amount of products in stock of node  $i$  at any instant  $t$ . Due to the lead time  $L^i$  for shipment, another state  $y_1^i(t)$  termed as inventory position is defined to track how the ordering decisions affect later inventory level. It includes goods in both of inventory and WIP. The ordering information is communicated instantaneously, but an order placed at time  $t$  can only be processed at time instant  $t+1$  because of administrative processing. Therefore, the standing order  $y_3^i(t)$  for each node is defined as the amount of order to be processed at time  $t+1$ . All these variables are assigned in a process control sense.

The end-customer demand  $d_1^{R_e}(t)$  enters the SC process as disturbance and it is modeled *a priori* as a bounded function of time:  $0 \leq d_1^{R_e}(t) \leq d_1^{\max}$ . Notice that this definition of the demand is quite general and it accounts for any standard distribution typically analyzed in the considered problem. It is assumed that the demand process takes an ARMA time series model driven by white noise. This treatment to demand model is to simplify the exposition of the thesis. However, the demand model in real-world has to be obtained from the recorded data via system identification techniques.

After the description of SC process, the next subsections address the problem of dynamics analysis for benchmark SC network. To achieve this goal, the following strategy consisting of three elements is proposed: (1) the characterization of supply chain from the process system engineering point of view; (2) the development of dynamic models to capture typical behavior of supply chain; (3) the switched system approach to describe the real-world supply chain dynamics.

### 2.2.2 The Supply Chain Node Dynamics

Supply chains operate as 'pull systems' driven by orders between their nodes.

## 2.2 Supply Chain Dynamics and Modeling

More specifically, the linkage among supply chain nodes consists of material, information, and financial flows. Thus overall supply chain dynamics obey the following principles. The end-customer demand arises at  $R_e$ ,  $R_e$  replenishes its stock from  $W_h$ ,  $W_h$  from  $D_i$ , and finally  $F_a$  manufactures the product. This sequential distribution process is managed by echelon managers: manager  $i$  decides how much to order from echelon  $(i+1)$ . Information in the form of replenishment orders spreads from downstream to upstream, triggering product flow in an opposite direction. Two flows are subjected to time delays: the orders placed by echelon manager  $i$  are processed by  $(i+1)$  at next review period, and all shipments of product from  $(i+1)$  to  $i$  arrive after a reliable transportation time  $L^i$ , both are deterministic and fixed over time horizon of interest. The lead time can be acquired from SC managers or can be estimated from the gathered data. With consideration of the nominal ordering delay, the orders submitted to node  $(i+1)$  at time  $t$  can only be available to node  $i$  at time  $t+L^i+1$ . Under assumption of a well-coordinated manufacturing plan with ideal raw material supply, the production model is approximated by a delay unit of  $L^{F_a}$  being the production lead time.

To illustrate micro-dynamics of supply chain network, take any echelon  $i \in \mathcal{N}$  and consider the information and material exchange with its supplier and customer. There are four events performed within every review period  $t$  under the following sequence of operations:

- The  $i$ th echelon receives goods in the amount of  $d_2^i(t-L^i)$  from its supplier.
- The demand  $d_1^i(t)$  from downstream echelon  $(i-1)$  is observed.
- The delivery  $u_2^i(t)$  is made to satisfy the standing order  $y_3^i(t-1)$ , any unfilled demand will not be lost but backlogged until deliveries can be made.
- The manager of the  $i$ th echelon updates standing order  $y_3^i(t)$  and inventory  $y_2^i(t)$ , and with the new information on echelon state, places an order  $u_1^i(t)$  to the upstream echelon  $i+1$ .

The shipment decision  $u_2^i(t)$  from  $i$ th echelon to  $(i-1)$ th echelon plays an important role in accurately representing system dynamic response. For the actual delivery ‘the best one can do’ policy [37] is adopted in practical supply chain operations, i.e. when sufficient inventory is available at current echelon  $i$  the actual delivery will be  $u_2^i(t) = y_3^i(t-1)$ . Otherwise, the whole stock has to be delivered  $u_2^i(t) = y_2^i(t-1)$ . Therefore, the actual delivery policy is defined as:

$$u_2^i(t) = \begin{cases} y_3^i(t-1) & 0 \leq y_3^i(t-1) \leq y_2^i(t-1) \\ y_2^i(t-1) & 0 \leq y_2^i(t-1) \leq y_3^i(t-1) \end{cases} \quad (2.1)$$

The customer satisfaction of current node  $i$  is indicated by backorder defined as the difference between the total standing order at  $(t-1)$  and the amount of goods actually delivered at  $t$  :

$$BO^i(t) = y_3^i(t-1) - u_2^i(t) \quad (2.2)$$

The larger the backorder, the poorer is the customer satisfaction.

The inventory position  $y_1^i(t)$ , inventory  $y_2^i(t)$  and standing order  $y_3^i(t)$  satisfy law of conservation according to product and order flows. The discrete time models that describe the  $i$  th echelon's dynamics are:

$$y_1^i(t) = y_1^i(t-1) + d_2^i(t) - u_2^i(t) \quad (2.3)$$

$$y_2^i(t) = y_2^i(t-1) + d_2^i(t-L^i) - u_2^i(t) \quad (2.4)$$

$$y_3^i(t) = y_3^i(t-1) + d_1^i(t) - u_2^i(t) \quad (2.5)$$

The difference equations (2.1)-(2.5) can be solved directly in the time domain or by using transformation techniques. In particular, the  $z$ -transform is the most desirable transformation because it converts the difference equations into algebraic ones. In this thesis,  $z^{-1}$  is used to refer to the backward shift operator and  $z$  complex variable in  $z$ -transform. When applied to time-dependent signal  $s(t)$ ,  $z^{-k}$  becomes backward shift operator (e.g.  $\mathcal{Z}\{s(t-k)\} = z^{-k} \mathcal{Z}\{s(t)\} = z^{-k} S(z)$ ). Then the supply chain models in the  $z$ -domain are obtained as follows:

$$y_1^i(z) = \frac{1}{1-z^{-1}}(d_2^i(z) - u_2^i(z)) \quad (2.6)$$

$$y_2^i(z) = \frac{1}{1-z^{-1}}(z^{-L^i} d_2^i(z) - u_2^i(z)) \quad (2.7)$$

$$y_3^i(z) = \frac{1}{1-z^{-1}}(d_1^i(z) - u_2^i(z)) \quad (2.8)$$

$$u_2^i(z) = \begin{cases} z^{-1} y_3^i(z) & 0 \leq z^{-1} y_3^i(t) \leq z^{-1} y_2^i(t) \\ z^{-1} y_2^i(z) & 0 \leq z^{-1} y_2^i(t) \leq z^{-1} y_3^i(t) \end{cases} \quad (2.9)$$

The difference equations (2.6)-(2.9) are derived by physical laws governing the dynamics of the SC systems. They describe the individual nodes' dynamics when complemented with the local ordering policy, i.e. the mechanism for determining  $u_1^i(t)$  from accessible information at time  $t$ . The detailed discussion of ordering policies is covered in Section 2.3. The ordering decision  $u_1^i(t)$  does not have an immediate effect on any of the current node's outputs. However, when considering the interactions between SC nodes, the current node's order affects the overall SC

## 2.2 Supply Chain Dynamics and Modeling

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dynamics. The exchange of information and material between SC nodes may cause cancellation of some input variables when describing the overall SC dynamics.

During the SC operations, the stock conditions (sufficient or insufficient) of the node  $i$  are changing at different time instant  $t$  over planning horizon. Hence, the SC is naturally a switched system [61, 62] where the general dynamics given by Eqs. (2.6)-(2.9) can become in particular one of the following cases through time [30]: ISHS, ISLS or LS. The particular subsystems that are associated to the corresponding cases described by Eq. (2.9) are specified in Sections 2.2.3.

### 2.2.3 Switched System

A switched system is a hybrid system that is composed of a family of continuous-time or discrete-time subsystems/modes and a set of rules orchestrating the switching between the subsystems [63]. The switched system can be found in various contexts in manufacturing, communication network, traffic control, even in chemical process. A good industrial example is the production line in which different products are assembled. It requires several dynamic models to describe its behavior under different production modes. In SCM context, at any given time a particular subsystem is chosen according to the actual SC operations condition, in which case the system is called ‘controlled’. In this section, different subsystems are investigated with respect to different expressions for the delivery of echelon  $i$ .

**Definition 2.1.** *The supply chain node  $\forall i \in \mathcal{N}$  is **Infinite Supply and High Stock (ISHS)** subsystem at a determinate time instant  $t$  if the following conditions are met: (1) the upstream suppliers have ample items in stock so that the demand of current node  $i$  is satisfied; (2) the stock of current node  $i$  is sufficient so that the downstream customers’ demand can be satisfied. The delivery policy corresponds to the first case of Eq. (2.9).*

**Theorem 2.1.** *When the supply chain node is in operation under **ISHS** subsystem, the expression of Eq.(2.6) for the inventory position of current node  $i$  is:*

$$y_1^i(z) = \frac{z^{-1}}{1-z^{-1}} [u_1^i(z) - d_1^i(z)] \quad (2.10)$$

*Proof.* According to the definition of the standing order  $y_3^i(t)$ , the delivery at review period  $t$  is determined by the standing order of time instant  $t-1$ :

$$u_2^i(t) = y_3^i(t-1) \quad (2.11)$$

Substitute the relation in Eq. (2.11) for Eq. (2.5) to yield

$$y_3^i(t) = d_1^i(t) \quad (2.12)$$

which means that, from Eqs.(2.11) and (2.12),  $u_2^i(t+1)=d_1^i(t)$ . Thus the current node's delivery is determined by its customer's order with one review period delay:

$$u_2^i(t) = d_1^i(t-1) = u_1^{i-1}(t-1) \quad (2.13)$$

From Eq. (2.13), the following is obtained immediately:  $d_2^i(t) = u_2^{i+1}(t) = u_1^i(t-1)$ . Thus applying this relation and Eq. (2.13) to model (2.3) and then taking the  $z$ -transform implies Eq. (2.6) to be Eq. (2.10).  $\square$

**Remark 2.1.** *The two conditions in ISHS case are frequently assumed in literature [64-68]. Under this condition, the IP of node  $i$  is determined by both its own ordering policy and the ordering policy of node  $(i-1)$ . The resulting Eq. (2.10) can be treated as the SC model to design various control strategies.*

**Definition 2.2.** *The supply chain node  $\forall i \in \mathcal{N}$  is **Infinite Supply and Low Stock (ISLS)** subsystem at a determinate time instant  $t$  if the following conditions are met: (1) the upstream suppliers have ample items in stock so that the demand of current node  $i$  is satisfied; (2) the stock of the current node  $i$  is insufficient so that the downstream customers' demand can not be satisfied. The current node's delivery policy corresponds to the second case of Eq. (2.9).*

**Theorem 2.2.** *When the supply chain node is in operation under ISLS subsystem, the expression of Eq.(2.6) for the inventory position of current node  $i$  is:*

$$y_1^i(z) = (z^{-1} + z^{-2} + \dots + z^{-(L^i+1)})u_1^i(z) \quad (2.14)$$

*Proof.* The second condition of ISLS operation indicates that current node  $i$  keeps a low stock and there is always less stock than the amount ordered by its customer. Thus the delivery is limited by the inventory:

$$u_2^i(z) = z^{-1}y_2^i(z) \quad (2.15)$$

Substituting Eq. (2.7) into Eq. (2.15) yields

$$u_2^i(z) = \frac{z^{-(L^i+1)}}{1-z^{-1}}d_2^i(z) - \frac{z^{-1}}{1-z^{-1}}u_2^i(z) = z^{-(L^i+1)}d_2^i(z) \quad (2.16)$$

The first condition assumes the supplier has sufficient stock so that the delivery to the current node  $i$  is determined according to what has been ordered:

$$d_2^i(z) = u_2^{i+1}(z) = z^{-1}y_3^{i+1}(z) = z^{-1}u_1^i(z) \quad (2.17)$$

## 2.2 Supply Chain Dynamics and Modeling

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Replacing Eq. (2.17) into Eq. (2.16) results in

$$u_2^i(z) = z^{-(L+2)} u_1^i(z) \quad (2.18)$$

Using Eqs. (2.17) and (2.18) on Eq.(2.6), inventory position becomes:

$$y_1^i(z) = \frac{z^{L+1} - 1}{z^{L+1}(z-1)} u_1^i(z) \quad (2.19)$$

Factoring the numerator of Eq. (2.19) leads to Eq. (2.14).  $\square$

**Remark 2.2.** In *ISLS* case of Eq. (2.14), the inventory position of node  $i$  depends only on its past orders. This result indicates that if the supplier of node  $i$  has high stock but the node  $i$  itself keeps low stock, then its full inventory has to be cleared to meet customer demand. Thus its inventory position is independent of the inventory but is solely determined by the products available in *WIP* which are ordered in the previous review periods.

**Definition 2.3.** The supply chain node  $\forall i \in \mathcal{N}$  is **Limited Supply (LS)** subsystem if its upstream suppliers does not have enough stock to supply current node  $i$ .

**Theorem 2.3.** When the supply chain is in operation under **LS** subsystem, the expression of Eq.(2.6) for the inventory position of current node  $i$  is:

$$y_1^i(z) = \begin{cases} \frac{1}{z-1} (y_2^{i+1}(z) - d_1^i(z)) & y_2^i(t) \geq z^{-1} d_1^i(t) \\ \frac{z^{L+1} - 1}{z^{L+1}(z-1)} y_2^{i+1}(z) & \text{otherwise} \end{cases} \quad (2.20)$$

*Proof.* Since the supplier of current node  $i$  has insufficient inventory, the delivery is limited by its stock instead of the demand from node  $i$ :

$$d_2^i(z) = u_2^{i+1}(z) = z^{-1} y_2^{i+1}(z) \quad (2.21)$$

The inventory position transfer function becomes  $y_1^i(z) = \frac{1}{z-1} (y_2^{i+1}(z) - z u_2^i(z))$ .

When the node  $i$  has a high stock, then as given in Eq. (2.17):

$$y_1^i(z) = \frac{1}{z-1} (y_2^{i+1}(z) - d_1^i(z)) \quad (2.22)$$

When the node  $i$  keeps a low stock, then its inventory position transfer function is:

$$y_1^i(z) = \frac{1}{z-1} (y_2^{i+1}(z) - y_2^i(z)) = \frac{1}{z-1} \left( y_2^{i+1}(z) - y_1^i(z) - \frac{z(z^{-L_i} - 1)}{z-1} d_2^i(z) \right) \quad (2.23)$$

Substituting Eq. (2.21) into Eq. (2.23) and rearranging it:

$$\left( 1 + \frac{1}{z-1} \right) y_1^i(z) = \frac{z^{L_i+1} - 1}{(z-1)^2 z^{L_i}} y_2^{i+1}(z) \quad (2.24)$$

Further simplification of Eq. (2.24) gives:

$$y_1^i(z) = \frac{z^{L_i+1} - 1}{z^{L_i+1}(z-1)} y_2^{i+1}(z) \quad (2.25)$$

Therefore, Eqs. (2.22), (2.25) are the relations given in Eq. (2.20) that describe the limited supply case.  $\square$

**Remark 2.3.** Under *LS* operation Eq. (2.20), the inventory position of node  $i$  is not determined by its ordering policy. This result is intuitive: if the supplier has low inventory no matter how node  $i$  orders, the inventory position is limited by the stock available in its supplier.

Based on the above analysis, the supply chain network can be regarded as a switched system due to its delivery policy, which switches among the three modes as shown by Eqs. (2.7)-(2.9) coupled conditionally with either of Eq. (2.10), (2.14) or (2.20). The operation of the supply chain depends on: (1) the relation of  $\text{MIN}(y_3^i(t), y_2^i(t))$ , which determines if node  $i$  has enough stock to meet the demand of its immediate customer  $(i-1)$ ; and (2) the relation  $\text{MIN}(y_3^{i+1}(t), y_2^{i+1}(t))$ , which determines if the immediate supplier of node  $i$  have sufficient existences to supply the current node. Thereby, the operation cases change as these relations varying through time. Thus the real dynamics of supply chain are analyzed based on the switched system technique in this section. In order to give complete description of the supply chain dynamics, the ordering policies are discussed in next section.

## 2.3 The Ordering Policies

In addition to delivery policy, the most frequently faced decisions by the operation managers is ‘how much’ or ‘how many’ of product to make or order so that external (e.g., customer demand) or internal requirements for some item can be satisfied. The purpose of ordering decisions is to regulate production/distribution in such a way that supply is matched to demand, inventory levels are kept at acceptable levels and capacity requirements are maintained to be minimum. The complete de-

### 2.3 The Ordering Policies

scription of SC dynamics is realized by the switched system when it is complemented with the ordering policies of each echelon. The ordering policy is the strategy for determining  $u_i^i(t)$  from accessible information at time  $t$ . The full information set at time  $t$  for the entire supply chain network is composed of the inventory records  $y_1^i(t)$  and  $y_2^i(t)$  for all nodes  $i \in \mathcal{N}$  up to time  $t$  and orders  $u_i^i(t)$  up to time  $t-1$ :

$$\mathcal{I}(t) := \left[ \bigcup_{i \in \mathcal{N}} \{y_1^i(t), \dots, y_1^i(1); y_2^i(t), \dots, y_2^i(1)\} \right] \cup \left[ \bigcup_{i \in \mathcal{N}} \{u_i^i(t-1), \dots, u_i^i(1)\} \right] \quad (2.26)$$

If information can be shared across the whole supply chain network [69, 70], a global manager makes all the ordering decisions for each node based on any subset of  $\mathcal{I}(t)$ . A decentralized configuration without information sharing is presumed in practice, where each local manager acts independently and places orders based on the private information  $\mathcal{I}_i(t)$ :

$$\mathcal{I}_i(t) := \{y_1^i(t), \dots, y_1^i(1); y_2^i(t), \dots, y_2^i(1)\} \cup \{u_i^i(t-1), \dots, u_i^i(1)\} \cup \{d_1^i(t-1), \dots, d_1^i(1)\} \quad (2.27)$$

**Remark 2.4.** It turns out that  $u_i^i$  in this set are sometimes redundant. To see this, note from Eq. (2.10) when in ISHS case  $u_i^i(t-1) = y_1^i(t) - y_1^i(t-1) + d_1^i(t-1)$ . Clearly, every  $u_i^i$  in this information set can be calculated with this formula from  $y_1^i$  and  $d_1^i$  that are also in the set. Similar conclusions can be drawn from Eq. (2.14) in ISLS case and from Eq. (2.20) in LS case. Thus, the following set is equivalent  $\mathcal{I}_i(t) = \{y_1^i(t), \dots, y_1^i(1); y_2^i(t), \dots, y_2^i(1)\} \cup \{d_1^i(t-1), \dots, d_1^i(1)\}$ . However, by using the expanded information set, one can in principle model all decentralized ordering strategies where each of the elements in  $\mathcal{I}_i(t)$  is allowed to play some role.

As in [66], this thesis focuses on ordering policies that are

- *proper*; i.e., When the orders received at any node  $i \in \mathcal{N}$  are constant over time, its inventory position tends to a constant equilibrium value that is independent of the initial conditions, and the ordering decision converges to the value of received orders. Improper policies amplify inventory position perturbations over time and eventually result in unbounded costs. Intuitively, most practical ordering policies chosen by the rational echelon managers should satisfy properness assumption.
- *LTI*; i.e.,  $u_i^i(t)$  is a time-invariant linear function of the elements in  $\mathcal{I}_i(t)$  because linear policies are often found in practice.

In practice, the echelon managers should design their ordering policies by taking into account these two appealing guidelines. Furthermore, managers are often



faced with alternative opportunities in managing a dynamic supply chain. To properly assess the impact of these alternatives, *model-based decision-making* are extremely useful because they are able to provide valuable input in support of management's decision process. The model-based ordering decision is covered in next subsections.

### 2.3.1 Control Strategies for Supply Chain System

As indicated in previous section, the SC system is essentially a dynamic balance of material and information flow with the ordering decision serving as the control effort. As a result, the ordering policy can be viewed as control strategy.

Two control scenarios are considered. The **first scenario** assumes that every echelon is viewed as *responsible center*, so each echelon is held accountable for a specified set of activities and decisions incurred by the local operations. As a matter of fact, decentralization of decisions is an inevitable facet of managing a large organization: (1) The organizational barriers between SC players often exist and information flows can be restricted such that complete centralized control of material flows may not be feasible or desirable. (2) Modern companies constantly face the challenge of making timely decisions using specialized information. An effective way of meeting such a challenge is to delegate decision rights to the 'person on the spot' who has the intimate knowledge of immediate surroundings [71]. Therefore, the decisions under this scenario are made based on the subset of full information set  $\mathcal{I}_i(t)$ . This control scenario is termed as the decentralized strategy. With slight abuse of the term 'subsystem', it refers to different notion from that in Section 2.2.3. Supply chains are large-scale systems that can be typically decomposed into a number of subsystems (nodes). Under this control strategy, a local controller is connected to each subsystem. There are two cases considered in the **first scenario**:

- If there is no signal transfer (information sharing) between different controllers, the supply chain network is treated as a purely decentralized control system as shown in Figure 2.4, where decisions are made locally due to the absence of global coordinator.

### 2.3 The Ordering Policies

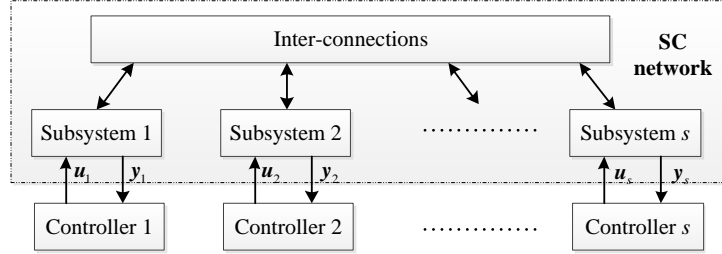


Figure 2.4: Decentralized control scheme of general supply chain network

- If there is certain information shared via LAN to achieve some degree of coordination among controllers, the supply chain network is treated as a distributed control system as shown in Figure 2.5.

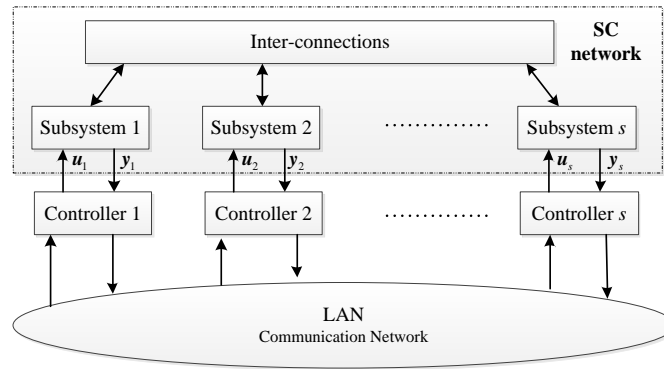


Figure 2.5: Distributed control scheme of general supply chain network

The **second scenario** assumes that all the echelon managers act as a *team*, and they have a communal goal as to optimize the global objective function. This is appropriate and feasible when all the players of the SC belong to one business entity, or they are allowed to share the information  $\mathcal{I}(t)$  whereby each echelon's objective function is a fixed proportion of the overall system cost. This control scenario shown in Figure 2.6 is termed as the centralized control schemes.

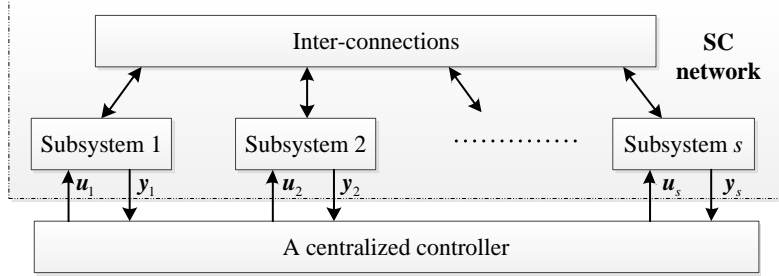


Figure 2.6: Centralized control scheme of general supply chain network

### 2.3.2 Classical EOQ Policies

Inventory position or inventory at each echelon  $i$  is the controlled variable, which must be maintained at a desired setpoint to cushion the current echelon against unpredictable demand variations. The demand at each echelon is treated as a disturbance variable. Echelon managers have to implement and evaluate control policies that reorder product to keep inventory position at a predetermined setpoint. The classical EOQ policies  $((s, Q), (s, S), (R, S), (R, s, S))$  are simple feedback policies relying on **IF-THEN** rules for ordering product. In EOQ policies, a decision to place an order is made when either the inventory position  $y_1^i(t)$  is lower than or equal to a threshold value  $s$ , or the current review period  $R = \delta$  runs out. The quantity of the order corresponds to either a fixed quantity  $Q$  or to the amount that will bring the IP to an ‘Order-Up-To’ (OUT) value  $S$ . The decision rules for these four standard EOQ policies are summarized as follows:

- $(s, Q)$ :** IF  $y_1^i(t) < s$ , THEN order quantity  $Q$ .
- $(s, S)$ :** IF  $y_1^i(t) < s$ , THEN order quantity  $S - y_1^i(t)$ .
- $(R, S)$ :** IF  $R = t \cdot \delta - (t-1) \cdot \delta, t = 1, 2, \dots$ , THEN order quantity  $S - y_1^i(t)$ .
- $(R, s, S)$ :** IF  $R = t \cdot \delta - (t-1) \cdot \delta, t = 1, 2, \dots$  or  $y_1^i(t) < s$ , THEN order quantity  $S - y_1^i(t)$ .

Among these conventional ordering policies, the third one  $(R, S)$  known as the ‘Order-Up-To’ is particularly interesting to inventory control purpose. The complete dynamic behavior of node  $i$  is represented in the following block diagram in Figure 2.7. Conceptually, the OUT policy is very easy to understand. The inventory position is reviewed periodically at any node  $i \in \mathcal{N}$ , and an ‘Order’  $u_1^i(t)$  is placed to bring the inventory position to ‘Up-To’ a defined level  $w^i(t)$ :

### 2.3 The Ordering Policies

$$u_1^i(t) = w^i(t) - y_1^i(t) \quad (2.28)$$

The OUT setpoint is updated at every review period with

$$w^i(t) = \hat{D}_{L+1}^i(t) + \gamma^i \hat{\sigma}_{L+1}^i(t) \quad (2.29)$$

where the estimate of mean demand  $\hat{D}_{L+1}^i(t)$  over  $L+1$  periods and the estimate of standard deviation  $\hat{\sigma}_{L+1}^i(t)$  of the demand over  $L+1$  periods are determined respectively by  $\hat{D}_{L+1}^i(t) = (L+1)\hat{d}_1^i(t)$  and  $\hat{\sigma}_{L+1}^i(t) = \sqrt{L+1}\hat{\sigma}^i(t)$ . The parameter  $\gamma^i$  is known as the safety factor, which is a constant chosen to meet desired service level. The supply chain managers usually set it equal to 0 and increase the lead time by one [27, 65]: the value of  $L^i$  is inflated and the extra inventory represents the safety stock. This approximation is made so that no new parameter is introduced into the setpoint design. Commonly, demand estimate is approached by filtering techniques such as moving average and exponential smoothing, whose formula are well-known to be:

$$\hat{d}_1^i(t) = \begin{cases} \alpha^i d_1^i(t) + (1-\alpha^i)\hat{d}_1^i(t-1) & \text{Exponential Smoothing} \\ \frac{\sum_{j=0}^{T_m-1} d_1^i(t-j)}{T_m} & \text{Moving Average} \end{cases} \quad (2.30)$$

where  $\alpha^i = 1/(1+T_a^i)$ ,  $T_a^i$  is the average age of the demand data for the exponential smoothing forecast and  $T_m$  is the number of review periods within the window size.

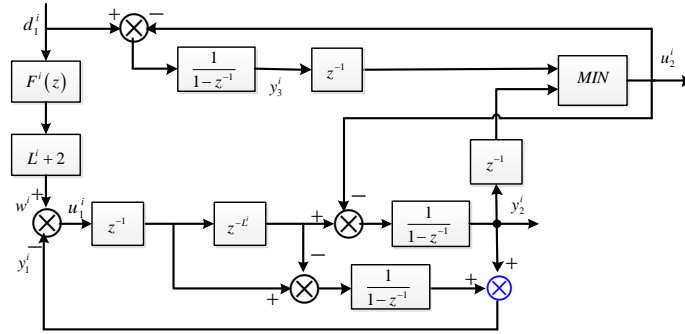


Figure 2.7: Block diagram of dynamics of node  $i$  with the Order-Up-To policy

**Remark 2.5.** The OUT ordering policy is the simplest and the most commonly used replenishment strategy both in practice and in academia. Obviously, in control

*engineering sense it is a proportional controller with gain equal to one. The classical OUT policy can be modified to be a proportional controller with gain adaption. This ordering policy gives considerably more flexibility to ‘tune’ the ordering process in order to exploit supply chain characteristics.*

### 2.3.3 Fractional OUT Ordering Policy

The OUT policy seems to unavoidably lead to a bullwhip effect [27, 65] when demand has to be forecasted. In this section a generalized OUT decision rule is presented to avoid that drawback. Moreover, as indicated in Figure 2.7, the effect of  $z^{-L}$  on the ordering policy is actually cancelled out in the blue sum junction for inventory and WIP due to the consolidated adjustment of the inventory position. The idea is to consider this effect by breaking the sum junction into two parts in order to form two sub-feedback control loops. Thus the following alteration to the OUT system is made:

$$u_1^i(t) = \hat{d}_1^i(t) + \beta^i (w_2^i(t) - WIP^i(t)) + \eta^i (w_1^i(t) - y_2^i(t)) \quad (2.31)$$

As noted in [27], fractional OUT (FO) ordering policy relies on control engineering principles with a block diagram for the policy shown in Figure 2.8. In the FO policy, the formulation of order quantity uses a user-adjustable parameter  $\eta^i$  to dictate the response to a discrepancy (or error) of inventory  $(w_1^i(t) - y_2^i(t))$  and a similar parameter  $\beta^i$  that define the response to WIP discrepancy  $(w_2^i(t) - WIP^i(t))$ . The inventory and WIP targets are  $w_1^i(t), w_2^i(t)$  respectively.

The primary difference in decision rule (2.31) with (2.28) is that both the inventory and WIP errors are as namesake fractionally taken into account. This is a full adjustment strategy based on APIOPBCS [26, 27, 72]. The APIOPBCS system is expressed as ‘Let the ordering targets be equal to the sum of: average demand, a fraction of the inventory difference in actual stock compared to target stock and a fraction of the difference between target WIP and actual WIP’. The two feedback loops in FO ordering policy are inventory feedback loop that determines the rate at which inventory deficit is recovered and supply line control, which regulates the rate at which WIP deficit is recovered.

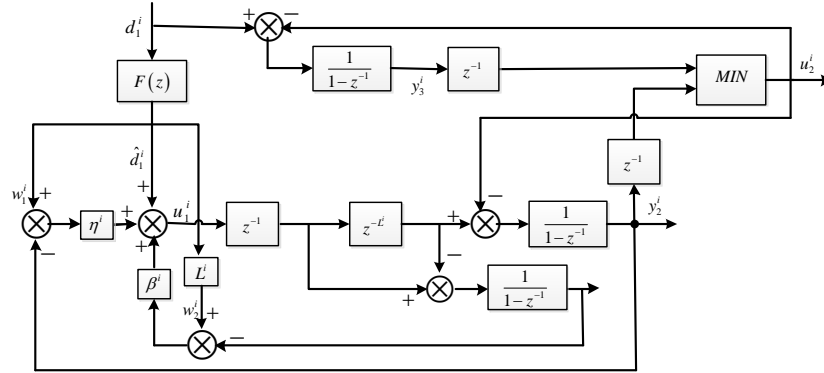


Figure 2.8: Block diagram of dynamics of node  $i$  with fractional ordering policy

**Remark 2.6.** It is important to see the difference between fractional OUT (2.31) and classical OUT (2.28). From Eqs. (2.29) and (2.28), the classical OUT decision is computed from  $u_1^i(t) = \hat{D}_{L+1}^i(t) + \gamma^i \hat{\sigma}_{L+1}^i(t) - y_1^i(t) := (L+2)\hat{d}_1^i(t) - y_1^i(t)$ . It can be converted to  $u_1^i(t) = \hat{d}_1^i(t) + (L\hat{d}_1^i(t) - WIP^i(t)) + (\hat{d}_1^i(t) - y_2^i(t))$ , i.e.  $u_1^i(t) = \hat{d}_1^i(t) + (w_2^i(t) - WIP^i(t)) + (w_1^i(t) - y_2^i(t))$ . It turns out to be the complete analogue to the fractional rule presented in (2.31) when parameters  $\beta^i = \eta^i = 1$ . Therefore, the key difference between fractional and classical OUT policy is that both the inventory and WIP errors are completely taken into account by classical OUT but the errors are included only fractionally in fractional OUT. Another difference is that there are two separate feedback loops (one for the inventory and one for the WIP) in FO policy, whereas in classical OUT policy, there is only one joint feedback loop for the IP.

### 2.3.4 PID-Based Ordering Policy

PID controllers are widely used in industry for a variety of applications [73, 74] and are arguably the most exhaustively studied controller structure in the engineering literature. However, the PID approach has not received much attention in SCM literature. The reason is that it does not correspond to what is actually performed in real SC systems, where forecast is presented explicitly. If the PID approach is adopted, the feed-forward forecasting unit is no longer used to eliminate the discrepancy between desired and actual IP. The PID is a feedback based strategy and any periodicity in demand can be for instance tackled with feedforward compensation schemes. This is now accomplished by the integral action offered by the  $I$  element of the controller. Moreover, the addition of  $D$  elements complicates the tuning effort, since an additional tuning parameter is included. The tuning of PID control-

lers also requires a time consuming trial-and-error procedure based on simulations, which is a challenge for SC engineers without control engineering background.

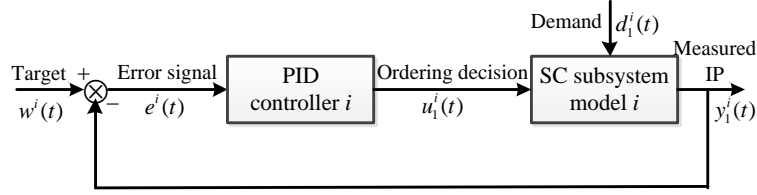


Figure 2.9: Block diagram representation of feedback PID control for node  $i$

It has been shown that node embedded with PID controllers can reduce excess stock levels by 80% and hence reduce cost [75]. This section deals with a feedback PID control that manipulates the ordering decisions to keep the IP close to the pre-specified target levels. Figure 2.9 presents a continuous-time representation of PID-based ordering policy. In this feedback control system the IP target  $w^i(t)$  is compared against the IP signal  $y_1^i(t)$  and the resulting error signal  $e^i(t)$  is used by the PID decision policy to calculate the ordering signal  $u_1^i(t)$  :

$$u_1^i(t) = K_p^i \left[ e^i(t) + \frac{1}{\tau_i^i} \int_0^t e^i(\varepsilon) d\varepsilon + \tau_D^i \frac{de^i(t)}{dt} \right] \quad (2.32)$$

where the tuning parameters  $K_p^i$  is the proportional gain of the controller  $i$ ;  $\tau_i^i$ , the integration time;  $\tau_D^i$ , the differentiation time of PID control. The practical implementation of the discrete-time PID algorithm can take incremental form or positional form, e.g. incremental form:

$$\Delta u_1^i(t) = K_c^i \left[ \left( 1 + \frac{\delta}{\tau_i^i} + \frac{\tau_D^i}{\delta} \right) e^i(t) - \left( 1 + 2\frac{\tau_D^i}{\delta} \right) e^i(t-1) + \frac{\tau_D^i}{\delta} e^i(t-2) \right] \quad (2.33)$$

in which  $\delta$  is the sampling time of discrete control and it equals to the decision review period of the supply chain network.

