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Macroeconomics and the Labor Market

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2015

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Submitted to the Faculty of Economics and Business Administration of Ghent University and to the Faculty of Economic Sciences of Göttingen University in fulfillment of the requirements of the degree Doctor in Economics

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Acknowledgements

In completing this dissertation, I have received advice and support from many people. This is the place to thank them.

First and foremost, I am grateful to my joint PhD supervisors Tino Berger and Gerdie Everaert. From the beginning of this PhD project, Tino has been a true mentor. He encouraged me to develop and pursue my own research interests, while offering guidance in the academic world. Through his advice, I learned to conduct my research thoroughly and to be patient, even in times when the finish line was out of sight.

Having Gerdie as a joint supervisor was invaluable for this PhD project. Not only is he an experienced researcher with a great expertise in econometrics, but he is also a dedicated supervisor who had an open door at all times. Cooperating with him was a unique experience. I am deeply thankful to him for making this joint PhD project possible.

I would also like to express my gratitude to the members of my reading committee: Freddy Heylen, Gert Peersman, Helmut Herwartz, and James Morley. Their comments and suggestions have greatly improved the papers and I appreciate their time and effort.

During the past years, I have encountered many people, whose support and friendship made my time as a doctoral student so much more enjoyable. A warm thank you goes to my PhD companion Sibylle Grabert and to my former colleagues at the Institute of Public Economics in Münster, with whom I shared the ups and downs of life and research. Moreover, I thank my colleagues at Ghent University for making the last year in Belgium such a remarkable experience.

This dissertation was financially supported by different institutions. I greatly appreciate three years of postgraduate funding from the Friedrich-Ebert-Stiftung. I also thank the Special Research Fund (BOF) at Ghent University for financing my research. Finally, I acknowledge support from the National Bank of Belgium for the last chapter of this dissertation.

Hauke Vierke

Ghent, September 2015

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1 General Introduction

1.1 Motivation of the research questions

The success of macroeconomic policy relies on precise and timely estimates of the state of the economy. Measuring the state of the economy often entails decomposing a series of aggregate data, such as the unemployment rate, into a long-run level and deviations thereof. In the long-run, the economy is assumed to be driven by supply-side factors, i.e. labor, capital, and its productivity. They determine the economy's potential to produce goods and services. Transitory deviations from the long-run path are usually associated with factors that influence the total demand for goods and services. In case of the unemployment rate, the long-run level is often referred to as the natural rate. Friedman (1968) and Phelps (1968) argued that when actual unemployment is equal to its natural level, total production coincides with the economy's potential and the inflation rate is stable. It is a stylized fact that the labor market fluctuates around this natural level along with the alternating phases of economic booms and recessions. The labor market gap can be used by central banks as a yardstick for monetary policy. If the unemployment gap is positive, i.e. current unemployment exceeds the natural level, the central bank might favor stimulative policies to drive down the unemployment rate. A negative unemployment gap can indicate an overheating economy and herald a rising inflation rate. The natural level and the respective gap provide also valuable information for fiscal policy, which should provide stimulus only if the labor market is temporarily below its potential. If, in contrast, unemployment is characterized as a structural problem, policy makers should turn to alternative strategies that focus on the employability of the workforce.

The preceding considerations establish linkages between unemployment, inflation and output (production). The relation between unemployment and output is referred to as Okun's law, named after Arthur Okun (1962), who observed a negative correlation between the deviations of unemployment and output from their natural levels. Specifically, he found that a negative output gap of 2% corresponds to a positive unemployment gap of 1% in the United States. The intuition is straightforward. An economy that produces goods and services below its potential will employ less people than needed at full potential. Consequently, the unemployment rate rises. Another important linkage is the famous Phillips (1958) curve. In its original form, the curve represented the empirical relationship between changes in the unemployment rate and wage growth, but economists soon began to replace the latter with price inflation. The stability of the relationships between these macroeconomic aggregates is essential for economists and policy makers. For example, central banks use more advanced versions of the Phillips curve to forecast inflation. In case of the United States Federal Reserve (FED), the trade-off between inflation and unemployment is directly incorporated in its monetary policy objectives. The so-called dual mandate states that the FED should pursue price stability and full unemployment, which requires the economy to be close to its potential.

There is an ongoing debate about whether the relationship between the labor market and other key variables is stable over time. In econometric terms, these relationships are potentially non-linear. In fact, many argue that they have changed substantially or have broken down completely. For example, the large and negative output gaps around the globe in the course of the recent Great Recession implied substantial deflation according to standard macroeconomic models. Although inflation slowed down, there was no notable deflationary period. This "deflation puzzle" could imply that the Phillips curve has broken down. The discussion has recently regained attention as signs of economic improvement in the United States have led many observers to call for a turnaround in the FED's policy.[1] Since its peak in late 2009, the unemployment rate has dropped from 10% by almost half to 5.1% in August 2015. Thus, the FED is expected to end the period of quantitative easing and begin to tighten monetary policy. Such a policy change would not only have consequences for the United States, but is likely to affect the world economy. Other central banks, including the European Central Bank (ECB), will have to adjust their policy accordingly. Hence, whether or not the FED believes that the economy is back to potential is downright crucial. However, the substantial decline in unemployment might not accurately reflect the state of the economy. A drop in the unemployment rate can also be caused by discouraged people, who stop actively seeking a job and are thus not counted in the unemployment statistics. Moreover, people might technically be employed, but work fewer hours than before the crisis - a phenomenon known as underemployment. In a recent speech, FED chair Janet Yellen (2015) elaborated on the role of labor force participation and underemployment:

> "*[...] it is my judgment that the lower level of the unemployment rate today probably does not fully capture the extent of slack remaining in the labor market - in other words, how far away we are from a full-employment economy. [...] I think*

[1] Just days before this dissertation went to press, the Federal Open Market Committee delayed the expect tightening for at least another month.

> *a significant number of individuals still are not seeking work because they perceive a lack of good job opportunities, and that a stronger economy would draw some of them back into the labor force. [...] Another factor we consider when assessing labor market slack is the elevated number of workers who are employed in part-time jobs but would prefer to have full-time work [...] I continue to think that it probably remains higher than it would be in a full-employment economy.*"

The information content of the unemployment rate for the state of the economy today is probably lower than some years or even decades ago, because other channels have become more important for the adjustment process. This observation is an important motivation for this dissertation. The first half will investigate the interaction of the labor market and the rest of the economy at the aggregate level in a stylized empirical macroeconomic model. The questions to be answered are: Has the relationship between the cyclical components in output, unemployment and inflation changed over time? How can these changes be linked to structural changes within the economy? How can an alternative labor market indicator be constructed, that is robust to the aforementioned problems?

So far, only the *level* of the economy's potential and the *size* of the business cycle gap have been discussed. However, there is evidence that also the *variance* of the business cycle has changed notably over time in many countries. The most prominent example is the Great Moderation - a term which describes the large and persistent decline in macroeconomic volatility during the 1980s, that lasted until the recent Great Recession. Changes in the volatility are equally important from a policy point of view, as a greater volatility implies greater uncertainty about the exact size of the business cycle gap and thus the state of the economy. Even if one is not interested in the volatility per se, accounting for changing volatility remains important for hypothesis testing. The second part of this dissertation focuses on the volatility of macroeconomic data. As a matter of course, economists are not only interested in measuring volatility, but also explaining it. If the driving forces can be revealed, this possibly opens up a way to also influence volatility. Again, the focus in the corresponding chapters will be on the interaction of the labor market with other macroeconomic aggregates. A new strand of literature gives preliminary evidence that the demographic composition, i.e. the share of old and young workers, might play an important role for economic volatility. However, as will be discussed later, one can raise doubts about the methodology used to establish the empirical relationship. The key questions answered in the second part of this dissertations is whether the previous findings in the literature on demographics and its effect on output volatility are spurious. Moreover, the aim is to re-estimate the effect more consistently.

1.2 Methodology

The methodology used in this dissertation arises as a natural choice due to the unobservability of the economy's potential and the business cycle gap. In macroeconomic data only the sum of the two components is observed. There exist different approaches to disentangle cyclical from trend components. Besides purely statistical approaches such as the Hodrick-Prescott (HP) filter, a widely used technique is to estimate a structural time series models as pioneered by Harvey (1985). These models are also known as Unobserved Components (UC) models. The data series, for example the unemployment rate, is decomposed into a trend and a cyclical component by imposing a stochastic law of motion for each of these two components. Identification can be achieved by assuming that shocks to the trend component have permanent effects on the economy, while shocks to the cyclical component are only transitory. The stochastic processes will therefore exhibit different degrees of persistence. Given an identifying assumption, the unobserved latent variables are extracted using the Kalman filter and the unknown coefficients are estimated via Maximum-likelihood techniques. In order to better pin down the natural rate, cyclical swings in the unemployment rate are allowed to co-move with other macroeconomic variables such as the cyclical component in output or inflation. The correlation parameters in this case represent the Phillips curve and Okun's law. They anchor the empirical time series model to macroeconomic theory. The structural time series approach is not necessarily limited to the unobserved components. It is also possible to assume a stochastic law of motion for the volatility and filter it in a similar way. Moreover, the latent components need not to be purely stochastic, but can also be linked to other explanatory variables.

A key contribution of this dissertation is to account for various forms of non-linearity on the class of structural time series models. This implies that the correlation and variance parameters are allowed to change over time, which results in highly nonlinear models that cannot be estimated in the classical (frequentist) way. Instead, the dissertation builds on Bayesian estimation techniques. When the classical approach is not feasible because the likelihood function is ill-behaved, the Bayesian approach offers an interesting an often intuitive alternative solution. In contrast to the likelihood principal, that makes a statement about the likelihood of the data conditional on the model's unknown parameters, the Bayesian approach turns the problem around and gives the probability of the unknown parameters given the observed data. Moreover, it allows to incorporate prior knowledge about the parameter, for example when some parameter regions can be ruled out according to economic theory. As such, it is attractive also for policy purposes. It offers an elegant way to incorporate a policy maker's belief about certain parameters. Finally, the Bayesian methods applied in this dissertation entail computing the complete distribution of the unknown components conditional on all other parameters. Hence, the results do not only give a point estimate, but also quantify the uncertainty around these estimates - something that is not straightforward within the classical approach.

1.3 Outline of the dissertation

Although the chapters of this dissertation are related, they represent self-contained papers, each tackling a specific research question. The outline of the dissertation is as follows:

- Chapter 2 starts with an illustration of how the unemployment rate has become a misleading indicator for the state of the labor market in the United States and in Germany. While in the United States recent movements in the unemployment rate appear to be partially driven by discouraged workers or early retirees, the German labor market has adjusted in the crisis mostly along the intensive margin, i.e. working hours. In order to cushion the impact of the economic and financial crisis, Germany introduced a country-wide short time work program. Thus, only looking at the unemployment rate when evaluating the state of the labor market would be misleading. In this chapter an alternative measure for the slack of the aggregate labor market is proposed. Instead of the conventional unemployment rate, the ratio of aggregate hours worked over potential hours is used. This measure takes into account the intensive margin and is robust to variations in labor force participation. An unobserved components model is estimated to disentangle cyclical swings in the labor market from its natural level. The resulting hours-based gap outperforms a conventional unemployment-gap when it comes to explaining monetary policy with historic data.

- Chapter 3 focuses on structural change in an empirical macroeconomic model for the United States economy. Structural change can affect the relation between the labor market and other macroeconomic aggregates, but also the volatility of the trend and gap components. Specifically, the model decomposes output, inflation and unemployment in their stochastic trend and business cycle gap components, with the latter linked through the Phillips curve and Okun's Law. A key feature in this chapter is the use of a novel Bayesian approach to testing which parameters are time-varying. Based on United States data from 1959 to 2014, the results point to substantial variation in Okun's law. The relation between unemployment and output is found to be stronger during recessions than recoveries. The slope of Phillips curve is found to be time invariant but flat, i.e. the link to the nominal sphere of the economy is weak. Changes in the volatility of the latent components are only found to be important for cyclical shocks, while shocks to the long-run or natural components are constant over time.

- Macroeconomic volatility, which has also been discussed in the previous chapter, is investigated more intensively in Chapter 4. In particular, the focus lies on explaining the observed changes in output volatility. Recent studies suggest a robust relationship between the share of young and old labor force participants and the volatility of aggregate production. This chapter casts doubt on the methodology used in these studies. The time series properties of the data are not taken into account correctly, so that regression results might be spurious. The data used in these regressions involve non-stationary data, i.e. data with trend-like behavior. It is a well-known phenomenon that this can lead to spurious correlation estimates, although there exists no meaningful relationship. This chapter replicates three well-published studies and investigates whether the results are robust to the aforementioned concerns.

- Chapter 5 investigates the determinants of output volatility in a panel of 22 OECD countries. The chapter adds important methodological contributions. Building on the observation of time series characteristics from the preceding chapter, the model explicitly accounts for the trend-like behavior of the data. An unobserved components model with stochastic volatility is chosen. As demonstrated by Everaert (2011), the relationship between integrated series, that do not constitute a cointegrating relationship, can still be consistently estimated in an UC framework. We follow this approach to investigate the driving forces behind business cycle volatility. Recent findings on the role of demographics are combined with the literature on fiscal policy and its effects on output stabilization. The results suggest indeed a robust effect of demographics.

1.4 Main findings, implications, and outlook

The dissertation follows two lines of research: The role of non-linearities in a structural time series model for the aggregate labor market and the interaction between the labor market and macroeconomic volatility. The main findings from the first part can be summarized as follows. When investigated in detail, the labor market in the United States and Germany underwent important structural changes. The unemployment rate, although still a key variable for macroeconomists and policy makers, does not capture all relevant dimensions of the labor market. Changes in hours worked and labor force participation are additional channels through which the labor market can adjust to cyclical shocks.

This does not imply that the unemployment rate should be abolished as a key indicator. Rather, the findings reinforce the need to also look at other dimensions of the labor market and take them into account when evaluating the state of the economy. The Taylor rule exercise in Chapter 2 suggests that the FED has already done this in the past, since the alternative indicator explains central bank behavior better than an unemployment-based rule.

When a standard structural time series model is extended to allow for various forms of structural change, important conclusions can be drawn about the changing characteristic of the United States economy. The potential growth rate of the economy has declined substantially. In addition to the "productivity slowdown" during the early 1970s, which has previously been documented in the literature, the results show a prolonged and persistent decline in the potential growth rate over the whole sample. This finding delivers support for the hypothesis of a "secular stagnation" in the United States. The period of slow growth, low interest rates, and sluggish employment growth is likely to be a permanent phenomenon. Now that a permanent slowdown is established as a stylized fact, more economic research is needed on the sources and consequences of this slowdown. Most economists would agree that the stagnation implies huge potential costs: From the problem of the zero lower bound related to overall low interest rates, to the millions of under- and unemployed who see their skills deteriorating. Hence, future research should disentangle the driving forces of the slowdown.

Another interesting finding are the apparent business cycle swings in the Okun relation. Over the whole sample the Okun coefficient is larger in recessions than in booms. This implies that when output falls below potential, job losses are big and occur relatively fast. During the following recovery, job growth is weaker and it takes more time for employment to return to its pre-recession level. The results from this dissertation show that this is not a recent phenomenon. Rather, the Okun relation has always been non-linear over the postwar business cycles. However, the Great Recession still stands out as an episode of exceptional labor market sensitivity. An interesting path for future research is the link between Okun's law and the recent literature on the financial cycle - a concept similar to the cycle in aggregate production. Borio (2014) demonstrated how to approximate the financial cycle based on data on credit and property prices. Incorporating this additional information substantially improves the real-time performance when estimating the state of the economy. As noted by Borio (2014), the turning points of the financial cycle do not necessarily match the conventional business cycle turning points. While the business cycle appears at the 1-8 year frequency, the financial cycle typically last up to 16 years. If the sensitivity of unemployment depends on financial conditions, this might help to explain the complex dynamics of the Okun coefficient found in this dissertation. The dependence of Okun's law on financial conditions should ideally not only be identified from the time series dimension but also from cross-country differences. In the United States, credit to private sector and market capitalization are historically higher than in Europe and these differences could explain the variation in cross-country Okun regressions.

The main results from the second half of this dissertation can be summarized as follows: The volatility of the business cycle has changed notably in many of the industrialized countries within the last 50-60 years. However, there exist important differences across countries. The demographic composition of the labor force can explain a large part of these changes. Another important factor is fiscal policy in the form of labor taxes. This is in line with the established concept of taxes as automatic stabilizers. It should be noted that stabilization might come at a cost. Distortionary taxes, such as labor taxes, can exert negative effects on the growth rate of the economy. These effects are likely to depend on the composition of the government spending as well. A possible path for future research is to investigate the determinants of output volatility and output growth simultaneously. The proposed framework from the final chapter offers a well-suited model. In Chapter 5 the average growth rate is allowed to change over time, but assumed to be purely stochastic. A straightforward model extension would be to link the growth rate to a set of explanatory variable. Such a study could pick up the discussion on the secular stagnation from Chapter 3 as well.

Bibliography

BORIO, C. (2014): "The Financial Cycle and Macroeconomics: What Have We Learnt?," *Journal of Banking & Finance*, 45, 182–198.

EVERAERT, G. (2011): "Estimation and Inference in Time Series with Omitted I(1) Variables," *Journal of Time Series Econometrics*, 2(2), 1–28.

FRIEDMAN, M. (1968): "The Role of Monetary Policy," *American Economic Review*, 58(1), 1–17.

HARVEY, A. C. (1985): "Trends and Cycles in Macroeconomic Time Series," *Journal of Business & Economic Statistics*, 3(3), 216–227.

OKUN, A. M. (1962): "Potential GNP: Its Measurement and Significance," *American Statistical Association, Proceedings of the Business and Economic Statistics Section*, pp. 98–104.

PHELPS, E. S. (1968): "Money-Wage Dynamics and Labor-Market Equilibrium," *Journal of Political Economy*, 76, 678–711.

PHILLIPS, A. W. (1958): "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957," *Economica*, 25(100), 283–299.

YELLEN, J. L. (2015): "Recent Developments and the Outlook for the Economy," *Speech at the City Club of Cleveland, July 10, Ohio.*

2 | Estimating the Natural Rate of Hours

with Tino Berger

Abstract: This paper proposes an alternative measure for the slack of the aggregate labor market. The natural rate of hours holds valuable information about the state of the labor market which are not reflected by conventional measures, such as the equilibrium rate of unemployment, since it takes into account the intensive margin and is robust to variations in labor force participation. We set up and estimate a multivariate unobserved components model using information in GDP, inflation, and hours worked, and apply it to the United States and Germany. The estimated hours gap outperforms conventional unemployment gap measures in a Taylor rule by means of formal model comparison.

JEL classification: C32, E24, E32, E52

For helpful comments we thank Freddy Heylen, Gerdie Everaert, Irina Panovska, and participants at the 20th SNDE Annual Symposium, the CMR Lunch Seminar at the University of Cologne, the 7th ifo Workshop on Macroeconomics and the Business Cycle, and the Baltic Sea Colloquium. Hauke Vierke acknowledges financial support from the Friedrich-Ebert-Stiftung and from Ghent University's Special Research Fund (BOF).

2.1 Introduction

Knowing how far a country's labor market deviates from its equilibrium level is of great relevance for various reasons. The sign and magnitude of the labor market gap provide valuable information to better predict output growth and the impact of monetary policy. Central banks take some form of natural rate estimate into account when evaluating the state of the economy. For instance, the Federal Reserve Open Market Committee (FOMC) estimates the "long-run normal rate of unemployment", as published quarterly in the FOMC's Summary of Economic Projections.

The most widely used indicators for the state of the labor market are the Non-accelerating inflation rate of unemployment (NAIRU) and the corresponding gap. The idea of a natural rate of unemployment has been pioneered by Friedman (1968) and Phelps (1968), who claim that unemployment is at its natural level when neither inflationary nor deflationary pressure emanates from the labor market. The existence of a constant NAIRU has been questioned after the oil price shocks of the 1970s as unemployment remained high even after inflation had stabilized. More recently, the NAIRU is assumed to be a function of labor market institutions and real macroeconomic variables such as real interest rates or productivity growth and hence is time-varying.

Using the NAIRU as an indicator for the state of the aggregate economy assumes that the unemployment rate captures the most relevant changes in the labor market. In this paper we argue that labor force participation (extensive margin) and hours worked (intensive margin) play an important role for the adjustment process, such that additional information, other than the unemployment rate, can help to estimate the natural level more precisely. During the Great Recession Germany has reacted to the severe decline in GDP with a widespread short-time work-program. In order to avoid layoffs, employees agreed with firms to work less and received a subsidized wage allowance. The scope of underemployment in Germany, with about 1.5 million short-time workers at maximum, was concealed by a relatively stable unemployment rate during the crisis. However, hours worked dropped by 3.3%, i.e. the adjustment occurred along the intensive margin. Moreover, changes in labor force participation can alter the unemployment rate, even if the overall employment level stays constant. If many discouraged workers exit the labor force after a severe recession, the unemployment rate overestimates the state of the labor market. There is an ongoing debate on the determinants of the decline

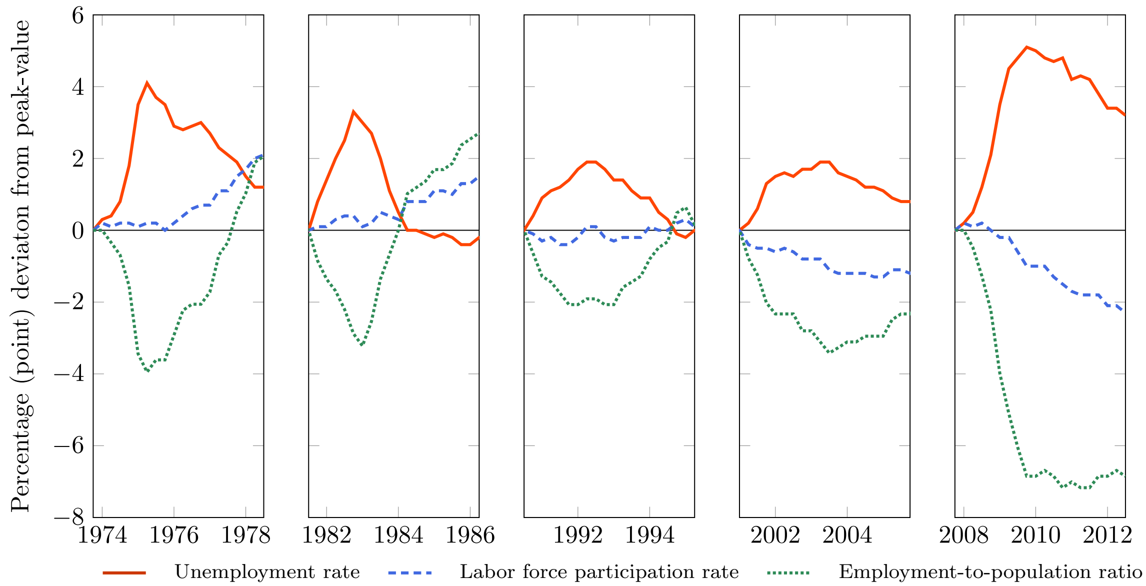
In January 2012, two and a half years after the end of the Great Recession, FED chairman Ben Bernanke argued whether or not an increase in long-term unemployment has caused a shift in the natural rate of unemployment. Bernanke concluded that the unemployment rate of 8.5% was well above any natural rate estimate, hence sustaining an accommodative stance of monetary policy would be within the scope of the FED's dual mandate (Bernanke, 2012).

in U.S. labor force participation and its impact on employment dynamics. Fujita (2014) argues that both long-run and cyclical factors have driven the decline and that the number of discouraged workers has certainly risen during the recent recession. Whether these marginally attached workers play an important role for the unemployment rate is not clear-cut: Davig and Mustre-del Rio (2013) find that reentry of the "shadow labor supply" will only have a modest impact on the unemployment rate. Ravikumar and Shao (2014) construct an alternative unemployment rate that accounts for the reentry of discouraged workers. This series is only slightly higher as the official series and exhibits a similar trend. However, according to Romero (2012) the number of around 900,000 discouraged workers at the end of 2013 is severely underestimated as this only includes people who have searched for a job within the past year. In contrast, 3.2 million workers generally want a job but stopped looking more than one year ago. Zandweghe (2012) argues that the current cyclical gap between the actual and trend labor force participation rate is likely to hold back the return of the unemployment rate to its long-run level. Hence, the dynamics of hours worked and labor force participation may have important implications for the state of the labor market. By exclusively focusing on unemployment as the labor market indicator these information are neglected.

In this paper we take a fresh look at the adjustment dynamics of various labor market variables over the business cycle in the United States and Germany. The main focus is on the estimation of an additional indicator for the state of the labor market that captures movements along the extensive and intensive margin. We estimate the natural rate of total hours over potential hours and find a meaningful correlation between the proposed indicator and other macroeconomic variables. We demonstrate the implications of our alternative labor market gap for the conduct of monetary policy. A Taylor rule based on the hours gap as the relevant labor market indicator leads to a better model fit and is not subject to parameter instability. We find strong support for the hours-based rule using formal model comparison.

The remainder of the paper is structured as follows: Section 2.2 shows the adjustment dynamics of key labor market variables over the business cycle. Section 2.3 introduces the hours ratio as an alternative variable of aggregate movements in the labor market. Section 2.4 lays down a stylized structural time series model used to estimate the natural rate of hours. Section 2.5 describes the data and the Bayesian estimation procedure. Results are given in section 2.6. We demonstrate the importance of our findings for monetary policy via a Taylor rule in section 2.7. Section 2.8 gives a conclusion.

Figure 2.1: Labor market indicators over business cycles: United States



2.2 Labor market adjustment

This section analyzes the adjustment dynamics of key labor market variables over the business cycle. Particular attention is paid to changes over time and across countries in the response of (i) the employment-to-population ratio, (ii) the unemployment rate, (iii) labor force participation, and (iv) hours worked after a recession. We focus on the United States and Germany. The United States is known to have a flexible labor market, while Germany represents the more rigid European labor market regime. The focus in the following graphical analysis is on the role of the extensive margin for the United States and the intensive margin for Germany.

Figure 2.1 shows the evolution of the United States unemployment rate, the labor force participation rate and the employment-to-population ratio during business cycles over the period from 1973 to 2012. Each graph plots the series until the 20^{th} quarter after the peak date. All variables are displayed in deviations from their value at the business cycle peak. After the first two recessions the unemployment rate and the employment-to-population ratio evolve symmetrically and follow a V-shaped pattern, i.e. both series return to their pre-recession levels shortly after a sharp turning point. Labor force participation stagnates for some time, but starts to increase right after the trough. This suggests that the decline in the unemployment rate during the recoveries was driven by additional job creation and not by people exiting the labor force. During the recession in the 1990s the unemployment rate and

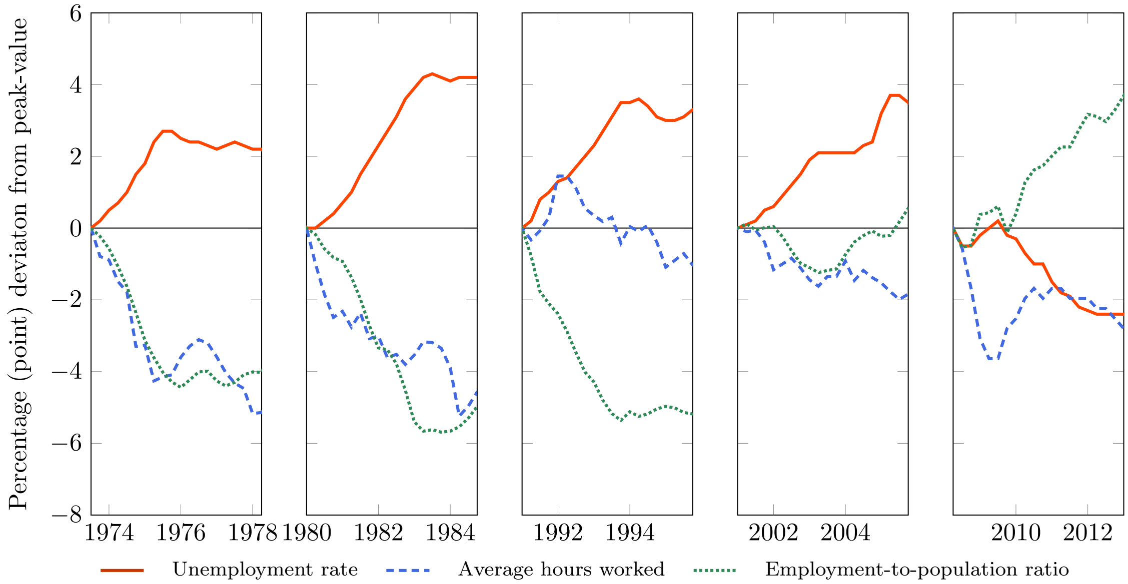
We use the peak and trough dates provided by the National Bureau of Economic Research for the United States and the Economic Cycle Research Institute for Germany throughout this paper.

the employment-to-population ratio exhibit a U-shaped pattern. Although this recession was less severe in terms of absolute changes, it took longer time for both series to return to their pre-recession level. The early 2000s recession shows an incomplete U-pattern, as neither the unemployment rate nor the employment-to-population ratio return to their pre-recession level during the recovery. However, the absolute change is larger for the employment-to-population ratio. In contrast to the recessions of the 1970s and 1980s, the labor force participation rate steadily decreased throughout this cycle. Thus, the rather moderate increase in the unemployment rate suggests that fewer people actively looked for a job. Regarding the most recent recession, all three labor market indicators exhibit changes that are substantially larger in magnitude as compared to all previous recessions in the sample. In contrast to former periods, the employment-to-population follows an L-shaped pattern, i.e. employment stagnates at its recession level and without any sign of recovery. The unemployment rate, however, evolved in a very different way. While the magnitude of changes in these two variables has been very similar over the previous recessions, changes in the unemployment rate are considerably smaller during the most recent recession. Moreover, the declining unemployment rate after 2010:Q1 coincides with a steep decline in labor force participation. Thus, the decline in the unemployment rate may not be interpreted as a recovery of the labor market as such, but as the result of fewer people joining the labor market. The increased importance of the extensive margin in the recent recession and to a lesser extend in the early 2000s recession implies that the deviation of the unemployment rate from a natural level such as the NAIRU is too small and does not reflect all dimensions of the aggregate labor market. Even if one still believes in the NAIRU as an important piece of information, its relation to other labor market variables such as the labor force participation rate and the employment-to-population ratio has changed over time. The latter point on its own is relevant for the conduct of monetary policy.