The PID type of control has been utilized in SCM for very good reasons [14, 76]: Proportional control improves performance but leaves an offset from target; the integral term eliminates the offset. However, the integral term has a destabilizing influence, so the derivative term is introduced to restore the necessary stability margin. Since the SC system has an integrating nature a step change in demand then becomes a ramp disturbance. To guarantee the offset-free control the PID tuning rules for this application must cause no offset for output ramp disturbance. Therefore, the controller must include an integral mode.

## 2.3 The Ordering Policies

**Remark 2.7.** *The use of a PID-based decision policy is a relatively simple yet challenging task. It can be used to illustrate to SC practitioners the benefit of an engineering-based approach in this non-traditional SCM application. Note that 90% of industrial PID controllers are tuned manually and the results are probably worse in practical implementation. The PID controller needs to be tuned for close set-point tracking (for example, quick response to a new imposed target level) and good disturbance rejection. However, the lead time is key factor that can greatly deteriorate dynamic performance of this decision policy. Careful tuning is required to avoid poor dynamic performance.*

### 2.3.5 MPC Ordering Policy

Within the broad spectrum of system theory and control engineering, this thesis particularly focuses on the application of model predictive control to the SCM and recent relevant studies have been found to provide an attractive solution to SCM problems [77, 78].

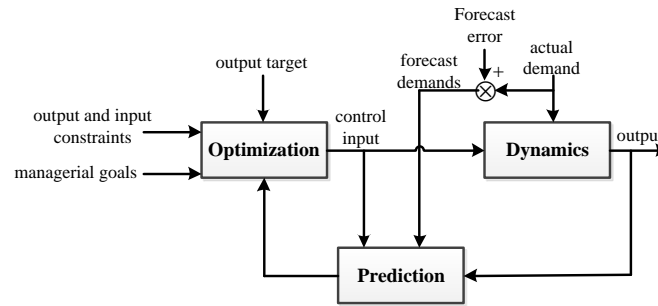


Figure 2.10: Block diagram of MPC scheme for node  $i$

The MPC is a natural solution to the SCM problems due to its intrinsic feed-forward compensation mechanism. Using prediction models, the demand can be anticipated and better performance can be achieved. Moreover, the complexity is comparable to that of PID due to the specific approach. In this section, a schematic MPC approach to managing the SC is introduced and the detailed description is left in the following chapters.

A typical MPC scheme is shown in Figure 2.10, where the Dynamics block accounts for the actual behavior of the SC system over time, which has been described in Section 2.2 and 2.3. The block of Prediction is responsible for estimating the future outputs by using the available forecasts of the demand and a model of the SC dynamics system. The availability of a dynamic model is crucial to predict the system behavior at future time instants. The Optimization block is the effective



‘engine’ of MPC approach, as it can take managerial goals into account and generate the control actions (i.e., the ordering decisions from nodes) that govern the SC. Thus the MPC approach to SCM is a multi-objective optimization-based control technique. It consists in computing the control inputs at each review period by solving an optimal control problem. The prediction and optimization steps are repeated at the successive time instants with a one-step-forward shift of the sliding window and by using new outputs and forecasts of the demands.

We conclude Section 2.3 after introducing different types of ordering policy including classical EOQ policy, modified OUT (FO) policy, PID-based and MPC ordering policies. The supply chain dynamics description in Section 2.2 is completed when either of these ordering policies is employed as managerial decision.

## 2.4 Demand Estimate Techniques

Estimating or forecasting highly uncertain customers’ demand is an important contributor to inventory control improvement. Without an estimate of future demand, it is impossible to plan the inventories that offer customers a desirable service level. Therefore, the estimation under a SCM environment is also called forecasting and the forecasting models are required to be relatively accurate and simple to operate. The following moving average and exponential smoothing are filters in essence and it is the same in principles as the identification task with filter but easier for the SC engineers to understand and operate.

Inventory control needs to cope with a variety of customer demand patterns. Because in stationary demand patterns, forecasting ahead are fixed in value and the forecast for one period ahead is the forecast for any number of periods ahead. The assumption of a stationary demand pattern is imposed in order to take full advantage of this benefit. The demand however may lose stationarity over some periods of time, e.g. monotonically increase towards sales period. The integrator of the PID controller or in the disturbance models of EPSAC will treat it as a stationary error.

### 2.4.1 Moving Average

The general form of the moving average is introduced in Eq. (2.30):

$$\hat{d}_1^i(t) = \frac{d_1^i(t)}{T_m} + \frac{d_1^i(t-1)}{T_m} + \dots + \frac{d_1^i(t-T_m+1)}{T_m} \quad (2.34)$$

where  $T_m = 2, 3, \dots$  and the sum of  $T_m$  weights will always sum to one, this equally weighted moving average is the definition of a true average. However, the practical use of moving average as a estimating model has some problems: difficult to start from a situation where no data exist; relatively large amounts of data to be stored (a

## 2.4 Demand Forecasting Techniques

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significant data storage problem if forecasts are provided for node with several thousands of stocked items); a sudden cut off in weighting for data not included; all data are weighted equally irrespective of their age (by a simple logic one would suggest that more recent data should be weighted more heavily).

The last problem would be overcome by extending the concept of equal weight to an exponentially weighted average. The definition of an average with weights declining exponentially with time would be of the general form of an infinite series defined as:

$$\hat{d}_1^i(t) = \alpha^i d_1^i(t) + \alpha^i(1-\alpha^i)d_1^i(t-1) + \alpha^i(1-\alpha^i)^2 d_1^i(t-2) + \dots \quad (2.35)$$

where  $0 < \alpha^i < 1$  since to produce a true average the sum of weights must sum to one. On first examination, a forecast based on Eq. (2.35) would appear to be relatively complicated to implement. However, it is possible to show that it can be modified to a much simpler statement as shown in next subsection.

### 2.4.2 Exponential smoothing

It is possible to show that Eq. (2.35) can be modified to a much simpler statement via  $\hat{d}_1^i(t) = \alpha^i d_1^i(t) + (1-\alpha^i)[\alpha^i d_1^i(t-1) + \alpha^i(1-\alpha^i)d_1^i(t-2) + \dots]$ , which immediately leads to the one-period ahead forecast based on exponential smoothing of the form:

$$\hat{d}_1^i(t) = \alpha^i d_1^i(t) + (1-\alpha^i)\hat{d}_1^i(t-1) \quad (2.36)$$

In contrast to the moving average, the exponential smoothing forecast model offers some advantages: easy to initialize because once an estimate for  $\hat{d}_1^i(t-1)$  is made, forecasting can proceed with all the unknowns on the right hand side of Eq. (2.35); data storage is economical since  $\hat{d}_1^i(t-1)$  embodies all previous data and hence only the value of  $\hat{d}_1^i(t-1)$  needs to be retained from one period to the next; produce no sudden cut off in weighting of demand data irrespective of age.

In exponential smoothing forecast model, the average age of demand data is  $1/\alpha^i$ . When  $\alpha^i$  is high, a good response to an upward demand change is anticipated. However, such a high value can cause an overreaction to a single period's high demand. Conversely, when  $\alpha^i$  is low, although the effect of an impulse will be ignored, the response to an upward demand change will be poor. For the extreme case of  $\alpha^i = 0$  the forecast is totally insensitive to changes in the demand; and for  $\alpha^i = 1$  the forecast is extremely sensitive to changes and can overreact to relatively small changes. Ideally the best value of  $\alpha^i$  would minimize the sum of squared forecasting errors, so in the majority of practical situations values of either 0.1 or 0.2 are useful compromise.

**Remark 2.8.** *The moving average models and exponential smoothing models assume that there is no trend of any kind in the demand data so they play well for stationary demand pattern. If the demand data displays long term growth or a seasonal influence that stands out clearly against the noise, a **Brown's linear exponential smoothing** model or **Holt's Linear Exponential Smoothing** model can compute local estimates of both level and trend. This falls out of the scope of this section.*

## 2.5 Summary

In this chapter the methods from control engineering are used to properly capture underlying SC dynamics. Applying systematic analysis yields well-understood mathematical expressions for the benchmark SC system dynamics. These mathematical expressions will serve as nominal models in the following chapters to formulate the inventory control and replenishment policies based on model predictive control principles. The contributions are as follows.

In response to these objectives, a detailed description of the SC dynamics is made. First, the topology of a benchmark supply chain is chosen in order to avoid exhaustively enumerating and modeling all types of supply chain networks. The incentives of analyzing a SC with this structure is validated and explained in four aspects: the fundamental nature of SCs is retained; extensive use in industrial applications and academic research; dynamic characteristics are maintained; it is a representation of popular ‘beer game’ or ‘Forrester model’.

Next, the SC dynamics are analyzed starting from the system interior microscopic structure, i.e. a generic node that forms the SC network. The structure, assumptions and variables are characterized from the process system engineering point of view to facilitate the dynamics analysis. The difference equations obtained from the conservation of mass and information flows are formulated to represent the dynamic features of the node. The model that captures the links between nodes and SC operational behavior is approached by the switched system theory. The switched system model is derived as a result of the actual operation requirements, namely the delivery policies determined by the changing conditions of stock levels of the supplier node and current node. Then three subsystems are proposed according to three operational conditions and the corresponding models are derived. The **ISHS** case is frequently assumed for the replenishment rules in the next chapters.

Another crucial decision to be made is the ordering policy and it is regarded as control strategies in the replenishment rules. The MPC-based ordering policy is the focus of this thesis and it will be elaborated in the following chapters. For the purpose of comparison with upcoming MPC-based replenishment, a series of ordering policies with increasing functionality and sophistication are also introduced since they are commonly used in SCM literature. Rather than using managerial expressions, control-oriented interpretation and formulations of these ordering policies are highlighted. Finally, these ordering policies require the forecast of demand to de-

## 2.5 Summary

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termine future inventory targets. The forecasting models of moving average and exponential smoothing are presented as they are simple to use and popular in SCM literature. The IP setpoints will be determined according to these forecasting models for the MPC-based replenishment in the following chapters.

Parts of this chapter have been published in:

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Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., “A centralized model predictive control strategy for dynamic supply chain management,” in *Manufacturing, Modelling, Management and Control*, 2013 7th IFAC Conference on, pp. 1630-1635, 2013.

# 3

## Supply Chain Performance Measures

It is widely acknowledged that there has been relatively little interest in developing measurement systems and metrics for evaluating supply chain performance [79, 80]. Many factors may contribute to this situation including: the complexity of capturing metrics across multiple organizations, the unwillingness to share information among SC partners. As [81] observes, one of the main problems with supply chain metrics is that “there is no evidence that meaningful performance measures that span the supply chain exist”.

Nevertheless, a significant aspect in supply chain design and analysis is the establishment of proper performance measures. A set of performance measures can be used to determine the efficiency and/or effectiveness (responsiveness) of the existing supply chain, or to compare competing alternative chains. Effectiveness measures the extent to which a customer’s requirements are met and efficiency is how economically SC partners’ resources are utilized when providing a pre-specified customer satisfaction level. Available literature identifies a number of important performance metrics to evaluate SC effectiveness and efficiency. These measures may be categorized as either qualitative or quantitative [82]. When analyzing the SC performance, qualitative evaluations such as ‘good’, ‘fair’, and ‘poor’ are fuzzy and difficult to use in a meaningful way. On the contrary, the quantitative measures highlight that they are objective rather than relying on the subjective interpretations of individual factor. As a result, quantitative performance measures are often preferred. There have been predominantly two categories of different performance measures:

1. Cost (Efficiency): includes inventory and operating costs.
2. Customer responsiveness: includes lead time, stock out/backorder probability, and fill rate.

The following mainly deals with the quantitative metrics or indicators that measure the SC performance. The metrics on the basis of a combination of cost and

### 3.1 Introduction

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customer responsiveness can evaluate the SC performance on minimizing demand variance amplification, also known as the bullwhip effect [83-89]. Being a comprehensive SC performance metric, the bullwhip will be discussed.

## 3.1 Introduction

A significant phenomenon in supply chain management, first observed by Forrester [49], indicates the amplification of orders' variability along the upstream direction in supply chain networks. Subsequently this observation is termed as bullwhip effect [90, 91]. In a serial supply chain with a factory, a distributor, a wholesaler, and a retailer, one can observe that the retailer's orders to wholesaler show greater variability than the end-consumers' demand, the wholesaler's orders display even more oscillation, and the factory's manufacturing plan is the most volatile. The common drawbacks of bullwhip effect could be bad demand forecast, insufficient or excessive inventory holding, poor customer service and uncertain production planning. The bullwhip effect has been recognized in SCM literature as one of the chief barriers in improving supply chain performance.

Earlier research has focused on many aspects of the bullwhip effect at least on a theoretical level, e.g. revealing its existence, identifying its various causes and providing remedies for the problem [60, 66, 92-94]. It is natural to measure and quantify the bullwhip effect first and then continue to mitigate it because "what gets measured gets done" as saying goes. The understanding thus gained can possibly be used for control and elimination purpose. In the past two decades researchers tried to tackle this problem from the perspectives of control theory [15, 27, 95, 96] on account of the resemblance of dynamical control systems to supply chain networks. A brief literature review is presented in the next section.

The body of literature on the bullwhip effect study is extensive and they can be roughly divided into three streams:

- 1) research that seeks to determine the impact of forecasting techniques employed by supply chain managers on the bullwhip effect [65, 97].
- 2) research that focuses on quantifying the impact of supply chain's design parameters (such as ordering policy, inventory management policy, and production variation and batching) on the bullwhip effect [98, 99].
- 3) research that examines the effect of supply chain dynamics (e.g. information sharing) on the bullwhip effect [28, 100-102].

This thesis advocates the approaches to achieving bullwhip mitigation by a control engineering oriented formulation and explores the impact of the novel replenishment policies on the bullwhip effect. The replenishment rules are designed according to the philosophy of model predictive control and our study falls into the second and third research streams.

Along the years many researches on quantifying the bullwhip effect only used statistical approach to compute the order VAR as the bullwhip metric under certain ordering policy and demand pattern [27, 30, 103, 104]. As more and more control methods and techniques have been utilized to design the replenishment policies, there is a missing link between bullwhip problem and diverse control engineering methods. For example, a few scientific reports have demonstrated qualitative improvement on damping the bullwhip effect by applying MPC strategy but none has mentioned quantification at all. Quantification of the bullwhip effect has been disregarded owing to structure limitations and implicit formulations of these MPC controllers. Motivated by the research gap, this chapter aims to contribute in the following aspects:

- In order to analyze the impact of these bullwhip damping strategies, the first task is to model the ordering policies both in time-domain and frequency-domain. Then it is possible to perform subsequently comparative analysis for three replenishment policies, i.e. classical OUT, FO policy, and MPC-based ordering policy.
- The bullwhip metric in frequency-domain is linked to its counterpart in time-domain via  $z$ -transform of the ordering policy transfer function. Other techniques like discrete Fourier transform are also discussed.
- The contribution of this chapter to the extensive literature lies in presenting an analytical expression of the bullwhip which is suitable for the situation when MPC-based ordering policy is used. This bullwhip measure is related to the control-theoretic metric via discrete Fourier transform or inverse  $z$ -transform. The closed-form solution is directly equivalent to the common statistical measures used in literature.

## 3.2 Review of Related Literature

A great amount of bullwhip effect related literature pertains to ‘irrational behavioral’ aspects of the supply chain partners rather than treating them as rational players, which is out of scope of this thesis. There is another large body of literature has focused on quantifying the bullwhip effect by *variance ratio*. Therefore, the dominant orientation in these studies to analyze the bullwhip effect is an estimation process, which clearly involves statistical considerations [105]. Of particular interest in this thesis is the use of control systems engineering concepts (transfer functions, frequency response,  $z$ -transform etc.) to quantify and mitigate the bullwhip effect. Therefore, the selection criteria of the reviewed papers are: 1) involving control-theoretic approach to quantification; 2) relying on control engineering techniques to mitigate the bullwhip effect.

*On quantifying the bullwhip effect:* The literature on quantifying the bullwhip effect is already vast. It typically involves a serial supply chain with one retailer

### 3.2 Review of Related Literature

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and one supplier being subjected to different information distorting factors that cause the bullwhip effect. The variance amplification ratio of orders is by far the most widely used statistical measure to detect its existence. Some researchers examined the single-echelon supply chain and compared the VAR of placed orders to demands [65, 90, 91]. The reports concerning the analysis in the multi-echelon supply chains exist for a specific family of demand (e.g., AR, ARMA) and traditional ordering policies (e.g., OUT policy or smoothing replenishment rule) [60, 67]. These results hold dependently of the customer demand, ordering policy, and supply chain configuration. The paradigmatic works of [26, 27] draw on control theory and z-transform techniques. They quantified the VAR as the noise bandwidth by employing the transfer function and frequency response plot. Moreover, they analyzed this transfer function to reach conclusions about the bullwhip effect for different ordering policies, demand, and forecasting techniques. This quantifying method has been the fundamental of their subsequent studies [93, 103, 106]. In the work of [66, 67], the bullwhip metric is the ratio of the root mean square errors of order and demand, instead of conventional VAR, because the modified metric makes the transfer function of ordering policy easier to derive via spectral analysis. The maximum modulus of the transfer function is defined as the bullwhip metric.

*On mitigating the bullwhip effect:* Whereas many works have been done on understanding and reducing the bullwhip effect in two echelon or multi-echelon supply chain, prior efforts for limiting VAR largely focused on information sharing. In fact information exchange has been regarded as one of the main ways for taming the bullwhip effect. A number of authors in [28, 64, 94, 99, 102, 107] explored and discussed the value of information sharing on reducing the bullwhip effect. The prior research shows that the bullwhip effect cannot be completely removed even with complete information sharing and with whatever forecasting methods when the OUT policy is used. The FO replenishment policy has been designed to avoid the side-effect of the OUT policy. This type of replenishment rules does not only increase the flexibility for decision-making, but also allows for balancing the target inventory costs and production fluctuations [108]. In the work [109], the authors summarized the contributions from 2003 to 2013 on the impact of information sharing and smoothing replenishment rules in terms of bullwhip mitigation. It should be emphasized that the OUT and FO replenishment rules are essentially PID controller, or P controller to be accurate, from control engineering perspective. It is popular and easy but time-consuming to tune. However, it is only suitable for tackling lower-level problems of control pyramid. The work [110] employed a two-degrees-of-freedom IMC structure with analytical tuning rules for bullwhip effect avoidance on a serial supply chain (i.e. the propagation of the demand fluctuation is defined as the ratio of orders to successive echelons).

The role of model predictive control in both of operational and tactical levels of SCM has been indicated in literature [78]. MPC was first applied to inventory management for a single manufacturing site problem in [33]. It has subsequently led to an increasing number of studies in the last two decades [78, 36, 42, 44-46, 48, 111]. Unfortunately, to the best of our knowledge, there is no report on quantifying the



bullwhip effect when MPC-based ordering policy is adopted. This is the main motivation of Chapter 3 & 4 to study the bullwhip problem in context of MPC applications.

### 3.3 The Bullwhip Effect Metrics

The bullwhip effect is a phenomenon that is repeatedly found in industry operations [49], in simulation such as Beer Game, and also in macroeconomic data [112-114]. Figure 3.1 illustrates the concept: the end-customer places an order (whip) and the order fluctuations build up upstream the supply chain. In multi-echelon SCs, even very steady customer demand can generate greatly fluctuating suppliers' orders several echelons upstream. The upstream suppliers feel as if they were at the end of a bullwhip, where a small perturbations at the handle (end-customers) cause huge movements at the tip (upstream suppliers).

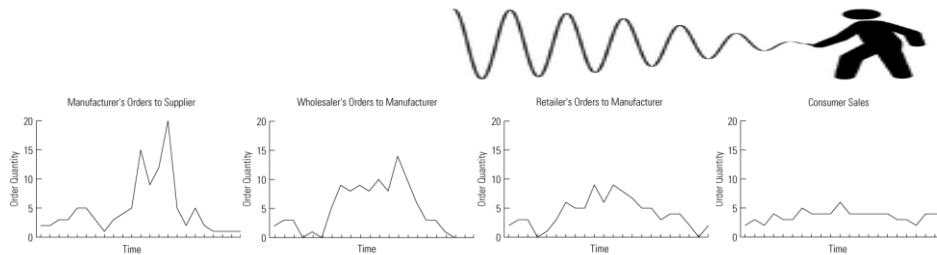


Figure 3.1: A conceptual illustration of the bullwhip effect and a time series showing how order varies occur between two neighboring echelons in a real-world SC(MIT Sloan Management Review[115])

The bullwhip effect was originally described as a form of ‘information distortion’ and measured by comparing the order variance with the demand variance. This definition captures the distortion of *information flow* that goes upstream (the downstream echelon’s order is the demand input to the upstream echelon). Another definition is mainly found in the empirical studies [116], which uses the ratio of the variance of order receipts or shipments and the variance of sales. This type of definition captures the distortion of *material flow* along downstream. The two bullwhip measurements are usually good approximation to each other since the material flow follows somewhat information flow. The first definition indicates that bullwhip is a cost driver: it directly links to the cost because the upstream inventory policy is driven by the downstream orders. In contrast, the second definition suggests bullwhip is the outcome of upstream order-fulfillment. Hence, the bullwhip measure-

### 3.3 The Bullwhip Effect Metrics

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ments based on *material flow* is the consequence of the information-based bullwhip measurement. Furthermore, in the first definition the bullwhip effect is a result of one decision maker, i.e. the echelon in question. However, in the second definition three decision effects are involved. First, sales data are determined by the actual demand and actual stock, but the actual stock is a result of the inventory policies made in previous review periods. Second, the echelon in question makes decisions taking structural and economic condition into account. Third, the actual order receipts are a result of the suppliers' previous inventory policies so the order receipts might not exactly equal the orders. In view of these differences, the information-based definition is more suitable for analysis purposes.

#### 3.3.1 Bullwhip Effect Metrics in Time Domain

As illustrated by Figure 3.1, the term bullwhip effect coined by Lee *et al.* [90, 91] refers to the situation where the orders to the supplier tend to have larger variation than the demands from the customer, and this distortion amplifies along upstream direction. Practical bullwhip measurement is an estimation process that obviously implies statistical methods. Leaving the SC setup and assumptions unspecified, the measurement of bullwhip effect seems quite straightforward and it is generally associated with the amplification mode:

**Definition 3.1.** *When the two sets of matching 'in' and 'out' time series data are available for demand and orders respectively and these two sets of data are related through an ordering policy, then the bullwhip effect ( $\mathcal{BW}$ ) can be defined as the ratio of two variances:*

$$\mathcal{BW} = \frac{\text{Variance (Orders)}}{\text{Variance (Demand)}}$$

**Remark 3.1.** *It should be noted that  $\mathcal{BW}$  in Definition 3.1 is not the only metric to measure the bullwhip but it is definitely the most basic one. This expression of the bullwhip effect is clearly a measure that is specific to a transformational mechanism. It reveals if the transformation mechanism is mitigating, neutral or amplifying in nature:*

- $\mathcal{BW} < 1$  implies demand signal damping
- $\mathcal{BW} = 1$  implies demand signal neutrality
- $\mathcal{BW} > 1$  implies demand signal amplification

**Remark 3.2.** *This metric is applicable to both of the following cases: a single decision-making unit with a specific ordering policy and a system which represents a more macro-type transformation mechanism, e.g. a subsection of a supply chain covering a whole sequence of ordering policies. In the system case, the demand*

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## SUPPLY CHAIN PERFORMANCE MEASURES

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*occurs at the downstream end and the order is generated by the ordering policy of the upstream end.*

The bullwhip metric in **Definition 3.1** clearly involves statistical methods, i.e. any computation of  $\mathcal{BW}$  based on an observational fixed sample is simply an estimate of some underlying true  $\mathcal{BW}$  measure [117]. Alternatively, new perspectives from control engineering are devoted to quantification in this section. Moreover, the phenomenon has multifaceted impacts on strategic, tactical and operational aspects of SC performance [101], and  $\mathcal{BW}$  in **Definition 3.1** is not by far the only relevant metric in relation to measuring the bullwhip effect. Thus, some key performance indicators are defined according to two criteria ‘customer service level’ and ‘operational performance’ [118].

- 1) *The order VAR* is the most commonly used metric in literature to represent the bullwhip effect. Intuitively, practical ordering policies selected by rational managers should be proper and LTI as suggested in Subsection 2.3. The ordering policy can transform a customer demand sequence  $\{n(t)\}_{t=1}^{\infty}$  into a unique set of order sequence  $\{u(t)\}_{t=1}^{\infty}$ . The order VAR is conventionally measured as the ratio of the CV (or variance or standard deviation) of placed orders  $u$  to the CV (or variance or standard deviation) of received demands  $n$  [28, 93, 118]:

$$\mathcal{BW} = \frac{\sigma_u^2 / \mu_u}{\sigma_n^2 / \mu_n} \quad (3.1)$$

where  $\sigma^2$  and  $\mu$  are variance and mean value respectively. When  $\mathcal{BW}$  is measured for a single decision-making unit,  $n$  is the demands that this unit has to fulfil and  $u$  is the orders generated by a specific ordering policy. When  $\mathcal{BW}$  is measured for a subsection of SC,  $n$  is the demands that downstream end node has to fulfil and  $u$  is the orders generated by the ordering policy of upstream end node. This version of metric is preferred because of its ability to monitor the scale of the orders variance amplification phenomenon.

- 2) *Inventory variance ratio* measures the inventory fluctuations indicating the degree of multi-echelon supply chain system instability. An increased inventory variance may result in higher holding and backlog costs, inflating the average inventory cost per period [119]. It has similar definition as order VAR:

$$InvVAR = \frac{\sigma_{Inv}^2 / \mu_{Inv}}{\sigma_n^2 / \mu_n} \quad (3.2)$$

### 3.3 The Bullwhip Effect Metrics

The order VAR (3.1) indicates whether the ordering policy is amplifying or attenuating the demand variations. It only represents one half of the bullwhip problems as the ordering rules also affect the inventory dynamics. Thus the inventory variance ratio (3.2) is used to measure the fluctuations of inventory.

- 3) *Bullwhip slope metric:*

$$SL = \frac{|\mathcal{N}| \sum_{i=1}^{|\mathcal{N}|} i \cdot \mathcal{B}\mathcal{W}^i - \sum_{i=1}^{|\mathcal{N}|} i \cdot \sum_{i=1}^{|\mathcal{N}|} \mathcal{B}\mathcal{W}^i}{|\mathcal{N}| \sum_{i=1}^{|\mathcal{N}|} (i)^2 - \left( \sum_{i=1}^{|\mathcal{N}|} i \right)^2} \quad (3.3)$$

It measures the geometric propagation of bullwhip in a multi-echelon SC system, where  $|\mathcal{N}|$  is the total number of echelons and  $i$  represents the position of the  $i$ th echelon. A large value informs a fast propagation of the bullwhip effect through the supply chain and a low value means a smoothed propagation. Similarly the *inventory slope metrics* can be defined.

- 4) *Average inventory*  $AInv^i = \frac{1}{T} \sum_{t=0}^T y_1^i(t)$  is the mean value of an echelon's inventory over the observation interval  $T$ . It is a complementary metric of *inventory variance ratio* and both of them are associated with the inventory holding cost and backorder cost. Summing the average inventory values of all echelons in a given SC results in another indicator called *systemic average inventory*  $SAInv = \sum_{i=1}^{|\mathcal{N}|} AInv^i$ , which is a well-established industrial practice [118]. Procter & Gamble and Compaq Computer routinely measure both their own inventory and downstream distributors' inventory [120].
- 5) The *zero-replenishment* phenomenon is the event when the echelon managers do not place orders in a review period  $t$ . If the demand is stationary signal during a time horizon of interest, the occurrence of the *zero-replenishment* means an erroneous excessive dimensioning of previous orders [101]. The *zero-replenishment* metric is:

$$ZR^i = \sum_{t=0}^T \kappa^i(t), \quad \kappa^i(t) = \begin{cases} 1, & u_1^i(t) = 0 \\ 0, & u_1^i(t) \neq 0 \end{cases} \quad (3.4)$$

which is used to test timely responsiveness and reactivity of the echelon's operation. This phenomenon could also be measured at systemic level as *systemic zero-replenishment*  $SZR = \sum_{i=1}^{|\mathcal{N}|} ZR^i$ , which is derived by sum-

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ming the *zero-replenishment* of individual echelon over the whole supply chain.

- 6) *Fill rate and backlog* are two metrics representative of the customer service level. Both metrics are evaluated at retailer level  $i=1$  every review period. The *fill rate* is the percentage of end-customer's demand fulfilled on time

$$FR^i = \frac{u_2^i(t)}{d_1^i(t)} \quad (3.5)$$

Backlog has been defined in (2.2) as a cumulative measure. Although the metrics are based on the information of one echelon, they capture the performance of all partners, since the service to the final customer is the end purpose of the entire supply chain.

**Remark 3.3.** *The order VAR (3.1) is the most widely used measure to quantify the bullwhip effect. Together with inventory variance ratio metric, they measure the internal process efficiency and indicate how each echelon performs. However, measuring the internal process efficiency at the individual echelon only explains the individual performance of each link in the SC network separately. To compensate the insufficiency, systematic measures are used as complementary metrics of order and inventory VAR. The bullwhip slope (3.3) and inventory slope metrics summarize all the ratios obtained for each echelon in a single measure (the slope of the linear interpolation) allowing a complete comparison between different SCs or same SC with distinct configurations. These two slope metrics and systematic measures give a significant and concise overview of the SC properties both in terms of bullwhip and inventory stability with just one value instead of the  $|\mathcal{N}|$  values.*

These performance indicators for evaluating the bullwhip effect are defined in time domain based on statistical approach. All or a subset of these indicators can be chosen to measure the bullwhip effect in a well-rounded way. In the following subsection, the order VAR metric is expressed in frequency domain.

### 3.3.2 Bullwhip Metrics in Frequency Domain Analysis

The researchers may face the challenge of lacking accurate customer demand information when investigating the bullwhip effect in practice. To this end, frequency domain based supply chain analysis is introduced to quantitatively estimate the bullwhip effect as a function of inventory management policies.

To facilitate quantification of the bullwhip effect, standard techniques from control engineering are applied in this section. The ordering policy for any echelon can be treated as a special 'system' with the customer demand  $n$  acting as the input

### 3.3 The Bullwhip Effect Metrics

and the order  $u$  acting as the output. Any real-life demand sequence  $\{n(t)\}_{t=0}^{\infty}$  can be decomposed into a series of pure harmonic components  $\mathcal{A}_n(w)e^{-j\omega t}$  with different angular frequencies  $w \in [-\pi, \pi)$  and different complex amplitudes  $\mathcal{A}_n(w)$  by applying discrete Fourier transform. The formula for complex amplitude of any component is  $\mathcal{A}_n(w) = \sum_{t=-\infty}^{\infty} n(t)e^{-j\omega t} = \sum_{t=0}^{\infty} n(t)e^{-j\omega t}$ . Because the ordering policy satisfies the requirements of properness and LTI function, the following two results are deduced immediately: 1) Any component  $\mathcal{A}_n(w)e^{-j\omega t}$  of the decomposed demand (as input to the ordering policy ‘system’) defines an order  $\mathcal{A}_u(w)e^{-j\omega t}$  (as output of the ordering policy ‘system’) with the same angular frequency but an altered amplitude; and 2) The combined output  $\{u(t)\}_{t=0}^{\infty}$  as the order sequence is the superposition of all the components  $\mathcal{A}_u(w)e^{-j\omega t}$  across frequency domain  $w \in [-\pi, \pi)$ . The FRF for the ordering policy is obtained such that  $T_{u/n}(e^{jw}) = \mathcal{A}_u(w) / \mathcal{A}_n(w)$ , which depends only on the specific ordering policy but not on the customer demand. It is well-known that the squared modulus of FRF  $|T_{u/n}(e^{jw})|^2$  is the variance amplification ratio of the order’s amplitude to the demand’s amplitude for particular component of frequency  $w$ . The maximum squared modulus across all frequencies  $w \in [-\pi, \pi)$  is the worst case over the superposition of all possible inputs. Thus a definition of bullwhip is given as:

**Definition 3.2.** *The supply chain system in Figure 2.3 described by models (2.1)-(2.5) subjected to predefined ordering policies is defined to have the bullwhip effect if the  $\mathcal{BW}$  measure for any echelon with demand  $n$  and order  $u$  satisfies the following condition:*

$$\mathcal{BW} = \sup_{w \in [-\pi, \pi)} |T_{u/n}(e^{jw})|^2 > 1 \quad (3.6)$$

**Remark 3.4.** *This metric is useful when checking the possibility of the bullwhip effect caused by a specified ordering policy. However, the practical demands are seldom perfectly harmonic signals, so bullwhip metric is to be developed in next subsection based on ordering policy transfer function.*

Consider the bullwhip situation of the multi-echelon supply chain in Figure 3.2, which is an illustrative variation of Figure 2.3. A harmonic component  $\mathcal{A}_n(w)e^{-j\omega t}$  of end-customer demand enters the chain at retailer echelon, then the output orders from each echelon is also harmonic with the same frequency but a different amplitude  $\mathcal{A}_u^i(w)e^{-j\omega t}$ ,  $i = 1, 2, \dots, 4$ . The factory orders  $\{u_1^4(t)\}_{t=0}^{\infty}$  is the superposition of the harmonic outputs at the current echelon  $\mathcal{A}_u^4(w)e^{-j\omega t}$ . The FRF from end-

customer's demand to the factory echelon's order is

$$T_{u/n}^4(e^{jw}) = \frac{\mathcal{A}_u^4(w)}{\mathcal{A}_n(w)} = \frac{\mathcal{A}_u^4(w)}{\mathcal{A}_u^3(w)} \cdots \frac{\mathcal{A}_u^i(w)}{\mathcal{A}_u^{i-1}(w)} \cdots \frac{\mathcal{A}_u^1(w)}{\mathcal{A}_n(w)} \quad (3.7)$$

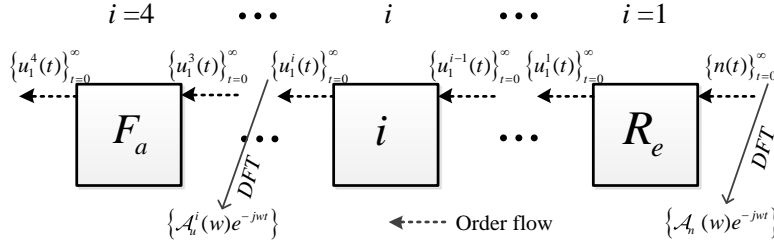


Figure 3.2: The demand/order flow within the multi-echelon supply chain and the Fourier transform of the demand/order sequences

The bullwhip can also be computed as the ratio of the order variance at a generic node  $i$  to the order variance of end-customer. An additional ‘echelon- $i$  FRF’ needs to be defined as  $T^i(e^{jw}) = \frac{\mathcal{A}_u^i(w)}{\mathcal{A}_u^{i-1}(w)}$ , where if  $i=1$ , then  $\mathcal{A}_u^0(w) = \mathcal{A}_n(w)$ .

Then the following relation stands

$$T_{u/n}^i(e^{jw}) = \prod_{k=1}^i T^k(e^{jw}) \quad (3.8)$$

The SC is said to be ‘homogeneous’ if all the echelons are alike, i.e. each echelon is subjected to the same configuration and ordering policy [121]. The behavior of the whole system can be inferred from the behavior of one echelon. Thus, the following theorem can be derived from relation (3.8):

**Theorem 3.1.** *In the homogeneous chains, if the bullwhip exists in the first echelon  $\mathcal{BW}^1 > 1$ , then  $\mathcal{BW}^i > 1$  for all  $i$ .*

*Proof.* In the homogeneous chains, Eq. (3.8) reduces to  $|T_{u/n}^i(e^{jw})|^2 = \left(|T_{u/n}^1(e^{jw})|^2\right)^i$ .