Turning to the German labor market, there is a remarkable difference in the magnitude between the different cycles within Germany and compared to the United States. Figure 2.2 shows the evolution of the German unemployment rate, hours worked and the employment-to-population ratio during business cycles over the period from 1973 to 2013. The first four cycles deliver clear evidence on how business cycle shocks left permanent "scars" on the German economy as the unemployment rate initially increased and then stagnated on a new plateau. Similarly, employment exhibits an L-shaped pattern during the first three recessions, but employment dynamics are very different for the recession in the early 2000s as job-losses

Juhn and Potter (2006) argue that unemployment during the early 2000s has led to a more or less permanent withdrawal from the labor market.

Figure 2.2: Labor market indicators over business cycles: Germany



are small and employment recovers quickly. The fact that the unemployment rate increases despite growing employment can be explained by German labor market reforms that led to a broader definition of the unemployed population. The picture for the 2008 recession is drastically different from previous cycles as the unemployment rate increases only slightly and only for a few quarters before it continues to decrease permanently. Similarly, the recession does not alter the upward trend in employment notably. Almost all of the adjustment occurs along the intensive margin of the labor market. Hours worked plunge sharply in Germany, but recover just as fast. The series for hours worked over all five recessions also points to the important distinction between trend and cyclical movements. The hours series exhibits a distinct downward trend movement in Germany over the full sample, but the very last recession is a distinct example for a temporary decrease in the number of hours worked. This is the result of short-time work arrangements during the crisis in Germany - a feature of the labor market not captured by the official unemployment rate.

The importance of the intensive margin for the adjustment process on the labor market also appears in cross-country comparisons. Ohanian and Raffo (2012) document that in many OECD countries about half of the adjustment occurs along the intensive margin. In fact, different dynamics in hours worked might be one reason why the unemployment rate reacted so differently across countries during the Great Recession. Table 2.1 shows the changes in

In January 2005 the number of unemployed persons shot up to about 5 million people, mainly because previous welfare recipients were classified as "capable of working" and, thus, counted as unemployed.

Table 2.1: Labor market adjustment during the Great Recession

	Change from peak to trough	
	Germany 08:Q1-09:Q2	United States 07:Q4-09:Q2
GDP %	-6.0	-4.1
Unemployment rate pp.	-0.1	4.4
Employment (level) %	0.3	-4.8
Avg. hours per empl. pers. %	-3.3	-1.5

GDP, the unemployment rate, the employment level, and hours per employed person from peak to trough. While the size of the GDP shock is larger for Germany, the United States experienced a stronger reaction of the labor market with a larger increase in the unemployment rate and a bigger drop in employment. The intensive margin appears to be less important for the adjustment process. In contrast, Germany experienced only a slight movement in the (un)employment series (with a counter-intuitive sign). Hours per employed person, however, have been reduced substantially during the recession. This can be explained by the short-time work program, which was set up in order to avoid mass layoffs during the crisis. Hence, solely looking at the NAIRU could be misleading for the case of Germany.

In sum, the adjustment of labor markets differs across countries and has changed over time. Using the unemployment rate as the predominant labor market indicator (i) leaves out the intensive margin and (ii) is sensible to shifts in labor force participation. Hence, using the natural rate of hours, as estimated from a structural time-series model, provides a more realistic picture about the state of the labor market.

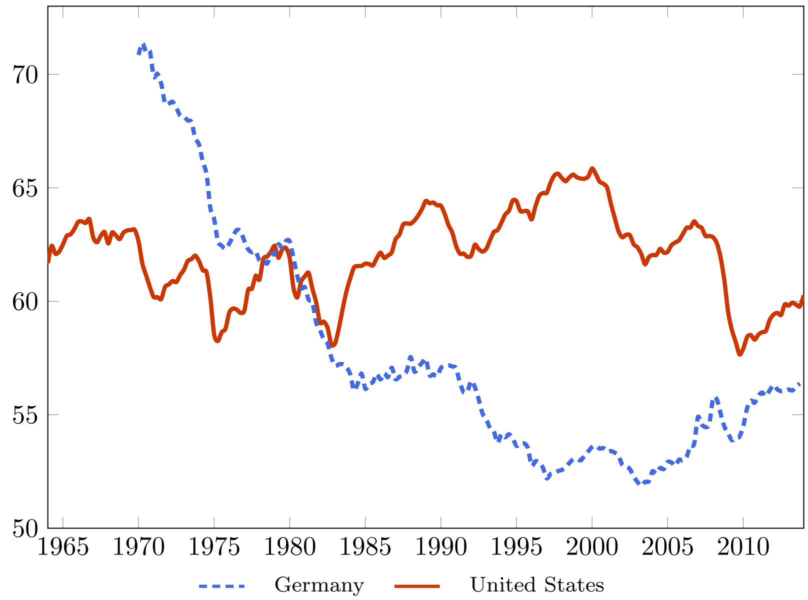
2.3 Measuring employment in hours worked

An indicator which takes into account the intensive margin and is robust to movements along the extensive margin is the ratio of aggregate hours worked to potential hours of an economy within one year. It is also known as the *employment rate in hours* and given by

$$\text{hours rate} = \frac{\text{hours per employed person} \times \text{employment}}{1920 \times \text{population at working age}}. \tag{2.1}$$

The employment rate in hours has previously been used as an indicator for the aggregate labor market by Dhont and Heylen (2008) and Berger and Heylen (2011).

Figure 2.3: Overall employment measured in hours



This indicator combines different dimensions of the labor market. Fewer hours worked show up as a lower number, thereby capturing movements along the intensive margin. Changes in the participation rate affect the numerator in eq. (1). However, changes in labor force participation that do not correspond to changes in the employment level, i.e. unemployed persons, who give up looking for a job, will not shift the hours rate. This is different from the unemployment rate, which would decrease if unemployed people exit the labor force. While changes in the unemployment can be induced by very different factors with very different implications, changes in the hours rate are always linked to an increase (or decrease) in overall labor utilization. For the calculation of potential hours, we assume a workweek of 40 hours and 48 workweeks, which results in 1920 potential hours per capita and per year. Figure (2.3) shows hours series for both countries considered here. Germany starts at a much higher employment level than the United States, but exhibits a persistent downward trend just until the early 2000s. For example, in 1970:Q1 Germans worked about 71% of potential hours while Americans worked about 63%. Since the early 1980s, the United States have reached a permanently higher employment level (in hours) than Germany. More recently, both series exhibit convergence. Differences in the hours rate are substantially smaller than

The number of potential hours per year may be affected by labor market legislation and thus varies over time and countries. Infrequent changes in potential hours would change the long-run mean of the hours rate. Since the aim of our econometric approach is to extract cyclical swings from long-run movements this should not affect our results. Nevertheless, we check the robustness of the results regarding the choice of potential hours.

20 years ago. Moreover, the United States exhibit more pronounced cyclical swings while the German series seems to be driven mainly by structural or long-run factors. The cross-country differences are the subject of a large literature. While some authors emphasize the role of labor and product market characteristics such as employment protection legislation, union power, wage bargaining systems, and barriers to entry, other studies highlight the influence of fiscal policy, particularly the differences in the level and composition of taxes and government expenditures (see Berger and Heylen, 2011). However, the focus here is not on the mean of the series but on the cyclical movements around its long-run trend. In particular, the series for hours is decomposed into a long-run trend and a cyclical component. To the best of our knowledge all previous studies on this subject have focused on the unemployment rate.

2.4 Empirical model

This section lays down a structural time series model to estimate the natural rate of hours. The empirical model borrows from the recent NAIRU literature, i.e. we estimate the time-varying natural rate within a multivariate unobserved components (UC) model that treats the equilibrium rate and its corresponding gap as latent variables. The latent variables as well as the model parameters are jointly estimated using a Gibbs sampling procedure.

Laubach (2001) introduced the structural unobserved components model in order to estimate the NAIRU. In his bivariate model, a Phillips curve is used to link the unobserved unemployment gap to changes in the rate of inflation. The model is structural in the sense that, by using a Phillips curve, the resulting equilibrium rate of unemployment is consistent with the stochastic law of motions for the latent variables that have been specified, and with a zero unemployment gap when inflation is stable. The latter point distinguishes NAIRU estimates based on an unobserved components model from purely statistical trend-cycle decompositions.

Since Laubach (2001) the literature has extended the modeling framework in several dimensions. Common to most of the recent NAIRU estimates is that they rely on multivariate unobserved components models, i.e. the latent variables are identified using information contained in various macroeconomic aggregates. In fact, Basistha and Startz (2008) show that using additional information from multiple indicators which share a similar cycle with the unemployment rate, cuts in half uncertainty about the estimate as measured by variance.

We will refer to the equilibrium level of the hours rate as the natural rate of hours.

Domenech and Gomez (2006) focus on the United States and estimate a model in which the latent variables are identified using information contained in inflation, unemployment, output, and investment. Berger (2011) estimates a trivariate unobserved components model for the aggregate Euro area NAIRU with special emphasis on correlated shocks and structural breaks in the trend components of output and unemployment.

This paper's model is based on the assumption that the series for total hours shares a common cyclical component with the two other variables considered, namely inflation and output. First, the employment-inflation relation is presented. A standard Phillips curve relation expresses realized inflation as the sum of current expectations of future inflation and the labor market gap. Denoting the inflation rate in period t by π_t and defining the labor market gap, h_t^c, as the deviation of total hours, h_t, from its natural level, h_t^*, the Phillips curve can be written as

$$\pi_t = E_t(\pi_\infty) + \theta(L)(h_t - h_t^*) + \varepsilon_t^\pi, \tag{2.2}$$

where ε_t^π is a Gaussian mean zero white noise error term, θ is the slope coefficient of the Phillips curve and the lag polynomial is defined as $\theta(L) = \theta_0 + \theta_1 L + ... + \theta_q L^q$. Inflation expectations are not observed and thus have to be proxied. Often, a backward-looking or accelerationist curve is assumed, i.e. lagged inflation is used as a proxy for inflation expectations (see e.g. Laubach, 2001; Fabiani and Mestre, 2004). Instead of assuming backward-looking expectations this paper follows Morley, Piger, and Rasche (2015) and proxies the expectation term by a stochastic trend. In theoretical work, Cogley and Sbordone (2008) and Goodfriend and King (2009) derive versions of the New Keynesian Phillips Curve (NKPC) that incorporate a time-varying inflation trend, such that the inflation gap rather than the level of inflation is influenced by the real activity gap. In empirical studies, the forward-looking NKPC can be reconciled with the data once inflation is allowed to have a stochastic trend (see Nelson and Lee, 2007; Piger and Rasche, 2008). The inflation trend, π_t^*, is assumed to follow a driftless random walk,

$$\pi_t^* = \pi_{t-1}^* + \eta_t^\pi, \tag{2.3}$$

where η_t^π is a Gaussian mean zero white noise error term. Similarly, real GDP is decomposed into a stochastic trend and a cyclical component. Trend output, g^*, is assumed to follow a random walk with drift. The cyclical component in output is linked to the hours gap through a stylized production function, which states that deviations of output from its trend

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We included the investment ratio as an additional variable in a previous version of the paper. However, as the series does not deliver much information beyond what is already contained in the output series, we stick to trivariate model.

We will refer to this as the hours gap.

Atkeson and Ohanian (2001) show that United States inflation is well described by a random walk.

are correlated with the hours gap,

$$g_t = g_t^* + \omega(L)h_{t-1}^c + \varepsilon_t^g, \tag{2.4}$$

$$g_t^* = \gamma + g_{t-1}^* + \eta_t^g, \tag{2.5}$$

where ε_t^g and η_t^g are Gaussian mean zero white noise error terms and the lag polynomial is defined as $\omega(L) = \omega_0 + \omega_1 L + ... + \omega_q L^q$.

In equation (2.4) the output gap reacts to previous periods' hours gaps. However, the direction of causality may also be the other way around. Indeed, most studies relating the unemployment rate to output assume a lagged reaction of the unemployment gap to the output gap. As Knotek (2007) has shown for the United States the correlation between current unemployment and past output has gained importance over time. However, since the labor market series used in our model incorporates both employment (in persons) and hours worked, the dynamic relationship is not clear-cut. In fact, hours worked are considered a leading indicator. For instance, Kydland and Prescott (1990) and Cooley and Prescott (1995) investigate the United States business cycle and find that while employment lags output, hours worked slightly lead. Fiorito and Kollintzas (1994) use data for the G7 and show that for most countries employment is a lagged and hours is a leading or coincident indicator.