Due to  $\sup_{w \in [-\pi, \pi]} \left(|T_{u/n}^1(e^{jw})|^2\right)^i = \left(\sup_{w \in [-\pi, \pi]} |T_{u/n}^1(e^{jw})|^2\right)^i$ , it follows that  $\mathcal{BW}^i = (\mathcal{BW}^1)^i$ . It is obvious that  $\mathcal{BW}^i > 1$ .  $\square$

**Theorem 3.1** shows that if an ordering policy causes bullwhip effect for one echelon in a homogeneous SC, it causes the bullwhip effect for the whole chain.

### 3.4 Quantifying the Bullwhip Effect in Frequency Domain

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Thus the magnitude of the bullwhip effect can be calculated around a single echelon (requiring echelon- $i$  TF/FRF  $T^i(e^{jw})$ ), a section of SC (requiring TF/FRF of this section of SC  $T_{u/n}^i(e^{jw})$ ), or the whole SC system (requiring TF/FRF  $T_{u/n}^4(e^{jw})$ ).

### 3.4 Quantifying the Bullwhip Effect in Frequency Domain

This section uses  $z$ -transform to derive analytical solution to bullwhip quantification problem. The  $z$ -transform of the demand and order sequences are respectively denoted by the following two expressions  $T_n(z) := Z\{n(t)\} = \sum_0^\infty n(t)z^{-t}$  and  $T_u(z) := Z\{u(t)\} = \sum_0^\infty u(t)z^{-t}$ . Therefore the  $z$ -transform of a given discrete sequence is essentially the discrete Fourier transform after the substitution of  $z = e^{jw}$ . Therefore,  $T_u(z)$  can be linked to  $\mathcal{A}_u(w)$  via  $T_u(e^{jw}) = \mathcal{A}_u(w)$ . Clearly, the following relation holds:  $T_{u/n}(e^{jw}) = \frac{T_u(e^{jw})}{T_n(e^{jw})} = \frac{\mathcal{A}_u(w)}{\mathcal{A}_n(w)}$ . In a considered echelon, the transfer function for an ordering policy is the linear mapping of the  $z$ -transform of received customer demands to the  $z$ -transform of placed orders, which takes the form as a ratio of two polynomials in  $z$ :

$$T_{u/n}(z) = \frac{T_u(z)}{T_n(z)} = \frac{b_n z^n + b_{n-1} z^{n-1} + \dots + b_0}{a_n z^n + a_{n-1} z^{n-1} + \dots + a_0} \quad (3.9)$$

The pulse-transfer function  $T_{u/n}(z)$  also usually manifests itself as the polynomial in backward shift operator  $z^{-1}$ :

$$H(z^{-1}) = \sum_{k=0}^\infty h(k) \cdot z^{-k} = h(0) + h(1)z^{-1} + h(2)z^{-2} + \dots \quad (3.10)$$

**Lemma 3.1.** *Under stationary stochastic demands, the means of demand sequence and order sequence cancel out as  $\mu_u = \mu_n$  over the long term due to properness and LTI property of the ordering policy.*

*Proof.* If the discrete-time stochastic demand  $n(t)$  is applied as input to a linear ordering ‘system’, the output order  $u(t)$  is also a discrete-time random process. The input and output are related by  $u(t) = \sum_{k=0}^\infty h(k)n(t-k)$ , where  $h(k)$  is the impulse response of ordering policy ‘system’.



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The demand is wide sense stationary stochastic process, thus  $\mu_n = \mu_n(t) = \mu_n(t-k)$ . The mean value of orders (output of the ordering policy ‘system’) is by its definition derived as  $\mu_u(t) = E\{u\} = \sum_{k=0}^{\infty} E\{n(t-k)\}h(k) = \mu_n \cdot \sum_{k=0}^{\infty} h(k)$ . Therefore, the mean of orders is  $\mu_u = \mu_u(t) = \mu_n T_{u/n}(1)$ . At 0 Hz the transfer function  $T_{u/n}(1)$  is equal to the DC gain. However, the ordering policies follow from the assumption of properness that if all of the echelons of a chain use proper policies and the customer places order of a size  $u^\infty$ , then a steady state or equilibrium must arise after a long run where all of the echelons of the chain place orders of same size  $u^\infty$ . Under this condition, the DC gain is 1. Thus,  $\mu_u = \mu_n$ .  $\square$

The next step of investigation is to quantify the bullwhip effect and there are two cases under consideration in the following subsections.

### 3.4.1 With Known Customer Demand

The first case is to consider that when customer demand has been obtained, which is demonstrated as follows:

- **Case I.** When the customer demand is observed as a time series  $\{n(t)\}_{t=0}^{\infty}$ , its representation in frequency domain can be approached by discrete Fourier transform or  $z$ -transform. The frequency domain representation of orders is computed using Eq. (3.9) since the transfer function for an ordering policy has been determined. The variance of the demand sequence, by Parseval’s theorem, is  $\frac{1}{2\pi} \int_{-\pi}^{\pi} |T_n(e^{jw})|^2 dw$ . Considering the **Lemma 3.1**, the magnitude of the bullwhip effect defined in Eq. (3.1) is given by:

$$\mathcal{BW} = \frac{\int_{-\pi}^{\pi} |T_u(e^{jw})|^2 dw}{\int_{-\pi}^{\pi} |T_n(e^{jw})|^2 dw} = \frac{\int_{-\pi}^{\pi} |T_{u/n}(e^{jw}) \cdot T_n(e^{jw})|^2 dw}{\int_{-\pi}^{\pi} |T_n(e^{jw})|^2 dw} \quad (3.11)$$

The real demands are seldom perfectly sinusoidals but rather a combination of different sine waves. In a practical way, the magnitude of the bullwhip effect can be quantified very accurately. The input demand is a time series with  $N$  observations and it can be decomposed as the sum of a constant and  $(N/2-1)$  sinusoidals with frequencies that start at 0 radians per review time and increase in multiples of a base frequency  $w_0 = 2\pi/N$ :  $n = c + x_1 + \dots + x_{(N/2-1)}$ , where  $x_k = A_k \sin(w_0 \cdot k \cdot t + \phi_k)$   $k=1, 2, \dots, (N/2-1)$ . The variance of  $k$  th sine wave is  $A_k^2/2$  and the covariance between two sinusoidals is zero if their

### 3.4 Quantifying the Bullwhip Effect in Frequency Domain

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frequencies are different [27], then the variance of input signal can be written as:

$$\text{var}\{n\} = 0.5 \sum_{k=1}^{N/2-1} A_k^2 \quad (3.12)$$

The meaning of (3.12) is that the spectral density estimate  $A_k^2$  represents the contribution of frequency  $k \cdot w_0$  to the total variance of the input signal. The output orders are a summation of  $N/2-1$  sinusoids  $y_k : u = c + y_1 + \dots + y_{(N/2-1)}$ . The sinusoidal  $y_k$  will have the same frequency as  $x_k$ , but the amplitude and phase angle may have changed. The amplitude of sine signal  $y_k$  is denoted by  $\tilde{A}_k$ , and thus  $y_k = \tilde{A}_k \sin(w_0 \cdot k \cdot t + \tilde{\phi}_k)$   $k=1, 2, \dots, (N/2-1)$ . The frequency response plot gives the amplitude ratio values  $AR_k, k=1, \dots, N/2-1$  for all frequencies from 0 to  $\pi$  radians per sample time. Then  $\tilde{A}_k = AR_k A_k$  for all  $k=1, \dots, N/2-1$ . Next, the variance of the generated orders is determine as

$$\text{var}\{u\} = 0.5 \sum_{k=1}^{N/2-1} A_k^2 AR_k^2 \quad (3.13)$$

The estimated order variance amplification is given by the ratio of (3.13) to (3.12)

$$\mathcal{BW} = \frac{\text{var}\{u\}}{\text{var}\{n\}} = \frac{\sum_{k=1}^{N/2-1} A_k^2 AR_k^2}{\sum_{k=1}^{N/2-1} A_k^2} \quad (3.14)$$

Thus the calculation of bullwhip is actually making a weighted average of all squared amplitude ratio  $AR_k^2$  values, with the weights being determined by the squared amplitude values  $A_k^2$  found in the periodogram from DFT. This method indicates 3 steps for quantifying the bullwhip effect:

- (1) To obtain the transfer function of the ordering policy  $T_{u/n}(z)$ . The input of the ordering system is the demand from customer and the output is the corresponding orders. With this transfer function, the amplitude ratio can be obtained from frequency response plot, which represents the ratio between the amplitude of the generated order and the amplitude of the sinusoidal demands.
- (2) Applying the Fourier transform to the demand time series can generate a set of sinusoids of different frequencies with a particular amplitude and phase

associated with each frequency. The plot (periodogram) of squared amplitude versus the frequencies of demand signal is obtained.

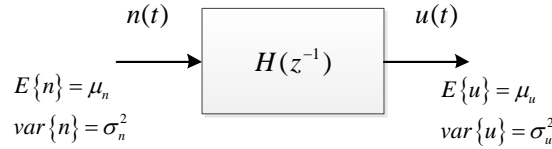
- (3) The application of above 2 steps results in (3.14) and then lead to final bullwhip measurement.

This methods use off-line data of SC operations to calculate the bullwhip. For any possible demand set, it is beneficial to analyze if the current ordering policy causes the demand amplification and to what extent it results in. When the demand set is not available, it is impossible to anticipate in advance whether a designated ordering policy can cause the bullwhip effect. The following subsection provides a solution to this predicament.

### 3.4.2 With Unknown Customer Demand

The result in **Case I** is accurate but requires full knowledge of the customer demand. The essential step to quantify the bullwhip effect is calculating the variances of demand and order. When demand is unknown the bullwhip can still be tested based on (inverse) z-transform of ordering policy transfer function. The quantification of the bullwhip effect is given in **Case II**.

- **Case II.** Firstly a method of calculating the output variance of linear system is given when the input is white noise. The proof and results are given in **Lemma 3.2**. Then this conclusion will be used as the basis for quantifying the bullwhip effect when the demand is an ARMA time series driven by white noise. The bullwhip quantification method is indicated in **Remark 3.5** and **Remark 3.6**.



*Figure 3.3: Output/Input mean and variance for stochastic input  $n(t)$  and normalized stochastic input  $n'(t)$*

Figure 3.3 considers the ordering policy as a special ‘system’ described by transfer function (3.10). When the demand as input to this system is a stationary stochastic process whose mean and variance are known, the mean and variance of the output are related to input mean and variance according to **Lemma 3.1** and the

### 3.4 Quantifying the Bullwhip Effect in Frequency Domain

following **Lemma 3.2** respectively.

**Lemma 3.2.** *When the demand is uncorrelated stochastic process, the ratio of output variance to input variance is a constant given by  $\sigma_u^2/\sigma_n^2 = \sum_{k=0}^{\infty} h^2(k)$ , where  $h(k)$  is the system response at time  $k$  to an impulse applied at  $k=0$ , and  $h(k)$  depends on the parameters of the system.*

*Proof.* The variance of output by definition can be computed from the following formula  $\sigma_u^2 = E\{[u(t)]^2 - [E\{u(t)\}]^2\}$ . The transfer function  $H(z^{-1})$  describes the relation between input signal and output signal as  $u(t) = H(z^{-1})n(t)$ , thus  $\sigma_u^2 = E\{[H(z^{-1}) \cdot n(t)]^2 - [E\{H(z^{-1}) \cdot n(t)\}]^2\}$ . After tedious but simple mathematical manipulation, it can be finally converted to

$$\sigma_u^2 = \left( \sum_{k=0}^{\infty} h^2(k) \right) \cdot \sigma_n^2 + \sum_{r \neq s} h(r)h(s) [E\{n(t-r) \cdot n(t-s)\} - E\{n(t-r)\} \cdot E\{n(t-s)\}]$$

When the input  $n(t)$  is uncorrelated stochastic signal, the second term (autocovariance function) in right side of this equation will be zero. Therefore, the variance of a linear system's output divided by the variance of the input (when the input is uncorrelated stochastic signal) is equal to the sum of the squared impulse response in time domain:  $\sigma_u^2/\sigma_n^2 = \sum_{k=0}^{\infty} h^2(k)$ .  $\square$

**Remark 3.5.** *The insight of this lemma is that if white noise/uncorrelated stationary stochastic signal is entering a transfer function, then the output variance is equal to the input variance multiplied by the sum of squared impulse-response coefficients. The assumption of customer demand as white noise input signal may not be practical in reality. However, it is conceptually useful in applications of using white noise as the basis to model the real-life customer demand. In fact, it plays an important role in quantifying the bullwhip effect as presented in the next chapter. In order to calculate the VAR (the bullwhip effect) from demand to the order, the insight will be exploited. It should be noted that the normalization of the input signal can not affect the variance ratio, i.e.  $\sigma_u^2/\sigma_n^2 = \sigma_u^2/\sigma_n^2$  because it will not change the uncorrelated input assumption and the linearity of system.*

**Remark 3.6.** *This lemma also suggests a procedure to quantify the bullwhip for any echelon of the SC. In Chapter 2, the end-customer demand is assumed to take an ARMA time series driven by white noise and the demand model is represented by  $T^0(z)$  in Figure 3.4. Here the bullwhip effect (demand VAR)  $\sigma_{u_{iRe}}^2/\sigma_{d_{iRe}}^2$  of retailer node is chosen as an example, but the procedure is essentially analogous for all the other ratios. From the block diagram in Figure 3.4, the transfer function*

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## SUPPLY CHAIN PERFORMANCE MEASURES

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from white noise to the retailer orders is  $T_{u/e}(z) = T^0(z) \cdot T^{R_e}(z)$  and from **Remark 3.5** the VAR of  $u_1^{Re}(t)$  to  $e(t)$  is  $\sigma_{u_1^{Re}}^2 / \sigma_e^2 = \sum_{t=0}^{\infty} T_{u/e}(t)^2$ , where  $T_{u/e}(t)$  is the impulse response for ‘system’ described by  $T_{u/e}(z)$ . Similarly, the VAR from  $e(t)$  to  $d_1^{Re}(t)$  can be obtained  $\sigma_{d_1^{Re}}^2 / \sigma_e^2 = \sum_{t=0}^{\infty} T^0(t)^2$ . Therefore, the bullwhip for retailer echelon is  $\mathcal{BN} = \sigma_{u_1^{Re}}^2 / \sigma_{d_1^{Re}}^2 = \sum_{t=0}^{\infty} T_{u/e}(t)^2 / \sum_{t=0}^{\infty} T^0(t)^2$ . The illustrative example of next chapter will show that the procedure is repeatedly exploited.

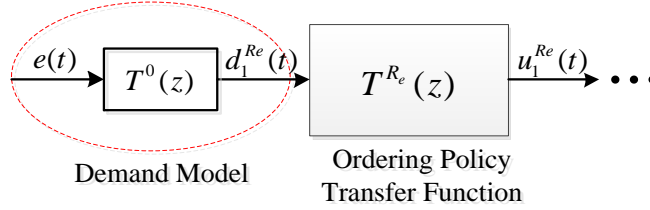


Figure 3.4: Block diagram of the ARMA demand generator and ordering policy system for retailer node

This section presents the methods to quantify the bullwhip effect in frequency domain using the control engineering techniques. Although it is an important quantitative metric, the bullwhip effect is impossible to describe the full picture of the SC performance. The next section introduces the economic cost as an important measure of the SC efficiency.

### 3.5 Supply Chain Economic Performance

The work of Chopra and Meindl [50] has identified major drivers of supply chain performance. They include three logistical drivers (facilities, transportation, and inventory) and three cross-functional drivers (information, sourcing, and pricing) that determine the performance of any supply chain. These drivers interact with each other to determine the supply chain's performance in terms of responsiveness and efficiency.

The *responsiveness* indicates a supply chain's ability to 1) respond to demand at a wide range of quantities; 2) meet a high service level; 3) handle a large variety of products; 4) handle supply uncertainty; 5) take care of short lead times. However,

### 3.5 Supply Chain Economic Performance

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supply chain responsiveness comes at a cost. For instance, to respond to a wider range of quantities demanded, capacity must be increased, which increases costs. This increase in cost leads to lower efficiency. Supply chain *efficiency* is the inverse of the cost of making and delivering a product to the customer.

#### 3.5.1 Drivers of Supply Chain Performance

First of all, three most important drivers and their impacts on the supply chain performance are defined and discussed.

1. **Inventory.** Changing the inventory policies can dramatically alter the supply chain efficiency and responsiveness. Designing the inventory policies is the frequently used strategy in this study to achieve desired performance.
2. **Transportation** entails moving inventory from one place to another in the supply chain. It can take the form of many combinations of modes and routes, each with its own performance characteristics. A reliable transportation is assumed in this work, i.e. a fixed lead time between SC players.
3. **Information** consists of data and analysis concerning the rest of drivers, costs, and customers throughout the supply chain. Information is potentially the biggest driver of performance it directly affects each of the other drivers. Information presents management with the opportunity to make supply chains more responsive and efficient. The effort of quantifying the bullwhip effect, i.e. information distortion is a prerequisite to proposing methods in order to mitigate this effect.

It is important to realize that these factors do not act independently but interact with each other to determine the overall supply chain performance. The goal of SCM is to strike the balance between responsiveness and efficiency that fits with the competitive strategy. To reach this goal, a company must structure a proper combination of the three logistical and three cross-functional drivers.

#### 3.5.2 Measures Based on Cost

In the previous sections, the information driver of supply chain performance is measured by quantifying the bullwhip effect. In the meantime, several non-cost based metrics suitable for the inference on bullwhip are introduced to provide a multi-faceted analysis and description of this information distortion effect. The measures based on cost are presented in the following section. The aim of this

## SUPPLY CHAIN PERFORMANCE MEASURES

phase is to select a set of financial metrics that can evaluate the supply chain performance in a well-rounded way.

The main goals of SCM are mainly to: 1) increase the customer satisfaction, and 2) maximize the profit by cost minimization and sales maximization. In the first target customer satisfaction is measured by the absolute value of the difference between delivery and demand around retailer echelon and the inventory holding profile. In the second objective, the sales maximization can be realized by a variety of business maneuvers such as promotion, pricing and advertisement campaign. However, sales maximization is not the main concern of SC engineers who want to maximize profit. The most widely used objective is reducing the cost, and it is typically minimized for an entire SC, or only for particular business units. Thus the second goal is attained by the minimization of the operating costs of whole SC that is reflected by the following terms.

### 3.5.2.1. Inventory Holding Cost

Let  $t$  to be the current time instant and  $T$  be the length of the management horizon respectively.

The first term is the average excess inventory (AEI)  $AEI = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^{|\mathcal{N}|} y_2^i(t)$ ,

which measures the level of overstock of the whole chain. The associated economic metric is the inventory holding cost:

$$Inv_c = \sum_{t=1}^T \sum_{i=1}^{|\mathcal{N}|} (InvSC^i(t) \cdot y_2^i(t) + InvHC^i(t) \cdot y_2^i(t)) \quad (3.15)$$

where the first element is the handling (salaries, handling facilities, etc.) and storage (maintenance, depreciation, insurance, etc.) costs related to keeping the inventory at the  $i$ th node. The second element accounts for the financial costs of having money invested in inventories, resulting from the fact that inventories are capital assets.

### 3.5.2.2. Shortage Cost

The second term is the average backorder (ABO)  $ABO = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^{|\mathcal{N}|} BO^i(t)$ ,

which indicates the depletion of safety stock and may affect customer satisfaction level. The related economic metric is the shortage cost:

$$BO_c = \sum_{t=1}^T \sum_{i=1}^{|\mathcal{N}|} BOC^i(t) \cdot BO^i(t) \quad (3.16)$$

The back order cost at the  $i$ th node is  $BOC^i$  due to shortage of product, which costs normally much more since it may endanger company's business reputation. This cost may vary greatly depending on whether the customers may agree to wait

### 3.5 Supply Chain Economic Performance

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until their orders are made available or choose from other options of suppliers. Moreover, extra costs occur in backlogging due to additional administrative work, price discounts for late delivery, handling and so on.

#### 3.5.2.3. Transportation Cost

The third term measures the cost of transferring material between nodes:

$$Tran_c = \sum_{t=1}^T \sum_{i=1}^{|\mathcal{N}|} TranC^i(t) \cdot u_2^i(t) \quad (3.17)$$

The transportation cost of sending product from the  $i$ th node to its downstream customer is  $TranC^i$ . It is time invariant and reliable. The transportation costs depend only on the shipment volume and do not consider truck sizes or discrete size shipment.

#### 3.5.2.4. Production Cost

The echelon of factory is special since it involves the production process. The cost of manufacturing the product is described by

$$Prod_c = \sum_{t=1}^T \left( up(t) \cdot FC + Ba(t) \cdot VC + u_1^{Fa}(t) \cdot \left( \sum_j RC^j(t) \cdot Raw^j \right) \right) \quad (3.18)$$

where the variable  $up(t)$  is 1 if the plant starts processing production task (the factory order is not null) at review period  $t$ , 0 otherwise;  $FC$  is the fixed cost. This cost consists of three terms for the plant cost: a fixed start-up cost, a variable manufacturing cost and raw material cost. So the first term in (3.18) considers the start-up costs of preparing the production process for current production orders  $u_1^i(t)$ , which is incurred each time product is produced regardless of the production quantity. The second term takes into account the variable costs  $VC$  associated with the size of batch such as utilities, materials, etc., which is dependent on the amount of production  $Ba(t)$ . The last term represents the cost of purchasing all the required raw materials  $\{Raw^j, j = 1, 2, \dots\}$  to fulfill  $u_1^i(t)$  units of product.

#### 3.5.2.5. Revenue

The revenue that will be generated from the sales at retailer level is given by

$$Revenue = \sum_{t=1}^T \left( Sale(t) \cdot u_2^i(t) \right) \quad i \in \{R_e\} \quad (3.19)$$

where  $Sale(t)$  is the unit price of product and  $u_2^i(t)$  is the amount of product sold by the retailer.

Selection of profit as the performance criteria for the supply chain is one of many options. Given the above terms, the profit function is given by



## SUPPLY CHAIN PERFORMANCE MEASURES

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$$Profit = Revenue - Prod_c - Tran_c - BO_c - Inv_c \quad (3.20)$$

The total value generated in the whole supply chain network is one of the most meaningful performance measures. The objective in SCM is thus the maximization of the profit, i.e. the difference between the revenue and all costs associated with the activities in SC operations.

It is worth noting that there is certain confliction between inventory control and the profit maximization because they represent different supply chain performances, i.e. supply chain responsiveness and supply chain efficiency respectively. The SC engineers should strike a balance when selecting the performance index to evaluate their management strategies.

### 3.6 Summary

This chapter deals with the SC performance measures used to evaluate the different replenishment rules, among which the bullwhip metrics are the focus. Thus the fundamental contributions are made to quantify the bullwhip effect using control engineering techniques.

First, a literature review is briefly conducted on the quantification and mitigation of the bullwhip effect with application of control techniques. Then the chief bullwhip metric together with several other complementary indicators is formulated to fully describe this phenomenon. The most important indicator is the order variance amplification ratio so the first part performs analysis and quantification in time domain with straightforward statistical methods. However, this approach may suffer inaccuracy as it computes  $BW$  based on a fixed observational sample and it is simply an estimate of some underlying true  $BW$  measure. This approach is unable to anticipate if a designed ordering policy can cause the bullwhip effect.

To avoid these drawbacks, new perspectives from control engineering are devoted to quantification. One of the original contributions is that the analytical expression of bullwhip metric can be derived when the transfer function for an ordering policy is known. This closed-form solution is developed based on frequency domain analysis rather than time domain statistical analysis. Two cases are discussed in the frequency domain approach: When the customer demand is known, demand and order representations in frequency domain can be achieved by discrete Fourier transform or ordering policy transfer function. The variance amplification ratio is derived using Parseval's theorem. When demand is unknown the bullwhip can still be tested based on inverse  $z$ -transform of ordering policy transfer function. According to Theorem 3.2 the bullwhip metric can be evaluated by VAR as long as the ordering policy transfer function is determined. To avoid inverse  $z$ -transform, an alternative but equivalent approach of arriving at VAR expression is presented and it is based on the matrices of the coefficients of the systems transfer function.

In addition to the non-cost based metrics that are suitable for the inference on the bullwhip effect, the economic cost measures are also introduced to depict the

### 3.6 Summary

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financial aspect (efficiency) of SC performance. These performance indexes will be used to evaluate the EPSAC-based replenishment rules and control strategies.

The results presented in this chapter have been published in:

Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., 2015. “Quantifying and mitigating the bullwhip effect in a benchmark supply chain system by an extended prediction self-adaptive control ordering policy.” *Computers & Industrial Engineering*, 81, 46-57.

# 4

## Mitigating the Bullwhip Effect within an MPC Framework

Although it has been more than two decades since the first introduction of MPC to inventory management problem in 1992 [33], there are only countable reports in literature concerning the application of MPC to SCM. To the best of our knowledge, none of the published papers is found to deal with quantifying the bullwhip effect in the context of MPC decision framework. The researchers ignored this challenging but significant issue by choice or unintentionally probably because they failed to establish a linkage between the field of control theory and bullwhip quantification. The previous chapter has cast the bullwhip quantification problem in the systems and control theory perspective. The following will fill the research gap by translating the SCM problem into a MPC formulation suitable for implementing the bullwhip quantification method.

This chapter is devoted to show how the adoption of the proposed quantification methods can help academics and practitioners to better understand and study the bullwhip effect under a MPC-based ordering policy. To do so, the MPC is presented as a flexible decision framework for both managing the inventory and meeting the customer requirements. This decision framework's ability to yield an analytical solution to the MPC-based replenishment rule facilitates quantification process using method in previous chapter. The application of this method to measure the bullwhip caused by the MPC-based ordering policy is tested through a comparative analysis with conventional OUT and FO ordering policies.

## 4.1 Introduction

### 4.1 Introduction

The undesired observation known as the bullwhip effect in supply chain operations leads to excessive oscillations of inventory and order levels. Quantifying and mitigating the bullwhip effect become crucial issue in SCM. In the last two decades, academics tried to tackle it from control engineering perspective on account of the resemblance between the dynamic control systems and SC networks. The SC networks are viewed as complex control systems and there are many aspects to study in SCM problem. Quantifying and mitigating the bullwhip effect is only one important part of the problem. Another one is the improvement of inventory management policy, the goal of which is to maintain the inventory level at each decision center to satisfy the customer demands by ordering from the upstream suppliers. It has to be admitted that different aspects of SCM are not completely independent but correlated either explicitly or implicitly, which may imply conflicting objectives and ambiguous preferences over control tasks. Thus the SC practioners need to take into account other aspects of SCM when focusing on one. To meet this SCM requirement, a possible approach could be using MPC technique. Recent studies utilizing MPC have been found to provide an attractive solution for SCM (refer to Chapter 1). As can be seen from literature there are several advantages of applying MPC to SCM. The MPC controller achieves the operational objectives of tracking inventory level target and meeting customers' demand. Moreover, it can optimize a cost function that indicates a proper measure for SC performance. In addition, MPC decision framework shows a qualitative improvement in reducing the bullwhip effect over conventional methods.

<p><b>Nomenclature</b></p> <p><b>Indices</b></p> <p><math>t</math> the supply chain planning period</p> <p><math>k</math> future time index based on current <math>t</math></p> <p><math>i</math> node/echelon index</p> <p><b>Statistics</b></p> <p><math>\sigma_*^2</math> the variance of stochastic variable *</p> <p><math>\mu_*</math> the mean of stochastic variable *</p> <p><b>Variables</b></p> <p><math>y</math> measured IP of the control system for retailer node</p> <p><math>u</math> orders placed by retailer</p> <p><math>\Delta u</math> orders of retailer in differenced form</p> <p><math>n</math> process disturbance of node subsystem</p>	<p><math>e</math> white noise that drives the end-customer</p> <p><math>d_1^{R_e}</math> end-customer demand</p> <p><math>w</math> setpoint for IP</p> <p><math>r</math> reference trajectory for <math>y</math></p> <p><b>Parameters/Notations</b></p> <p><math>\mathcal{B}_*</math> the bullwhip expression caused by the ordering policy *</p> <p><math>*(z^{-1})</math> polynomial * in terms of backward shift operator <math>z^{-1}</math></p> <p><math>N_u</math> control horizon of MPC controller</p> <p><math>N_1</math> minimum prediction horizon of MPC</p> <p><math>N_2</math> prediction horizon of MPC controller</p> <p><math>\gamma</math> desired service level for retailer node</p> <p><math>p^i</math> penalty on excessive movement of the</p>
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## MITIGATING BULLWHIP EFFECT WITHIN AN MPC FRAMEWORK

orders of retailer node	$\beta^i$ adjustable parameter to regulate the response to WIP discrepancy in FO policy for the $i^{th}$ node
$U$ vector of the optimizing future control increments, i.e. the predictive future orders in differenced form	$\eta^i$ adjustable parameter to regulate the response to inventory discrepancy in FO for the $i^{th}$ node
$T_{u/n}$ pulse transfer function of the MPC ordering policy	$\mu$ mean value of ARMA demand
$T_a^i$ the average age of the demand data	$\varphi$ auto regressive coefficient of demand
$\alpha^i$ exponential smoothing parameter for demand forecaster of the $i^{th}$ node	$\theta$ moving average coefficient of demand

This chapter outlines how to quantify and mitigate the bullwhip effect when a MPC decision framework is devised for the benchmark supply chain network. Instead of applying the commonly used statistical estimation methods, the control-theoretic approach presented in Chapter 3 is exploited to derive an analytic expression of bullwhip metric. The approach is able to predict whether the intended ordering policies can cause the bullwhip effect but it requires ordering policy transfer function. To achieve the goal, a closed-form solution for the MPC is derived by introducing Diophantine equations, which is an equivalent realization of EPSAC strategy [122]. This allows of obtaining polynomial expression for the multi-step predictor and the closed loop formulation of MPC strategy in the unconstrained case. Consequently, the transfer function for MPC-based ordering policy can be deduced graphically from a corresponding representation of this MPC formulation. The comparative study is performed in an illustrative example, where the bullwhip effect caused by conventional OUT policy and MPC-based ordering policy is quantified respectively following the control-theoretic approach. In addition, the numerical simulation compares the performance of MPC decision framework with two conventional ordering policies in terms of demand tracking, inventory holding profile and bullwhip evaluation indicators. The simulation shows that MPC decision framework outperforms the other two ordering policies on these terms.

### 4.2 MPC Principles

Model predictive control is a methodology developed around certain common key principles. Given a model of the system, one needs to construct an objective function that incorporates the control goals to formulate an optimal control problem. Usually MPC implements and repeats optimal control in a receding horizon way. In each control step, only the first control sample of the optimal control sequence is used, subsequently the horizon is shifted one step and the optimization process is restarted with new measurements information. MPC has gained wide acceptance in both academic community and modern industries as a useful tool for advanced multivariable control. The literature review in Chapter 1 has shown its applications

## 4.2 MPC Principle

and potential ability to deal with various aspects of SCM. The philosophy of MPC strategy is illustrated in Figure 4.1.

**Prediction:** Any model that can predict the future system states could be selected as the process model. It is an internal model running in parallel with the system, by which the process outputs are predicted over a time period of  $N_2$  (prediction horizon) samples  $y(t+k|t)$ . In the thesis, discrete-time material and information flow models of SC network are used of the following expression:

$$y(t+1) = f(y(t), y(t-1), \dots; u(t), u(t-1), \dots; n(t), n(t-1), \dots) \quad (4.1)$$

where  $y(t)$  is the output (e.g., the inventory position in SC setting) used for evaluating the objective function;  $u(t)$  is the control input (e.g., the ordering decisions);  $n(t)$  is the disturbance (e.g., customer demand).

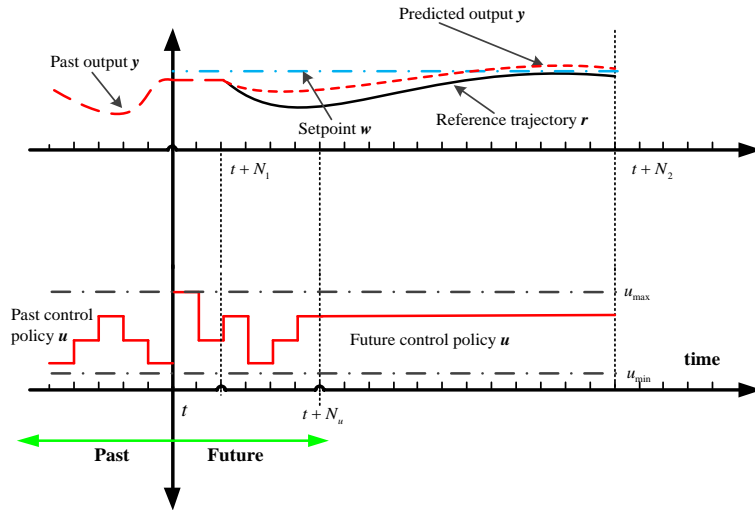


Figure 4.1: Visualization on MPC principles

**Reference Trajectory:** The control goal is to lead the system outputs along a desired path  $\{r(t+k|t), k=1, \dots, N_2\}$  to the final target. Such a path called reference trajectory describes how the future outputs are guided from the current value  $y(t)$  to its setpoint  $w(t)$ . If the future setpoints are not specified, a predictor can be used to predict the desired trajectory that the outputs are forced to track.

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## MITIGATING BULLWHIP EFFECT WITHIN AN MPC FRAMEWORK

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**Control Law:** An essential task of prediction is the calculation of the predicted control errors between output predictions and output targets  $\{[r(t+k|t) - y(t+k|t)], k = N_1, \dots, N_2\}$ . The error plays an important role in compensating the cumulative effect of model mismatch and external disturbance. The predicted output  $y(t+k|t)$  also depends on the postulated control input  $u(t+k|t)$  at  $k=0, \dots, N_2-1$ . In most MPC algorithms there are some structuring of the future control law, thus reducing the degrees of freedom in the control vector  $\{u(t+k|t), k=0, \dots, N_2-1\}$ . This can be done by defining a control horizon  $N_u$  after which the postulated control strategy remains constant:  $\{u(t+k|t) = u(t+N_u-1|t), k \geq N_u\}$ . The control horizon affects how aggressive or conservative the control action is. In many practical applications, a default value of 1 for the control horizon has led to amazingly good results [122]. The tuning of the control horizon is normally approached via *trial and error*.

**Objective Function:** Having these, the MPC algorithm obtains the control vector  $\{u(t+k|t), k=0, \dots, N_u-1\}$  by minimizing the objective function. The flexibility to define objective function is a significant advantage of MPC. It can easily optimize and coordinate different control measures at the same time. Note that penalizing the rate of change on variables yields a more robust but less active controller. Whatever form of control measure is involved, the overall optimization problem can be expressed as

$$\min_{u(t)} J = J(y(t), u(t)) \quad (4.2)$$

$$\begin{aligned} s.t. \quad & y(t) = f(y(t), u(t), n(t)) \\ & u_{\min} \leq u(t) \leq u_{\max} \\ & \Delta u_{\min} \leq \Delta u(t) \leq \Delta u_{\max} \\ & y_{\min} \leq y(t) \leq y_{\max} \end{aligned}$$

in which  $y(t)$  is the system output at time  $t$ . The predicted future outputs

$$y(t) = [y^T(t+1|t), \dots, y^T(t+N_2|t)]^T,$$

are based on the predicted customer demands at time instant  $t$

$$n(t) = [n^T(t|t), \dots, n^T(t+N_2-1|t)]^T,$$

and the future control inputs at time instant  $t$

$$u(t) = [u^T(t|t), \dots, u^T(t+N_u-1|t)]^T.$$

In optimization problem (4.2), there might also be equality and inequality constraints for the system variables. As for SCM problem, a variety of objective func-

### 4.3 An MPC-Based Replenishment

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tions can be chosen aiming at achieving different managerial goals. Some researchers prefer to maximize the profit (3.20) since it is straightforward to understand and formulate. Nevertheless, the objective function is generally attractive when it considers both economic factors and dynamic performance characteristics.

**Receding Horizon Strategy:** Only the first element  $u(t|t)$  of the optimal control  $\{u(t+k|t), k=0, \dots, N_2-1\}$  is actually applied. The remainder can be discarded, because at next review period  $t+1$  as new output  $y(t+1)$  is obtained the whole procedure is repeated after an appropriate backward time shifting. This leads to a new control input and the approach is called ‘the receding horizon’ principle.

## 4.3 An MPC-Based Replenishment

The SC system is characterized by a dynamic balance of material and information flow with the ordering decision serving as control effort. As a result, the ordering policy can be viewed as control strategy. Intuitively, SCM aims to hold the inventory of each node at pre-specified target so it can meet customers’ demand via a replenishment rule (ordering products from upstream suppliers). Next, the formulation of MPC-based replenishment is detailed following principles in Section 4.2, which facilitates bullwhip quantification. As with literature on conventional ordering policies, the MPC is implemented in decentralized style.

### 4.3.1 End-Customer Demand Model

The dynamics of Retailer node are treated individually because the end-customer demand  $d_1^{R_e}(t)$  enters SC network as disturbance. The demand is generally described as a bounded function of time in Section 2.2 to account for any demand pattern. When quantifying the bullwhip effect, a demand model has to take a specific form. This treatment to demand process also aims to make the ensuing exposition of MPC formulation easier. Assume that  $d_1^{R_e}(t)$  is an ARMA time series driven by white noise, of the form in (4.3)

$$D^{R_e}(z^{-1})d_1^{R_e}(t) = C^{R_e}(z^{-1})e(t) \quad (4.3)$$

where the term  $e(t)$  is the white noise. The orders and coefficients of polynomials  $C^{R_e}(z^{-1})$  and  $D^{R_e}(z^{-1})$  are determined by the specific demand pattern under consideration. The accuracy in the prediction of future demand depends on the identification of an accurate ARMA model. Usually, the order and parameters of real-world demand model have to be recognized from the recorded historical data via system identification techniques.



### 4.3.2 The Multistep Predictor

According to SC dynamics analyzed in Section 2.2-2.3, each node is described as a hybrid system switching between different modes. Assuming the two conditions in **Definition 2.1** is an approximation to the real dynamics of the current node. The **Remark 2.1** reveals that Eq. (2.10) is usually treated as the SC model to design various control strategies. The following exposition of MPC formulation is given for retailer node as an example. However, the procedures are essentially analogous for the other nodes. For the sake of simplicity, a new set of notations is used to demonstrate the procedure. Using the relation in Eq. (2.10), the retailer node is modeled in discrete time with  $y$ ,  $u$ ,  $n$  as inventory position, ordering decision, end-customer demand respectively:

$$A(z^{-1})y(t) = B(z^{-1})u(t) + n(t) \quad (4.4)$$

$$D(z^{-1})n(t) = C(z^{-1})e(t) \quad (4.5)$$

where the polynomials  $A(z^{-1})$ ,  $B(z^{-1})$  of IP model have orders  $n_a$ ,  $n_b$  respectively. The polynomials of ARMA demand model are  $C(z^{-1})$  and  $D(z^{-1})$  of orders  $n_c$ ,  $n_d$  respectively.

The fundamental step in MPC is the  $k$ -step-ahead prediction of IP using the model (4.4) and (4.5):

$$y(t+k|t) = \frac{B(z^{-1})}{A(z^{-1})} \cdot u(t+k|t) + \frac{1}{A(z^{-1})} \cdot \frac{C(z^{-1})}{D(z^{-1})} \cdot e(t+k|t) \quad (4.6)$$

Now consider the first Diophantine equation [123]:

$$\frac{C(z^{-1})}{A(z^{-1}) \cdot D(z^{-1})} = E_k(z^{-1}) + z^{-k} \frac{F_k(z^{-1})}{A(z^{-1}) \cdot D(z^{-1})} \quad (4.7)$$

where the polynomials  $E_k(z^{-1})$ ,  $F_k(z^{-1})$  have the orders  $n_e = k-1$ , and  $n_f = \max(n_a + n_d - 1, n_c - k)$  respectively. Then using Eq. (4.7), the second term in RHS of Eq. (4.6) can be rewritten as:

$$\frac{1}{A(z^{-1})} \cdot n(t+k|t) = E_k(z^{-1}) \cdot e(t+k|t) + \frac{1}{A(z^{-1}) \cdot D(z^{-1})} \cdot F_k(z^{-1}) \cdot e(t) \quad (4.8)$$

Note that  $E_k(z^{-1}) \cdot e(t+k|t)$  consists of only future terms and hence the best prediction is 0. Now using model (4.4), (4.5) and (4.7) leads to

### 4.3 An MPC-Based Replenishment

$$\frac{1}{A(z^{-1}) \cdot D(z^{-1})} \cdot F_k(z^{-1}) \cdot e(t) = \frac{F_k(z^{-1})}{C(z^{-1})} \cdot \left( y(t) - \frac{B(z^{-1})}{A(z^{-1})} \cdot u(t) \right) \quad (4.9)$$

which can be substitute back into Eq. (4.8) to have

$$\frac{1}{A(z^{-1})} \cdot n(t+k|t) = \frac{F_k(z^{-1})}{C(z^{-1})} \cdot \left( y(t) - \frac{B(z^{-1})}{A(z^{-1})} \cdot u(t) \right). \quad (4.10)$$

Another Diophantine equation (4.11) is introduced in order to separate the predictor into two parts that are known at time  $t$  and future signals (with  $\Delta = 1 - z^{-1}$ ):

$$\frac{B(z^{-1})}{A(z^{-1}) \cdot \Delta} = G_{k+1}(z^{-1}) + z^{-k-1} \cdot \frac{H_{k+1}(z^{-1})}{A(z^{-1}) \cdot \Delta} \quad (4.11)$$

in which  $G_{k+1}(z^{-1})$ ,  $H_{k+1}(z^{-1})$  are of orders  $n_g = k-1$ ,  $n_h = \max(n_a, n_b - k - 1)$ . Substituting Eq. (4.11) and Eq. (4.10) into the predictor (4.6) results in:

$$\begin{aligned} y(t+k|t) = & \underbrace{G_{k+1}(z^{-1}) \cdot \Delta u(t+k|t)}_{y_{optimize}(t+k|t)} \\ & + \underbrace{\frac{H_{k+1}(z^{-1})}{A(z^{-1})} \cdot u(t-1) + \frac{F_k(z^{-1})}{C(z^{-1})} \left( y(t) - z \cdot \frac{B(z^{-1})}{A(z^{-1})} \cdot u(t-1) \right)}_{y_{base}(t+k|t)} \end{aligned} \quad (4.12)$$

Note that using Diophantine equations to derive the predictor is equivalent to that in EPSAC [124]. This is because: the term  $y_{optimize}(t+k|t)$  is the step response  $G_{k+1}(z^{-1})$  due to (4.11) convolved with the optimizing future control increments; and the rest of the terms constitute the base response  $y_{base}(t+k|t)$  in EPSAC approach, which is determined by (i) the free response (second term in RHS of (4.12)) of the system because of past inputs and outputs and (ii) disturbance prediction (rest of the terms). Using the matrix notations, the predictors in (4.12) form into  $Y = GU + \bar{Y}$  where  $Y = [y(t+N_1|t), \dots, y(t+N_2|t)]^T$  and  $U = [\Delta u(t|t), \dots, \Delta u(t+N_u-1|t)]^T$ ;  $N_1$  is the minimum prediction horizon; the step response matrix  $G$  and base response  $\bar{Y}$  are formed by respectively collecting the polynomials  $G_{k+1}$  and terms  $y_{base}(t+k|t)$  for  $k \in [N_1, N_2]$ .

### 4.3.3 MPC Algorithm

The MPC control purpose is to guide IP  $y(t)$  from its current value to the final target  $w(t)$  along a specified reference trajectory  $\{r(t+k|t), k=N_1, \dots, N_2\}$  while attempting to mitigate the bullwhip effect. The IP target is updated every review period according to  $w(t) = \hat{n}^{L+1}(t) + \gamma \hat{\sigma}^{L+1}(t)$ , the notations of which have the similar inferences as in Eq. (2.29). The MPC algorithm calculates the future ordering decisions  $u$  by optimizing a specified cost function of the following quadratic form:

$$J = \sum_{k=N_1}^{N_2} \left\{ [r(t+k|t) - y(t+k|t)]^2 + p [\Delta \cdot u(t+k - N_1|t)]^2 \right\} \quad (4.13)$$

$$= (R - Y)^T (R - Y) + p \cdot U^T U$$

subject to  $\Delta u(t + \bar{k}|t) = 0$ ,  $\forall \bar{k} \in [N_u, N_2]$ , where  $R = [r(t+N_1|t), \dots, r(t+N_2|t)]^T$  and  $p$  is penalty on control increment. The cost function (4.13) is a multi-objective expression that addresses the main SC operation goals. From the managerial point of view, the selection of a quadratic cost structure allows for a satisfactory trade-off between fast reaction to demand changes and smoothness of ordering decisions [125]. As a result, the MPC controller can track the target *IP* with desired dynamic characteristics, yet it will prevent rapid demand fluctuations from propagating in the SC at the same time. The main goals are: 1) the first setpoint tracking term intend to maintain the inventory positions at user-specified targets over time. It should be noted that the regulation of inventory position implies the control of inventory since the setpoint of inventory position is calculated by considering the inventory holding and WIP level simultaneously; 2) the second term attempts to alleviate the bullwhip effect by eliminating abrupt and aggressive ordering decisions and subsequently prevent SC from undesired variability. The bullwhip reduction is anticipated if the change of orders  $\Delta u$  is suppressed. From control-theoretic perspective, the second term introduces robustness in presence of uncertainty.

Thus the closed-form solution  $U^* = (G^T G + p \cdot I)^{-1} G^T (R - \bar{Y})$  is obtained by minimizing (4.13). Only the first value from  $U^*$  is applied as the ordering decision:

$$\Delta u(t|t) = \sum_{k=N_1}^{N_2} \lambda_k [r(t+k|t) - y_{base}(t+k|t)] \quad (4.14)$$

where  $\lambda_k$  are the elements in the first row of matrix  $(G^T G + p \cdot I)^{-1} G^T$ . Substituting for base response  $y_{base}(t+k|t)$  from (4.12) in Eq. (4.14) yields:

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$$\begin{aligned}
A(z^{-1}) \cdot C(z^{-1}) \cdot \sum_{k=N_1}^{N_2} \lambda_k z^{-N_2+k} r(t+N_2|t) &= A(z^{-1}) \cdot \sum_{k=N_1}^{N_2} \lambda_k \cdot F_k(z^{-1}) \cdot y(t) \\
+ \left( A(z^{-1}) \cdot C(z^{-1}) \cdot \Delta + C(z^{-1}) \cdot \sum_{k=N_1}^{N_2} \lambda_k \cdot z^{-1} \cdot H_{k+1}(z^{-1}) - B(z^{-1}) \cdot \sum_{k=N_1}^{N_2} \lambda_k \cdot F_k(z^{-1}) \right) u(t|t) \\
\Leftrightarrow K(z^{-1}) \cdot r(t+N_2) &= J(z^{-1}) \cdot y(t) + I(z^{-1}) \cdot u(t|t)
\end{aligned} \tag{4.15}$$

where polynomials  $K$ ,  $J$ ,  $I$  are the coefficients of  $r(t+N_2|t)$ ,  $y(t)$ ,  $u(t)$  respectively as indicated above. Relation (4.15) represents the closed-form of polynomial representation of MPC, depicted in Figure 4.2.

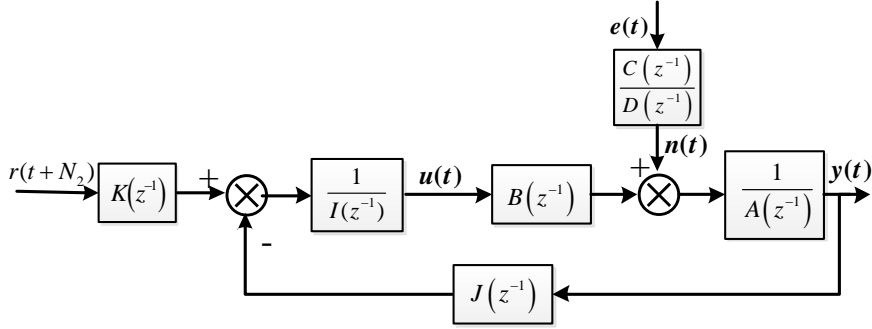


Figure 4.2: The block diagram of closed-loop formulation of MPC-EPSAC decision framework

The transfer function from end-customer demand  $n$  to retailer's orders  $u$  for MPC-based ordering policy is derived graphically from the block diagram:

$$T_{u/n}(z^{-1}) = \frac{-J(z^{-1})}{\varphi_c(z^{-1})} \tag{4.16}$$

with the characteristic polynomial  $\varphi_c(z^{-1}) = A(z^{-1}) \cdot I(z^{-1}) + B(z^{-1}) \cdot J(z^{-1})$ .

The retailer node uses above MPC-EPSAC approach to determine its ordering policy transfer function  $T^1(z)$  as indicated in Eq. (4.16). With the available ordering policy transfer function, **Remark 3.6** indicates that the bullwhip effect for any node, section or the whole SC can be quantified. The following quantifies the bullwhip effect caused by OUT/FO and MPC ordering policies illustratively on a particular ARMA demand.

#### 4.4 An Illustrative Example

The procedure of resolving the closed-form expressions of the bullwhip effect is illustrated by applying as an example to retailer echelon. Consider the mean-centred ARMA demand signal:

$$\begin{cases} d_1^{R_e}(0) = e(0) + \mu \\ d_1^{R_e}(t) = \varphi(d_1^{R_e}(t-1) - \mu) + e(t) - \theta e(t-1) + \mu \end{cases} \quad (4.17)$$

where  $\mu$  is the known mean of demand signal and  $e(t)$  is white noise with the mean of 0 and the variance of  $\sigma_e^2$ . For stability, the values of  $\varphi, \theta$  should be  $|\varphi| < 1$ ,  $|\theta| < 1$ . It is assumed that  $\mu \gg 4\sigma_e$  so negative order is extremely rare. The ARMA(1,1) demand process assumption is in line with literature, e.g. real sets of demand data from retail SCs [64, 70] and demand process for Procter & Gamble products [93].

The transfer function that describes completely the demand pattern in frequency domain is:

$$T^0(z) = \frac{d_1^{R_e}(z)}{e(z)} = \frac{z - \theta}{z - \varphi} \quad (4.18)$$

Taking the inverse  $z$ -transform on Eq. (4.18) leads to:

$$T^0[n] = \varphi^{n-1}(\varphi - \theta \cdot h[n-1]) \quad (4.19)$$

where  $h[n-1]$  is the unit step function. The VAR between ARMA demand and white noise is calculated by using the relation in **Lemma 3.2**:

$$ARMA_{VAR} = \frac{\sigma_{n,ARMA}^2}{\sigma_e^2} = \sum_{n=0}^{\infty} (\varphi^{n-1}(\varphi - \theta \cdot h[n-1]))^2 \quad (4.20)$$

When  $n$  approaches to infinite,  $ARMA_{VAR}$  converges to  $1 + \frac{(\theta - \varphi)^2}{1 - \varphi^2}$ . Thus the VAR from white noise  $e(t)$  to ARMA demand can be obtained with the analytical expression (4.20). This is the technique indicated by **Lemma 3.2** and **Remark 3.6** which will be repeatedly exploited to derive the bullwhips.

##### 4.4.1 Bullwhip for Classical Policy

Recall from Subsection 2.3.2 that the conventional OUT ordering policy of retailer node is defined by

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$$u_1^i(t) = w^i(t) - y_1^i(t) = (L+2)\hat{d}_1^i(t) - y_1^i(t) \quad (4.21)$$

and that the policy requires an estimate or forecast of demand for the coming periods of lead time. The estimation techniques of Eq. (2.30) are well understood and popular with practitioners. For uncorrelated demand signal, the best forecast  $\bar{d}_1^i$  is well-known to be the average of all previous demands. However, for ARMA the correlated demand, an estimate  $\hat{d}_1^i$  can be produced with less estimate error than  $\bar{d}_1^i$  by using a filter such as the exponential smoothing [26, 65, 91]:

$$\hat{d}_1^i(t) = \hat{d}_1^i(t-1) + \alpha^i (d_1^i(t) - \hat{d}_1^i(t-1)) \quad (4.22)$$

where  $\alpha^i = 1/(1+T_a^i)$ . Figure 4.3 gives the relationships among various signals.

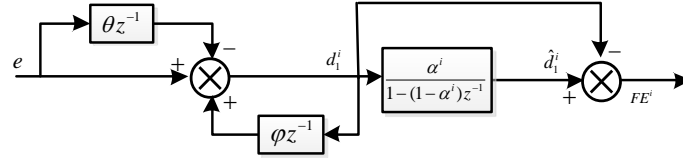


Figure 4.3: Block diagram of the ARMA demand generator, exponential smoothing of ARMA demand and estimate error

First to determine the optimum smoothing parameter  $T_a^i$  that minimizes the MSE of the one-step ahead forecast for particular  $\varphi, \theta$ . Minimizing the variance of one-step ahead forecast error  $FE^i = d_1^i(t+1|t) - d_1^i(t+1) = \hat{d}_1^i(t) - d_1^i(t+1)$  is equivalent to minimizing its MSE [93]. Thus the optimum  $T_a^i$  is derived by differentiating the variance of forecast error and solving for zero gradient. **Remark 3.6** gives the procedure to find the VAR from white noise to forecast error:

$$\frac{\sigma_{FE}^2}{\sigma_e^2} = \frac{2(T_a^i)^2 \cdot [\varphi\theta - \theta - T_a^i(1 + \theta^2 - 2\theta\varphi) - 1 - \theta^2]}{(1 + 2T_a^i)(1 + T_a^i)(T_a^i(\varphi - 1) - 1)(\varphi + 1)} \quad (4.23)$$

Minimizing Eq. (4.23) leads to the optimum  $T_a^i$ :

$$Optim\_T_a^i = \frac{\left( (\theta + 1)^2 - 6\varphi\theta - 2(1 - \theta)^2\varphi + 2\varphi^2\theta + (\theta + 1)\sqrt{\varphi(\varphi - \theta)(1 - \varphi\theta)} \right)}{-4(\varphi - 1)^2 + 4(1 - \theta)(\varphi - 1)^2 + (1 - \theta)^2(3\varphi - 1)} \quad (4.24)$$

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The tuning parameter  $T_a^i$  is determined by (4.24) to be  $Optim\_T_a^i$  in order to minimize the MSE of the one-step ahead forecast by exponential smoothing method. When (4.24) results in negative or complex values, the optimal value  $T_a^i = \infty$  should be used.

It is convenient to derive the transfer function from Figure 2.7 for OUT policy with the known optimal value  $T_a^i$ . The analytical expression of bullwhip is developed by exploiting **Remark 3.6**:

$$\mathcal{B}_{OUT} = \frac{\sigma_{u_1^{Re}, OUT}^2}{\sigma_{d_1^{Re}, ARMA}^2} = 1 - \frac{\left( \frac{2L(\varphi-1)(L+2T_a+1) \cdot (T_a + \theta - \theta\varphi + T_a\theta^2 + \theta^2 - 2T_a\theta\varphi + 1)}{(1+T_a)(1+2T_a)(T_a(1-\varphi)+1)(\theta^2 - 2\theta\varphi + 1)} \right)}{(4.25)}$$

where  $L = \bar{L} + 2$  and the value of  $T_a$  is determined in Eq. (4.24).

Now recall FO policy formulated by (2.31) where two parameters  $\beta^i, \eta^i$  are involved. Setting  $\beta^i$  equal to  $\eta^i$  is particularly advantageous to system robustness as demonstrated by [126]. The analytical expression of bullwhip for FO policy can be derived analogously according to above procedure: rearrange the block diagram (2.28) to get the transfer function of FO ordering policy and multiply by the transfer function of demand signal; take the inverse z-transform and find the sum of the squared impulse response coefficients; divide the result by variance of demand signal. The expression is very lengthy:

$$\mathcal{B}_{FO} = \frac{\sigma_{u_1^{Re}, FO}^2}{\sigma_{d_1^{Re}, ARMA}^2} = \frac{(1-\varphi^2)(\eta^i \cdot \rho_1 \cdot \rho_4 - 2 \cdot \rho_2 \cdot \rho_3 \cdot \rho_5)}{(\theta^2 - 2\theta\varphi + 1) \cdot \rho_2 \cdot \rho_4} \quad (4.26)$$

where  $\rho_1 = (2\theta - \varphi - \varphi\theta^2)\eta^i + \varphi - 2\theta - 2\varphi\theta + \varphi\theta^2 + \theta^2 + 1$ ,  $\rho_2 = (\varphi^2 - 1)(\eta^i - 2)(\varphi\eta^i - \varphi + 1)$ ,  $\rho_3 = (\theta - \varphi T_a^i + 2T_a^i\theta + \varphi\theta - T_a^i\varphi\theta^2)\eta^i + T_a^i(1 + \varphi - 2\theta + \theta^2 - 2\theta\varphi + \varphi\theta^2) - 2\varphi\theta + \theta^2 + 1$ ,  $\rho_4 = (2T_a^i + 1)(T_a^i\eta^i + 1)(1 + \varphi)(\eta^i - 2)(\varphi\eta^i - \varphi + 1)(T_a^i - \varphi T_a^i + 1)$ ,  $\rho_5 = (\eta^i + L^i\eta^i + 1)(L^i\eta^i + 2\eta^i + 2T_a^i\eta^i + 1)$ .

### 4.4.2 Bullwhip for MPC Policy

In the MPC ordering policy the tuning parameters default value  $N_1=1$  is chosen. The prediction horizon  $N_2=3$  is chosen because it is sufficient for the retailer's controller to predict IP over future 3 review periods, i.e.  $L^{Re} = 2$  periods of lead time and 1 period of nominal ordering delay time. The control horizon is  $N_u=N_2$  because it allows the controller to make several changes in  $\Delta u$  before reaching reference

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and a smoother optimal control action is expected. The longer control horizon is beneficial in bullwhip reduction consideration.

Above choices of tuning parameters are used and according to MPC-EPSAC strategy in Section 4.3, the ordering policy transfer function is obtained following two steps: (1) the  $k$ -step-ahead prediction of IP is approached by solving two Diophantine equations as specified in Section 4.3.2; (2) the closed-form of polynomial representation of MPC is obtained by optimizing the cost function (4.13) as stated in Section 4.3.3. Then the transfer function from retailer demand  $n$  to retailer orders  $u$  is derived from Figure 4.2 under this specific choice of parameters:

$$T^1(z^{-1}) = \frac{\left( (\varphi - \kappa_1 p^2 - \kappa_2 p) z^{-2} + ((6\theta - 6 + \kappa_1) p^2 + (9\theta - 9 + \kappa_2) p - \varphi + \theta - 1) z^{-1} \right)}{-\theta \kappa_3 z^{-3} + (\kappa_3 - \theta \kappa_4) z^{-2} + (\kappa_4 + \theta \kappa_5) z^{-1} - \kappa_5} \quad (4.27)$$

where  $\kappa_1 = 5\varphi\theta + 3\varphi^2\theta - 6\varphi - 5\varphi^2 - 3\varphi^3$ ,  $\kappa_2 = 3\varphi\theta - \varphi^2\theta - 9\varphi - 3\varphi^2 + \varphi^3$ ,  $\kappa_3 = 1 + 8p - p^3$ ,  $\kappa_4 = 2p^3 + 14p^2 - 5p - 1$  and  $\kappa_5 = p^3 + 20p^2 + 12p + 1$ .

With the available ordering policy transfer function, the bullwhip effect for retailer node can be quantified analogously following procedures in Subsection 4.4.2:

$$\mathcal{B}_{MPC} = \frac{\sigma_{u_i^{Re}, MPC}^2}{\sigma_{d_i^{Re}, ARMA}^2} = \frac{\sigma_{u_i^{Re}, MPC}^2 / \sigma_e^2}{\sigma_{d_i^{Re}, ARMA}^2 / \sigma_e^2} = \frac{Z^{-1} [T^1(z^{-1}) \cdot T^0(z^{-1})]}{Z^{-1} [T^0(z^{-1})]} \quad (4.28)$$

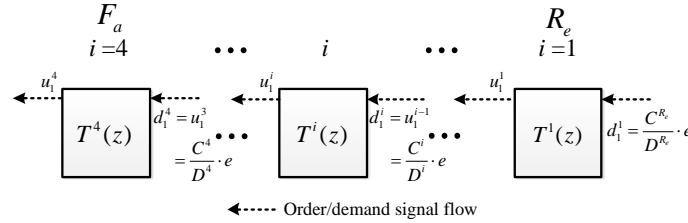


Figure 4.4: The block diagram of demand/order flow within the multi-echelon supply chain and ordering policy transfer function for each echelon

The results were obtained by using MATLAB (MathWorks, Inc.) to assist in the algebraic manipulation. The analytic bullwhip expression is too lengthy to present the details here but it was crosschecked by comparison in a numerical simulation.

According to Figure 4.4, the ordering policy transfer function for the other nodes can be analogously derived. Under decentralized control scheme, each of them employs a MPC-based replenishment that is in line with formulation in Sec-



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tion 4.3 except for modification of demand model (e.g., demand model of node  $i$  becomes  $d_1^i = u_1^{i-1} = \left( \prod_{k=1}^{i-1} T^k(z) \right) \cdot d_1^1 = \frac{C^i}{D^i} \cdot e$ ). For the homogeneous SC at equilibrium state, all the echelons use the same ordering policy: the whole system behavior can be inferred from the behavior of one echelon.

### 4.4.3 Numerical Test

This section focuses on the validation of obtained bullwhip expressions by testing them on the real world demand patterns from Procter and Gamble's home-care and family-care products [93]. Among these 15 different product demand, 10 are chosen to compare their bullwhips calculated by analytic expressions ( $BW_0$ ) and numerical simulation ( $BW_1$ ) respectively.

A typical numerical example is developed where  $L^R = 2$ ,  $\mu = 8$  and  $T_a^i$  is set to minimize one-step ahead forecast error via (4.24). In [93], the researchers also give the empirical value of  $\eta^R$  for different products that minimize the average inventory holding while achieving the 99.5% customer service level. The comparative results in Table 4.1 show that the analytic expression for bullwhip is directly equivalent to the common statistical measure often used in simulation. It can also assist the SC managers to foresee the bullwhip presence of a designed ordering policy and subsequently to determine a more suitable one.

As can be seen from Table 4.1, the bullwhip effect for a single retailer node is very small ( $BW_0 < 1$ ) in theory under MPC-based ordering policy and it matches the numerical simulation results ( $BW_1 \approx BW_0$ ). The bullwhip results both in theory and in numerical simulation show a significant reduction of the bullwhip effect by the MPC-based ordering policy compared to the traditional ordering policies. Next, the simulation is performed on the SC in Figure 2.3 to examine the bullwhip effect caused by three ordering policies. The real-world demand pattern in the fourth row of Table 4.1 is selected as the end-customer demand.

#### 4.4 An Illustrative Example

Table 4.1: The comparisons for the bullwhip effect caused by classical OUT, FO and MPC-base ordering policies of 10 real-life demand patterns, calculated by theoretical analytic expressions and numerical simulation respectively

$\theta$	$\varphi$	$T_a^{R_s}$	OUT Policy		FO Policy			MPC Policy		
			BW <sub>0</sub>	BW <sub>1</sub>	$\eta^i$	BW <sub>0</sub>	BW <sub>1</sub>	$p$	BW <sub>0</sub>	BW <sub>1</sub>
0.128	0.629	0.896	7.1155	7.2767	0.8030	5.4593	5.8121	0.7	0.7854	0.7702
0.342	0.673	2.383	4.1317	4.1954	1.0590	4.4842	4.4352	0.6	0.7561	0.6488
-0.597	0.611	-0.325	15.0322	14.9230	0.001	1.1856	1.1889	3	0.8949	0.9046
-0.133	0.711	5.130	2.1895	2.2577	0.4220	4.0477	4.1697	1	0.9342	0.9360
0.074	0.371	$\infty$	1.0080	1.0080	1.3657	1.7448	1.7163	1.5	0.5163	0.5553
-0.296	0.607	-0.075	12.9150	12.5856	0.001	1.0508	1.050	2.5	0.8817	0.8492
0.459	0.641	23.39	1.3477	1.3468	1.2370	1.9797	1.9874	0.6	0.6354	0.5291
-0.072	0.694	0.149	9.6383	9.8009	0.4170	3.9489	4.2467	1.5	0.8970	0.8805
-0.454	-0.35	$\infty$	1.0080	1.0080	1.2740	1.5190	1.5050	0.8	0.3741	0.3507
-0.295	-0.018	$\infty$	1.0080	1.0080	1.0815	1.1672	1.1731	0.7	0.4985	0.4987

## MITIGATING BULLWHIP EFFECT WITHIN AN MPC FRAMEWORK

The operations of supply chain are evaluated over 100 weeks, i.e. the base time for review is one week and there are 100 simulation samples. The end-customer demand follows the fourth pattern in Table 4.1 and it is generated according to Eq. (4.17), where the mean  $\mu$  is assumed to be 10 units and  $\varphi = 0.711$ ,  $\theta = -0.133$ . The same demand is applied to the SC under three different ordering policies.

*Table 4.2: Simulation data and initial states of supply chain operation*

<i>Supply Chain echelon</i>	<i>PTT <math>L^i</math> (Weeks)</i>	<i>iIL (Units of Product)</i>	<i>iWIP (Units of Product)</i>
Retailer	2	10	5
Wholesaler	2	10	5
Distributor	2	10	5
Factory	2	10	5

Table 4.2 reports SC configurations on PTT, iIL and iWIP. All echelons start with the same initial conditions in this homogeneous SC in order to understand the dynamics of the system and the origin of bullwhip effect. For OUT policy, the optimal  $T_a^i$  is determined according to the procedure in Subsection 4.4.1. The tuning parameters  $\eta^i$  for FO policy are chosen manually to keep a balance between bullwhip reduction and good customer service level. For MPC policy, the minimum prediction horizon  $N_1=1$  because the system time-delay only involves one period of nominal order process time. The maximum prediction horizon  $N_2=3$  is sufficient long for the local controller to predict IP over future 3 weeks, i.e. 2 weeks of PTT and 1 week nominal ordering delay. The control horizon is  $N_u=2$  and the weighting factor is  $p=1$  in the cost function. These design parameters are tuned to obtain a balance between reasonable control effort and acceptable IP tracking error.

The time behaviors of control variables (ordering decisions) and controlled variables (inventory positions) regarding three control policies are presented respectively in Figure 4.5--Figure 4.10. The figures start from the 15<sup>th</sup> week onward when the initialization effect of SC operations disappears.

Figure 4.5 shows that in OUT ordering policy there is obvious amplification of the variations on orders from retailer to factory and the bullwhip quantities in Table 4.3 confirms this observation. Using the same exponential smoothing technique to calculate the setpoints for FO policy, it succeeds in generating smooth ordering patterns as shown in Figure 4.7. FO policy reduces the order variability compared with the OUT policy but it is less responsive to changes in the demand than the latter. As a result, as shown in Figure 4.6 and Figure 4.8, it takes longer time for the IP to recover from demand input signal and consequently the inventory related costs will be larger for FO than they are for OUT. The amplification of order variance is not obviously found from retailer to factory afterward and the ordering decisions between 15<sup>th</sup> and 100<sup>th</sup> week keep good tracking of customer demand variation. Clearly the magnitude of variance of orders is suppressed.