To close the model we have to specify the stochastic law of motion for the natural rate and the hours gap. The latter is specified as a stationary autoregressive process. Denoting the natural rate by h_t^* and cyclical hours by h_t^c, the dynamics for employment can be written as

$$h_t = h_t^* + h_t^c, \tag{2.6}$$

$$h_t^* = \mu + h_{t-1}^* + \eta_t^h, \tag{2.7}$$

$$h_t^c = \phi(L)h_{t-1}^c + \nu_t, \tag{2.8}$$

where η_t^h and ν_t are Gaussian mean zero white noise error terms and the lag polynomial is defined as $\phi(L) = \phi_0 + \phi_1 L + ... + \phi_q L^q$.

This corresponds to the Okun's law approach used in the NAIRU literature in which the cyclical in output is linked to the unemployment gap (see e.g. Fabiani and Mestre, 2004).

We also estimated the model with a standard specification, i.e. where the labor market gap reacts to lagged values of the output gap and find the results are nearly identical. The results of this exercise are not reported but are available upon request.

2.5 Data and estimation methodology

2.5.1 Data

We use quarterly data from 1964:Q1 to 2013:Q4 for the United States and from 1970:Q1 to 2013:Q4 for Germany. The inflation series is obtained by taking first differences of the log of the seasonally adjusted Consumer Price Index (CPI) at an annualized rate. Output is the log of real gross domestic product at constant local prices. The hours series is calculated according to equation (2.1). Details on the data sources are given in 2.A.

2.5.2 Estimation methodology

The outlined model can in principle be estimated by using the Kalman filter and maximum likelihood (ML) technique. Instead of using ML, we estimate the model using Gibbs sampling. For our purposes, the Gibbs sampler has a number of advantages over standard ML estimation. First, it directly provides an entire distribution of all parameters and states allowing us to analyze the uncertainty around our state estimates. Second, by specifying prior distributions for the variance parameters which are strictly positive, we avoid the so-called pile-up problem. Third, by using prior information we down-weight the likelihood function in regions of the parameter space that are inconsistent with out-of-sample information and/or in which the structural model is not interpretable.

The Gibbs sampler splits the model parameters and unobserved components into blocks that are conditional on each other in order to draw sequentially from the conditional distribution. After a sufficiently large number of iterations, the algorithm produces draws from the joint posterior distribution of all parameters and states. Thus, the credible bands around the states combine filter and parameter uncertainty.

Gibbs Sampling

Denote the model parameters by $\psi = \left\{\phi, \theta, \omega, \sigma_{\varepsilon^\pi}, \sigma_{\varepsilon^g}, \sigma_{\eta^h}, \sigma_{\eta^\pi}, \sigma_{\eta^g}, \sigma_\nu, \gamma, \mu\right\}$. The posterior density of interest is $p(h^c, h^*, \pi^*, g^*, \psi | h, \pi, g)$. Given an arbitrary set of starting values $(h^c_{\{0\}}, h^*_{\{0\}}, \pi^*_{\{0\}}, g^*_{\{0\}}, \psi_{\{0\}})$, the algorithm consists of the following blocks:

1. Sample the unobserved components $(h^c_{\{1\}}, h^*_{\{1\}}, \pi^*_{\{1\}}, g^*_{\{1\}})$ from $p(h^c, h^*, \pi^*, g^* | h, \pi, g, \psi_{\{0\}})$ according to observation equations (2), (4) and (6) and state equations (3), (5), (7) and

In the classical ML approach filter and parameter uncertainty can be calculated, but it is not obvious how to combine them.

See Kim and Kim (2013) for simulation based evidence that support Bayesian estimation for UC models over ML.

(8).

2. Sample the parameters ψ from $p(\psi_{\{1\}}|h, \pi, g, h^c_{\{1\}}, h^*_{\{1\}}, \pi^*_{\{1\}}, g^*_{\{1\}})$.

Sampling from these blocks can then be iterated J times and, after a sufficiently long burn-in period B, the sequence of draws $(B+1, ..., J)$ approximates a sample from the virtual posterior distribution. Details on the exact implementation can be found in 2.B.

Priors

Normal priors are used for all slope parameters, while inverted gamma-2 distributions are used for all variance parameters. As stated above, the main motivation for setting these priors is to down-weight the likelihood function in regions of the parameter space that are inconsistent with out-of-sample information and/or in which the structural model is not interpretable. Previous estimates as well as economic theory give us an idea about the approximate value of the model's parameters. However, using previous studies to set priors should be done with caution, particularly if these studies consider the same time period. We therefore use previous estimates only as a rough indication for the prior means but choose the prior variance fairly loose. Prior beliefs about the parameters and the strength of these beliefs are given in Tables 2.2-2.5. We express the strength of the priors as a fraction of the sample precision. For many parameters the strength is set to 0.0001, implying that prior information is proportionate to 0.01% of the in-sample-information. We apply slightly more informative priors only for the standard deviations of trend and cycle components, which are proportionate to 10% of the sample information. For example, the prior mean of 0.7 for $\sigma_{\eta g}$ implies that 95% of all shocks to potential output lie between -1.4% and +1.4% per quarter. The prior belief about the AR-coefficients imply a hump-shaped pattern of the labor market gap. Moreover, we assume positive signs on the hours-output relation and the slope of the Phillips curve, implying that above-trend inflation is associated with a positive output gap and a positive inflation gap.

When estimating the model, a lag order q of two is chosen, which is in line with the literature (see e.g. Laubach, 2001; Domenech and Gomez, 2006; Basistha and Startz, 2008) and sufficient for quarterly data.

Alternatively, we estimated the model with more than two lags. As additional lags have not found to be significant and the results were similar, we only report results for the AR(2) model.

Table 2.2: Parameter estimates for the United States: Hours-based model

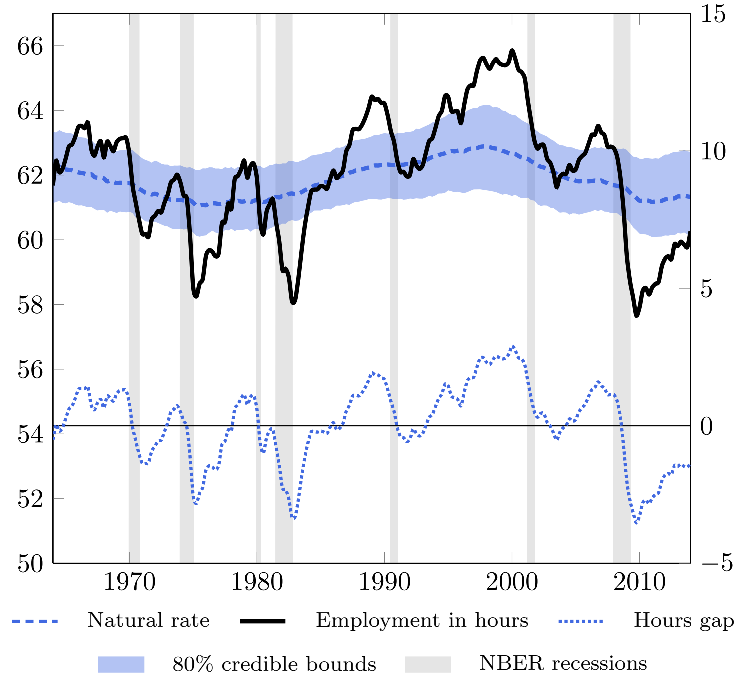
		Prior		Posterior		
	Parameter	Belief	Strength	Mean	5%	95%
Employment	ϕ_1	1.4	0.0001	1.322	1.154	1.447
	ϕ_2	-0.6	0.0001	-0.411	-0.489	-0.318
	σ_ν	0.7	0.1	0.416	0.377	0.460
	$\sigma_{\eta h}$	0.1	0.1	0.128	0.091	0.184
Inflation	$\Sigma\theta_i$	1.0	0.0001	0.951	-0.160	2.024
	$\sigma_{\eta\pi}$	0.25	0.1	0.541	0.388	0.729
	$\sigma_{\varepsilon\pi}$	1.0	0.0001	1.748	1.576	1.947
Output	$\Sigma\omega_i$	1.5	0.0001	1.538	0.717	2.254
	$\sigma_{\eta g}$	0.7	0.1	0.651	0.547	0.774
	$\sigma_{\varepsilon g}$	0.1	0.0001	0.815	0.684	0.961
	γ	0.7	0.0001	0.745	0.665	0.822

2.6 Results

2.6.1 United States

Table 2.2 shows the posterior mean for all model parameters along with the 5^{th} and 95^{th} percentile of the posterior distributions. The AR coefficients imply a highly persistent cyclical component as the posterior of the sum is close to one. The estimated standard deviation for cyclical shocks is substantially larger than for permanent shocks. The slope of the Phillips curve is 0.95 which states that a labor market gap of one percent leads to a deviation of actual from trend inflation of roughly the same size. However, the 90% credible interval also contains the value of zero. The output-employment relation is somewhat stronger with a mean estimate of 1.5. The drift in trend output, which represents the average growth rate of potential GDP, is very close to what is commonly found in the literature and the credible interval is quite narrow. Figure 2.4 plots the hours series, the mean estimate of the natural rate (left scale) and the corresponding hours gap (right scale). Shaded areas indicate recessions of the U.S. economy. The natural rate exhibits moderate long-run swings with two turning points. The rather steep decline within the first years of the sample can be explained by a large-scale withdrawal from prime-age men from the U.S. labor force. This withdrawal is associated with a rapid expansion of the social security programs (see Parsons, 1980). The first turning-point occurs in the late 1970s when the initial downward trend reversed and the natural rate started to increase for about two decades. The continuing decrease in male labor force participation

Figure 2.4: Natural rate and hours gap: United States



was then offset by a growing participation of women, who increased their hours devoted to market work rapidly until the late 1980s. This expansions slowed down in the 1990s, as described by Juhn and Potter (2006). Despite important shifts in hours worked between different labor market subgroups, aggregate hours appear relatively stable over the sample (see McGrattan and Rogerson, 2004). The second turning point lies in the late 1990s when the labor force participation of women started to stagnate and failed to offset the ongoing decline of male participation. The impact of the Great Recession on the natural rate is rather moderate, since the model ascribes most of the movements in the original hours series to the cyclical component. Recently, the natural rate of hours is at roughly the same level as in the 1970s. The estimated labor market gap of 3.5% during the trough of the last recession is as large as never before. We compare our alternative measure of the labor market gap to a conventionally estimated unemployment gap. In order to estimate the unemployment gap we replaced the hours series with the U.S. civilian unemployment rate and estimated the model as outlined in Section 2.4. Parameter estimates for the unemployment-based model are given in Table 2.3. A visual comparison of the gap measures is given in Figure 2.5. Both gap series pick

Alternatively, we used NAIRU series from the CBO or OECD to calculate the unemployment gap. This leads to even more pronounced differences between the hours and unemployment gap.

For a better comparison the hours gap has been multiplied by minus one. Thus, positive gaps imply that the labor market performs below its trend level.

Table 2.3: Parameter estimates for the United States: Unemployment-based model

		Prior		Posterior		
	Parameter	Belief	Strength	Mean	5%	95%
Unemployment	ϕ_1	1.4	0.0001	1.437	1.273	1.570
	ϕ_2	-0.6	0.0001	-0.528	-0.612	-0.429
	σ_ν	0.7	0.1	0.355	0.321	0.393
	$\sigma_{\eta u}$	0.1	0.1	0.105	0.081	0.142
Inflation	$\Sigma\theta_i$	-1.5	0.0001	-1.028	-2.247	0.139
	$\sigma_{\eta\pi}$	0.25	0.1	0.554	0.409	0.744
	$\sigma_{\varepsilon\pi}$	1.0	0.0001	1.740	1.574	1.923
Output	$\Sigma\omega_i$	-1.5	0.0001	-1.595	-2.443	-0.729
	$\sigma_{\eta g}$	0.7	0.1	0.628	0.529	0.753
	$\sigma_{\varepsilon g}$	0.1	0.0001	0.787	0.663	0.934
	γ	0.7	0.0001	0.736	0.664	0.808

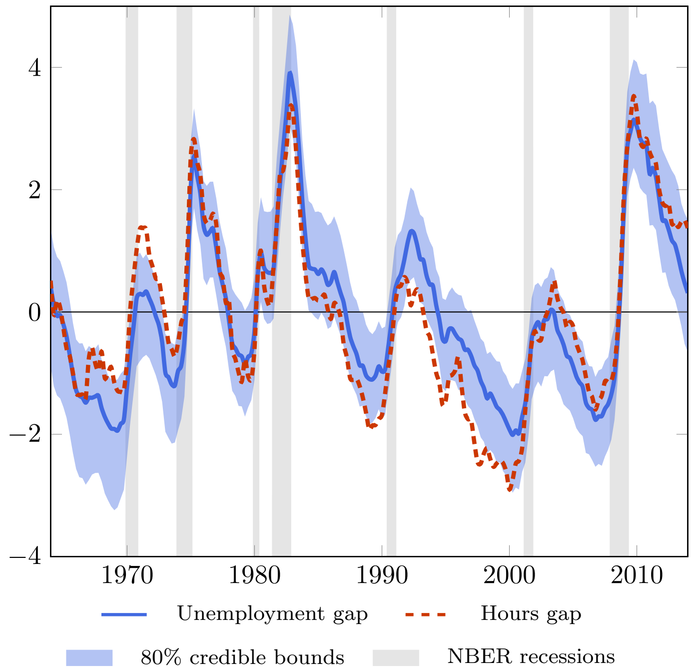
up the same business cycle turning points and match with the NBER recession dates. While for some periods the gaps are almost identical, the hours gap moves outside the 80% credible bound of the unemployment gap in other periods. These differences are large for the recession in the early 1970s as well as for most of the 1990s. The hours gap implies that the deviation from the long-run labor market level in the 1990s was larger than standard unemployment measures suggested. Regarding the Great Recession, both gap measures peak at about the same level but diverge during the most recent observations. While the unemployment gap is almost zero by the end of 2013, the hours gap is still as big as 1.5%. As differences in the gaps can lead to substantially different policy choices, one would like to have a statistical measure for these differences. Thus, we compute the difference of the unemployment and the hours gap in each iteration of the Gibbs sampler, resulting in an empirical distribution of the gap difference at each quarter. In times where the intensive and the extensive margin of the labor market become more important we find that the 90% credible interval does not include zero.

However, the fact that the credible interval contains zero in many other periods should not be interpreted as evidence that the gaps are not different from each other. In fact, while non-overlapping confidence bounds insure significant differences, the reverse is not true as shown by Schenker and Gentleman (2001).

Central banks might in general be interested in the complete distribution of relevant variables, but they will ultimately base their policy choice on some type of average value,

This is the case for the time around 1992 and 1997.