#### 4.4 An Illustrative Example

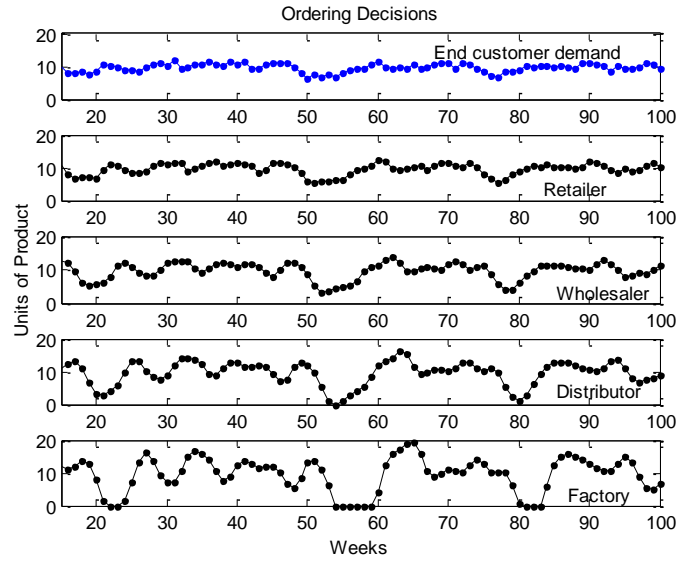


Figure 4.5: The ordering decisions (control efforts) of four nodes over 15th–100th review periods for the OUT ordering policy

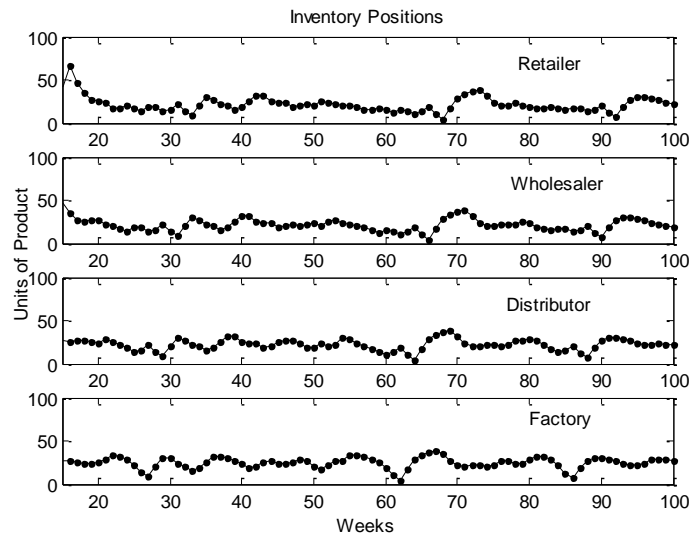


Figure 4.6: The inventory positions (outputs) of four nodes over 15th–100th review periods for the OUT ordering policy

## MITIGATING BULLWHIP EFFECT WITHIN AN MPC FRAMEWORK

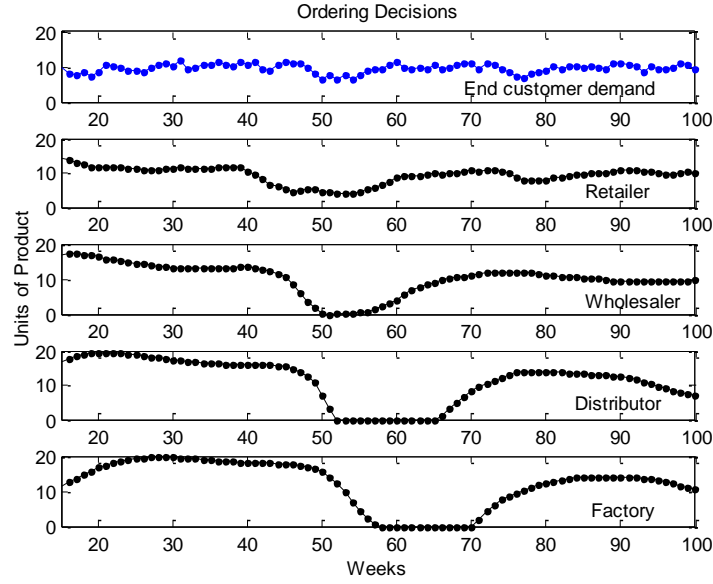


Figure 4.7: The ordering decisions (control efforts) of four nodes over 15th—100th review periods for the FO ordering policy

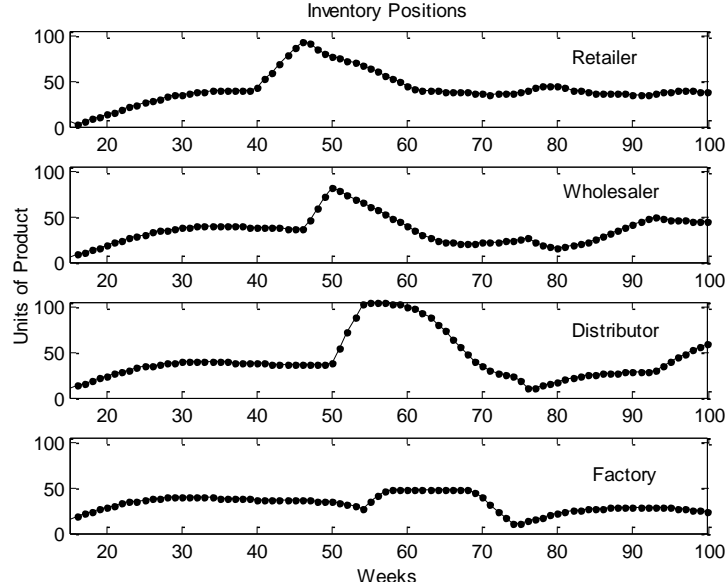


Figure 4.8: The inventory positions (outputs) of four nodes over 15th—100th review periods for the FO ordering policy

#### 4.4 An Illustrative Example

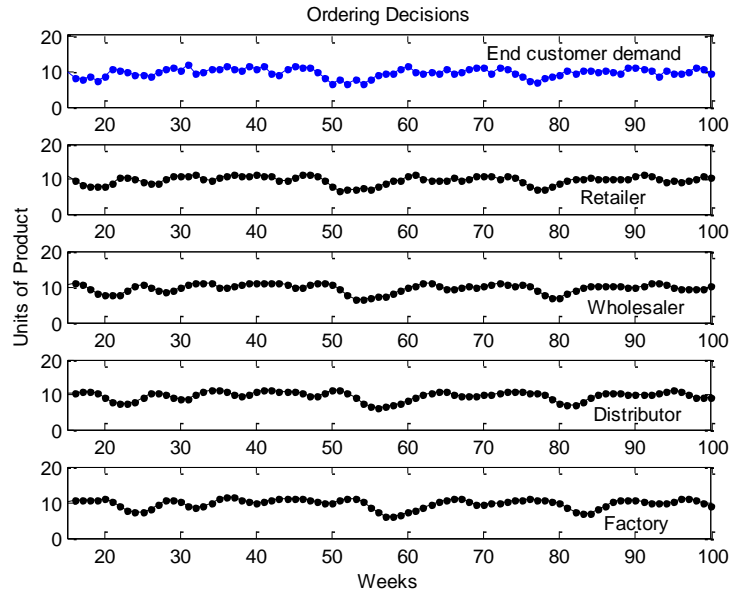


Figure 4.9: The ordering decisions (control efforts) of four nodes over 15th—100th review periods for the MPC ordering policy

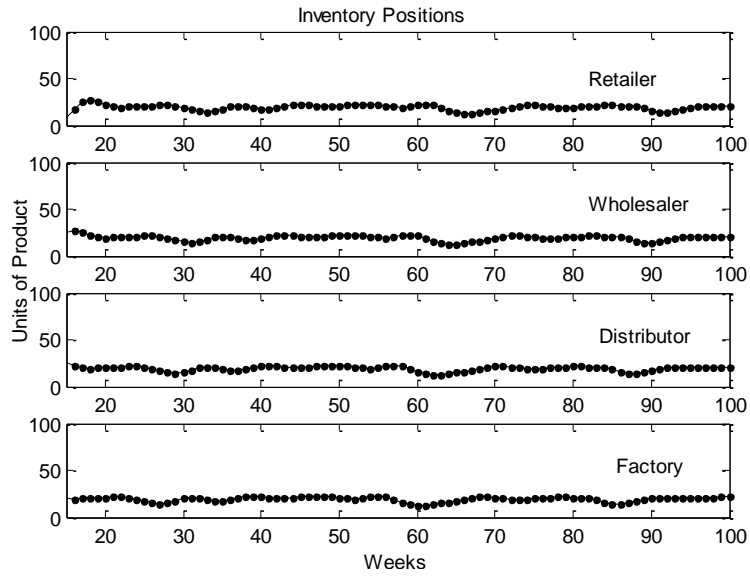


Figure 4.10: The inventory positions (outputs) of four nodes over 15th—100th review periods for the MPC ordering policy

## MITIGATING BULLWHIP EFFECT WITHIN AN MPC FRAMEWORK

*Table 4.3: Bullwhip quantities caused by three ordering policies*

<i>Ordering Policies</i>	<i>Retailer</i>	<i>Wholesa.</i>	<i>Distrib.</i>	<i>Factory</i>	<i>Overall BW</i>	<i>Bullwhip Slope</i>
MPC-based Ordering Policy	0.9858	1.0402	1.0599	1.0624	1.1547	0.06
Fractional Ordering Policy	4.0992	2.4131	1.6916	1.0771	19.5878	4.86
Order-Up-To Ordering Policy	2.0464	2.0761	2.1236	2.0236	18.2567	5.34

Table 4.3 reports the bullwhip quantities (calculated after 15<sup>th</sup> week) for each node and the whole SC caused by the three ordering policies. The *overall BW* shows that the MPC ordering policy outperforms the other 2 in the sense of their abilities to reduce the order variations along the whole SC. The last column according to Eq. (3.3) indicates the speed of propagation of the bullwhip effect through the SC. The MPC policy has smoothed propagation along the echelons, which shows that it is capable of limiting the propagation of bullwhip shockwave and it has less sensitivity to bullwhip than the other 2 policies.

*Table 4.4: Average inventory and zero-replenishment*

	<i>Average inventory (units)</i>			<i>Zero-replenishment</i>		
	<i>AInv, T=100</i>			<i>ZR</i>		
	<i>OUT</i>	<i>FO</i>	<i>MPC</i>	<i>OUT</i>	<i>FO</i>	<i>MPC</i>
Retailer	0.86	17.52	0.14	0	0	0
Wholesaler	1.54	10.50	0.49	0	3	0
Distributor	3.05	12.99	0.84	1	14	0
Factory	5.49	7.47	1.18	11	13	0
Systemic indexes	10.94	48.49	2.65	12	30	0

As the complementary indices of bullwhip metric introduced in Subsection 3.3.1, the average inventory and zero-replenishment are presented in Table 4.4. The high zero-replenishment of FO means that current inventory level exceeds the target so it is indicative of erroneous excessive placement of previous orders. The low zero-replenishment of each node for MPC policy is indicative of its responsive operations. It also has the lowest and most smoothed average inventory along the up-

## 4.6 Summary

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stream echelons. As a result of this, the inventory related costs will be smaller than they are for the other 2 policies. Aggregating individual performance indices into a systemic index of overall performance gives a concise overview on the properties of ordering policies.

## 4.5 Discussion

The proposed quantification method is suitable for any ordering policy as long as its transfer function has been derived. The numerical test proves that the metric of bullwhip effect calculated with control engineering techniques is mathematically equivalent to statistical measure commonly used in literature.

The superiority of MPC in mitigating the bullwhip effect is a result of its formulation. Firstly, the MPC uses the on-line model to calculate the predictions of the future inventory position and to optimize the future ordering decisions. The techniques to build the model of a real-world SC are approached by either theoretical analysis or system identification [127]. Second, the MPC approach calculates the ordering decisions based on the minimization of objective function (4.13), the second term of which penalize the control increment ( $\Delta u$ ). This helps to limit the excessive movement of the manipulated variable and hence to reduce the ordering variability. In practice, the MPC control of the supply chain has the flexibility to add or modify the terms in objective function to fulfill operational requirements.

The conventional OUT, FO ordering policies and proposed MPC ordering policy should be treated as complementary rather than competitive techniques. For example, 1) the MPC offers an ideal decision framework for the tactical level execution to address combinatorial aspect of the problem (financial aspect can also be included in the optimization framework), while OUT and FO are more suitable for regulating operations at low level control problems; 2) the factory node prefers smooth demand pattern otherwise the production switching cost would be large, then the distributor could use FO to generate smooth ordering pattern. The retailer needs to be responsive to the end-customer demand thus OUT or MPC ordering policy is desirable. Integrating different ordering policies to control a SC system may be a better solution.

Finally, the planning and inventory control of the supply chain can be achieved with the help of MPC principles. As pointed by [21], although supply chains have resemblance to the control systems, they have some other unique properties which do not allow direct application of control methods. For example, the MPC formulations rely on the formal mathematic models to make prediction. Therefore, the possibility and accuracy of the SC model have an important effect on the quality of SCM. In this setting, extended cooperation between control engineers and SCM experts can greatly improve supply chain control performances by combining techniques in MPC and practical requirements from SC operations.



## 4.6 Summary

Although there have been numerous reports on the bullwhip effect related research, none of them is concerned with quantifying this phenomenon under MPC decision framework. The contributions in this chapter fill the research gap in SCM literature. First of all, an overview of MPC principles is provided for the proposed new decision framework to follow. This MPC-based ordering policy facilitates to quantify the bullwhip effect using the methodology presented in last chapter. The main content involves three parts: 1) adapting the SC model developed in Chapter 2 to suit the purpose of control in this study; and 2) the Diophantine approach to MPC formulation is applied because it can lead to a closed-form solution and then determine the transfer function for MPC-based ordering policy; and 3) the analytical expression of the ordering policy transfer function is used to quantify the bullwhip effect, using the method indicated by **Theorem 3.2** and **Remark 3.6**. This expression of the bullwhip measure is developed based on control theoretic approach rather than statistical analysis. It can be applied directly to obtain the bullwhip quantities if the knowledge of demand patterns and ordering policies are known. Similarly, the bullwhips caused by conventional OUT, FO ordering policies can be quantified.

In an illustrative example, the bullwhip effect caused by OUT/FO and MPC ordering policies are quantified in the retailer node for a particular ARMA demand. The numerical test proves the equivalence of control theoretic approach to the commonly used statistical analysis in literature. In the simulation, three ordering policies are applied to the SC of structure in Figure 2.3 and their simulation results (the time behaviors of system outputs and control efforts at different echelons, the bullwhip results) are exhibited. In terms of bullwhip mitigation, MPC-based ordering policy results in better performance in general than the other two policies.

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Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., 2014. “Decentralized and centralized model predictive control to reduce the bullwhip effect in supply chain management.” *Computers & Industrial Engineering*, 73, 21–31.

Fu, D., Dutta A, Ionescu, C., De Keyser, R., “Reducing the bullwhip effect in supply chain management by applying a model predictive control ordering policy,” in *Information Control Problems in Manufacturing*, 2012 *14th IFAC Symposium on*, pp. 481-486, 2012.

## 4.6 Summary

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# 5

## MPC Schemes for Decentralized & Centralized Replenishment

Modern supply chains are highly interconnected facilities operating over multiple locations and organizations as they grow larger and larger in structure and become more and more complex in connectivity. Therefore, an ever increasing competitive economy demands an optimal decision-making process and calls for the efficient SCM methods. The operational decisions and control strategies for the SC networks can be addressed by a variety of methods, among which the adoption of model predictive control scheme is the focus of the thesis. MPC does not stand for a single specific control strategy but rather a family of control methods that select the control actions based on on-line optimization of an objective function. The EPSAC strategy is a typical member of the MPC family that is developed by adaptive control community. Its popularity over traditional control approach is attributed to, besides the relative straightforward design procedure, its ability to explicitly handle the physical constraints on system input & output variables and to deal with multivariable control problems. The formulation of EPSAC-based decision algorithm offers an ideal replenishment policy for SCM.

When SC networks become large and complex, a proper control structure is necessary to make the SC more efficient and responsive. A well-structured SC control systems may help the SCs achieve better performances. The following describes the decentralized & centralized replenishment rule for the benchmark supply chain based on the MPC-EPSAC formulation.

### 5.1 Introduction

Traditional heuristics methods (intelligent rules) employed by the SC practitioners are easy to implement and require less data, but they lead to not the best results. It

## 5.1 Introduction

is becoming increasingly difficult for the companies to compete on a global level with only heuristic decision-making tools. Due to the increasing complexity of global supply network, almost every practioner is facing with the issue of SC uncertainty [128]: i) these decision-makers often do not know definitely what to decide under SC uncertainty as they are indistinct about the objectives; ii) they need effective control actions due to lacking information about or understanding of SC dynamics; iii) they are unable to anticipate exactly the impacts of potential control actions on SC behaviour. Thus their decision-making task is prone to systematic errors.

<b>Nomenclature</b>	
<b>Indices/sets</b>	
$i \in \mathcal{N}$	node/echelon/subsystem index
$j$	inputs index for centralized model
$t \in T$	the supply chain planning period
$k$	future time horizon index based on current $t$
$\mathcal{N}$	set of echelons/nodes
$ \mathcal{N} $	number of echelons/nodes
$ T $	the inventory planning horizon
$\mathcal{I}(t)$	full information set for the whole SC at time instant $t$
$\mathcal{I}_i^0(t)$	information subset for echelon $i$ at time instant $t$
<b>Variables</b>	
$L^i$	lead-time of echelon $i$
$y_2^i$	measured inventory at echelon $i$
$x_2^i$	inventory, model output of $i$
$y^i$	measured IP of the control system for echelon $i$
$x^i$	IP, model output of echelon $i$
$u_1^i$	orders placed by the $i$ th echelon
$u_{base}^i$	basic future control actions
$\delta u_1^i$	optimizing future control actions
$u_2^i$	delivery to echelon $i-1$
$d_1^i$	demand from echelon $i-1$
$d_2^i$	received product by the current echelon from echelon $i+1$
$d_1^{R_c}$	end-customer demand
$e^i$	white noise
$n^i$	process disturbance of subsystem $i$
$r^i$	reference trajectory of subsystem $i$
$w^i$	IP setpoint of subsystem $i$
<b>Parameters/Notations</b>	
$\mathcal{S}$	the whole SC network
$\mathcal{S}^i$	the subsystem $i$ of SC network
$\mathcal{C}^i$	controller/agent for subsystem $i$
$*(z^{-1})$	polynomial $*$ in terms of backward shift operator $z^{-1}$
$M$	transfer function matrix for centralized controller model
$\gamma^i$	service level for subsystem $i$
$q^i$	penalty on IP tracking error of subsystem $i$
$p^i$	penalty on excessive movement of orders of subsystem $i$
$N_u^i$	control horizon of $\mathcal{C}^i$
$N_1^i$	min. prediction horizon of $\mathcal{S}^i$
$N_2^i$	prediction horizon of $\mathcal{S}^i$
$U^i$	vector of the optimizing future control actions, i.e. future orders
$\Delta U^i$	vector of the future optimizing control increments of $\mathcal{C}^i$ , i.e. future orders in differenced form
$\underline{Y}^i, \bar{Y}^i$	vector of lower and upper limit of inventory position for $\mathcal{S}^i$
$\Delta \underline{U}^i, \Delta \bar{U}^i$	vector of lower and upper limit of

## MPC SCHEMES FOR DECENTRAL. & CENTRAL. REPLENISHMENT

$\underline{U}^i, \bar{U}^i$	future optimizing control vector of lower and upper limit of the optimizing future control	$\mu$	mean value of ARMA demand
$n_u$	number of inputs (orders)	$\varphi$	auto regressive coefficient of ARMA demand
$n_y$	number of outputs (IP)	$\theta$	moving average coefficient of ARMA demand
$\alpha^i$	filter parameter for reference trajectory	$K_p^i$	proportional gain of PID controller
		$\tau^i$	integral constant of PID controller

The traditional approaches have limited power in tackling the problems inherent in SC uncertainties and dynamics. As such, there has been a growing interest in the subject of modelling, analysis and control on SC, both from research and practical perspectives. As [10] rightly highlighted, the future research direction should focus on developing computational tools and optimization-based methodologies that enable SC managers and engineers to analyse, design and evaluate SCs as dynamic systems. In response, this chapter is particularly interested in the application of MPC-EPSAC to the SCM problems because:

- the decision algorithm can be tuned to reach acceptable performance in terms of both control-theoretic and economic objectives;
- the wide acceptance of its ability to deal with constraints and multivariable control of large-scale SC network.

EPSAC can be applied to SC planning and control problems subject to SCM domain-specific modifications for the unique characteristics of SCs when formulating the replenishment rules. In addition, in technical control systems, the controllers are devices that adjust the systems' responses, which lead to automatic control. Very differently in SC systems the EPSAC controllers are devised to provide decision-making support for the human beings (the real decision-makers).

The methodological benefits of MPC strategy in supporting operational decision-making process have been investigated in Chapter 4. The problems of quantifying and mitigating the bullwhip effect also have been addressed with EPSAC formulation by means of Diophantine equations. However, this approach is developed under unconstrained assumption and it is difficult to include physical restrictions in the algorithm. Several constraints are commonly encountered in actual operations so SC managers are required to take these restrictions into account when designing the decision policies. To cope with this limitation, given that handling constraints efficiently for manipulated and/or controlled variables is one of important abilities of MPC, physical restrictions that arise from practical SC operations will be incorporated in the following EPSAC-based replenishment policies. Moreover, the choice of control structures (centralized, decentralized or distributed) is determined by informational and organizational factors among the interrelated SC players. This chapter discusses the centralized and decentralized implementation of EPSAC to the SC in Figure 2.3. It can be taken as a proof-of-concept indus-

## 5.2 Network-wide Control Structures

trial model of SC replenishment, leading to general rules of investigating the dynamics, inventory control, and bullwhip mitigation for even more complex SCs. Next chapter will deal with distributed EPSAC problem.

## 5.2 Network-wide Control Structures

Since the advent of MPC, the process industry has witnessed a transition from multi-loop PID control strategies to centralized multivariable MPC control [129]. Drawbacks of using one centralized controller led to the idea of partitioning the control system into smaller units and applying MPC controllers to each one of them. The following defines three specific implementations of MPC to benchmark SC.

### 5.2.1 Centralized MPC Control Structure

The SCs are highly interconnected networks operating over multiple locations and products, neglecting these interactions may lead to lower profits. A central coordinator that controls the SC operations can account for these interactions. Modern enterprises tend to expand their scales and interactions, thus it is not rare for them to own a whole supply chain. In principle, if an MPC controller is applied in a centralized fashion, it derives and guarantees the globally optimal control actions for the whole SC. Moreover, one frequently suggested method for reducing the bullwhip effect is to centralize the demand information. Therefore, the necessity of applying a fully centralized MPC control scheme is confirmed.

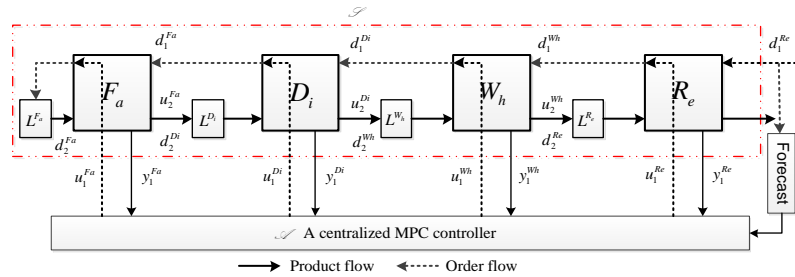


Figure 5.1: Implementation of centralized MPC scheme to supply chain network

In centralized control in Figure 2.6, any system is regulated by a single controller. This controller supervises the whole system and accesses full information about the system. Based on the full information, the controller makes the decisions that are optimal for the entire system and sends them back to the system. A specific implementation of centralized MPC scheme to the benchmark SC network is graphically represented by Figure 5.1.

However, a disadvantage for centralized MPC is related to high computational complexity which grows about exponentially in practice. The high time complexity

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is not the key issue in SC replenishment schemes because the review period is usually in units of days, weeks, months etc. whereas the algorithms are reasonably computed on modern PC to produce results in a time scale of seconds or minutes. Another problem of centralized control is its sensitivity to model error and low resilience with respect to operational changes [130, 131]. Due to these drawbacks and the fact that SCs are traditionally managed in decentralized style, the idea of using separate MPC controller for each subsystem is natural.

### 5.2.2 Decentralized MPC Control Structure

Decentralized control schemes are usually employed for systems composed of  $s$  interacting subsystems. For each of these subsystems a local controller is devised on the basis of corresponding local model and information (Figure 2.4). By dividing the whole system into subsystems and by implementing independent controllers, the complete information of the entire system is also separated.

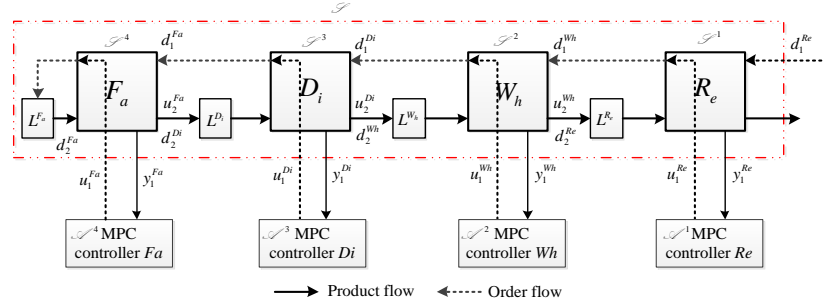


Figure 5.2: Implementation of decentralized MPC scheme to supply chain network

In SC case, today's business environment tends to involve multiple sites working together to deliver product, while reporting to different organizational units. There often exist organizational barriers between the SC facilities so that the information flows can be restricted or costly. Consequently, complete centralized control of material flows may not be feasible or desirable. Even when all facilities belong to the same company the organizational boundaries might seem artificial, some companies have intentionally decentralized operations of their business functions. Each of them has its unique culture, constraints, and objectives. These are motivations of most companies have utilized decentralized management. A specific implementation of the decentralized MPC scheme to the SC is represented by Figure 5.2.

In a pure decentralized control scheme, some or all of the interactions between subsystems are neglected or simply regarded as disturbance. Thus the controllers

## 5.2 Network-wide Control Structures

may become unstable when the interactions are strong. The control policy of each controller does not guarantee global performances. The use of some form of coordination between the controllers for different subsystems is a remedy, which results in the distributed control.

### 5.2.3 Distributed MPC Control Structure

The distributed control strategy is shown in Figure 2.5. Similar to decentralized control, distributed control also uses independent local controllers for different subsystems. Different from decentralized control, the required coordination can be achieved by information exchange among local controllers. Therefore, each local controller will make its own decisions based on both information from the subsystem itself and the information obtained from other subsystems.

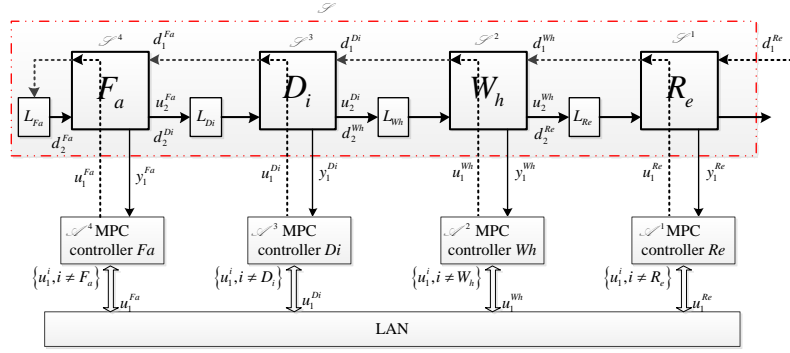


Figure 5.3: Implementation of distributed MPC scheme to supply chain network

Standing in the middle ground between the centralized and decentralized control, distributed control retains the computational efficiency of decentralized control, and at the same time considers the status of other subsystems and tries to reach the performance of the centralized control. A specific implementation of the distributed MPC scheme to the benchmark SC is given in Figure 5.3.

Having these ideas in mind, the following presents the first two MPC schemes for replenishment rules of SC. The distributed MPC scheme will be discussed in next chapter.

## 5.3 Problem Description

In this work, the benchmark multi-echelon SC indicated in the square box of Figure 5.1-Figure 5.3 is under study. To be in consistence with the SC dynamics analysis introduced in Chapter 2, the same process description is assumed but the notations



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are restated for simpler exposition (refer to nomenclature in Section 5.1). As illustrated in Figure 5.4, it would provide SC engineers with solid base to work from when dealing with practical operational problems.

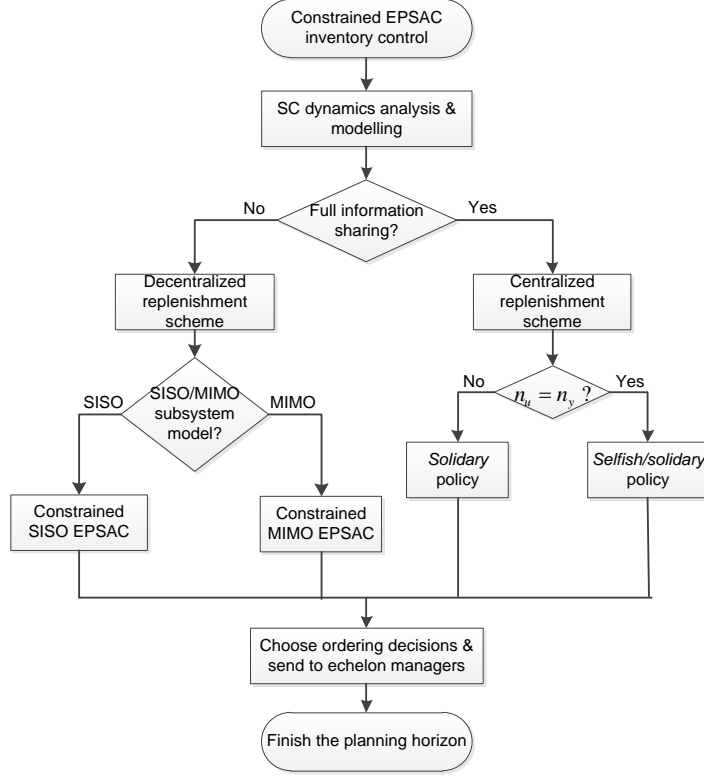


Figure 5.4: Flow chart of EPSAC control problems for benchmark SC network

### 5.3.1 IP Control Models

First recall the micro-dynamics analysis of any node  $i \in \mathcal{N}$  in Section 2.2, the discrete time models describing the  $i$ th node's inventory  $y_2^i(t)$  and inventory position  $y^i(t)$  are repeated here:

$$y_2^i(t) = y_2^i(t-1) + d_2^i(t-L) - u_2^i(t) \quad (5.1)$$

$$y^i(t) = y^i(t-1) + d_2^i(t) - u_2^i(t) \quad (5.2)$$

The relations (5.1) and (5.2) describe the inventory and IP dynamics of individ-

### 5.3 Problem Description

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ual node  $i$ . The full profile of dynamics requires the definition of two operational decisions in practical SC operations. The first one is the ordering policy, i.e. the method of determining  $u_1^i(t)$  from accessible information at time  $t$ . The full information set  $\mathcal{I}(t)$  for the whole SC network is composed of records on  $y_2^i(t)$ ,  $y^i(t)$  up to time  $t$  and  $u_1^i(t)$  up to time  $t-1$  for all echelons  $i \in \mathcal{N}$ .

$$\mathcal{I}(t) := \left[ \bigcup_{i \in \mathcal{N}} \{y_2^i(t), \dots, y_2^i(1); y^i(t), \dots, y^i(1)\} \right] \cup \left[ \bigcup_{i \in \mathcal{N}} \{u_1^i(t-1), \dots, u_1^i(1)\} \right]$$

Another important decision to make is the amount of goods to dispatch in response to downstream customers' orders. The actual delivery is conditional on if the available stock can satisfy customer demand, so the shipping policy must be described by an algorithm:

```

 $\forall i \in \mathcal{N}$ 
if       $y_2^i(t-1) + d_2^i(t-L^i) \geq d_1^i(t)$ 
then     $u_2^i(t) = d_1^i(t)$ 
else,
         $u_2^i(t) = y_2^i(t-1) + d_2^i(t-L^i)$ 
end

```

However, the stock conditions (sufficient or insufficient) of the current node  $i$  are changing during the operational process. It has been reasoned in Section 2.2 that the SC is naturally a switched system. Uncertainty is introduced in SC models by assuming the **ISHS** operation of the node subsystem. According to **Theorem 2.1**, the expressions of model outputs become:

$$x_2^i(t) = \frac{z^{-1}}{1-z^{-1}} \left( z^{-L^i} u_1^i(t) - d_1^i(t) \right) \quad (5.3)$$

$$x^i(t) = \frac{z^{-1}}{1-z^{-1}} \left( u_1^i(t) - d_1^i(t) \right) \quad (5.4)$$

The relations (5.3) and (5.4) will be used as basic models in multi-step predictor design of the following MPC formulation. Being one of the earliest predictive controllers, some key elements of EPSAC are presented via design of decentralized and centralized replenishment rules in the following sections.

### 5.3.2 Control Problems for SC Network

In the first control scenario indicated in Figure 5.2, each echelon is partitioned as a subsystem  $\mathcal{S}^i$ ,  $i=1,2,\dots$ , and is controlled by an autonomous controller  $\mathcal{S}^i$  implementing an MPC-EPSAC control strategy subject to local information to determine its own operational decisions. This is the decentralized control scenario assuming that every subsystem is a *cost center*, so each echelon is responsible for the cost incurred by its own decisions. These decisions are made based on  $\mathcal{I}_i^0(t)$ :

$$\mathcal{I}_i^0(t) := \{y_2^i(t), \dots, y_2^i(1); y^i(t), \dots, y^i(1)\} \cup \{u_1^i(t-1), \dots, u_1^i(1)\} \cup \{d_1^i(t), \dots, d_1^i(1)\}$$

The second control scenario in Figure 5.1 assumes that all the echelons act as a *team*, and they have a communal goal as to optimize the global cost function defined by a global controller  $\mathcal{S}$ . This is appropriate when all parties of  $\mathcal{S}$  can share the information  $\mathcal{I}(t)$  whereby every single echelon's cost function is a definite proportion of the whole system's cost. The MIMO version of EPSAC is used to design the centralized replenishment rule.

## 5.4 Decentralized Replenishment Rule

### 5.4.1 Prediction Model for SC Subsystem

In the block diagram of MPC scheme in Figure 5.5, the difference between measured IP and the model output serves as feedback signal to prediction block. With the feedback signal and input orders  $u_1^i$ , the prediction block computes the future IP. Based on these predicted IP, the MPC optimizer calculates the future optimal ordering decisions.

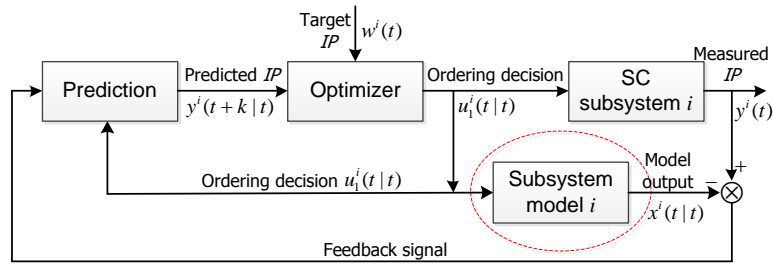


Figure 5.5: SISO EPSAC approach to decentralized MPC control of IP

#### 5.4 Decentralized Replenishment Rule

The EPSAC strategy is based on the following process model of subsystem  $i$  :

$$y^i(t) = x^i(t) + n^i(t) \quad (5.5)$$

the detail of which is illustrated in Figure 5.6, with the control input  $u_1^i$ , measured output  $y^i$  of subsystem, model output  $x^i$ , subsystem  $y^i$  vs. model  $x^i$  disturbance  $n^i$ , model disturbance  $d_1^i$ .

The subsystem model output  $x^i$  represents the effect of the control input  $u_1^i$  on the subsystem's actual output  $y^i$  and it is a signal hard to measure. IP model (5.4) indicates a dynamic relationship between  $x^i$  and  $u_1^i$ , i.e. the current inventory position  $x^i(t)$  does not depend on the current order  $u_1^i(t)$  but on the previous orders and inventory positions. The subsystem model is also subjected to demand variation  $d_1^i$  as load disturbance.

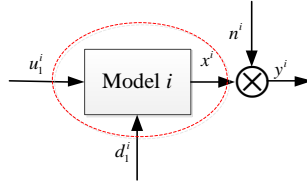


Figure 5.6: Generic SISO model of subsystem  $i$  in decentralized EPSAC formulation

The disturbance  $n^i$  includes the effects in the measured output  $y^i$ , which do not come from the model input via the available model  $i$ . These non-measurable disturbances can be modeled by a colored noise process:

$$n^i(t) = \frac{C^i(z^{-1})}{D^i(z^{-1})} e^i(t) \quad (5.6)$$

where  $e^i(t)$  is white noise and  $C^i(z^{-1}) / D^i(z^{-1})$  is considered to be a design filter.

At each review period  $t$ , Eqs.(5.4)-(5.6) are used to predict IP  $y^i(t+k|t)$ , where  $k=1, \dots, N_2^i$ . In this formulation, the controlled variable is the measured inventory position  $y^i(t)$  of the subsystem  $i$ . The manipulated (control or decision) variable is the order  $u_1^i(t)$  placed by subsystem  $i$ . The predicted values on  $y^i(t)$  at review period  $t$  over  $k$  samples are defined as:

## MPC SCHEMES FOR DECENTRAL. & CENTRAL. REPLENISHMENT

$$y^i(t+k|t) = x^i(t+k|t) + n^i(t+k|t) \quad (5.7)$$

Predictions of  $x^i(t+k|t)$  and  $n^i(t+k|t)$  are done respectively by recursion of subsystem model (5.4) and by using filtering techniques on the noise model (5.6).

The purpose of control is to guide the inventory position  $y^i(t)$  from its current value to the final target  $w^i(t)$  along a desired and generally smooth reference trajectory  $\{r^i(t+k|t), k=1, \dots, N_2^i\}$ . It might be obtained by the first-order lag model:

$$r^i(t+k|t) = \alpha^i \cdot r^i(t+k-1|t) + (1-\alpha^i)w^i(t+k|t) \quad k=1, \dots, N_2^i \quad (5.8)$$

where if  $\alpha^i \approx 1$  it means a slow transition from the current inventory position  $y^i(t)$  to the target  $w^i(t)$ . The target is updated with  $w^i(t) = \hat{D}_{L+1}^i(t) + \gamma^i \hat{\sigma}_{L+1}^i(t)$  at each review period, the notations of which have the similar inferences as in Eq. (2.29).

### 5.4.2 SISO EPSAC Approach to Decentralized Replenishment

Many individual decisions of different importance have to be made and coordinated at a typical SC every review period. The job of *planning* supports decision-making by identifying alternatives of the future activities and selecting good ones or even the best one. Chapter 1 classifies the planning tasks into three levels and proves that MPC as an optimization method offers ideal solutions to the operational level planning problems. The crucial decision to make for a replenishment plan is the ordering. The natural way is to find the alternative ordering decisions, to compare them with respect to a given criterion, and to select the best one. This part achieves the planning by EPSAC approach.

With the available reference trajectory, the objective is to drive the future *IP*  $y^i(t+k|t)$  as close to targets  $w^i(t+k|t)$  as possible. From the point of view of optimizing the system dynamics, the aim of the control action is to bring the system output to the target value without excessive control effort. The ordering decisions are obtained by optimizing a cost function of the following quadratic form:

$$J^i = \sum_{k=N_1^i}^{N_2^i} q^i \cdot [r^i(t+k|t) - y^i(t+k|t)]^2 + \sum_{k=0}^{N_1^i-1} p^i \cdot [\Delta u_1^i(t+k|t)]^2 \quad (5.9)$$

From the managerial point of view, there are often multiple criteria which imply conflicting objectives and ambiguous preferences between alternatives. For example, the bullwhip effect ought to be as small as possible while, at the same time inventories are to be minimized. In this case no 'optimal' solution exists that can accomplish both objectives to the highest possible degree. The main goals of the cost function (5.9) are: 1) to regulate and match the inventory position to its

## 5.4 Decentralized Replenishment Rule

planned target via the first term, where the penalty weight reflects the inventory holding cost and it can be designed to regulate the IP tracking errors; 2) the bull-whip effect mitigation via the second term by penalizing the abrupt change of ordering decisions and subsequently prevent SC from undesired variability. Therefore, the controller will track the target  $IP$  with good dynamics, yet it will prevent rapid demand fluctuations from propagating in the SC at the same time. There are no typical values of  $q^i$  and  $p^i$  which can be suggested for these parameters when tuning the controller. The use of them makes the MPC tuning heuristic but a balance has to be made between acceptable control effort and acceptable control performance.

The fundamental steps in the EPSAC formulation consist of the prediction and the optimization. In prediction, the future response of subsystem  $i$  is the cumulative result of two effects:

$$y^i(t+k|t) = y_{base}^i(t+k|t) + y_{opt}^i(t+k|t) \quad (5.10)$$

The two contributions have the following origins:

- $y_{base}^i(t+k|t)$  is the effect of past control  $\{u_1^i(t-1), u_1^i(t-2), \dots\}$  and past model disturbance  $\{d_1^i(t-1), d_1^i(t-2), \dots\}$ , a pre-specified future base control scenario  $\{u_{base}^i(t+k|t), k=0, 1, \dots\}$ , which is defined *a priori*; and future (predicted) disturbances  $n^i(t+k)$ .
- $y_{opt}^i(t+k|t)$  is the effect of the optimizing future control actions  $\{\delta u_1^i(t+k|t), k=0, 1, \dots\}$  with  $\delta u_1^i(t+k|t) = u_1^i(t+k|t) - u_{base}^i(t+k|t)$ , where  $u_1^i(t+k|t)$  is the optimal control input that the algorithm is looking for. The optimizing control actions  $\delta u_1^i$  can be considered as a series of impulses  $\{h_k^i, h_{k-1}^i, \dots\}$  and a final step  $g_{k-N_u+1}^i$  of input  $\{\delta u_1^i(t+k|t), k=0, \dots, N_u^i-1\}$  to output  $y_{opt}^i(t+k|t)$ .

In brief, the key EPSAC prediction equation can be expressed in matrix notation:

$$Y^i = Y_{base}^i + Y_{optimize}^i = \tilde{Y}^i + G^i U^i \quad (5.11)$$

where  $Y^i = [y^i(t+N_1^i|t), \dots, y^i(t+N_2^i|t)]^T$ ,  $\tilde{Y}^i = [y_{base}^i(t+N_1^i|t), \dots, y_{base}^i(t+N_2^i|t)]^T$ ,  
 $U^i = [\delta u_1^i(t|t), \dots, \delta u_1^i(t+N_u^i-1|t)]^T$ ,

$$G^i = \begin{bmatrix} h_{N_1^i}^i & h_{N_1^i-1}^i & \cdots & h_{N_1^i-N_u^i+2}^i & g_{N_1^i-N_u^i+1}^i \\ h_{N_1^i+1}^i & h_{N_1^i}^i & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{N_2^i}^i & h_{N_2^i-1}^i & \cdots & h_{N_2^i-N_u^i+2}^i & g_{N_2^i-N_u^i+1}^i \end{bmatrix}.$$

The guidelines on choosing  $u_{base}^i$  (a simple example being chosen here is 0) and the common practice to structure future control scenario  $\{\delta u_1^i(t+k|t), k=0,1,\dots\}$ , together with the detailed procedure on how to obtain prediction (5.11) are reported in [122]. A relationship is established between  $\Delta u_1^i(\cdot|\cdot)$  and  $\delta u_1^i(\cdot|\cdot)$  expressed in matrix notations:  $\Delta U^i = A^i U^i + b^i$  with

$$\Delta U^i = [\Delta u_1^i(t|t), \dots, \Delta u_1^i(t+N_u^i-1|t)]^T,$$

$$A^i = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}, b^i = \begin{bmatrix} u_{base}^i(t|t) - u_1^i(t-1) \\ u_{base}^i(t+1|t) - u_{base}^i(t|t) \\ \vdots \\ u_{base}^i(t+N_u^i-1|t) - u_{base}^i(t+N_u^i-2|t) \end{bmatrix}.$$

The cost function (5.9) of quadratic form in  $\Delta U^i$  has the following structure using the notations from Eq.(5.11):

$$J^i(\Delta U^i) = (R^i - Y^i)^T Q^i (R^i - Y^i) + (\Delta U^i)^T P^i \Delta U^i \quad (5.12)$$

where  $Q^i = q^i \cdot I_{N_2^i-N_1^i+1}$ ,  $P^i = p^i \cdot I_{N_u^i}$  and the vector  $R^i$  is defined analogously as  $Y^i$ . Equation (5.12) is easily converted to the standard quadratic cost in  $\Delta U^i$ :

$$J^i(\Delta U^i) = (\Delta U^i)^T H^i \Delta U^i + 2(f^i)^T \Delta U^i + c^i \quad (5.13)$$

in which  $H^i = (A^i)^{-T} (G^i)^T Q^i G^i (A^i)^{-1} + P^i$ ,

$$f^i = -[(A^i)^{-T} (G^i)^T (Q^i)^T (R^i - \tilde{Y}^i) + (A^i)^{-T} (G^i)^T (Q^i)^T G^i (A^i)^{-1} b^i],$$

$$c^i = (R^i - \tilde{Y}^i)^T Q^i (R^i - \tilde{Y}^i) + 2(R^i - \tilde{Y}^i)^T Q^i G^i (A^i)^{-1} b^i + (b^i)^T (A^i)^{-T} (G^i)^T Q^i G^i (A^i)^{-1} b^i$$

### 5.4.3 Ordering Decision via Optimization

Now the control of SC is formulated as an optimization problem in which the control actions  $U^i$  are computed on the basis of cost function (5.13) subject to con-

## 5.4 Decentralized Replenishment Rule

straints. Some practical requirements in SCM operations are posed as constraints on the system variables. Three types are considered in most of studies:

- (1) *Constraints on output variables.* The EPSAC controller minimizes the deviation of inventory position of each subsystem from its target. The inventory position consists of inventory and WIP, both of which have capacity constraints. Thus the inventory position can only stay within high and low limit due to these restrictions.

$$\underline{Y}^i \leq Y^i \leq \bar{Y}^i, i \in \mathcal{N} \quad (5.14)$$

- (2) *Constraints on control increment.* There are some hard low and high bounds for changes (or moves) on orders of each subsystem. If proper constraints on changes of orders are imposed, the limitation of demand variation can be expected and this leads to mitigated demand fluctuation in upstream end of SC.

$$\Delta \underline{U}^i \leq \Delta U^i \leq \Delta \bar{U}^i, i \in \mathcal{N} \quad (5.15)$$

- (3) *Constraints on control variables.* In addition to the constraints on change of orders, there are some high limits on the ordering quantities on account of transportation capacity limitation.

$$\underline{U}^i \leq U^i \leq \bar{U}^i, i \in \mathcal{N} \quad (5.16)$$

These constraints in Equations (5.14)-(5.16) can be constant or changing over the prediction horizon  $N_2^i$ . They can all be expressed in terms of  $\Delta U^i$ , resulting in the matrices  $\tilde{A}^i$ ,  $\tilde{b}^i$  of linear matrix inequality  $\tilde{A}^i \Delta U^i \leq \tilde{b}^i$  in the optimization:

$$\tilde{A}^i = \begin{bmatrix} I_{N_u} \\ -I_{N_u} \\ (A^i)^{-1} \\ -(A^i)^{-1} \\ G^i (A^i)^{-1} \\ -G^i (A^i)^{-1} \end{bmatrix}, \tilde{b}^i = \begin{bmatrix} \Delta \bar{U}^i \\ -\Delta \underline{U}^i \\ \bar{U}^i + (A^i)^{-1} b^i \\ -\underline{U}^i - (A^i)^{-1} b^i \\ \bar{Y}^i - \tilde{Y}^i + G^i (A^i)^{-1} b^i \\ \tilde{Y}^i - \underline{Y}^i - G^i (A^i)^{-1} b^i \end{bmatrix} \quad (5.17)$$

For an EPSAC formulation relying on the models (5.5) and (5.6) to describe the SC subsystem dynamics with a cost function (5.9) subject to linear inequality constraints  $\tilde{A}^i \Delta U^i \leq \tilde{b}^i$ , the numerical solutions can be approached by QP optimization techniques. The purpose of this chapter is to formulate the replenishment rules for SC instead of focusing on how to practically obtain the numerical solution to optimization problem. In fact, the literature on optimization and numerical integration is already vast regarding the last issue.



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## MPC SCHEMES FOR DECENTRAL. & CENTRAL. REPLENISHMENT

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*Table 5.1 The decentralized replenishment scheme based on EPSAC algorithm*

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**Algorithm** *Decentralized replenishment rule*

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*Data:* Inventory position  $\{y^i(t), y^i(t-1), \dots\}$ , past orders  $\{u_1^i(t-1), u_1^i(t-2), \dots\}$  and past demand,  $\{d_1^i(t), d_1^i(t-1), \dots\}$ , the parameters  $N_1^i, N_2^i, N_u^i, p^i, q^i, i=1, 2, \dots$ .

SISO EPSAC controllers: For  $i=1$ , and  $t$

1. *Initialization:* At time  $t$ , controller  $\mathcal{S}^i$  receives the current output  $y^i(t|t)$  from subsystem  $\mathcal{S}^i$ .
  2. *Prediction:* The future response of  $\mathcal{S}^i$  is predicted as Equation (5.11).
  3. *Local optimization:* Controller  $\mathcal{S}^i$  solves the QP optimization problem (5.13) for subsystem  $\mathcal{S}^i$  subjected to linear constraints (5.17).
  4. *Implement control action:* Convert control increment  $\Delta U^i$  to control action  $U^i$ . Implement the calculated actual ordering decision  $u_1^i(t|t)$  to  $\mathcal{S}^i$ .
  5. **if**  $i \neq |\mathcal{N}|$   
     Increase  $i$  by one  $i \leftarrow i+1$ , go to Step 1  
   **else**  
     Go to Step 6
  6. Increase  $t$  by one  $t \leftarrow t+1$  and go to step 1 and repeat.
- 

Table 5.1 describes in detail the procedure of how decentralized replenishment rule is performed. At each planning period  $t$ , the prediction (5.10) and the QP optimization are performed for each subsystems in parallel and they are solved  $|\mathcal{N}|$  times. The whole procedure will be repeated at next planning period taking into account the new information on measured inventory position. As a result,  $|\mathcal{N}| \times |T|$  QP optimization instances in total are solved during the whole planning horizon.

## 5.5 Centralized Replenishment Rule

Continued from the previous section, the EPSAC approach to centralized replenishment rule will be introduced. Similar to the method in previous study, the replenishment rule relies on the generic model and incorporates the physical constraints to form the constrained optimization problems. Different from previous study, as indicated by the flow chart in Figure 5.4, the centralized replenishment

## 5.5 Centralized Replenishment Rule

rule is approached by MIMO EPSAC algorithm with two alternatives, namely *selfish* and *solidary* control, both of which will be presented.

### 5.5.1 Prediction Model for SC System

Using the relation (5.2), the controlled variables (inventory position  $y^i$ ) and manipulated variables (ordering decisions  $u_i^j$ ) are collected for  $\forall i \in \mathcal{N}$  to constitute an overall models for the whole SC. It is used as the basic model in multi-step predictor design of EPSAC formulation:

$$\begin{bmatrix} x^{R_e}(t) \\ x^{W_h}(t) \\ \vdots \\ x^{F_e}(t) \end{bmatrix} = M \begin{bmatrix} u_1^0(t) \\ u_1^{R_e}(t) \\ \vdots \\ u_1^{F_e}(t) \end{bmatrix} \quad (5.18)$$

where end-customer demand  $u_1^0$  is treated as measured disturbances and  $M$  is a transfer function matrix:

$$M = \begin{bmatrix} \frac{-z^{-1}}{1-z^{-1}} & \frac{z^{-1}}{1-z^{-1}} & 0 & \cdots & 0 \\ \vdots & \frac{-z^{-1}}{1-z^{-1}} & \frac{z^{-1}}{1-z^{-1}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \frac{-z^{-1}}{1-z^{-1}} & \frac{z^{-1}}{1-z^{-1}} \end{bmatrix} \quad (5.19)$$

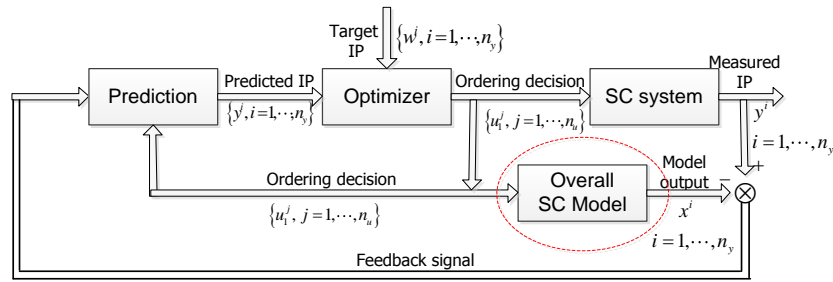


Figure 5.7: MIMO EPSAC approach to the centralized MPC control of IP

In the centralized replenishment scheme represented by Figure 5.7, the differ-

## MPC SCHEMES FOR DECENTRAL. & CENTRAL. REPLENISHMENT

ence between measured  $IPs$  and the model outputs serves as feedback signal to prediction block. With this feedback signals and inputs (orders  $u_1^i$ ), the prediction block predicts the future  $IPs$ . Then the controller calculates the future optimal ordering decisions.

For the SC system with  $n_u$  inputs and  $n_y$  outputs, the structure of the whole SC model (5.18) becomes:

$$y^i(t) = x^i(t) + n^i(t) \quad i = 1, \dots, n_y \quad (5.20)$$

the detail of which is shown in Figure 5.8, with model output  $x^i$  for a series-parallel model  $x^i(t) = f^i[y^i(t-1), y^i(t-2), \dots, u_1^i(t-1), u_1^i(t-2), \dots, u_{n_u}^i(t-1), u_{n_u}^i(t-2), \dots]$ , system inputs  $u_1^j$ , ( $j = 1, \dots, n_u$ ), system output  $y^i$ , system output  $y^i$  vs. model output  $x^i$  disturbance  $n^i(t) = \frac{C^i(z^{-1})}{D^i(z^{-1})} e^i(t)$ .

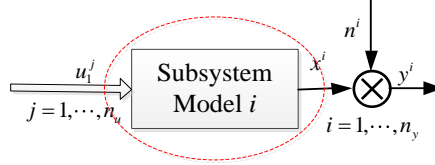


Figure 5.8: Generic MIMO model of subsystem  $i$  in centralized EPSAC formulation

An essential step consists of the predictions for the system outputs calculated by:

$$y^i(t+k|t) = x^i(t+k|t) + n^i(t+k|t) \quad i = 1, \dots, n_y \quad (5.21)$$

over  $k = N_1^i, \dots, N_2^i$ , where  $N_1^i$ ,  $N_2^i$  are minimum and maximum prediction horizons for the  $i$ th output. The future behavior of the IP  $y^i(t+k|t)$  triggered by the inputs  $u_1^j(t+k|t)$ ,  $j = 1, \dots, n_u$ ,  $k = 0, \dots, N_u^j - 1$  is given in its matrix form:

$$Y^i = Y_{base}^i + Y_{optimize}^i = \tilde{Y}^i + \sum_{j=1}^{n_u} G^{ij} U^j \quad i = 1, \dots, n_y \quad (5.22)$$

in which the vectors of future response and the optimizing control actions are  $Y^i = [y^i(t+N_1^i|t) \dots y^i(t+N_2^i|t)]^T$ , and  $U^j = [\delta u_1^j(t|t) \dots \delta u_1^j(t+N_u^j-1|t)]^T$ , the vector of base response is  $\tilde{Y}^i = [y_{base}^i(t+N_1^i|t) \dots y_{base}^i(t+N_2^i|t)]^T$ , and  $G^{ij}$  is a lower triangu-

## 5.5 Centralized Replenishment Rule

lar Toeplitz matrix containing the coefficients of the impulse and step responses of the  $j$  th input to the  $i$  th output starting from the minimum prediction horizon  $N_1^i$  until the maximum prediction horizon  $N_2^i$ :

$$G^{ij} = \begin{bmatrix} h_{N_1^i}^{ij} & h_{N_1^i-1}^{ij} & \cdots & h_{N_1^i-N_d^j+2}^{ij} & g_{N_1^i-N_d^j+1}^{ij} \\ h_{N_1^i+1}^{ij} & h_{N_1^i}^{ij} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{N_2^i}^{ij} & h_{N_2^i-1}^{ij} & \cdots & h_{N_2^i-N_d^j+2}^{ij} & g_{N_2^i-N_d^j+1}^{ij} \end{bmatrix} \quad (5.23)$$

The number of future control samples that can be changed is set by the corresponding control horizon  $N_u^j$ .

### 5.5.2 MIMO EPSAC Approach to Centralized Replenishment

The MIMO EPSAC calculates the optimal control vectors  $U^j$  in two different ways referred to as *solidary* and *selfish* policy. These two alternatives are different on the choice of specified cost functions, implementation conditions, algorithm complexity and control performance. Experience shows that the *selfish* policy leads to better performance under constrained control but it is less robust than *solidary* policy.

- *Solidary Policy*: The objective is to find the optimal ordering-decision vectors  $U^j$ ,  $j = 1, \dots, n_u$  by minimizing the following cost function:

$$J = \sum_{i=1}^{n_y} \sum_{k=N_1^i}^{N_2^i} q^i \cdot [r^i(t+k|t) - y^i(t+k|t)]^2 + \sum_{j=1}^{n_u} \sum_{k=0}^{N_d^j-1} p^j \cdot [\Delta u_1^j(t+k|t)]^2 \quad (5.24)$$

With this control policy being the same structure of objective function as in decentralized control scheme, the cost is summed over all SC subsystems. The objective is to minimize the total cost of all subsystems together, but not just to minimize the individual cost of each subsystem separately. Thus, the performance of one partner can be deliberately decreased with the purpose of increasing the performance of another. This means that the echelon managers have to make their ordering decisions collaboratively. Using the compact matrix notations, the objective function (5.24) is rewritten as  $J = \sum_{i=1}^{n_y} (R^i - Y^i)^T Q^i (R^i - Y^i) + \Delta U^T P \Delta U$ , where the penalty matrices are respectively organized as  $P = \text{blkdiag}[P^1 \dots P^j \dots P^{n_u}]$  with block matrix entries  $P^j = p^j \cdot I_{N_d^j}$  for  $j = 1, \dots, n_u$ , and  $Q^i = q^i \cdot I_{N_2^i - N_1^i + 1}$ . Next step is to define compound matrices  $G^i = [G^{i1} \dots G^{ij} \dots G^{in_u}]$  for  $i = 1, \dots, n_y$  and the compound

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vector  $U$  analogously as  $\Delta U = [(\Delta U^1)^T \cdots (\Delta U^j)^T \cdots (\Delta U^{n_u})^T]^T$ , and then the cost function becomes:

$$J = \sum_{i=1}^{n_y} (R^i - \tilde{Y}^i - G^i U)^T Q^i (R^i - \tilde{Y}^i - G^i U) + \Delta U^T P \Delta U \quad (5.25)$$

The relation between  $\Delta U$  and  $U$  is established via  $U = \Phi(\Delta U - \beta)$ , in which the compound matrix is defined as  $\Phi = \text{blkdiag}((A^1)^{-1} \cdots (A^j)^{-1} \cdots (A^{n_u})^{-1})$  and the compound vector is defined as  $\beta = [(b^1)^T \cdots (b^j)^T \cdots (b^{n_u})^T]^T$ , with

$$A^j = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}_{N_u^j \times N_u^j}, \quad b^j = \begin{bmatrix} u_{base}^j(t|t) - u_1^j(t-1) \\ u_{base}^j(t+1|t) - u_{base}^j(t|t) \\ \vdots \\ u_{base}^j(t+N_u^j-1|t) - u_{base}^j(t+N_u^j-2|t) \end{bmatrix}_{N_u^j \times 1}.$$

The cost function (5.25) is then transformed into standard quadratic cost in  $\Delta U$ :

$$J(\Delta U) = (\Delta U)^T H \Delta U + 2f^T \Delta U + c \quad (5.26)$$

where  $H = \sum_{i=1}^{n_y} \Phi^T (G^i)^T Q^i G^i \Phi + P$ ,

$$f = -\sum_{i=1}^{n_y} [\Phi^T (G^i)^T (Q^i)^T (R^i - \tilde{Y}^i) + \Phi^T (G^i)^T Q^i G^i \Phi \beta],$$

$$c = \sum_{i=1}^{n_y} [(R^i - \tilde{Y}^i)^T Q^i (R^i - \tilde{Y}^i) + 2(R^i - \tilde{Y}^i)^T Q^i G^i \Phi \beta + \beta^T \Phi^T (G^i)^T Q^i G^i \Phi \beta].$$

The constraints for *solidary* control policy are the same as posed as in relations (5.14)-(5.16) so the inequality constraints can be converted into  $\tilde{A}\Delta U \leq \tilde{b}$  with  $\Delta \bar{U}$ ,  $\Delta \underline{U}$ ,  $\bar{U}$ ,  $\underline{U}$ ,  $\bar{Y}$ ,  $\underline{Y}$  of analogous matrix expressions as defined in  $\tilde{Y} = [(\tilde{Y}^1)^T \cdots (\tilde{Y}^i)^T \cdots (\tilde{Y}^{n_y})^T]^T$  and

$$\tilde{A} = \begin{bmatrix} T_r \\ -T_r \\ \Phi \\ -\Phi \\ G\Phi \\ -G\Phi \end{bmatrix}, \quad \tilde{b} = \begin{bmatrix} \Delta \bar{U} \\ -\Delta \underline{U} \\ \bar{U} + \Phi\beta \\ -\underline{U} - \Phi\beta \\ \bar{Y} - \tilde{Y} + G\Phi\beta \\ \tilde{Y} - \underline{Y} - G\Phi\beta \end{bmatrix}, \quad \text{subject to } T_r = \begin{bmatrix} I_{N_u^1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \ddots & I_{N_u^j} & \mathbf{0} & \vdots \\ \vdots & \ddots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & I_{N_u^{n_u}} \end{bmatrix}, \quad G = \begin{bmatrix} G^1 \\ \vdots \\ G^i \\ \vdots \\ G^{n_y} \end{bmatrix}.$$

- *Selfish Policy*: In this case, the objective is to find the optimal control vectors

## 5.5 Centralized Replenishment Rule

$U^i, i=1, \dots, n_u$  which minimize the following quadratic cost function:

$$J^i = \sum_{k=N_t^i}^{N_u^i} q^i \cdot [r^i(t+k|t) - y^i(t+k|t)]^2 + \sum_{k=0}^{N_u^i-1} p^i \cdot [\Delta u_i^i(t+k|t)]^2 \quad (5.27)$$

This control policy is only applicable when the number of control variables equals to the number of controlled variables, i.e.  $n_u = n_y$ . Contrary to the *solidary* control policy, the goal of this strategy is not to minimize the overall cost of all SC subsystems, but just to minimize the individual cost of each subsystem separately. However, every subsystem also takes into account the effect of control actions of all other subsystems (possibly with competing interests). This objective does not result in degenerated performance because it will lead to a MIMO controller with internal cross compensation through dynamic interactions within the SC system. With the compact matrix notations as similarly defined in *Solidary Policy*, the cost function (5.27) is rewritten as  $J^i = (R^i - \tilde{E}^i - G^{ii}U^i)^T Q^i (R^i - \tilde{E}^i - G^{ii}U^i) + (\Delta U^i)^T P^i \Delta U^i$ , which is also a quadratic form in  $\Delta U^i$ :

$$J^i(\Delta U^i) = (\Delta U^i)^T H^i \Delta U^i + 2(f^i)^T \Delta U^i + c^i \quad (5.28)$$

where  $H^i = (A^i)^{-T} (G^{ii})^T Q^i G^{ii} (A^i)^{-1} + P^i$ ,

$$f^i = -(A^i)^{-T} (G^{ii})^T (Q^i)^T (R^i - \tilde{E}^i) - (A^i)^{-T} (G^{ii})^T (Q^i)^T G^{ii} (A^i)^{-1} b^i,$$

$$c^i = (R^i - \tilde{E}^i)^T Q^i (R^i - \tilde{E}^i) + 2(R^i - \tilde{E}^i)^T Q^i G^{ii} (A^i)^{-1} b^i + (b^i)^T (A^i)^{-T} (G^{ii})^T Q^i G^{ii} (A^i)^{-1} b^i, \text{ and}$$

the ‘base response’ projection vector is  $\tilde{E}^i = \tilde{Y}^i + \sum_{j=1, j \neq i}^{n_u} G^{ij} (A^i)^{-1} (\Delta U^j - b^j)$ .

The *selfish* control policy may be also subjected to constraints (5.14)-(5.16) as well and these inequality constraints can be converted into  $\tilde{A}^i \Delta U^i \leq \tilde{b}^i$ , subject to

$$\tilde{A}^i = \begin{bmatrix} I_{N_u^i} \\ -I_{N_u^i} \\ (A^i)^{-1} \\ -(A^i)^{-1} \\ G^{ii} (A^i)^{-1} \\ -G^{ii} (A^i)^{-1} \end{bmatrix}, \quad \tilde{b}^i = \begin{bmatrix} \Delta \bar{U}^i \\ -\Delta \underline{U}^i \\ \bar{U}^i + (A^i)^{-1} b^i \\ -\underline{U}^i - (A^i)^{-1} b^i \\ \bar{Y}^i - \tilde{E}^i + G^{ii} (A^i)^{-1} b^i \\ \tilde{E}^i - \underline{Y}^i - G^{ii} (A^i)^{-1} b^i \end{bmatrix}.$$

On the basis of overall SC model (5.18), the EPSAC controller calculates future control actions  $U$  via optimization problem (5.26) or (5.28), and returns the corresponding element of  $U$  as ordering decisions at current time  $t$  to each echelon. Above optimization problem formulation is standard QP (quadratic cost function with linear inequality constraints). The main steps of implementing centralized

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## MPC SCHEMES FOR DECENTRAL. & CENTRAL. REPLENISHMENT

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replenishment scheme are reported in Table 5.2.

*Table 5.2: The centralized replenishment scheme based on EPSAC algorithm*

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**Algorithm** *Centralized replenishment scheme*

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*Data:* Past inventory position  $\{y^i(t), y^i(t-1), \dots\}$ , past orders and external customer demand  $\{u_1^j(t-1), u_1^j(t-2), \dots\}$ ,  $\{u_1^0(t), u_1^0(t-1), \dots\}$ , parameters  $N_1^i$ ,  $N_2^i$ ,  $N_u^j$ ,  $p^j$ ,  $q^i$ ,  $i=1, \dots, n_y$ ,  $j=1, \dots, n_u$ .

*MIMO EPSAC controllers:*

1. *Initialization:* At review time  $t$ , centralized controller receives the current output  $y^i(t|t)$  from the SC system  $\mathcal{S}$ ,  $i=1, \dots, n_y$ .
  2. *IP prediction:* The future inventory position of the SC system is predicted as (5.22).
  3. *Global optimization:* The global controller  $\mathcal{A}$  solves the QP optimization problem (5.26) 1 time for *solidary control* policy or (5.28) with  $i=1, \dots, n_u$  for *selfish control* policy subjected to respective constraints.
  4. *Control action implementation:* Convert control increment  $\Delta U$  to control action  $U$ . Implement the actual ordering decision  $u_1^j(t|t)$  to corresponding node,  $j=1, \dots, n_u$ .
  5. **if**  $t < |T|$ , Increase  $t$  by one  $t \leftarrow t+1$ , go to Step 1 and repeat  
**else**, Stop
- 

Instead of  $|\mathcal{N}|$  separate controllers being implemented in decentralized control scheme, a single controller is employed to make ordering decisions for each SC echelon. In this case, orders and *IP* information at each echelon are fed to central controller as illustrated in Figure 5.1. At each planning period  $t$ , the prediction and QP optimization are performed for the global system, in which the prediction are performed 1 time whereas the QP optimization is solved 1 time in *solidary* policy and  $iter(t) \times |\mathcal{N}|$  times in *selfish* policy with respective time complexity. In general, their computation load tends to be larger compared to decentralized scheme. The whole procedure is repeated in the next planning period, taking into account the new measured inventory position information. As a result, there are  $|T|$  (in *solidary* policy) or  $\sum_{t=1}^{|T|} (iter(t) \times |\mathcal{N}|)$  (in *selfish* policy) QP problems in total are

## 5.6 An Illustrative Example

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solved within the whole planning horizon.

## 5.6 An Illustrative Example

### 5.6.1 Initialization of Simulation

The replenishment policies are evaluated over 100 weeks and the end-customer demand is generated according to the fourth row of Table 4.1 and it is generated according to Eq. (4.17), where the mean  $\mu$  is assumed to be 10 units and  $\varphi=0.711$ ,  $\theta=-0.133$ . The same demand is applied to the SC under 4 different ordering policies. In the initial state, each subsystem's *iIL* has 10 units of product, *iWIP* level has 10 units and *PTT* is 2 weeks. The same start-up conditions are assumed to eliminate their effects on the bullwhip results. Therefore, only different replenishment rules' effect on the bullwhip effect are considered and compared.

In this work, a PI controller (OUT policy is essentially a P controller with  $K_p=1$ , a specific case of PI control) is compared with the MPC schemes:

$$u_i^i(t) = K_p^i \left( 1 + \frac{1}{\tau^i(1-z^{-1})} \right) (w^i(t) - y^i(t)) \quad (5.29)$$

The calculation of setpoints by using Eq. (2.29) is referred to the method proposed in Section 2.3.2. The constraints imposed on the control actions and IP are constant over the planning horizon:  $\Delta u_i^1(t) \leq 3$ ,  $\Delta u_i^2(t) \leq 4$ ,  $\Delta u_i^3(t) \leq 5$ , and  $\Delta u_i^4(t) \leq 6$ ;  $0 \leq y^1(t) \leq 45$ ,  $0 \leq y^2(t) \leq 45$ ,  $0 \leq y^3(t) \leq 50$ ,  $0 \leq y^4(t) \leq 50$ . The PID control cannot handle constraints. In addition, there is no direct link between PI parameters and bullwhip reduction. The PI parameters have to be tuned manually to balance a lower economic cost and the avoidance of violation of the constraints on  $y^i$ :  $K_p^1 = 0.48, \tau^1 = 500$ ;  $K_p^2 = 0.56, \tau^2 = 600$ ;  $K_p^3 = 0.75, \tau^3 = 700$ ;  $K_p^4 = 0.88, \tau^4 = 800$ .



## MPC SCHEMES FOR DECENTRAL. & CENTRAL. REPLENISHMENT

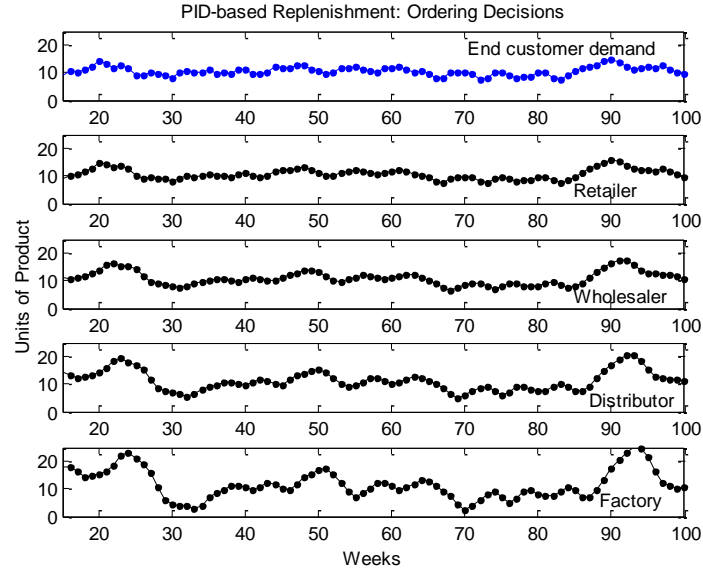


Figure 5.9: Time behavior of ordering decisions for PI control scheme

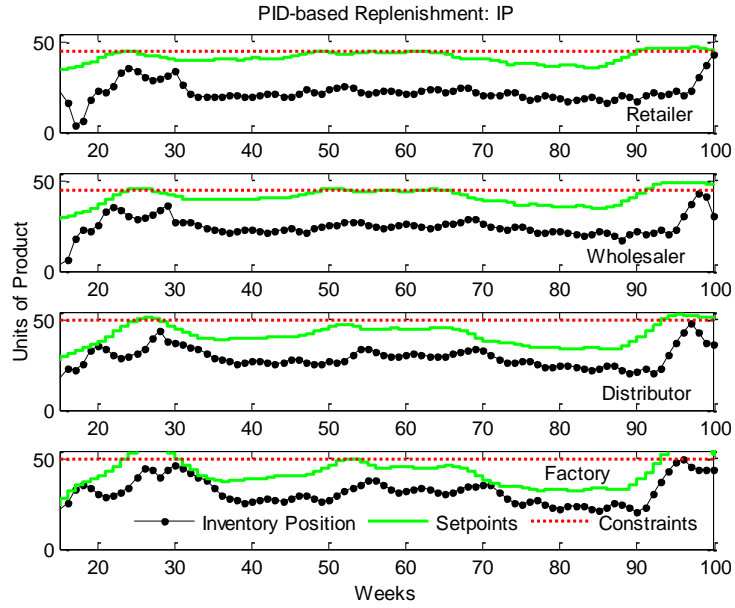


Figure 5.10: Time behavior of IP, setpoints and constraints for PI control scheme

Through simulation it is found that bullwhip increases with the increase of both

## 5.6 An Illustrative Example

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$K_p^i$  and  $\tau^i$  but the cost decreases as  $K_p^i$  is increased. The PID controller is unable in the same time to reduce bullwhip and keep low cost while satisfying the constraints. The result reported in Figure 5.9 and Figure 5.10 illustrates this observation. The figures start from the 15<sup>th</sup> week onward when the initialization effect of SC operations disappears.

### 5.6.2 Simulation for EPSAC Replenishment Rules

In decentralized MPC replenishment rule, each SISO EPSAC controller has the tuning parameters of the default values  $N_1^i=1$ ,  $N_u^i=1$ . The prediction horizon is chosen as  $N_2^i=5$  because it is sufficient for the local controller to predict outputs over future 3 weeks, i.e. 2 weeks of PTT and 1 week ordering delay. The penalties are assumed to be equal:  $q^i=1$ ,  $p^i=1$ ,  $\forall i \in \mathcal{N}$ .