Figure 2.5: Hours gap and unemployment gap: United States



i.e. the mean or mode of the distribution. Thus, differences in the mean estimate can have different policy implications. We demonstrate the policy implications of the two different gap estimates via a Taylor-rule exercise in Section 2.7.

2.6.2 Germany

The posterior distributions of the model parameter for Germany are given in Table 2.4. In contrast to the United States, permanent shocks to the labor market are larger, reflecting more rigid labor market institutions. The slope of the Phillips curve is similar to the United States, but again the credible intervals are large. The estimated production function coefficients, which display the correlation between the output and the labor market gap, sum up to 2.5 on average with the estimate being significantly different from zero. One notable feature of the German economy is the lower output drift, which implies a yearly growth rate of potential GDP of 2.2% as compared to 3% for the United States. Figure 2.6 shows the German hours series, the mean estimate of the natural rate of hours (left scale) and the corresponding hours gap (right scale). The natural rate follows the three decade-long decline in total hours that ended in the mid-2000s. Since then, trend hours have picked up again and are now back to the pre-1991 recession-level. While most of the cyclical swings in total hours appear rather persistent, the downturn in hours during the latest recession stands out as severe but

Table 2.4: Parameter estimates for Germany: Hours-based model

		Prior		Posterior		
	Parameter	Belief	Strength	Mean	5%	95%
Employment	ϕ_1	1.4	0.0001	0.747	0.518	0.960
	ϕ_2	-0.6	0.0001	0.123	-0.037	0.291
	μ	0	0.0001	-0.083	-0.120	-0.047
	σ_ν	0.7	0.1	0.409	0.358	0.465
	$\sigma_{\eta h}$	0.1	0.1	0.266	0.187	0.344
Inflation	$\Sigma\theta_i$	1.0	0.0001	1.176	0.178	2.212
	$\sigma_{\eta\pi}$	0.3	0.1	0.372	0.273	0.488
	$\sigma_{\varepsilon\pi}$	1.0	0.0001	1.165	1.047	1.296
Output	$\Sigma\omega_i$	1.5	0.0001	2.451	1.074	3.948
	$\sigma_{\eta g}$	0.7	0.1	0.889	0.699	1.089
	$\sigma_{\varepsilon g}$	1.0	0.0001	1.229	1.039	1.451
	γ	0.5	0.0001	0.541	0.427	0.656

short-lived. Almost all of the dynamics during this recession are explained by the transitory component, while the natural rate continues to increase at pre-recession pace. In order to compare the hours gap to the unemployment gap we re-run the model and substitute hours by the rate of unemployment. Parameter estimates for the unemployment-based model are given in Table 2.5. The resulting gap is compared to the hours gap in Figure 2.7. During the first two downturns and the following recoveries the unemployment and hours gap evolve symmetrically, implying that cyclical changes in hours worked and labor force participation had less of a role to play. The picture is somewhat different for the last three recessions. During the cycles in the 1990s and early 2000s the unemployment gap is larger (by as much as 2 percentage points) than the hours gap. Conversely, the hours gap is larger during the most recent recession, which is the result of nation-wide short-time work arrangements. Only looking at the unemployment rate as the relevant labor market indicator would not provide a complete picture of the state of the German labor market. This becomes even more clear if one compares the magnitude of the two gap measures over time. The cyclical increase in the unemployment rate during the latest recession is small compared to former periods. The recessions in the 1980s, 1990s, and early 2000s led to much larger shifts of the unemployment rate away from its long-run trend. In contrast, the hours gap reacts much stronger. This finding does not come as a surprise. As outlined in Section 2.2 unemployment stayed almost constant during the Great Recession in Germany, but aggregate hours worked declined.

Figure 2.6: Natural rate and hours gap: Germany

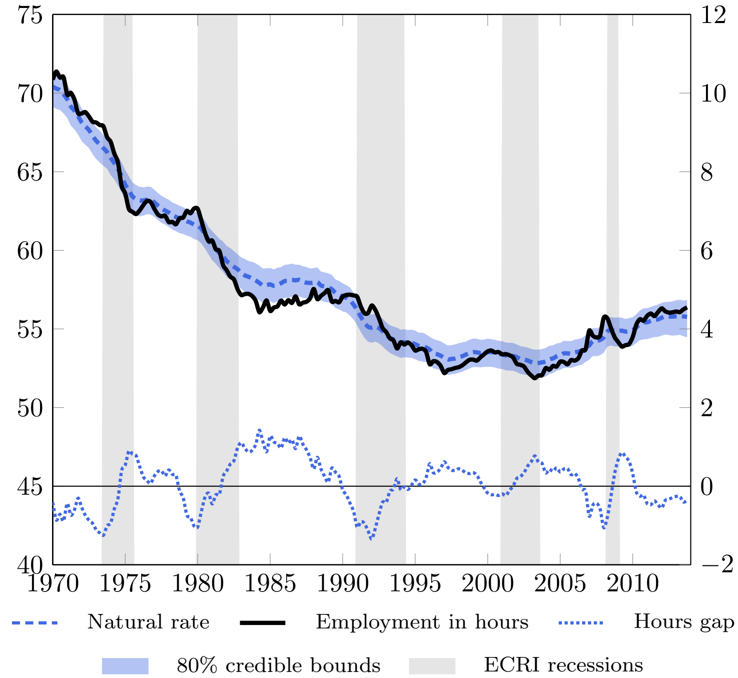
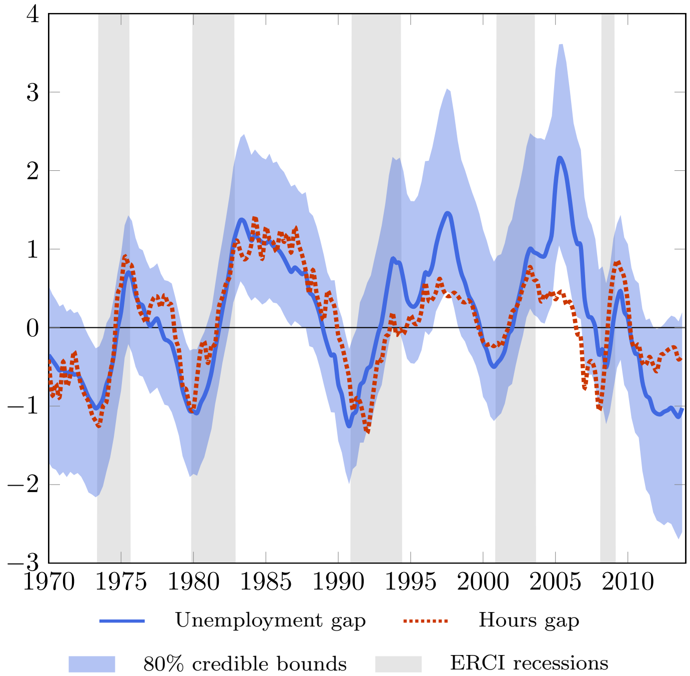


Table 2.5: Parameter estimates for Germany: Unemployment-based model

		Prior		Posterior		
	Parameter	Belief	Strength	Mean	5%	95%
Unemployment	ϕ_1	1.4	0.0001	0.971	0.657	1.258
	ϕ_2	-0.6	0.0001	-0.083	-0.338	0.180
	μ	0	0.0001	0.032	0.006	0.058
	σ_ν	0.7	0.1	0.349	0.308	0.395
	$\sigma_{\eta u}$	0.1	0.1	0.182	0.118	0.271
Inflation	$\Sigma\theta_i$	-1.5	0.0001	-0.713	-2.140	0.675
	$\sigma_{\eta\pi}$	0.3	0.1	0.391	0.285	0.525
	$\sigma_{\varepsilon\pi}$	1.0	0.0001	1.251	1.116	1.402
Output	$\Sigma\omega_i$	-1.5	0.0001	-1.731	-3.806	0.319
	$\sigma_{\eta g}$	0.7	0.1	0.962	0.770	1.177
	$\sigma_{\varepsilon g}$	1.0	0.0001	1.395	1.176	1.650
	γ	0.5	0.0001	0.532	0.405	0.655

Figure 2.7: Hours gap and unemployment gap: Germany



2.7 An hours-based Taylor rule

In a seminal paper, Taylor (1993) argued that changes in the federal funds rate (FFR) can be explained by a simple policy rule. According to this rule, central bankers set the nominal interest rate as a reaction to deviations of inflation from the inflation target and deviations of output from potential output. Given the assumption of a constant inflation target and a constant equilibrium real-interest rate, such a rule can be estimated as a regression of the FFR on a constant, some measure of inflation and a measure of the real activity gap. The latter variable is often proxied by the unemployment gap (see e.g. Mankiw, 2001; Ball and Moffitt, 2001; Lansing, 2006, 2008; Rudebusch, 2009, 2010).

In order to demonstrate policy implications of the hours gap, we estimate a standard Taylor rule relationship for the United States and analyze whether the hours gap can provide additional information to explain central bank behavior. The hours-based Taylor rule is compared to an unemployment-based model using Bayes factor. The Taylor rule in a regression equation takes the following form:

$$r_t = \beta_0 + \beta_1 \pi_t^{PCE} + \beta_2 gap_t + \varepsilon_t, \tag{2.9}$$

where r_t is the United States FFR, π_t^{PCE} is inflation measured by Core Personal Consumption

Table 2.6: OLS Taylor rule estimates

		Hours gap	Unemployment gap
Constant	β_0	2.415^{***}	3.269^{***}
Inflation	β_1	1.800^{***}	1.805^{***}
Gap	β_2	1.676^{***}	-1.428^{***}
$\overline{R}^2$		0.88	0.65
Log-likelihood		-93.31	-138.65
		QA-breakpoint test on β_2	
Date		1994:Q2	2001:Q2
MaxF		4.41	47.66^{***}
ExpF		1.07	20.08^{***}
AveF		1.69	13.30^{***}

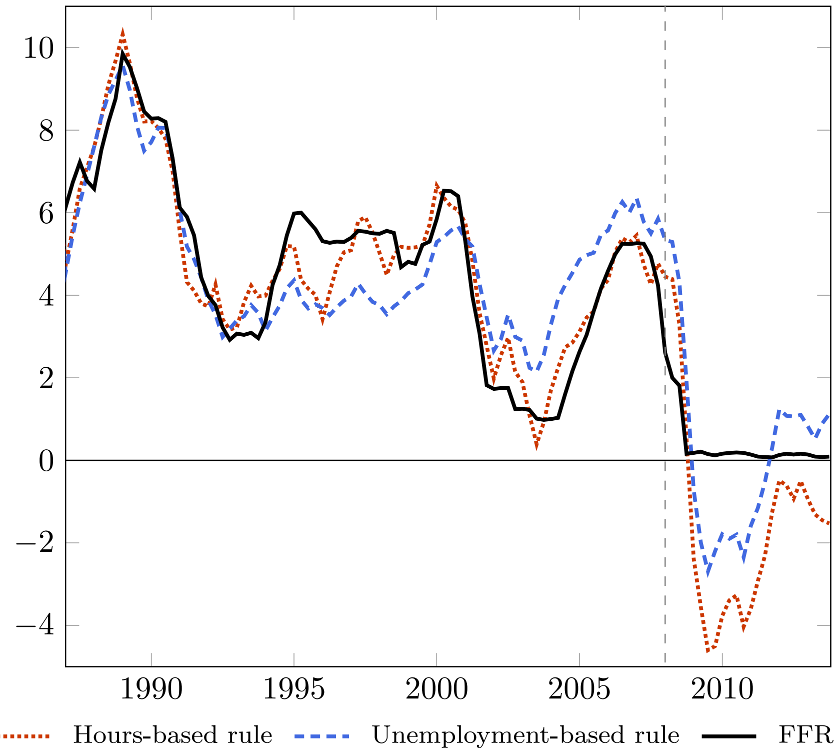
$***$ denote significance at the 1% level.

Expenditure, gap_t is either the hours or unemployment gap estimated in Section 2.6.1, and ε_t is an i.i.d. error term. We run the regression using OLS for data from 1987:Q1 to 2007:Q4, i.e. we choose the same starting date as Taylor, but extend the sample just until the start of the Great Recession. The estimated coefficients are given in Table 2.6. All coefficients are statistically significant and have the expected sign, i.e. the FED raises the target rate when the labor market is above its natural level. This is the case for negative unemployment gaps or positive hours gaps. The estimated coefficients for the unemployment based rule are similar to values typically found in the literature. For example, Rudebusch (2009) estimates an inflation coefficient of 1.3 and an unemployment gap coefficient of -2.0 while Ball and Moffitt (2001) report the inflation coefficient to range between 1.3 and 2.0 and the gap coefficient between -1.7 and -2.0. Importantly, the hours based Taylor rule has considerable higher explanatory power regarding the FED policy with an adjusted R^2 of 0.88 compared to 0.65 when the unemployment gap is used.

Finally, we test whether the hours gap outperforms the unemployment gap in the Taylor rule. The two models are compared using Bayes factor. Specifically, we denote the hours based Taylor rule as model M_1 and the unemployment gap based Taylor rule as model M_2. We follow Kass and Raftery (1995) and use the Schwarz criterion to approximate the (log)

We do not include more recent observations as we would have to deal with the zero lower bound in our analysis. Moreover, we estimated the same equation with a smaller sample starting in 1995, which led to very similar results.

Figure 2.8: Taylor rule estimates



Bayes factor given by

$$\log B_{12} \approx ll(D|M_1) - ll(D|M_2) - \frac{1}{2}(d_1 - d_2)\log(n), \tag{2.10}$$

where ll denotes the maximum of the log-likelihood function, d the number of parameters, and n the sample size. When we evaluate the hours-based model, M_1, against the unemployment-based model, M_2, we find $2 \times \log B_{12} = 90.7$. According to Kass and Raftery (1995), any value greater than 10 represents strong evidence against the alternative model. Thus, the Taylor rule with the hours gap is the model favored by the data.

The better fit of the hours based rule is visible in Figure 2.8, which shows the FFR along with the fitted series from both Taylor rules. The vertical dashed line marks the end of the sample, hence observations right from this line shed light on the policy implications of both rules during the recent crisis and recovery. We emphasize that the hours gap based rule performs superior in explaining monetary policy during most of the 1990s and 2000s. The

Bayes factor is a ratio of marginal likelihoods which are often difficult to calculate. In many cases, the marginal likelihood may not have a closed form solution.