The constraints imposed on the control actions and inventory positions are constant over prediction horizon  $N_2^i$ : for  $k=0, \dots, N_u^i$ ,  $\Delta u_1^i(t+k|t) \leq 3$ ,  $\Delta u_1^i(t+k|t) \leq 4$ ,  $\Delta u_1^i(t+k|t) \leq 5$ , and  $\Delta u_1^i(t+k|t) \leq 6$ ; for  $k=1, \dots, N_2^i$ ,  $0 \leq y^1(t+k|t) \leq 45$ ,  $0 \leq y^2(t+k|t) \leq 45$ ,  $0 \leq y^3(t+k|t) \leq 50$ ,  $0 \leq y^4(t+k|t) \leq 50$ . Only upper limit for  $\Delta u_1^i$  is imposed based on practical consideration: the suppliers have severer situation when they face sudden increase of orders than when they have to deal with a sudden decrease of demand.

The time behaviors of orders, IP and target IP at each echelon over planning periods are depicted in Figure 5.11 and Figure 5.12. It clearly shows that constraints on  $\Delta u_1^i$  are enforced and variations of demand are tracked well.

In the centralized MPC replenishment rule, the control horizon is  $N_u^j=1$ ,  $j=1, \dots, n_u$ , and the prediction horizon is  $N_2^i=10$ ,  $i=1, \dots, n_y$ , which exceeds the collective sum of the nominal ordering and PPT delays along four serial echelons. These long prediction horizons are required by the centralized decision-making approach in order to execute necessary feed-forward anticipations. The output weight matrix  $Q^i$ ,  $i=1, \dots, n_y$  can be chosen to put different emphasis on tracking the target IP of 4 echelons. If the target tracking ability for one echelon is stressed, larger weight should be put on this output. In this simulation no priority of controlling IP for particular echelon is taken:  $Q^i = 10 \cdot I_{N_2^i - N_1^i + 1}$ ,  $\forall i \in n_y$  and analogous argument applies to  $P^j = I_{N_u^j}$ .

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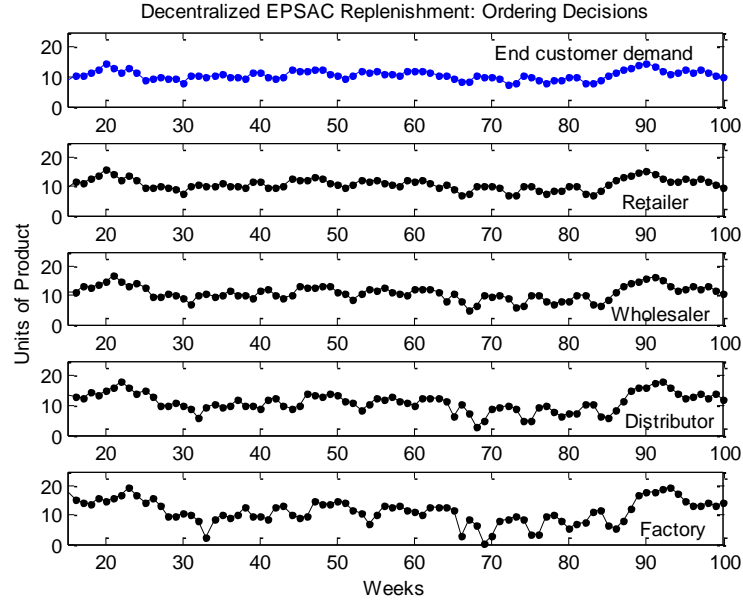


Figure 5.11: Time behavior of orders for decentralized EPSAC control scheme

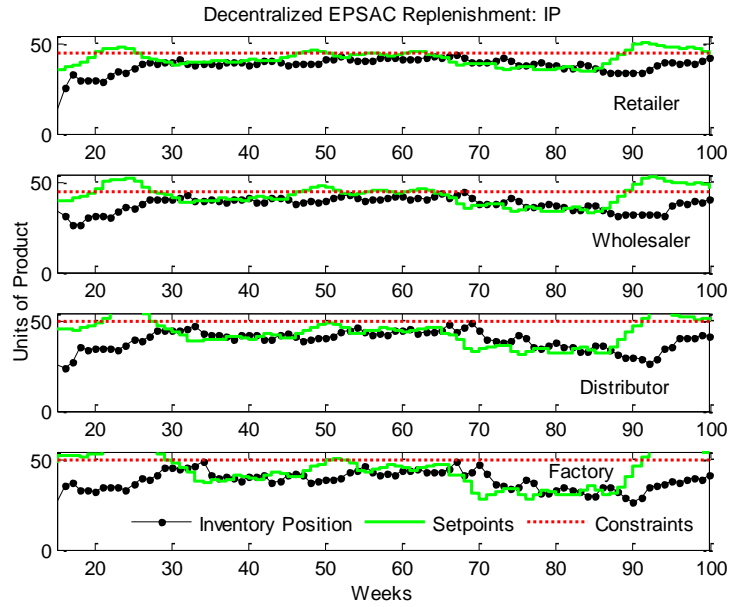


Figure 5.12: Time behavior of IP, setpoints and constraints for decentralized EPSAC control scheme

## 5.6 An Illustrative Example

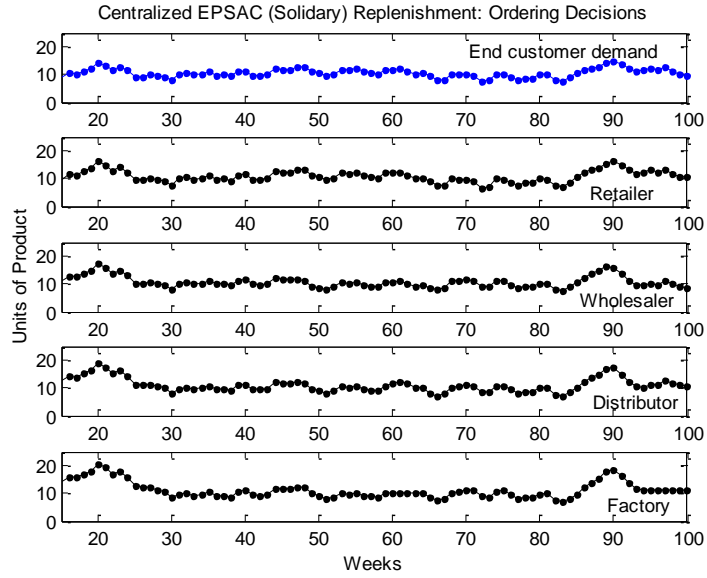


Figure 5.13: Time behavior of orders for centralized EPSAC control scheme with solidary policy

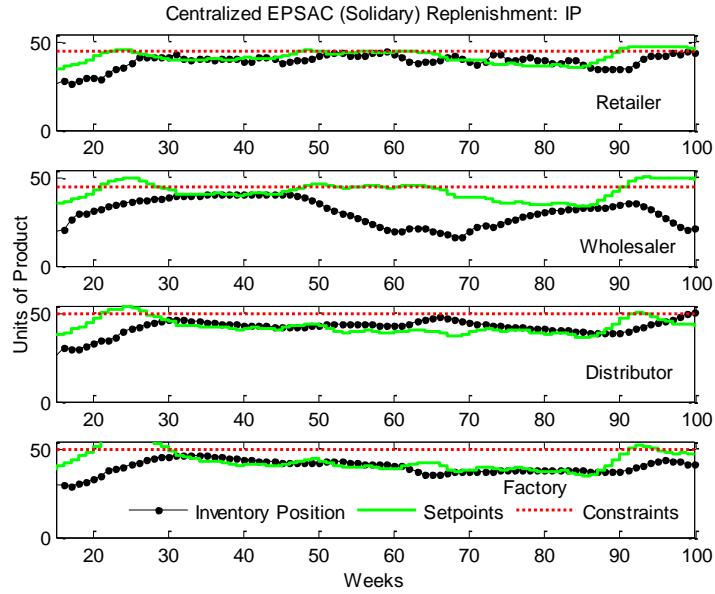


Figure 5.14: Time behavior of IP, setpoints and constraints for centralized EPSAC control scheme with solidary policy

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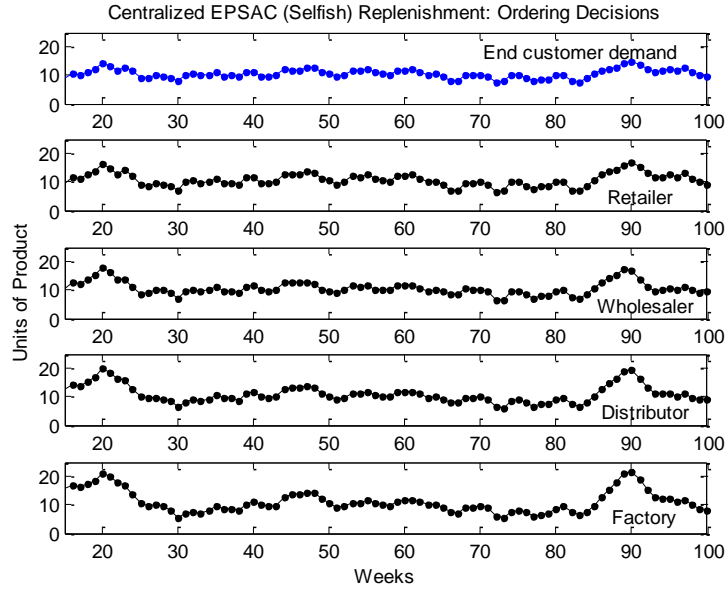


Figure 5.15: Time behavior of orders for centralized EPSAC control scheme with selfish policy

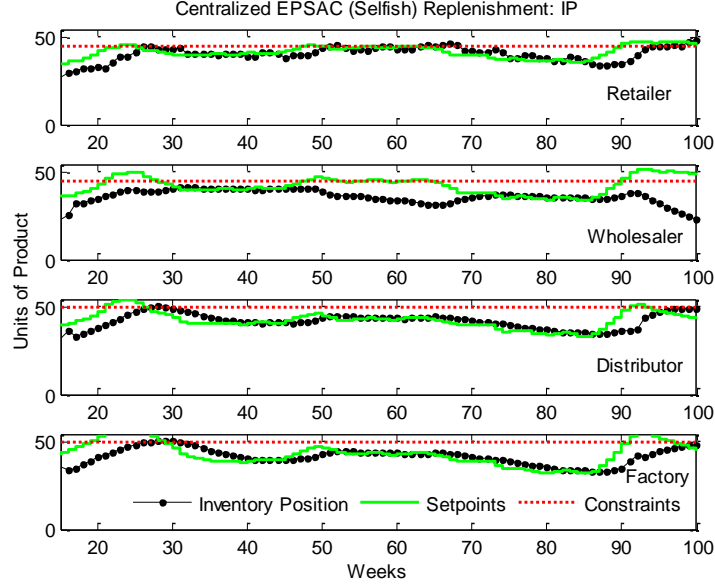


Figure 5.16: Time behavior of IP, setpoints and constraints for centralized EPSAC control scheme with selfish policy

## 5.6 An Illustrative Example

It is found that the difference of control performances between *solidary* and *selfish* policy is very small when no constraints are considered. It becomes very clear when the constraints are enforced. In *solidary* policy, the controller tries to achieve a faster response even at expense of creating bigger errors in some of the outputs. All the control actions strive to work together to control all the outputs and this results in a large deviation between  $y^2$  and their targets (Figure 5.14) because the change of tracking target for any of the outputs will disturb the other outputs. On the other hand, Figure 5.16 shows that when using the *selfish* policy the decoupling is improved because the deviation between  $y^2$  and its setpoints is decreased compared to *solidary* case, the deviations between  $y^3$ ,  $y^4$  and their setpoints almost disappear.

### 5.6.3 Performance Comparisons

The ultimate goals of controlling SC are mainly: 1) maximize customer satisfaction, and 2) minimize SC operating costs.

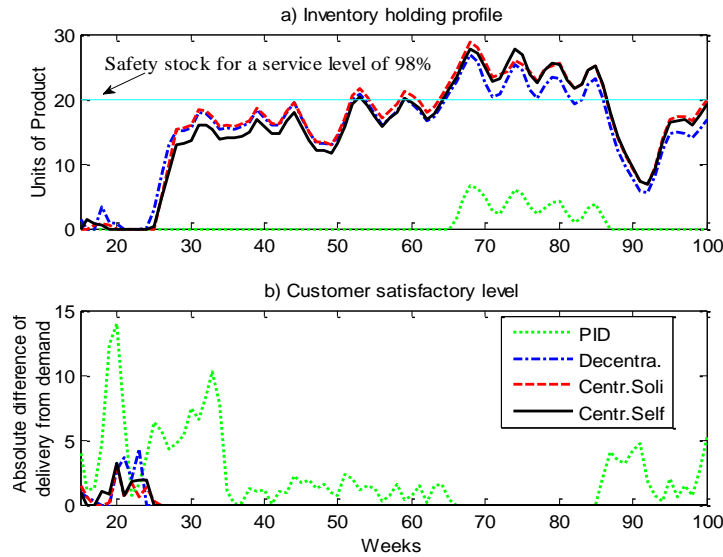


Figure 5.17: Comparisons of inventory holding profile and end-client satisfaction level

The first target is assessed by measuring absolute values of the difference between delivery and demand around retailer echelon. The smaller of the values

### MPC SCHEMES FOR DECENTRAL. & CENTRAL. REPLENISHMENT

means the higher level of end-customer satisfaction. Figure 5.17b and the average measure of customer satisfaction (*AMCS*) in Table 5.4 show that the EPSAC rules result in better customer satisfaction profiles (after 25<sup>th</sup> week the demands are satisfied continually) while PID control constantly has unsatisfied demand. The inventory holding profile indicates that the EPSAC rules keep inventory of retailer around the safety stock for a 98% service level but the PID control is unable to maintain a desirable inventory. The bullwhip is calculated around each echelon and the whole SC using the formula (3.1). The *overall BW* in Table 5.3 shows that centralizing information can reduce the order variation so the centralized EPSAC rules outperform the other 2 in mitigating the bullwhip effect. The last column indicates the centralized EPSAC rules have the most smoothed propagation of order variation along the SC echelons, which means they are capable of limiting the propagation of ordering shockwave and it has less sensitivity to bullwhip than the other 2 policies.

Table 5.3: Comparisons of the bullwhip effect for different replenishment rules

Control Scheme	Bullwhip at each echelon				Overall BW	BW Slope
	Retailer	Wholesaler	Distributor	Factory		
PI Ordering Policy	1.18	2.43	1.80	1.65	8.56	2.45
Decentralized EPSAC	1.54	1.62	1.53	1.42	5.46	1.31
Centralized EPSAC <i>Solidary</i>	1.62	1.41	1.31	1.29	3.93	0.76
Centralized EPSAC <i>Selfish</i>	1.58	1.21	1.29	1.30	3.24	0.55

Table 5.4: Comparisons of the economic performance indices for different replenishment schemes

Control Scheme	AMCS	AEI	ABO	TC	TC reduction (%)
PI Ordering Policy	2.17	20.00	35.75	17322	--
Decentralized EPSAC	0.37	57.96	1.94	14777	14.69
Centralized EPSAC <i>Solidary</i>	0.33	56.83	1.26	14409	16.81
Centralized EPSAC <i>Selfish</i>	0.38	56.39	0.98	14292	17.49

The second goal is evaluated by the calculation of the operating costs of whole

## 5.6 An Illustrative Example

SC. Among the measures based on costs in Section 3.5.2, the operating costs are reflected by the following two important terms. The first is the *AEI*

$$AEI = \frac{1}{|T|} \sum_{t=1}^{|T|} \sum_{i=1}^{|\mathcal{N}|} y_2^i(t)$$

which measures the cost of overstock. The second term is the *ABO*, which indicates depletion of the inventory and this can affect customer satisfaction level:

$$BO(t) = \sum_{i=1}^{|\mathcal{N}|} [d_1^i(t) - y_2^i(t-1) - d_2^i(t-L)]$$

$$ABO = \frac{1}{|T|} \sum_{t=1}^{|T|} BO(t)$$

To assess the performances of the different replenishment rules, the total cost  $TC = I_c + BO_c$  is defined following the strategy in [95], where  $I_c = \sum_{t=1}^{|T|} \sum_{i=1}^{|\mathcal{N}|} c' \cdot y_2^i(t)$  and  $BO_c = \sum_{t=1}^{|T|} c'' \cdot BO(t)$ . Consider a specific case where inventory holding cost is  $c' = \text{€}1$  per unit per period, the backorder costs much more since it may endanger company reputation:  $c'' = \text{€}2$  per unit per period. The reduction in cost by application of EPSAC control schemes is obvious when compared to PID control (*TC reduction* column in Table 5.4).

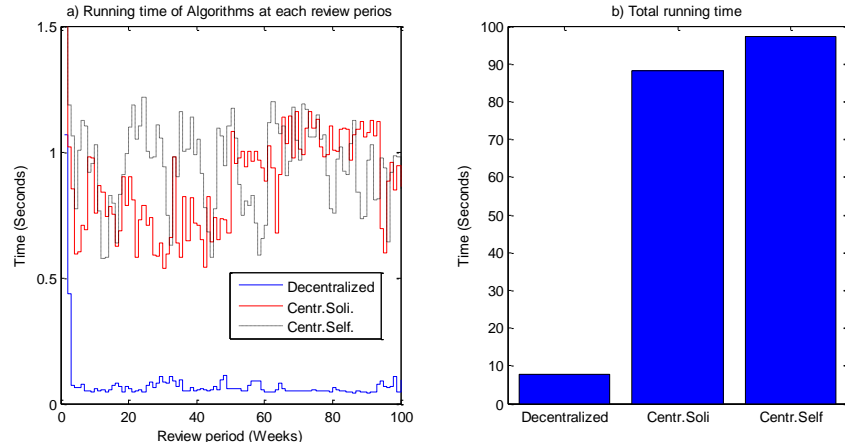


Figure 5.18: Running time of the replenishment rules per review period and total running time

Finally, time complexity analysis is not the major issue in SC replenishment schemes because the review period is usually in units of days, weeks, months etc.



whereas the algorithms are reasonably executed on modern PC to produce results in a time scale of seconds or minutes. A brief comparison of time complexity between different replenishment algorithms is conducted in Figure 5.18. The time consumption for the algorithms matches the analysis on QP problems in Section 5.4 and 5.5.

## 5.7 Summary

This study presents and discusses pragmatic MPC-EPSAC solutions to replenishment issues for the benchmark SC system with consideration of practical operation constraints. The original contribution to the SCM literature is that the novel replenishment rules are achieved through inventory position control by designing the ordering policies in the context of constrained EPSAC strategy. The EPSAC control schemes rely on SISO subsystem model in decentralized replenishment or MIMO model of overall SC in centralized replenishment to obtain the key IP prediction equations. The formulation of the performance index considers both economic factors and dynamic performance characteristics, e.g. inventory holding cost vs. IP target tracking and bullwhip mitigation vs. move suppression for the control variables. Moreover, there are two alternative control policies (*solidary* and *selfish*) applied to centralized replenishment rule, resulting in different control performances under practical operation constraints. Such paradigms as decentralized and centralized control structures provide the options for SC managers to choose particular rule that is suitable to their specific SC operation conditions and requirements.

Finally, the simulation results indicate that the proposed schemes are feasible and outperformed the conventional policy in terms of reduction on operating cost, mitigation of the bullwhip effect and increase of customer satisfaction level.

The results presented in this chapter have been published in:

Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R., 2016. "A constrained EPSAC approach to inventory control for a benchmark supply chain system." *International Journal of Production Research*, 54(1), 232-250.

## 5.7 Summary

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# 6

## MPC Scheme for Distributed Replenishment

The control of large-scale SC composed of finite number of interacting subsystems is a challenging problem, especially in presence of a strong coupling between subsystems, communication requirements and constraints. Distributed model predictive control appears as a promising method to address the issue. In this chapter, operation management is stated as an inventory tracking control problem, where IP levels should follow desired target over time. To achieve this goal, a distributed model predictive control scheme based on Nash equilibrium is proposed in which different EPSAC controllers communicate in order to regulate these constrained subsystems coupled through inputs, i.e. the ordering decisions. Each controller exchanges information with the rest of them to make a non-cooperative decision. The results indicate that the proposed control scheme may be appropriate for industrial practice, bringing a reduction of operating cost with a significant increase in customer satisfaction level, when compared with the PID-based replenishment policy.

### 6.1 Introduction

The ever increasing complexity of SC systems due to growing spatial distribution and interactions between the network elements poses many challenges to the operations and management of SCs. In this context it requires novel methods to support operational decisions, leading to an effective cooperation between various SC players. The cooperation is realized via information exchange between controllers, which results in different control structures [132] depending on degrees of information integration. Most of operational decisions have been traditionally made by managers at each node, leading to decentralized decision-making structure for

## 6.1 Introduction

large-scale SC system composed of interacting subsystems. Disregarding the strong interactions among subsystems may lead to unreliable decentralized control [133]. Recognition of this fact gives rise to a significant change in the way of managing these interacting SC partners, which is characterized by distributed decision-making process. The distributed control structure has received increasing attention in recent years [32, 48] since it can maintain the topology and flexibility of decentralized operations yet provides improved performance, such as reduction of the bullwhip effect via information sharing mechanism.

<b>Nomenclature</b>		$e^i$	white noise that drives the disturbance model of subsystem $i$
<b>Indices/sets</b>		$e$	white noise that drives the end-customer demand
$n$	number of subsystems in SC	$n^i$	process disturbance of subsystem $i$
$i \in \mathcal{N}$	node/echelon/subsystem index	$r^i$	reference trajectory for $y^i$
$j \in \mathcal{N}$	index for the neighboring subsystems of subsystem $i$	$w^i$	setpoint for $y^i$
$t \in T$	the supply chain planning period	<b>Parameters/Notations</b>	
$k$	future time index based on $t$	$\mathcal{S}$	the whole SC network
$\mathcal{N}$	set of echelons/nodes	$\mathcal{S}^i$	the subsystem $i$ of SC network
$ \mathcal{N} $	number of echelons/nodes	$\mathcal{S}^i$	controller/agent for subsystem $i$
$ T $	the inventory planning horizon	$*(z^{-1})$	polynomial $*$ in terms of backward shift operator $z^{-1}$
<b>Variables</b>		$\alpha^i$	exponential smoothing parameter
$L^i$	lead-time of echelon $i$	$\gamma^i$	service level for subsystem $i$
$y_3^i$	the standing order at echelon $i$	$q^i$	penalty on IP tracking error of subsystem $i$
$y_2^i$	measured inventory at echelon $i$	$p^i$	penalty on excessive movement of orders of subsystem $i$
$y^i$	measured IP of the control system for echelon $i$	$p_{\max}(t)$	maximum number of iterations at period $t$ for iterative DMPC
$x^i$	IP, model output of echelon $i$	$u_1^{i*}$	final ordering decision made by local MPC controller $\mathcal{S}^i$
$u_1^i$	orders placed by the $i$ th echelon	$U^i$	vector of the optimizing future control actions of i.e. future orders
$u_{base}^i$	basic future control actions	$\Delta U^i$	vector of the future optimizing control increments of $\mathcal{S}^i$ , i.e. future orders in differenced form
$\delta u_1^i$	optimizing future control actions	$\Delta U^{i*}$	vector of the optimized future
$\Delta u_1^i$	orders of $\mathcal{S}^i$ in differenced form		
$u_2^i$	delivery to downstream echelon $i-1$		
$d_1^i$	demand from echelon $i-1$		
$d_2^i$	received product by the echelon $i$ from upstream echelon $i+1$		
$d_1^{R_c}$	end-customer demand		

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$\Delta U_{(p)}^i$	control increments of $\mathcal{S}^i$ , vector of the optimized future control increments of $\mathcal{S}^i$ computed at the $p^{th}$ iteration	$N_1^i$	min. prediction horizon of $\mathcal{S}^i$
$N_u^i$	control horizon of $\mathcal{S}^i$	$N_2^i$	prediction horizon of $\mathcal{S}^i$
		$\mathcal{E}^i$	error accuracy of iteration at $\mathcal{S}^i$

The DMPC is one of the most suitable frameworks for operations and management of SCs because it has an essential role to play in distributed decision-making process with intrinsic predictions being exchanged fully or partially between the interacting MPC controllers. The DMPC is featured by two aspects: the interactions between subsystems are considered explicitly in local prediction models, the information is transmitted among controllers for interacting subsystems. The wide set of DMPC strategies that appeared in the literature can be classified according to different criteria [134]. The information exchange can be performed according to either of the following protocols:

- *non-iterative*, i.e. information is transmitted/received once by local controllers within each review period;
- *iterative*, i.e. information is transmitted/received multiple times in each review period until all the local controllers have reached consensus.

The cost function can be formulated as either:

- *non-cooperative*, i.e. each controller minimizes a local cost function;
- *cooperative*, i.e. each controller minimizes a global cost function.

In this chapter, a *non-cooperative, iterative* DMPC approach is proposed to determine the optimal ordering policy for a benchmark SC. In this *non-cooperative* approach, each subsystem (node) makes simultaneously the decisions and its local cost is obtained. Such approach is characterized by the Nash equilibrium [135], i.e. the set of ordering decisions for which no subsystem has a unilateral incentive to change its decision. Under Nash equilibrium, no subsystem can improve its performance by changing its decision when the other subsystems' choices of decisions remain unchanged. The possibility of iteratively exchanging information with the rest of controllers in the local optimization makes the globally optimized decisions available, by which the whole SC reaches effective coordination.

Having these ideas in mind, the chapter elaborates the DMPC scheme as follows. Section 6.2 presents the detailed modeling and control problems after dynamic analysis. Distributed replenishment schemes based on EPSAC algorithm are discussed in Section 6.3. The presented approach is then validated by an illustrative simulation in Section 6.4 to show the effectiveness of the control schemes. Finally, the conclusions are drawn in Section 6.5.

## 6.2 Problem Description

### 6.2.1 Partition of SC Network

The philosophy of DMPC is that the model predictive control for a large-scale system is transformed into several small-scale MPC control problems for autonomous subsystems. The SC network indicated in the square box of Figure 5.3 can be divided into  $n$  subsystems according to either the analysis on mathematical modeling or on the basis of its physical structure properties, or a combination of both. In this chapter, each echelon is partitioned as one subsystem ( $n=4$ ) and each is managed by an independent controller (also known as agent in distributed control literature) implemented with the MPC algorithm. In theory, the framework of bi-direct graph  $G=(V, S)$  is used to represent the topology and interactions among subsystems, where a set of vertexes  $V=\{1, \dots, n\}$  matches the subsystems and a set of directed arcs  $S \subseteq \{(i, j) \in V \times V \mid i \neq j\}$  represents the interactions or couplings between subsystem  $i, j$ . An arc  $(i, j) \in S$  indicates the control input for subsystem  $i$  directly affect the outputs of subsystem  $j$ . Once the SC configuration is fixed, the graph  $G$  is determined whereby the structural aspects of SC network and dynamic couplings can be obtained.

### 6.2.2 Distributed Controller Model

The same supply chain process description and node dynamics are performed as presented in Section 2.2. For sake of simplicity, the detailed analysis is not repeated here. Relations (2.1)-(2.5) are derived by physical laws governing the dynamics of the SC systems. They describe the individual nodes' dynamics when complemented with the local ordering policy, i.e. the mechanism for determining  $u_1^i(t)$  from accessible information at time  $t$ . The decision-making scheme illustrated in Figure 5.3 is called distributed control. Similar to fully decentralized control, distributed control implements independent local agents for different subsystems. Different from decentralized control, the required coordination can be achieved by information exchange between agents via local area network. Therefore, each local agent makes its own decisions based on both information from the subsystem itself and the information obtained from coupled subsystem. The detailed design of distributed ordering policy is covered in Section 6.3.

During the operational process, the stock conditions (sufficient or insufficient) of the current node  $i$  are changing at different time instants, as is modeled in Eq. (2.9). Hence, the SC is naturally a switched system where the general dynamics given by Eqs. (2.6)-(2.9) can become in particular one of the three cases through time: **ISHS**, **ISLS** or **LS**.

**Theorem 6.1.** *When the supply chain is in operation under **ISHS** mode, the transfer function for the inventory position of current node  $i$  can be restated as:*

$$x^i(z) = \frac{z^{-1}}{1-z^{-1}} [u_1^i(z) - u_1^{i-1}(z)] \quad (6.1)$$

Proof. Refer to the proof of **Theorem 2.1** and also note that  $d_1^i(z) = u_1^{i-1}(z)$ .

**Remark 6.1.** *Under this context, the inventory position of node  $i$  depends both on its ordering policy and the ordering policy of node  $(i-1)$ . In the IP control sense, the ordering decision  $u_1^i$  of node  $i$  can be manipulated by its local controller whereas  $u_1^{i-1}$  can not be manipulated. Then the resulting Eq. (6.1) is treated as the SC model to design DMPC control strategies. The other two cases of ISLS and LS result in different SC models, which can be referred to Section 2.2.3.*

The supply chain network can be regarded as a switched system due to its delivery policy, which switches among different modes depending on  $\text{MIN}(y_2^i(t), y_3^i(t))$  and  $\text{MIN}(y_2^{i+1}(t), y_3^{i+1}(t))$ . Thereby, the operational cases change as these relations are varying through time. Thus the real SC dynamics are described based on the switched system technique, but the output prediction model used in the following MPC formulation is approached by the time-domain version of the relation (6.1). The second term accounts for the interactions from the other subsystems  $(i-1)$  with the subsystem  $(i)$ . The relation between inventory position and control decisions is utilized as basic model in designing the multi-step predictor of MPC formulation. As one of the pioneering MPC controllers, EPSAC follows a standard procedure, which is presented in the following section via design of distributed replenishment policy.

## 6.3 Distributed MPC Replenishment Rule

### 6.3.1 The Principle of DMPC

The general overview of DMPC replenishment framework is shown in Figure 6.1. The whole SC network  $\mathcal{S}$  is divided into a couple of subsystems  $\mathcal{S}_i$ ,  $i=1, \dots, n$ , an autonomous agent  $\mathcal{A}_i$  implemented with a MPC algorithm is assigned to each of them. The agent uses both local information of  $\mathcal{S}_i$  and the transmitted information from its neighboring subsystems  $\mathcal{S}_j$ ,  $j=1, \dots, n, j \neq i$ . Thus, the information needs to be exchanged via LAN among agents of interacting subsystems. In this section, the detailed DMPC controller design is presented after exploring the principles of Nash Equilibrium [136].

### 6.3 Distributed MPC Replenishment Rule

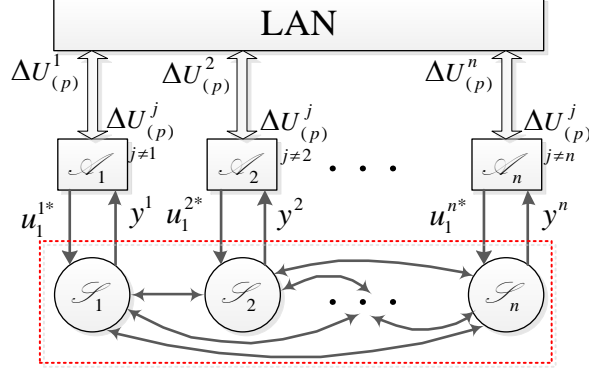


Figure 6.1: The general schematic representation of the DMPC strategy

A distributed MPC scheme referred to as non-cooperative DMPC algorithm is developed, which minimizes a local cost  $J^i$  for each subsystem  $\mathcal{S}_i$ . The local optimization by  $\mathcal{S}_i$  does not consider the performance of other subsystems, but the output of local subsystem  $\mathcal{S}_i$  is still related to other subsystems through input coupling. This DMPC scheme has different goals to meet so the problem can be resolved by seeking Nash equilibrium [137, 138], where each controller minimizes its cost function  $J^i$  only with respect to local control actions  $\Delta U^i(t)$  (in MPC replenishment rule, it is the future optimizing control increment, i.e. future orders in differenced form; in this iterative algorithm, the control actions  $\Delta U_{(p)}^i(t)$  are calculated at the  $p^{\text{th}}$  iteration) assuming that other agents' optimal solutions have been obtained:

$$\left. \frac{\partial J^i}{\partial \Delta U^i(t)} \right|_{\Delta U^{j*}(t), (j=1, \dots, n, j \neq i)} = 0 \quad (i=1, \dots, n) \quad (6.2)$$

The iteration stops and the resulted control input  $\Delta U^*(t) = (\Delta U^{1*}(t), \dots, \Delta U^{n*}(t))$  is called Nash optimal solution if for all  $\Delta U^i(t)$ ,  $i=1, \dots, n$ , the following condition is satisfied:

$$J^{i*}(\Delta U^{1*}(t), \dots, \Delta U^{i*}(t), \dots, \Delta U^{n*}(t)) \leq J^i(\Delta U^{1*}(t), \dots, \Delta U^i(t), \dots, \Delta U^{n*}(t)) \quad (6.3)$$

When the optimal solution  $\Delta U^*(t)$  is reached, each agent does not change its input  $\Delta U^i(t)$  because the local optimality has been obtained under condition (6.3), otherwise the performance index  $J_i$  degrades.



**Remark 6.2:** According to above definition, the overall system reaches the Nash equilibrium if every agent has obtained the Nash optimal solution to its local optimization. For the agent  $\mathcal{A}_i$ , it requires Nash optimal solutions  $\Delta U^{j*}(t)$  from other agents  $\mathcal{A}_j$ ,  $j \neq i$  in order to obtain its own optimal solution  $\Delta U^{i*}(t)$ . Note that the Nash optimal solution to the local control problem collectively is not the same thing as the global optimal control for the overall system.

### 6.3.2 EPSAC Formulation for Local Agent

The simplifying hypothesis is considered before introducing the DMPC algorithm:

**Assumption 6.1:** All the agents are synchronous.

**Remark 6.3:** This is not a strong assumption since the review period (sampling time) for supply chain operations is relatively long compared to the time required for computation, even in the case of iterative algorithm.

**Assumption 6.2:** The agents use the same control horizon and prediction horizon, i.e.  $N_u^i = N_u^j = N_u$ ,  $N_2^i = N_2^j = N_2$ ,  $\forall i, j = 1, \dots, n$ .

**Assumption 6.3:** The agents communicate and exchange information adequately under reliable LAN communication channels in order to accomplish the whole objective.

**Remark 6.5:** This assumption ensures the information can be transmitted and received from any local agent to all the others many times within the review period. It is an important hypothesis when the DMPC algorithm takes an iterative procedure.

The behavior of the whole supply chain network  $\mathcal{S}$  is reflected by the dynamics of  $n$  subsystems  $\mathcal{S}_i$ ,  $i = 1, \dots, n$ . The local agent  $\mathcal{A}_i$  is implemented with an EPSAC algorithm, which is based on the following model of local subsystem  $i$ :

$$y^i(t) = x^i(t) + n^i(t) \quad (i = 1, \dots, n) \quad (6.4)$$

with  $x^i(t) = f^i[x^i(t-1), x^i(t-2), \dots, u_1^i(t-1), u_1^i(t-2), \dots, u_n^i(t-1), u_n^i(t-2), \dots]$  as the model output (inventory position calculated from model (6.1)),  $u_1^i(t)$  as control input (ordering decision) and  $y^i(t)$  as SC output (actual inventory position) of the  $i^{th}$  subsystem. This means that the subsystem  $\mathcal{S}_i$  is coupled with the neighboring subsystems through inputs  $u_j^i(t)$ ,  $j \neq i$ . Agent  $\mathcal{A}_i$  has access to the model and the measured IP  $y^i$  of  $\mathcal{S}_i$ , receives the optimized input trajectory from other subsystems and then makes the ordering decision  $u_1^i$ . The disturbance  $n^i(t)$  accounts for the effects in the observed IP  $y^i(t)$  that are not from the control input  $u_1^i$  via the model output  $x^i(t)$ . This is non-measurable signal exhibiting a stochastic character

### 6.3 Distributed MPC Replenishment Rule

and it is depicted by a colored noise process:

$$n^i(t) = \frac{C^i(z^{-1})}{D^i(z^{-1})} e^i(t) \quad (i=1, \dots, n) \quad (6.5)$$

which is excited by white noise  $e^i(t)$  through a design filter  $C^i(z^{-1})/D^i(z^{-1})$ .