In order to check the robustness of this result, we estimated the Taylor rule using the Gibbs sampler with uninformative priors and calculated Bayes factor directly via the marginal likelihood as described in Chib (1995). We find that $2 \times \log B_{12} \approx 93$, confirming the results based on the Schwarz criterion.

unemployment gap based rule leads to substantially lower rates between 1994 and 2000 and higher rates for the period 2001 to 2008. This is in fact not new to the literature: Rudebusch (2006) estimates a simple Taylor rule model for 1987:Q4-2004:Q4 and finds similar deviations between the fitted and the actual series. He corrects for this fact by adding lagged endogenous variables. Lansing (2006) presents a Taylor rule with fixed coefficients which also leads to lower rates during the 1990s and higher rates the early 2000s. When the coefficients are estimated using OLS, Lansing (2008) finds that the FFR is consistently above the estimated rule from 1995 to 1998 and below the rule from 2003 to 2008. Adding stock market variables leads to a better fitting rule. Similar findings on the classic Taylor rule are presented by Orphanides (2003) as well as Rudebusch (2009).

Large deviations from the policy rule could also point to non-linearities in the form of parameter instability. Lee, Morley, and Shields (2015) estimate a "Meta-Taylor rule" via Bayesian model averaging techniques, where the weights on inflation and the output gap are allowed to change over time. The authors report a doubling of the gap coefficient for the mid-Greenspan era that started in 1994. This is seen as the result of the central bank's desire to avoid overheating of the economy. However, parameter stability can be restored by using the hours gap as a slack measure. We perform a Quandt-Andrews breakpoint test on the gap coefficient for one single break at an unknown point in time. Results are given in the lower half of Table 2.8. The null hypothesis of no break is rejected for the unemployment gap based rule. Results point to a significant change in the reaction of the central bank to unemployment deviations in 2001. For the hours gap based rule the null cannot be rejected at the usual significance levels.

A Taylor rule with the hours gap as the relevant measure of real activity does not suffer from parameter instability as the unemployment gap based rule does. Measuring labor market slack not only along the employment margin, but along the intensive margin helps to explain central bank behavior even on the basis on a simple two-variable Taylor rule.

Additional evidence in favor of the hours gap as an alternative indicator is given by the performance of the two rules during the recent crisis and recovery. Both series fail terribly in explaining the FFR during the recession, as the FED hit the zero lower bound (ZLB). However, since the end of 2011 the unemployment gap based rule would have called for

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In principal, these deviations could also be driven by the fact that our model for estimating unemployment gap is misspecified. Therefore, we ran an alternative Taylor regression and replaced our model-based unemployment gap with the official CBO gap. However, the fitted series are almost identical and results are not reported. We conclude that the deviations of the FFR from the unemployment-based rule are driven by factors not captured by the unemployment gap.

A Bai-Perron test for an unknown number of breaks cannot reject the null of no break for the hours-based rule, but confirms a significant breaks for the unemployment-based rule in 2001.

monetary tightening while the hours gap based rule still favors accommodative policies. In 2013:Q4 both rules differ by about 2.5 percentage points, which is tremendous in central bank terms. We notice that this number should be interpreted with caution in the presence of the ZLB, as important non-linearities may arise. However, the fact that both rules differ in the sign of the desired policy rate is already revealing. We take the Taylor rule exercise as evidence that central banks in fact consider more measures than the unemployment rate when evaluating the state of the labor market. As James Bullard (2013), President and CEO of the Federal Reserve Bank of St. Louis, explained in January 2013:

> *"Although some focus on the unemployment rate, it is only one aspect of the labor market. By itself, this indicator is an incomplete measure of overall labor-market health.* [...] *Along with payroll employment and the unemployment rate, the FOMC monitors the labor force participation rate, which has been a very important factor in recent years.* [...] *Changing practices in labor markets could bring more people into part-time and temporary work; from that point of view, hours might be a better indicator of the state of the labor market than simply counting the number of jobs."*

2.8 Conclusion

This paper argues that due to changes in the adjustment process to shocks of key labor market variables, the unemployment rate does not capture all important dimensions of the aggregated labor market. We propose the hours rate as an alternative indicator which takes into account adjustments along the intensive margin and is robust to changes along the extensive margin. The natural rate of hours and the corresponding hours gap is estimated from a multivariate unobserved components model for the United States and Germany. We utilize additional information contained in inflation and output in order to reduce the uncertainty around the natural rate estimate. A Phillips curve is used to derive the amount of hours worked at which inflation stabilizes. The hours gap is linked to the output gap via a reduced form production function equation.

For both countries the natural rate of hours evolves smoothly and picks up long-run trends in employment. The estimated labor market gaps are very persistent and follow the usual business cycle turning points. Results for Germany point to a strong impact of the recent crisis on the labor market. The widespread use of short-time work arrangements, concealed by a relatively stable unemployment rate, is picked up as a cyclical drop in hours. For the United States, we find that the labor market gap due to the crisis is severe, although our model assigns most of it to cyclical and not structural factors.

We demonstrate the policy implications of our findings via a Taylor rule estimation. A policy rule based on the hours gap as the relevant labor market indicator outperforms an unemployment gap based rule in explaining the FED. Bayesian model comparison favors the hours gap based model. Depending on whether the unemployment or the hours gap is taken into account, policy rules give very different advice on whether to end expansionary monetary policy in the United States.

Bibliography

Atkeson, A., and L. E. Ohanian (2001): "Are Phillips Curves Useful for Forecasting Inflation?," *Federal Reserve Bank of Minneapolis Quarterly Review*, 25(1), 2–11.

Ball, L., and R. Moffitt (2001): "Productivity Growth and the Phillips Curve," NBER Working Papers 8421, National Bureau of Economic Research, Inc.

Basistha, A., and R. Startz (2008): "Measuring the NAIRU with Reduced Uncertainty: A Multiple-Indicator Common-Cycle Approach," *The Review of Economics and Statistics*, 90(4), 805–811.

Bauwens, L., M. Lubrano, and J.-F. Richard (1999): *Bayesian Inference in Dynamic Econometric Models*. Oxford University Press, Oxford.

Berger, T. (2011): "Estimating Europe's Natural Rates," *Empirical Economics*, 40(2), 521–536.

Berger, T., and F. Heylen (2011): "Differences in Hours Worked in the OECD: Institutions or Fiscal Policies?," *Journal of Money, Credit and Banking*, 43(7), 1333–1369.

Bernanke, B. (2012): "Press Conference of the Chairman of the Federal Reserve, January 25, 2012," http://www.federalreserve.gov/mediacenter/files/FOMCpresconf20120125.pdf.

Bullard, J. (2013): "The Fed's Latest Balance-Sheet Policy: What Constitutes Substantial Labor-Market Improvement," *The Regional Economist*, (Jan).

Carter, C. K., and R. Kohn (1996): "Markov Chain Monte Carlo in Conditionally Gaussian State Space Models," *Biometrika*, 83(3), 589–601.

Chib, S. (1995): "Marginal Likelihood from the Gibbs Output," *Journal of the American Statistical Association*, 90(432), 1313–1321.

Cogley, T., and A. M. Sbordone (2008): "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review*, 98(5), 2101–2126.

Cooley, T., and E. Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, pp. 1–38. Princeton University Press, Princeton, NJ.

Davig, T. A., and J. Mustre-del Rio (2013): "The Shadow Labor Supply and its Implications for the Unemployment Rate," *Economic Review*, (Q III), 5–29.

Dhont, T., and F. Heylen (2008): "Why Do Europeans Work (Much) Less? It Is Taxes And Government Spending," *Economic Inquiry*, 46(2), 197–207.

Domenech, R., and V. Gomez (2006): "Estimating Potential Output, Core Inflation, and the NAIRU as Latent Variables," *Journal of Business & Economic Statistics*, 24, 354–365.

Fabiani, S., and R. Mestre (2004): "A System Approach for Measuring the Euro Area NAIRU," *Empirical Economics*, 29(2), 311–341.

Fiorito, R., and T. Kollintzas (1994): "Stylized Facts of Business Cycles in the G7 from a Real Business Cycles Perspective," *European Economic Review*, 38(2), 235–269.

Friedman, M. (1968): "The Role of Monetary Policy," *American Economic Review*, 58(1), 1–17.

Fujita, S. (2014): "On the Causes of Declines in the Labor Force Participation Rate," *FRB Philadelphia Research Rap Special Report*.

Goodfriend, M., and R. G. King (2009): "The Great Inflation Drift," NBER Working Papers 14862, National Bureau of Economic Research, Inc.

Juhn, C., and S. Potter (2006): "Changes in Labor Force Participation in the United States," *The Journal of Economic Perspectives*, 20(3), 27–46.

Kass, R. E., and A. E. Raftery (1995): "Bayes Factors," *Journal of the American Statistical Association*, 90(430), 773–795.

Kim, C.-J., and J. Kim (2013): "The 'Pile-up Problem' in Trend-Cycle Decomposition of Real GDP: Classical and Bayesian Perspectives," MPRA Paper 51118, University Library of Munich, Germany.

Knotek, E. S. (2007): "How Useful is Okun's Law?," Discussion Paper Q IV, Federal Reserve Bank of Kansas City.

Kydland, F. E., and E. C. Prescott (1990): "Business Cycles: Real Facts and a Monetary Myth," *Quarterly Review*, (Spr), 3–18.

Lansing, K. J. (2006): "Will Moderating Growth Reduce Inflation?," *FRBSF Economic Letter*.

——— (2008): "Monetary Policy and Asset Prices," *FRBSF Economic Letter*.

Laubach, T. (2001): "Measuring the Nairu: Evidence from Seven Economies," *The Review of Economics and Statistics*, 83(2), 218–231.

Lee, K., J. Morley, and K. Shields (2015): "The Meta Taylor Rule," *Journal of Money, Credit and Banking*, 47(1), 73–98.

Mankiw, N. G. (2001): "U.S. Monetary Policy During the 1990s," NBER Working Papers 8471, National Bureau of Economic Research, Inc.

McGrattan, E. R., and R. Rogerson (2004): "Changes in Hours Worked, 1950-2000," *Federal Reserve Bank of Minneapolis Quarterly Review*, 28(1), 14–33.

Morley, J., J. M. Piger, and R. H. Rasche (2015): "Inflation in the G7: Mind the Gap(s)?," *Macroeconomic Dynamics*, 19(04), 883–912.

Nelson, C. R., and J. Lee (2007): "Expectation Horizon and the Phillips Curve: The Solution to an Empirical Puzzle," *Journal of Applied Econometrics*, 22(1), 161–178.

Ohanian, L. E., and A. Raffo (2012): "Aggregate Hours Worked in OECD Countries: New Measurement and Implications for Business Cycles," *Journal of Monetary Economics*, 59(1), 40–56.

Orphanides, A. (2003): "Historical Monetary Policy Analysis and the Taylor Rule," *Journal of Monetary Economics*, 50(5), 983–1022.

Parsons, D. O. (1980): "The Decline in Male Labor Force Participation," *Journal of Political Economy*, 88(1), 117–134.

Phelps, E. S. (1968): "Money-Wage Dynamics and Labor-Market Equilibrium," *The Journal of Political Economy*, 76, 678–711.

Piger, J. M., and R. H. Rasche (2008): "Inflation: Do Expectations Trump the Gap?," *International Journal of Central Banking*, 4(4), 85–116.

Ravikumar, B., and L. Shao (2014): “Accounting for Discouraged Workers in the Unemployment Rate,” *Economic Synopses*, (9), 1–2.

Romero, J. (2012): “Where Have All the Workers Gone? Why are more people leaving the labor force, and what are they doing?,” *Econ Focus*, (2Q/3Q), 12–16.

Rudebusch, G. D. (2006): “Monetary Policy Inertia: Fact or Fiction?,” *International Journal of Central Banking*, 2(4), 85–135.

——— (2009): “The Fed’s Monetary Policy Response to the Current Crisis,” *FRBSF Economic Letter*.

——— (2010): “The Fed’s Exit Strategy for Monetary Policy,” *FRBSF Economic Letter*.

Schenker, N., and J. F. Gentleman (2001): “On Judging the Significance of Differences by Examining the Overlap Between Confidence Intervals,” *The American Statistician*, 55(3), 182–186.

Shephard, N. (1994): “Partial Non-Gaussian State Space,” *Biometrika*, 81(1), 115–131.

Taylor, J. B. (1993): “Discretion Versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.

Zandweghe, W. V. (2012): “Interpreting the Recent Decline in Labor Force Participation,” *Economic Review*, (Q I), 5–34.

Appendices

2.A Data description

All data are on quarterly basis. In case seasonal adjusted series were not available, we followed the X-12-ARIMA approach.

- Hours worked: The United States data on average hours worked are taken from the Bureau of Labor Statistics, BLS (via Datastream, code: $USHKIP..O$). For Germany we make use of a dataset provided by Ohanian and Raffo (2012) and add more recent data provided by the Federal Statistical Office (via Datastream, code: $BDHOURPBQ$).

- Employment: Data on United States civilian employment are taken from the BLS (via Datastream, code: $USEMPTOTO$). Data on German employment are collected from the German Bundesbank (via Datastream, code: $BDUSBA14O$).

- Population at working age: Data on United States civilian non-institutional population are taken from the BLS (via Datastream, code: $USCV....P$). For Germany we use population data provided by Ohanian and Raffo (2012) and add more recent data from OECD Main Economic Indicators (via Datastream, code: $BDQLFT32P$).

- Inflation: We use the Consumer Price Index from the BLS (via FRED, code: *CPIAUCSL*). German data are taken from the OECD's Main Economic Indicators (via Datastream, code: *BDQCP009F*).

- Gross Domestic Product: Real GDP data for the United States are taken from the U.S. Bureau of Economic Analysis (via FRED, code: *GDPC1*). In case of Germany, data are taken from the IMF International Financial Statistics (via Datastream, code: *BDI99BVRG*)

- Unemployment rate: The United States civilian unemployment rates are taken from the BLS (via Datastream, code: *USUN%TOTQ*). The German unemployment rates are taken from the OECD Main Economic Indicators (via Datastream, code: *BDQLRT28Q*).

- Recession dates: For the United States the peak and trough dates are defined by the NBER Business Cycle Dating Committee. For Germany we use dates from the Economic Cycle Research Institute (ECRI).

2.B Gibbs sampling algorithm

In this paper we follow a Bayesian approach and apply a Gibbs sampling procedure to estimate our model. This appendix gives details of the algorithm, which relies on step-wise sampling of the unobserved states (h^*, π^*, y^*, h^c) and the model's hyperparameters $(\phi, \theta, \omega, \sigma_{\varepsilon^\pi}, \sigma_{\varepsilon^g}, \sigma_{\eta^h}, \sigma_{\eta^\pi}, \sigma_{\eta^g}, \sigma_\nu, \gamma, \mu)$. The model's general state space form is given by

$$y_t = Z\alpha_t + \varepsilon_t, \qquad \varepsilon_t \sim \text{i.i.d.N}(0, H), \tag{2.11}$$

$$\alpha_t = d + T\alpha_{t-1} + K\eta_t, \qquad \eta_t \sim \text{i.i.d.N}(0, Q), \tag{2.12}$$

where y_t is a $p \times 1$ vector of observations and α_t an unobserved $m \times 1$ state vector. The matrices Z, T, K, H, Q and the vector d are assumed to be known (conditioned upon) and the error terms ε_t and η_t are assumed to be serially uncorrelated and independent of each other at all points in time. As equations (2.11)-(2.12) constitute a linear Gaussian state space model, the unknown state variables in α_t can be filtered using the standard Kalman filter. Within the classical approach the hyperparameters could be estimated via Maximum likelihood based on a prediction error decomposition. As these estimates are taken as true values when filtering the unobserved components, confidence intervals do not reflect parameter uncertainty, but only filtering uncertainty. The Bayesian approach allows for jointly estimating the states and hyperparameters and, thus, leads to credible intervals which take into account both sources of uncertainty. Given an initial guess for the hyperparameters, we start by filtering and sampling the unobserved components.

Block 1: Filtering and sampling the unobserved components

The model given by eq. (2) - (8) can be cast into the following state space form:

$$\underbrace{\begin{bmatrix} h_t \\ \pi_t \\ g_t \end{bmatrix}}_{y_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & \theta_1 & \theta_2 \\ 0 & 0 & 1 & \omega_1 & \omega_2 \end{bmatrix}}_{Z} \overbrace{\begin{bmatrix} h_t^* \\ \pi_t^* \\ g_t^* \\ h_t^c \\ h_{t-1}^c \end{bmatrix}}^{\alpha_t} + \underbrace{\begin{bmatrix} 0 \\ \varepsilon_{\pi t} \\ \varepsilon_{gt} \end{bmatrix}}_{\varepsilon_t}, \tag{2.13}$$

$$\underbrace{\begin{bmatrix} h_t^* \\ \pi_t^* \\ g_t^* \\ h_t^c \\ h_{t-1}^c \end{bmatrix}}_{\alpha_t} = \underbrace{\begin{bmatrix} \mu \\ 0 \\ \gamma \\ 0 \\ 0 \end{bmatrix}}_{\text{d}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{T}} \underbrace{\begin{bmatrix} h_{t-1}^* \\ \pi_{t-1}^* \\ g_{t-1}^* \\ h_{t-1}^c \\ h_{t-2}^c \end{bmatrix}}_{\alpha_{t-1}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{K}} \underbrace{\begin{bmatrix} \eta_t^h \\ \eta_t^\pi \\ \eta_t^g \\ \nu_t \end{bmatrix}}_{\eta_t}, \tag{2.14}$$

with $H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma^2_{\varepsilon^\pi} & 0 \\ 0 & 0 & \sigma^2_{\varepsilon^g} \end{bmatrix}$ and $Q = \begin{bmatrix} \sigma^2_{\eta^h} & 0 & 0 & 0 \\ 0 & \sigma^2_{\eta^\pi} & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta^g} & 0 \\ 0 & 0 & 0 & \sigma^2_\nu \end{bmatrix}$.

We make use of the standard Kalman filter to compute the vector of unobserved components at every point in time. Initial values are given by the unconditional distribution in case of the stationary component and are set to arbitrary values with large initial variance in case of non-stationary components. The state vector α_t is sampled from its conditional distribution via the multimove Gibbs sampler of Shephard (1994) and Carter and Kohn (1996).

Block 2: Sampling the hyperparameters

Conditioning on the unobserved components sampled in Block 1, the hyperparameters can be expressed as unknown parameters in the standard static linear regression model

$$y_t = b'x_t + u_t, \qquad u_t \sim \mathcal{N}\left(0, \sigma^2\right), \tag{2.15}$$

where x_t and b are $(\ell \times 1)$ vectors. The matrix version of (2.15) is $y = Xb + u$ with obvious notations X ($T \times \ell$ matrix), y and u ($T \times 1$ vectors). We follow the approach outlined in Bauwens, Lubrano, and Richard (1999) (pages 56-61). Prior information is represented through the following normal-inverted gamma-2 density

$$\varphi\left(b,\sigma^2\right) = f_{NIg}\left(b,\sigma^2|b_0,M_0,s_0,V_0\right), \tag{2.16}$$

with the prior information being summarized by the hyperparameters $(b_0, m_0, \sigma_0^2, v_0)$. First, b_0 is the prior belief about the coefficient vector b with corresponding prior strength $M_0 = m_0 M$ such that m_0 is defined as being the prior precision proportional to the sample precision matrix $M = X'X$. Second, σ_0^2 is the prior belief about the error variance σ^2, such that $s_0 = \sigma_0^2 V_0$ is the prior belief about the residual sum of squares s with V_0 being the corresponding prior strength defined as $V_0 = v_0 T$ where v_0 is the prior degrees of freedom proportional to the sample size T.

The posterior density of b and σ^2 in the linear regression model (2.15) with prior density (2.16) is a normal-inverted gamma-2 distribution

$$\varphi\left(b,\sigma^2|y,X\right) = f_{NIg}\left(b,\sigma^2|b_*,M_*,s_*,V_*\right), \tag{2.17}$$

with hyperparameters defined by

$$\begin{aligned}
M_* &= M_0 + X'X, \\
b_* &= M_*^{-1}\left(M_0 b_0 + X'X\widehat{b}\right), \\
s_* &= s_0 + s + \left(b_0 - \widehat{b}\right)'\left(M_0^{-1} + \left(X'X\right)^{-1}\right)^{-1}\left(b_0 - \widehat{b}\right), \\
V_* &= V_0 + T,
\end{aligned}$$

where $\widehat{b}$ is the LS estimator for b in (2.15). Sampling b and σ^2 from the posterior distribution (2.17) can then be done separately from

$$b \sim \mathcal{N}\left(b_*, \frac{s_*}{V_* - 2} M_*^{-1}\right), \tag{2.18}$$

$$\sigma^2 \sim IG_2\left(V_*, s_*\right). \tag{2.19}$$

If $X = [.]$, the posterior density in (2.17) reduces to

$$\varphi\left(\sigma^2|y,X\right) = f_{Ig}\left(\sigma^2|s_*,V_*\right), \tag{2.20}$$

with $s_* = s_0 + s$ and V_* as defined above.

The hyperparameters can now be sampled as:

- Obtain the posterior distribution of γ and $\sigma_{\eta g}^2$ in (2.5) conditioning on g_t^* by using (2.17) setting $y_t = g_t^* - g_{t-1}^*$ and $x_t = 1$ in (2.15). Next, sample γ and $\sigma_{\eta g}^2$ from (2.18) and (2.19) respectively.

- Obtain the posterior distribution of ϕ and σ_ν^2 in (2.8) conditioning on h_t^c by using (2.17) setting $y_t = h_t^c$ and $x_t = [h_{t-1}^c, h_{t-2}^c]$ in (2.15). Next, sample ϕ and σ_ν^2 from (2.18) and (2.19) respectively. Resample ϕ and σ_ν^2 in case the coefficients imply a non-stationary process.

- Obtain the posterior distribution of μ and $\sigma_{\eta h}^2$ in (2.7) conditioning on h_t^* by using (2.17) setting $y_t = h_t^* - h_{t-1}^*$ and $x_t = 1$ in (2.15). Next, sample $\sigma_{\eta h}^2$ from (2.18) and (2.19) respectively. For the model with $\mu = 0$ set $x_t = [.]$ and sample $\sigma_{\eta h}^2$ from (2.19).

- Obtain the posterior distribution of $\sigma_{\eta\pi}^2$ in (2.3) conditioning on π_t^* by using (2.20) setting $y_t = \pi_t^* - \pi_{t-1}^*$ and $x_t = [.]$ in (2.15). Next, sample $\sigma_{\eta\pi}^2$ from (2.19).

- Obtain the posterior distribution of θ and $\sigma_{\varepsilon\pi}^2$ in (2.2) conditioning on π_t^* and h_t^c by using (2.17) setting $y_t = \pi_t - \pi_t^*$ and $x_t = [h_t^c, h_{t-1}^c]$ in (2.15). Next, sample θ and $\sigma_{\varepsilon\pi}^2$ from (2.18) and (2.19) respectively.

- Obtain the posterior distribution of ω and $\sigma_{\varepsilon g}^2$ in (2.4) conditioning on g_t^* and h_t^c by using (2.17) setting $y_t = g_t - g_t^*$ and $x_t = [h_t^c, h_{t-1}^c]$ in (2.15). Next, sample ω and $\sigma_{\varepsilon g}^2$ from (2.18) and (2.19) respectively.

We repeat these steps iteratively 10,000 times and discard the first 5,000 draws as a burn-in sample.

3 | Testing for Time Variation in an Unobserved Components Model for the U.S. Economy

with Tino Berger and Gerdie Everaert

Abstract: This paper analyzes the amount of time variation in the parameters of a reduced-form empirical macroeconomic model for the U.S. economy. We decompose output, inflation and unemployment in their stochastic trend and business cycle gap components, with the latter linked through the Phillips curve and Okun's Law. A novel Bayesian approach is employed to test which parameters are time-varying and which components exhibit stochastic volatility. Using data from 1959Q2 to 2014Q3 we find substantial time variation in Okun's Law, while the Phillips curve slope appears to be stable. Stochastic volatility is found to be important for cyclical shocks to the economy, while the volatility of permanent shocks remains stable.

JEL classification: C32, E24, E31

The authors would like to thank Freddy Heylen, Thomas Kneib, James Morley and Joris Wauters for their constructive comments and suggestions. The paper has also benefited from discussions with participants at the 22nd and 23rd Symposium of the Society for Nonlinear Dynamics and Econometrics and various seminars and workshops. Hauke Vierke acknowledges financial support from the Friedrich-Ebert-Stiftung and from Ghent University's Special Research Fund (BOF).

3.1 Introduction

Over the last decades the U.S. economy has experienced a number of notable structural changes. Well documented are the productivity slowdown in the early 1970s and the reduction in the volatility of key macroeconomic variables in the mid 1980s, known as the Great Moderation. More recently, due to the experience of the 2001 recession and the Great Recession, the interest in the academic literature in analyzing structural changes has been renewed. In particular, during the Great Recession, with unemployment being very high, most Phillip curve estimates imply that prices should have fallen much more than what the actual data show. This case of missing deflation has cast doubt on the stability of the Phillips curve. Moreover, in the aftermath of the last two recessions, job growth was substantially lower than what the level of output growth would have implied. These episodes, known as 'jobless recoveries', have let many observers to conclude that the trade-off between unemployment and output has changed. Finally, the severeness of the Great Recession and the related increases in the volatility of key macroeconomic variables may herald the end of the Great Moderation.

A growing literature investigates time variation in macroeconomic relationships. First, the necessity for empirical models to account for changes in the volatility of macroeconomic variables has been emphasized by Hamilton (2008) and Fernández-Villaverde and Rubio-Ramírez (2010), the former showing that not accounting for volatility changes can lead to biased estimates and false hypothesis testing. Second, regarding the relation between inflation and real economic activity, the literature has collected growing evidence for a change in the slope of the Phillips curve. Ball and Mazumder (2011) forecast inflation over the period 2008-2010 using backward-looking Phillips curve estimates for the period 1960-2007. The model predicts substantial deflation, which is not in line with the slightly positive actual inflation rate observed over this period. Hall (2011) also emphasizes the case of missing deflation during the Great Recession and notes that inflation remained remarkably stable at a small but positive rate despite the large and persistent slack in real activity. Roberts (2006) analyzes data prior to the Great recession and finds that the Phillips curve slope of a reduced-form equation for the U.S. fell by nearly half between the periods 1960-1983 and 1984-2002. Similar results can be found in Atkeson and Ohanian (2001) and Mishkin (2007). Third, regarding the relationship between unemployment and real economic activity, a related literature investigates the stability of Okun's Law. Daly, Hobijn, Şahin, and Valletta (2012) note that if Okun's Law had held in 2009, the U.S. unemployment rate would only have risen by about half of the actual rise. Owyang and Sekhposyan (2012) conclude that the relation between unemployment and output fluctuations changes significantly during the most recent recessions. Lee (2000) reports international evidence for structural breaks in the Okun coefficient during the 1970s. Contradicting evidence is given by Ball, Leigh, and Loungani (2013), who find that Okun's Law is a 'strong and stable' relationship.

Measuring these various types of time variation is challenging as it relates to variables that are not directly observed. The Phillips curve links inflation to expected inflation and to a measure for the deviation of real economic activity from its potential, such as the output gap or the unemployment gap. These determinants are unobserved. The same argument holds for Okun's Law which models the interaction between the output gap and the unemployment gap. To proxy these unobserved factors, many studies rely on purely statistical trend-cycle decompositions based on filtering techniques such as the Hodrick-Prescott filter or use external estimates provided by a statistical bureau such as the Congressional Budget Office's (CBO) series for the U.S. economy. The first approach suffers from a lack of structural interpretation while the second entails the risk of falling into an endogeneity trap. The CBO for instance follows a growth model for calculating potential output thereby relying on constant values for the slope of the Phillips curve and the Okun's Law coefficient. As such, these slopes and their stability are artificially imposed on the data from the outset.

In this paper, we set up and estimate a multivariate unobserved components model for the U.S. economy to jointly estimate a time-varying NAIRU, trend inflation, potential output, and the respective gaps. Important model parameters are allowed to change over time. Specifically, we allow the forward-looking New Keynesian Phillips curve slope, the Okun's Law coefficient, the growth rate of potential output and the variances of the innovations to all unobserved components to vary over time.

The model in our paper is most closely related to the following recent papers. First, Stella and Stock (2012) estimate the time-varying trend inflation and the NAIRU using a bivariate unobserved components (UC) model with stochastic volatility (SV). While the Phillips curve slope is treated as constant in the forward-looking inflation equation, the implied backward-looking Philips curve has a time-varying slope parameter which is found to vary considerably. Second, Chan, Koop, and Potter (2015) build on this model and use a bounded random walk specification for the trend components. However, their analysis can be understood as a forecasting exercise as less emphasis is put on time variation in the parameters. They stick to a bivariate model of inflation and unemployment. Third, Kim, Manopimoke, and Nelson (2014) allow for two structural breaks in the slope of the U.S. New Keynesian Phillips curve. The sensitivity of inflation to the CBO output gap is found to be small but significant prior

An alternative version of Okun's Law relates the change in the unemployment rate to output growth. This framework, however, rests on the restrictive assumption of a constant natural rate of unemployment and a constant growth rate of potential output.

to 1971, while being insignificant from 1971 onwards.