An essential aspect in EPSAC algorithm is composed of the prediction for the output of the  $i^{th}$  subsystem:

$$y^i(t+k|t) = x^i(t+k|t) + n^i(t+k|t) \quad (i=1, \dots, n) \quad (6.6)$$

over a prediction horizon  $k=1, \dots, N_2$ . The predictions of  $x^i(t+k|t)$  and  $n^i(t+k|t)$  are respectively approached by recursion of IP model (6.1) in time-domain and by applying filtering techniques on the disturbance model (6.5). The future behavior of the  $i^{th}$  subsystem's IP  $y^i(t+k|t)$  triggered by the ordering decisions  $u_1^j(t+k|t)$ ,  $j=1, \dots, n$  can be given in compact form:

$$Y^i = Y_{base}^i + Y_{optimize}^i = \tilde{Y}^i + G^{ii}U^i + \sum_{\substack{j=1 \\ j \neq i}}^n G^{ij}U^{j*} \quad (i=1, \dots, n) \quad (6.7)$$

The future inventory position and the optimizing future control actions for the  $i^{th}$  subsystem are expressed in vector form  $Y^i = [y^i(t+N_1|t) \dots y^i(t+N_2|t)]^T$  and  $U^i = [\delta u_1^i(t|t) \dots \delta u_1^i(t+N_u-1|t)]^T$  with  $\delta u_1^i(t+k|t) = u_1^i(t+k|t) - u_{1base}^i(t+k|t)$  where  $u_1^i(t+k|t)$  is the optimal ordering decision that the optimizer is looking for. The *base response*  $\tilde{Y}^i = [y_{base}^i(t+N_1|t) \dots y_{base}^i(t+N_2|t)]^T$  is computed as the cumulative effect of the past control inputs/outputs, the pre-specified future base control scenario  $U_{base}^i = [u_{1base}^i(t|t) \dots u_{1base}^i(t+N_u-1|t)]^T$  and the predicted disturbances. The number of future control samples that can be adapted is determined by control horizon  $N_u$ . The *optimizing response*  $Y_{optimize}^i$  is an effect of the optimizing future control actions  $U^i$ . The hints on choosing  $U_{base}^i$  and the common practice to structure  $U^i$  are reported in [122].

Figure 6.1 and (6.7) indicate that the optimizing term depends not only on the control actions  $U^i$  of the  $i^{th}$  subsystem but also on the received optimized control actions  $U^{j*}$  from agents of its interacting subsystems. The last term in (6.7) is discrete time convolution of the interacting subsystems' optimized control actions  $U^{j*}$ ,  $j \neq i$  with the corresponding impulse and step response coefficients, i.e.  $G^{ij}$  is a Toeplitz matrix containing the coefficients of impulse/step responses of the optimized control inputs for  $\mathcal{S}_j$  to the  $i^{th}$  subsystem's IP starting from  $N_1$  to  $N_2$ . A relationship can be made between  $U^j$  and  $\Delta U^j = [\Delta u_1^j(t|t), \dots, \Delta u_1^j(t+N_u-1|t)]^T$ ,

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which is expressed in matrix notations  $\Delta U^j = A^j U^j + b^j$ ,

$$\text{with } A^j = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix} \text{ and } b^j = \begin{bmatrix} u_{base}^j(t|t) - u_1^j(t-1) \\ u_{base}^j(t+1|t) - u_{base}^j(t|t) \\ \vdots \\ u_{base}^j(t+N_u-1|t) - u_{base}^j(t+N_u-2|t) \end{bmatrix}.$$

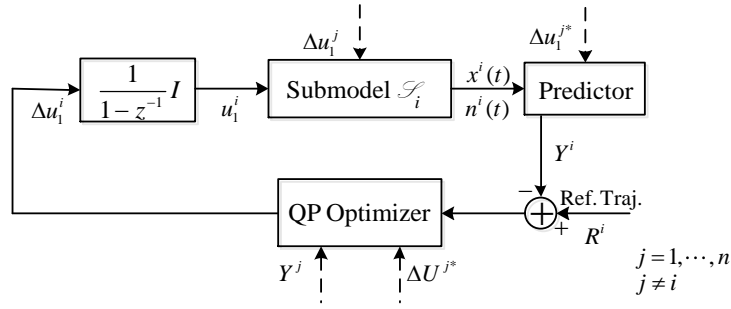


Figure 6.2: Block scheme of EPSAC unit for subsystem  $\mathcal{S}_i$  and local agent  $\mathcal{A}_i$

Next, each agent optimizes a local performance index with respect to its own control input  $U^i$  while assuming the optimal control inputs from other agents have been obtained. The local optimization problem is illustrated in Figure 6.2. The control inputs for  $\mathcal{A}_i$  is obtained by minimizing:

$$J^i = \sum_{k=N_1}^{N_2} q^i \cdot [r^i(t+k|t) - y^i(t+k|t)]^2 + \sum_{k=0}^{N_u-1} p^i \cdot [\Delta u_1^i(t+k|t)]^2 \quad (6.8)$$

where  $q^i$ ,  $p^i$  are output penalty and control penalty respectively. The control goal is to steer the inventory position  $y^i(t)$  from the current state to the setpoint  $w^i(t)$  following a reference trajectory  $\{r^i(t+k|t), k=N_1, \dots, N_2\}$ . The setpoint is given externally and updated with  $w^i(t) = \hat{D}_{L+1}^i(t) + \gamma^i \hat{\sigma}_{L+1}^i(t)$  at each review period (refer to Eq. (2.29)). The cost function (6.8) of quadratic form in  $\Delta U^i$  takes the following expression:

$$J^i(\Delta U^i) = \|(R^i - Y^i)\|_{Q^i}^2 + \|\Delta U^i\|_{P^i}^2 \quad (6.9)$$

in which  $Q^i = q^i \cdot I_{N_2-N_1+1}$ ,  $P^i = p^i \cdot I_{N_u}$ , and  $R^i = [r^i(t+N_1|t) \cdots r^i(t+N_2|t)]^T$  is reference trajectory. The motivations of selecting a quadratic cost structure have been discussed in Section 5.3.2.

### 6.3 Distributed MPC Replenishment Rule

Finally, the DMPC approach to the replenishment rule for the SC network can be defined as follows:

**Definition 6.1 (DMPC replenishment):** Given the whole SC system consisting of  $n$  subsystems  $\mathcal{S}_i$ ,  $i=1, \dots, n$ , the DMPC approach to replenishment uses IP model (6.1) to describe the dynamics and predict the future behaviors of the local subsystem. At each review period, the local agent  $\mathcal{S}_i$ ,  $i=1, \dots, n$  formulates an iterative MPC problem with horizons  $N_2$  and  $N_u$  ( $N_2 > N_u$ ), minimizing a desired cost function (6.9) under predictive output (6.10) and existing constraints (6.11)-(6.13):

$$\text{predictive output} \quad Y^i = \tilde{Y}^i + G^{ii}U^i + \sum_{j=1, j \neq i}^n G^{ij}U^{j*} \quad (6.10)$$

$$\text{output variable constraints} \quad \underline{Y}^i \leq Y^i \leq \bar{Y}^i \quad (6.11)$$

$$\text{control increment constraints} \quad \Delta \underline{U}^i \leq \Delta U^i \leq \Delta \bar{U}^i \quad (6.12)$$

$$\text{control variable constraints} \quad \underline{U}^i \leq U^i \leq \bar{U}^i \quad (6.13)$$

Through this mechanism, each agent is responsible for determining order decisions for respective subsystem when all the agents exchange information about their current and future decisions to reach Nash Equilibrium.

**Remark 6.6:** The local optimization problem in above definition for subsystem  $\mathcal{S}_i$  can be converted to standard QP problem:

$$\min_{\Delta U^i} \frac{1}{2} (\Delta U^i)^T \mathcal{H}^i \Delta U^i + (f^i)^T \Delta U^i \quad (6.14)$$

subject to  $\mathcal{A}^i \Delta U^i \leq \mathcal{B}^i$  with  $\mathcal{A}^i$  and  $\mathcal{B}^i$  are specified matrix and vector respectively depending on the types of constraints. Each EPSAC unit indicated in Figure 6.2 is composed of three linked function blocks: a QP optimizer, a model for subsystem  $\mathcal{S}_i$ , and an output predictor.

#### 6.3.3 Non-cooperative DMPC Control Architecture

In this DMPC scheme the efficient cooperations among different agents have to be assured by exchanging information regarding the optimized input trajectory  $U^{j*}$ . Within the same review period, all agents work coordinately in the proposed control strategy and solve their problems using the renewed information from all control agents. The EPSAC unit in Figure 6.2 and the relation (6.2) indicate the necessity to receive the interacting agents' optimal control actions  $\Delta U^{j*}$  ( $j \neq i$ ) in order to calculate the optimal solution  $\Delta U^i$  of agent  $\mathcal{S}_i$ , so that the whole SC network could reach Nash Equilibrium. Each agent  $\mathcal{S}_i$  uses the iterative algorithm reported

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in Table 6.1 to find the Nash optimal control decisions for  $\mathcal{S}_i$  in every review period.

*Table 6.1: Algorithm of iterative DMPC*

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**Algorithm** Iterative DMPC

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*Data:* Inventory position  $\{y^i(t), y^i(t-1), \dots\}$ , past orders and demand  $\{u_1^i(t-1), u_1^i(t-2), \dots\}$ ,  $\{d_1^i(t), d_1^i(t-1), \dots\}$ , parameters  $N_u$ ,  $N_2$ ,  $q^i$ ,  $p^i$ ,  $i=1, 2, \dots, n$ ; termination conditions: maximum number of allowable iterations  $p_{\max}(t) > 0$ , error accuracy  $\varepsilon^i > 0$ .

*DMPC controller:* At review time  $t$ ,

1. *Initialization:* Each agent  $\mathcal{S}_i$  receives the current output  $y^i(t)$  from subsystem  $\mathcal{S}_i$ , initializes the optimal local control actions  $\Delta \hat{U}^i$  to be zero. Then  $\mathcal{S}_i$  exchanges  $\Delta \hat{U}^i$  with agents of its interacting subsystems.  
Set iterative index  $p \leftarrow 0$ ,  $\Delta U_{(p)}^i \leftarrow \Delta \hat{U}^i$ ,  $i=1, \dots, n$
  2. *Local optimization:* Each agent  $\mathcal{S}_i$  solves the optimization problem in **Definition 6.1** for subsystem  $\mathcal{S}_i$  simultaneously resulting in the updated control action  $\Delta U_{(p+1)}^i$ ,  $i=1, \dots, n$ .
  3. *Check and update:* Each agent  $\mathcal{S}_i$  checks if the termination conditions for the iteration are satisfied, e.g. for  $i=1, \dots, n$ :  
**if**  $\|\Delta U_{(p+1)}^i - \Delta U_{(p)}^i\| \leq \varepsilon^i$  or  $p+1 \geq p_{\max}(t)$   
     set  $\Delta U^{i*} \leftarrow \Delta U_{(p+1)}^i$ ,  $p^*(t) \leftarrow p+1$ ;  
     end the iteration, and proceed to step 4  
**else**  
     set  $p \leftarrow p+1$ ;  
      $\mathcal{S}_i$  communicates and exchanges the new control decision  $\Delta U_{(p)}^i$  with agents of its interacting subsystems, and return to step 2  
**end**
  4. *Compute the control law* at time  $t$ ,  $\Delta u_1^{i*}(t|t) = [1, 0, \dots, 0] \cdot \Delta U^{i*}$  and implement the control action  $u_1^{i*}(t|t) = u_1^{i*}(t-1) + \Delta u_1^{i*}(t|t)$  to  $\mathcal{S}_i$ ,  $i=1, \dots, n$ .
  5. The initial estimation of optimal control actions for  $\mathcal{S}_i$  at  $k+1$  will be reasigned:  
 $\Delta \hat{U}^i \leftarrow \Delta U^{i*}$ ,  $i=1, \dots, n$ .
  6. Increase  $t$  by one sampling time  $t \leftarrow t+1$ , go to step 1 and repeat.
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### 6.3 Distributed MPC Replenishment Rule

**Remark 6.7:** Because of the DMPC principle introduced in Section 6.3.1, information exchange on newly computed control actions  $\Delta U_{(p)}^i$  is performed among interacting subsystems in step 3. If the absolute difference between the newly calculated optimal control and the one obtained in last iteration is smaller than a threshold  $\varepsilon^i$ , the iteration can stop. When the algorithm is convergent, the termination conditions will be met and the Nash equilibrium will be achieved. The maximum permissible iterate at each sampling time  $p_{\max}(t) > 0$  is a design limit and it is included in termination conditions to stop the iteration prior the sampling time runs out. As for supply chain system, it can be designed as a large number because the review period is in a time scale of days, weeks, or months, which is much longer than the required computation time. The Nash optimal solution for  $\mathcal{S}_i$  at time  $t$  will be used in step 1 as the initial estimation of control decision at next time instant  $t+1$ .

This DMPC approach to replenishment has converted the optimization of the whole SC to several small-scale optimization problems for subsystems, thus the computational burden can be eased. The main difference of DMPC to decentralized MPC is that there exists information exchange among agents via LAN and it takes an iterative algorithm, thus the computational convergence is taken into consideration in the following discussion. The exclusion of constraints does not affect the convergence analysis, and the constraints can be addressed directly in the local QP optimization problems of **Definition 6.1**.

At each review period  $t$ , the following output prediction for  $\mathcal{S}_i$  in the  $p^{\text{th}}$  iteration is obtained from (6.7):

$$Y_{(p)}^i = \tilde{Y}^i + H^{ii} \Delta U^i + \sum_{j=1, j \neq i}^n H^{ij} \Delta U_{(p)}^j + \zeta^i \quad (6.15)$$

where  $H^{ij} = G^{ij} (A^j)^{-1}$ ,  $\zeta^i = -\sum_{j=1}^n H^{ij} b^j$ . An explicit solution to Nash Equilibrium can

be derived by taking  $\left. \frac{\partial J^i}{\partial \Delta U^i} \right|_{\Delta U_{(p)}^i, (j=1, \dots, n, j \neq i)} = 0$ , which results in the following analytical expression of control actions for  $\mathcal{S}_i$

$$\Delta U_{(p+1)}^i = T^{ii} \left[ W^{ii} - \sum_{j=1, j \neq i}^n H^{ij} \Delta U_{(p)}^j \right] \quad (6.16)$$

with  $T^{ii} = \left( (H^{ii})^T Q^i H^{ii} + P^i \right)^{-1} (H^{ii})^T Q^i$  and  $W^{ii} = R^i - \tilde{Y}^i - \zeta^i$ . In the iterative algorithm it describes the relationship of future control actions between the  $p^{\text{th}}$  iteration and the  $(p+1)^{\text{th}}$  iteration for the local subsystem. Suppose that the algorithm

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satisfies convergent conditions, and then the whole system arrives at the Nash equilibrium if all the control agents of subsystems have achieved the Nash optimal solutions. The integral control actions of the whole system is

$$\Delta U_{(p+1)} = T^1 W + T^0 \Delta U_{(p)} \quad (6.17)$$

in which  $\Delta U = [(U^1)^T, \dots, (U^n)^T]^T$ ,  $T^1 = \text{blkdiag}(T^{11}, \dots, T^{nn})$ ,

$$W = [(W^{11})^T, \dots, (W^{nn})^T]^T, T^0 = \begin{bmatrix} 0 & -T^{11}H^{12} & \dots & -T^{11}H^{1n} \\ -T^{22}H^{21} & 0 & \dots & -T^{22}H^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -T^{nn}H^{n1} & -T^{nn}H^{n2} & \dots & 0 \end{bmatrix}.$$

It can be observed that  $W$  is already known at review time  $t$  hence the first term in (6.17) is a constant contributing no effect to the iteration. Therefore, the convergence in control actions (6.17) is equivalent to the following restated expression:

$$\Delta U_{(p+1)} = T^0 \Delta U_{(p)} \quad (6.18)$$

The spectral radius of the matrix  $T^0$  is closely related to the behavior of the convergence of this power sequence of  $T^0$ . Obviously, the convergent condition of the iterative algorithm to DMPC is

$$|\rho(T^0)| < 1 \quad (6.19)$$

where the spectral radius should be less than 1 if the convergent computation of the algorithm is required.

**Remark 6.8:** In fact, (6.19) is not only a necessary condition, but also a sufficient condition [139]. It should be noticed that the convergence of the algorithm is local because the convergence of the distributed computation is only analyzed at the current review time instant. If the distributed computation is convergent over the whole receding horizon, then the algorithm is globally convergent. Once the prediction horizon and control horizon are selected the matrices  $H^i, H^j$  are determined and thus entries in  $T^0$  only depend on  $Q^i, P^i$ , i.e. the convergence of the algorithm are directly affected by tuning parameters of penalty matrices  $Q^i, P^i$ . Therefore, when choosing the penalty matrices both control performance and convergence conditions should be examined.

## 6.4 An Illustrative Example

In the simulation study, the distributed EPSAC control scheme proposed in Section 6.3 is applied to the operations of SC. The objective is to satisfy the end-customers' demand and to minimize the cost by tracking the target IP level whilst taming the bullwhip effect.

In this section, the same initialization and configuration as that used in Section 5.6 are considered. The replenishment policy is examined over  $T=100$  review periods and the end-customer demand is generated following Eq. (4.17) with the same parameters. The constraints are set as  $\Delta u_1^1(\cdot) \leq 3$ ,  $\Delta u_1^2(\cdot) \leq 4$ ,  $\Delta u_1^3(\cdot) \leq 5$ , and  $\Delta u_1^4(\cdot) \leq 6$ ;  $0 \leq y^1(\cdot) \leq 45$ ,  $0 \leq y^2(\cdot) \leq 45$ ,  $0 \leq y^3(\cdot) \leq 50$ ,  $0 \leq y^4(\cdot) \leq 50$ . Due to practical consideration, only upper bound for  $\Delta u_1^i$  is imposed: the managers have severer condition when they have to deal with sudden increasing change of incoming orders than when they face a decrease of demand.

First, the design of PI loops and decentralized EPSAC for local subsystems are used for comparison purpose. The simulation results shown in Figure 5.9-Figure 5.10 for PI replenishment and Figure 5.11-Figure 5.12 for decentralized EPSAC replenishment are not repeated here.

In the DMPC control scheme, the prediction horizon is assigned as  $N_2^i = 5$ ,  $i=1, \dots, n$ . They are sufficient long for the controller  $\mathcal{S}_i$  to encompass the IP responses which are significantly affected by the current ordering decisions. These ordering decisions include  $u_1^i(t)$  that affects the inventory of  $\mathcal{S}_i$  after 3 weeks (i.e. 2 weeks of PTT and 1 week nominal ordering delay) and the neighboring subsystem's  $u_1^{i-1}(t)$  that affects the IP of  $\mathcal{S}_i$  after 1 week nominal ordering delay. These prediction horizons are reasonable in the distributed decision-making scheme. The control horizon is  $N_u^i = 1$ ,  $i=1, \dots, n$  to avoid introducing too much control effort. The output weight matrix  $Q^i$ ,  $i=1, \dots, n$  can be designed to place different stresses on tracking abilities of 4 subsystems. If one subsystem's target IP tracking ability is emphasized, larger value has to be put on its output weight. In the simulation, the IP control for all the subsystems are equally important:  $Q^i = I_{N_2^i - N_1^i + 1}$ ,  $\forall i \in n$  and analogous argument applies to  $P^i = I_{N_u^i}$ . The chosen  $P^i$  and  $Q^i$  also guarantee the convergence of the iterative algorithm.

It is found in Figure 6.4 that the DMPC outperforms the PI and decentralized EPSAC control, generating a much smoother IP variation compared with the rather oscillatory responses of the other two replenishment rules. It becomes very straightforward when they are compared in terms of performance indicators presented in the ensuing tables and figures.



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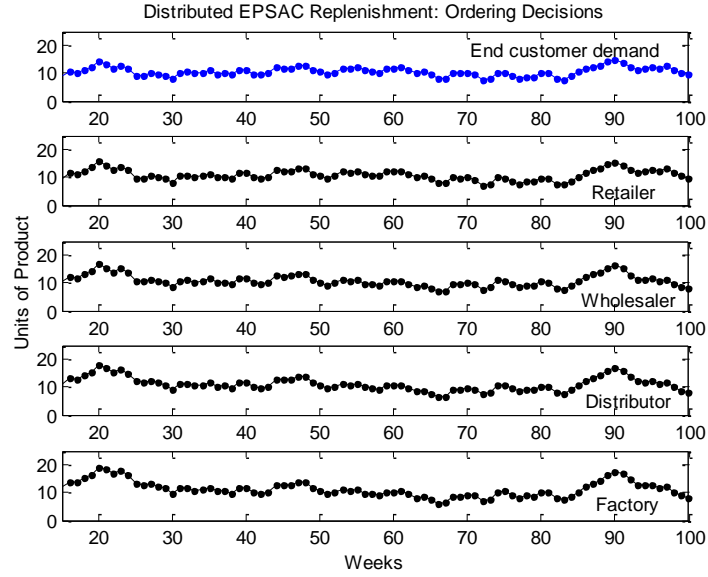


Figure 6.3: Time behavior of orders for distributed EPSAC control scheme

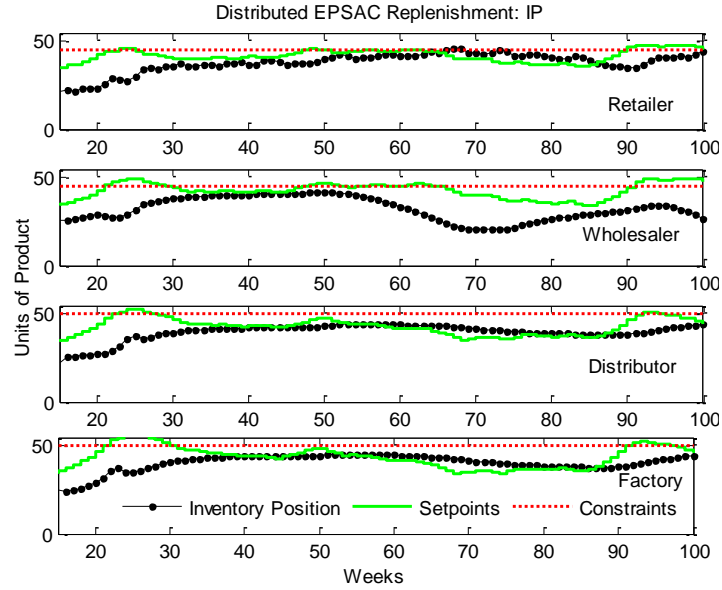


Figure 6.4: Time behavior of IP, setpoints and constraints for distributed EPSAC control scheme

## 6.4 An Illustrative Example

Table 6.2: Comparisons of the economic performance indices for different replenishment schemes

Control Scheme	AMCS	AEI	ABO	TC	TC reduction (%)
PI Ordering Policy	2.17	20.00	35.75	17322	--
Decentralized EPSAC	0.37	57.96	1.94	14777	14.69
Distributed EPSAC	0.14	58.22	0.92	14490	16.35

Similar economic performance evaluation is carried out as in Section 5.6.3. The cost reduction (TC & TC reduction columns in Table 6.2) by applying DMPC replenishment is obvious compared to PI approach.



Figure 6.5: Retailer's inventory holding profile and end-customer satisfaction level for three control schemes

Figure 6.5 and AMCS in Table 6.2 indicate that the EPSAC replenishment rules have better customer satisfaction level (after 25<sup>th</sup> week the demands are able to be met continually) and PID approach constantly has unsatisfied demand. Figure 6.5a) shows that the MPC replenishment can maintain the retailer's inventory around the

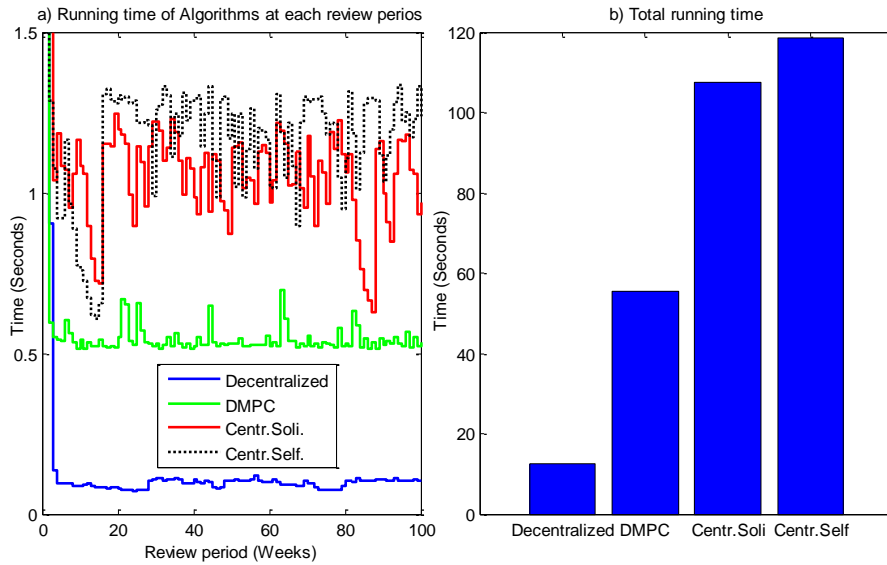
### MPC SCHEME FOR DISTRIBUTED REPLENISHMENT

safety stock of 98% service level and the PI approach is unable to keep that inventory level.

*Table 6.3: Comparisons of bullwhip for different replenishment schemes*

Control Scheme	Bullwhip at each echelon				Overall BW	BW Slope
	Retailer	Wholesaler	Distributor	Factory		
PI Ordering Policy	1.18	2.43	1.80	1.65	8.56	2.45
Decentralized EPSAC	1.54	1.62	1.53	1.42	5.46	1.31
Distributed EPSAC	1.62	1.23	1.32	1.36	3.58	0.65

The *overall BW* in Table 6.3 for the whole SC confirms that information sharing mechanism in DMPC control scheme can reduce the order variation and mitigate SC instability. The *BW slope* shows that DMPC replenishment rule results in a smoothed transfer of order variation along the SC echelons, the speed of bullwhip propagation is slow.



*Figure 6.6: Running time of four replenishment rules per review period and total running time*

A brief comparison of time complexity between different replenishment algo-

## 6.5 Summary

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rithms is conducted in Figure 6.6. The time consumption for the algorithms matches the analysis on QP problems in Section 5.4 and 5.5, the DMPC replenishment rule stands in the middle ground between centralized and decentralized EPSAC replenishment rules in terms of algorithm complexity.

## 6.5 Summary

Distributed approach is a natural choice for decentralized management of supply chains where the nodes are spread geographically or managed by different companies. This chapter presents an original solution to inventory control of the benchmark SC system based on DMPC principles. The modeling and control problems of SC are discussed firstly and the inventory position is controlled by designing ordering policies in the context of EPSAC algorithms. The whole SC network is decomposed into a series of interacting subsystems and each of them is assigned to a control agent. The local agent determines the optimal ordering decisions in order to achieve the best performance by iteratively solving optimization problems at each review period. The formulation of cost function considers both economic factors and dynamic performance characteristics, e.g. inventory holding cost versus IP target tracking and bullwhip mitigation versus move suppression for the control variables. Different control agents have to exchange information regarding their current and future decisions in order for the whole SC to reach Nash Equilibrium. Through this mechanism, the DMPC scheme maintains the flexibility and topology of the decentralized MPC scheme yet provides better control performances. Finally, the simulation results prove that the proposed scheme outperforms the conventional policy, e.g. a reduction of operating cost, mitigation of the bullwhip effect and increased customer satisfaction level.

The results presented in this chapter have been submitted to *IEEE Trans. Control Systems Technology* for possible publication:

Fu, D., Ionescu, C., Aghezzaf, E-H., De Keyser, R.. “An efficient, bullwhip reducing, predictive control strategy for inventory management in supply chains.” *IEEE Trans. Control Systems Technology*, under review.

# 7

## Conclusions and Perspectives

To conclude, the main contributions of this thesis are emphasized and suggestions for possible further research directions are outlined.

### 7.1 Summary

In this thesis, the system approach and model-based predictive control techniques are engaged to find the inventory control policies for the benchmark SC system. The replenishment relies on ordering policies that follow the principles of EPSAC control strategy. These policies pose the characteristics of having different information sharing scenarios, suggesting decentralized, centralized and distributed control structures. To adequately manage inventory in the presence of customer demand uncertainty, it is important for a decision-maker to understand the underlying dynamics. The SC network that shares the same structure of the well-known Beer Game SC benchmark is selected to test the proposed decision policies. A thorough investigation of the SC dynamics in Chapter 2 yields well-understood mathematical expressions that form the basis for the inventory control and replenishment policies based on MPC principles. The dynamics description consists of three elements:

- 1) the characterization of SC from the process control point of view
- 2) the mathematical expressions capturing the exchange of material and information within SC members and the switched system approach to describe the real-world SC operations
- 3) the links between conventional ordering policies (classical OUT, FO) and control-oriented expressions are build, PID-based decision policy is also introduced. These decision policies are introduced for comparison purpose.

Further on, following the dynamics analysis, the performance measures are

## 7.1 Summary

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proposed in Chapter 3 to assess the impacts of different decision policies on the SC operations. Due to its multifaceted effect on SC performance, the bullwhip metrics are especially emphasized on the analysis and quantification in either time domain or frequency domain. To fully describe the bullwhip effect, several other complementary performance indicators are also presented. For quantifying the bullwhip effect, new perspectives from control theory are introduced. One of the original contributions is that the analytical expression of bullwhip metric is derived if the knowledge of the transfer function for any ordering policy is known. This closed-form solution is developed based on frequency domain analysis rather than statistical analysis in time domain. The cost-based metrics are also presented to depict the financial aspect (efficiency) of supply chain performance. All the presented metrics together describe the two major features of SC operations: supply chain responsiveness and supply chain efficiency.

A Diophantine equation based MPC formulation is established in Chapter 4 to facilitate the derivation of analytical expressions of bullwhip by directly employing the quantification methods proposed in previous chapter. Under unconstrained case, this approach results in closed-form MPC solution that is equivalent to the EPSAC approach. Thus the expression of the bullwhip measure can be developed based on control theory analysis rather than statistical analysis.

As SC network becomes very large and complex, it is necessary to control it with a suitable control structure. A well-structured SC control system may help to achieve a better performance. Moreover, the considerations on physical structures, system dynamics, control objectives and information sharing scenarios in SC makes the selection of control structures a crucial decision, as outlined in Chapters 5 and 6.

Chapter 5 presents and discusses the traditional decentralized inventory control for the benchmark SC system with consideration of practical operational constraints. The formulation of EPSAC-based decision algorithm offers an ideal replenishment rule in SCM. As a control-oriented framework, the decision algorithm can be tuned to reach acceptable performance in terms of both control theory and economic objectives.

With a centralized control implementation, inventory position is regulated by designing ordering policies in the context of constrained EPSAC algorithms. The proposed schemes (*selfish* and *solidary* policies) outperform the decentralized EPSAC decision algorithm and all other conventional decision policies.

As a compromise being made when full information sharing is unavailable, a distributed EPSAC approach is proposed in Chapter 6 to resolve the inventory control and replenishment problem. Simulation results prove that it perfectly preserves the topology and flexibility of decentralized control scheme, with a better performance.

The EPSAC based replenishment policies have been demonstrated to be capable of managing the SC network. The use of flexible control structures allows for determining control designs that lead to desired results from either an operational or an economic standpoint. The work presented in this thesis provides a *proof-of-*

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## CONCLUSIONS AND PERSPECTIVES

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*concept* strategy for future SCM. It may be applicable to SC industrial practice because 1) it relies on explicit mathematical basis rather than traditionally heuristical methods which require SC managers have expert knowledge; 2) some typical performance indices show the proposed replenishment policies have good agreement with practical SCM goals.

### 7.2 Highlights

Here we provide to the readers the most important features of this original and novel work which make it outstands current state-of-art practice of SCM.

- the benchmark SC system is modeled from control engineering perspectives
- a novel method to derive analytic expression of bullwhip effect is proposed based on control theory concepts
- novel (decentralized and centralized) replenishment rules are designed in the context of constrained EPSAC strategy
- an original approach to SC replenishment by DMPC strategy is presented

### 7.3 Perspectives

The methodologies and frameworks presented here are essential steps forward that will assist the continuous improvement of SCM strategies. Some possible directions for further research are outlined.

- The insights into control engineering perspectives:
  - 1) there are several design parameters in MPC strategy such as the prediction horizon  $N_2$ , control horizon  $N_u$ , reference trajectory design parameter  $\alpha$  and penalties in the cost function. These parameters should be systematically tuned to obtain a balance between *acceptable control effort* and *acceptable control error*.
  - 2) the choice of control structures is determined by the specific information sharing scenario of the considered SC. Once the control structure is selected, the practical implementation of SISO EPSAC or MIMO EPSAC has to be determined by the SC model. Although the *selfish* EPSAC results in better performance than *solidary* EPSAC, it is only applicable to the case when the number of controlled variables equals the number of manipulated variables.
  - 3) SCM is a natural problem for distributed implementations of MPC-based replenishment policies. This draws the attention of future re-

### 7.3 Perspectives

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searchers to the DMPC frameworks. The last chapter discusses an iterative, non-cooperative distributed EPSAC control algorithm, which is a special case among many possible DMPC strategies. The future research directions in this area may include cooperative DMPC, economic DMPC [140] where SC managers seek to optimize economic benefits instead of current control objective of minimizing the deviations from target IP level. In DMPC, each agent works independently to achieve its local objective, but requires exchange of information via LAN in order to accomplish the overall task. So the study on the effect of communication delay for the DMPC strategy is an interesting research topic.

- 4) the factory is viewed the same as the other nodes in current work. However, as a production facility it may be proper to be modeled separately. In multi-product SC, the same factory processes multiple products. Thus, its control should incorporate a scheduling model to optimize the production sequence. The integration of control with scheduling will involve hybrid MPC problems in the next step of research.
- 5) some relevant extensions to current benchmark SC are considered as prospects of future research. The complex interactions within SC indicate the need to increase the number of nodes at each echelon and the types of product. The dynamics between different echelons is necessarily to be considered. Accordingly, the extension of MPC for these more complex SC networks is to be investigated. In fact, the SCs are hybrid systems in nature governed by continuous/discrete dynamics and logic rules. It is more acceptable to model them within the framework of the MLD/hybrid systems approach. Application and development of hybrid MPC is a challenging problem to solve.

- The insights into SCM pragmatism:

- 1) a future research direction is the implementation of the decision-making methods described in this thesis for a large-scale SC with real data. The efficiency of SC mostly depends on many decisions. The study in this thesis proves that MPC strategies provide a good decision making support to the SC managers. However, the final decisions are made by the human planners-SC engineers. It requires in-depth collaboration between control engineers and SC engineers in the future. A better way to implement the MPC strategies is probably to develop a software simulator. It should be user friendly for non MPC experts. In this way, other researchers may systematically test their control policies.
- 2) for heterogeneous SC, different replenishment policies should be allowed for different echelon nodes. The investigation of effects of dif-



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ferent start-up conditions on the bullwhip effect for each node and the whole SC should reveal interesting dynamics.

- 3) to be industrially relevant, the EPSAC-based decision policy must be numerically efficient to handle a large number of manipulated variables; i.e. it must be scalable when new members are added to the SC network, which is commonly found in practical SC operations.
- 4) the current decision policies are mostly used for supporting the human managers. Hence, an interesting research in the future is that of enhancing or fully automating the decision making process to eliminate the bias caused by human planners and shift the role of manager to other decision-making processes.

### 7.3 Perspectives

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