We contribute to this literature in the following way. Our unrestricted model nests important empirical models with time-varying parameters. In contrast to the existing literature, we start from a more general framework which allows most of the model's parameters to vary according to a random walk process. This allows for a very flexible evolution over time. We then select a parsimonious model by testing the relevance of the estimated time variation in each of the model's components. To this end, we use the Bayesian stochastic model specification search for state space models as outlined in Frühwirth-Schnatter and Wagner (2010). The Bayesian approach is well-suited to deal with the non-regular testing problem of deciding whether a component is fixed or time-varying. To the best of our knowledge, this is the first study to allow and explicitely test for a wide range of time-varying parameters in a macroeconomic time-series model. A such, our results will provide new evidence on the form and the degree of structural change in the U.S. economy.

Our main findings can be summarized as follows. First, the correlation between cyclical unemployment and cyclical output varies over time. The responsiveness of unemployment to output appears to be more pronounced in recessions. Second, the slope of the Phillips curve is constant over time. This finding is robust over a forward and backward-looking specification. Third, the growth rate of potential output has decreased from a quarterly growth rate of 1% in the 1960s to 0.4% in the 2000s. The most substantial decreases are observed over the 1970s and 2000s. Fourth, shocks to the output gap and to the transitory inflation component exhibit stochastic volatility while shocks to the NAIRU, potential output and trend inflation appear to be homoskedastic.

The remainder of the paper is structured as follows: The next section introduces our empirical model and explains how we test for time variation. Results are presented in section 3.3. In section 3.4 we perform several robustness checks and discuss model extensions. The final section concludes.

3.2 Empirical approach

This section explains our econometric approach. First, it lays out a multivariate unobserved components model with time varying parameters and stochastic volatilities, designed to fit U.S. macroeconomic data. Second, the Bayesian stochastic model selection approach is explained followed by a description of the Markov Chain Monte Carlo (MCMC) algorithm employed to estimate the model.

3.2.1 An unobserved components model

Output: trend/cycle decomposition

Consider a decomposition of real GDP y_t into a stationary cycle y_t^c and a non-stationary trend y_t^τ referred to as potential output

$$y_t = y_t^\tau + y_t^c + \varepsilon_t^y, \qquad \varepsilon_t^y \sim i.i.d.\mathcal{N}(0, \sigma_{\varepsilon,y}^2), \tag{3.1}$$

where ε_t^y is included to capture measurement error and non-persistent shocks. Potential output is modeled as a random walk process with stochastic drift κ_t

$$y_{t+1}^\tau = \kappa_t + y_t^\tau + \exp\{h_t^y\}\,\psi_t^y, \qquad \psi_t^y \sim i.i.d.\mathcal{N}(0,1), \tag{3.2}$$

$$\kappa_{t+1} = \kappa_t + \psi_t^\kappa, \qquad \psi_t^\kappa \sim i.i.d.\mathcal{N}(0,\sigma_\kappa^2). \tag{3.3}$$

The stochastic drift is included to capture permanent changes in the growth rate of potential output. The productivity slowdown in the early 1970s for instance is likely to have lowered the growth rate of potential output. Demographic changes as well as potential long-run effects of the Great Recession are other potential drivers of κ_t. The output gap y_t^c is modeled as a stationary autoregressive (AR) process of order two

$$y_{t+1}^c = \rho_1 y_t^c + \rho_2 y_{t-1}^c + \exp\{h_t^c\}\,\psi_t^c, \qquad \psi_t^c \sim i.i.d.\mathcal{N}(0,1). \tag{3.4}$$

This AR(2) specification allows the output gap to exhibit the standard hump-shaped pattern. The stochastic volatility terms $\exp\{h_t^y\}$ and $\exp\{h_t^c\}$ in the innovations to the trend and the cycle are included to account for changes in macroeconomic volatility such as the Great Moderation or the recent increase in volatility due to the financial crises. These components are specified below.

Inflation: a time-varying New-Keynesian Phillips curve

In contemporary macroeconomic models, the New-Keynesian Phillips Curve (NKPC) relates actual inflation to expected inflation and some measure for excess demand. It can be derived from a micro-founded theoretical model with Calvo (1983) pricing in which firms seek to set their price as a mark-up over marginal costs but are only randomly allowed to change their prices. However, in its pristine form the empirical performance of the NKPC is disappointing as the slope of the NKPC is often found to be small and insignificant. Moreover, it fails to match important stylized facts of inflation dynamics. The purely forward-looking specification implies that current inflation is the discounted present value of expected future activity gaps. As the activity gap is a stationary process inflation should be stationary as well. This is at odds with the observed high degree of persistence in inflation, which is typically found to be non-stationary. Fuhrer and Moore (1995), Mankiw (2001), Rudd and Whelan (2005, 2007) and Mavroeidis, Plagborg-Møller, and Stock (2014) discuss these failures in greater detail.

An appealing way to match the NKPC with the data is the introduction of stochastic trend inflation as in Kim, Manopimoke, and Nelson (2014); Morley, Piger, and Rasche (2015); Stella and Stock (2012). Cogley and Sbordone (2008) derive a NKPC that allows for a time-varying trend inflation rate. By incorporating trend inflation their purely forward-looking NKPC fits the data well without the need to include backward-looking components. We follow this literature and use an inflation gap NKPC in which inflation is modeled in deviation from trend inflation π_t^τ and the output gap is used as a measure of real activity, i.e.

$$\pi_t - \pi_t^\tau = \omega E_t(\pi_{t+1} - \pi_{t+1}^\tau) + \beta_t^\pi y_t^c + \widetilde{\zeta}_t, \tag{3.5}$$

where ω is a discount factor. As shown by Beveridge and Nelson (1981), in the presence of a zero-mean transitory component the trend component π_t^τ equals the long-run inflation forecast $\lim\limits_{h\to\infty} E(\pi_{t+h})$. Following Cogley and Sbordone (2008) and Kim, Manopimoke, and Nelson (2014), the term $\widetilde{\zeta}_t$ is included to capture variation in the inflation gap that is not explained by the conventional forward-looking NKPC. According to Mavroeidis, Plagborg-Møller, and Stock (2014) this term can be interpreted as a combination of cost-push shocks, such as shocks to the markup or to input (e.g. oil) prices. As this term is potentially serially correlated, it allows for an additional source of inflation persistence not related to expectations or real activity. Hence, our model resembles alternative hybrid NKPC models that explicitly add lagged inflation or supply shock variables. As we want to analyze whether the slope of the Phillips curve is time-varying we allow β_t^π to be a time-varying parameter. Our specification of the Phillips curve is most closely related to that of Kim, Manopimoke, and Nelson (2014) who allow β_t^π to vary using a three-state Markov switching model. Iterating equation (3.5) forward and rearranging yields

$$\begin{aligned} \pi_t &= \pi_t^\tau + \lim_{j\to\infty} \omega^j E_t(\pi_{t+j} - \pi_{t+j}^\tau) + \sum_{j=0}^{\infty} \omega^j E_t(\beta_{t+j}^\pi y_{t+j}^c) + \zeta_t, \\ &= \pi_t^\tau + \sum_{j=0}^{\infty} \omega^j E_t(\beta_{t+j}^\pi y_{t+j}^c) + \zeta_t, \end{aligned} \tag{3.6}$$

The trend may be attributed to shifts in monetary policy (see e.g. Woodford, 2008; Cogley and Sbordone, 2008; Goodfriend and King, 2012).

with $\zeta_t = \sum_{j=0}^{\infty} E_t\left(\widetilde{\zeta}_{t+j}\right)$ and $\lim_{j\to\infty} \omega^j = 0$. Equation (3.6) implies that inflation has a trend/cycle representation, i.e.

$$\pi_t = \pi_t^\tau + \pi_t^c + \varepsilon_t^\pi, \qquad \varepsilon_t^\pi \sim i.i.d.\mathcal{N}(0, \sigma_{\varepsilon,\pi}^2), \tag{3.7}$$

where π_t^c is the inflation gap given by

$$\pi_t^c = \sum_{j=0}^{\infty} \omega^j E_t(\beta_{t+j}^\pi y_{t+j}^c) + \zeta_t. \tag{3.8}$$

The idiosyncratic term ε_t^π is added in equation (3.7) to capture measurement error and non-persistent shocks. Trend inflation π_t^τ is modeled as a driftless random walk

$$\pi_{t+1}^\tau = \pi_t^\tau + \exp\{h_t^\pi\}\, \psi_t^\pi, \qquad \psi_t^\pi \sim i.i.d.\mathcal{N}(0, 1), \tag{3.9}$$

where the innovations ψ_t^π are allowed to exhibit stochastic volatility to capture changes in the dynamics of long-run inflation, possibly driven by different monetary policy regimes (see e.g. Stock and Watson, 2007, for a similar specification). The slope of the Phillips curve β_t^π is allowed to change over time according to a random walk

$$\beta_{t+1}^\pi = \beta_t^\pi + \eta_t^\pi, \qquad \eta_t^\pi \sim i.i.d.\mathcal{N}(0, \sigma_{\eta,\pi}^2). \tag{3.10}$$

We model the temporary inflation component ζ_t in equation (3.8) as an AR(1) process

$$\zeta_{t+1} = \varrho\zeta_t + \exp\left\{h_t^\zeta\right\} \psi_t^\zeta, \qquad \psi_t^\zeta \sim i.i.d.\mathcal{N}(0, 1). \tag{3.11}$$

Given the DGPs of y_t^c and β_t^π in equations (3.4) and (3.10) and the assumption that ψ_t^c and η_t^π are mutually uncorrelated error terms, the output gap term $\sum_{j=0}^{\infty} \omega^j E_t(\beta_{t+j}^\pi y_{t+j}^c)$ in equation (3.8) can be expressed as

$$\begin{aligned}
\pi_t^c &= \beta_t^\pi \begin{bmatrix} 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \omega \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} y_t^c \\ y_{t-1}^c \end{bmatrix}, & (3.12)\\
&= \frac{\beta_t^\pi}{1 - \omega\rho_1 - \omega^2\rho_2} \left(y_t^c + \omega\rho_2 y_{t-1}^c\right). & (3.13)
\end{aligned}$$

Hence, the model for inflation in equation (3.7) can be rewritten as

$$\pi_t = \pi_t^\tau + \beta_t^\pi \widetilde{y}_t^c + \zeta_t + \varepsilon_t^\pi, \tag{3.14}$$

where $\widetilde{y}_t^c = \frac{1}{1-\omega\rho_1-\omega^2\rho_2}\left(y_t^c + \omega\rho_2 y_{t-1}^c\right)$.

Unemployment: a time-varying Okun's Law relation

We assume that the unemployment rate u_t has the following trend/cycle representation

$$u_t = u_t^\tau + \beta_t^u y_t^c + \varepsilon_t^u, \qquad \varepsilon_t^u \sim i.i.d.\mathcal{N}(0, \sigma_{\varepsilon,u}^2), \tag{3.15}$$

where ε_t^u captures measurement error and non-persistent shocks. Following, among others, Staiger, Stock, and Watson (1997) and Laubach (2001) we model trend unemployment u_t^τ as a random walk process

$$u_{t+1}^\tau = u_t^\tau + \exp\left\{h_t^u\right\}\psi_t^u, \qquad \psi_t^u \sim i.i.d.\mathcal{N}(0, 1). \tag{3.16}$$

We give this component a NAIRU interpretation, i.e. as long as the observed unemployment rate equals this long-run trend, no inflationary pressure emanates from the labor market. Again, we allow for stochastic volatility in the trend component so that the variance of permanent shocks to the labor market can differ over time. The strength of Okun's Law β_t^u is allowed to change over time according to a random walk process

$$\beta_{t+1}^u = \beta_t^u + \eta_t^u, \qquad \eta_t^u \sim i.i.d.\mathcal{N}(0, \sigma_{\eta,u}^2). \tag{3.17}$$

Stochastic volatilities

All stochastic volatilities are modeled as random walks

$$h_{t+1}^k = h_t^k + \gamma_t^k, \qquad \gamma_t^k \sim i.i.d.\mathcal{N}(0, \sigma_{\gamma,k}^2), \tag{3.18}$$

for $k = y, \pi, u, c, \zeta$. A key feature of the stochastic volatility components $\exp\left\{h_t^k\right\}\psi_t^k$ is that they are nonlinear but can be transformed into linear components by taking the logarithm of their squares

$$\ln\left(\exp\left\{h_t^k\right\}\psi_t^k\right)^2 = 2h_t^k + \ln\left(\psi_t^k\right)^2, \tag{3.19}$$

where $ln\left(\psi_t^k\right)^2$ is log-chi-square distributed with expected value -1.2704 and variance 4.93. Following Kim, Shephard, and Chib (1998), we approximate the linear model in (3.19) by an offset mixture time series model as

$$g_t^k = 2h_t^k + \epsilon_t^k, \tag{3.20}$$

where $g_t^k = \ln\left(\left(\exp\left\{h_t^k\right\}\psi_t^k\right)^2 + c\right)$ with $c = .001$ being an offset constant, and the distribution of ϵ_t^k being the following mixture of normals

$$f\left(\epsilon_t^k\right) = \sum_{i=1}^{M} q_i f_N\left(\epsilon_t^k | m_i - 1.2704, \nu_i^2\right), \tag{3.21}$$

with component probabilities q_i, means $m_i - 1.2704$ and variances ν_i^2. Equivalently, this mixture density can be written in terms of the component indicator variable ι_t^k as

$$\epsilon_t^k | \left(\iota_t^k = i\right) \sim \mathcal{N}\left(m_i - 1.2704, \nu_i^2\right), \qquad \text{with} \qquad Pr\left(\iota_t^k = i\right) = q_i. \tag{3.22}$$

Following Omori, Chib, Shephard, and Nakajima (2007), we use a mixture of $M = 10$ normal distributions to make the approximation to the log-chi-square distribution sufficiently good. Values for $\{q_i, m_i, \nu_i^2\}$ are provided by Omori, Chib, Shephard, and Nakajima in their Table 1.

3.2.2 Stochastic model specification search

The empirical model outlined in the Subsection 3.2.1 nests a number of model specifications used in the recent literature. The univariate unobserved components model for inflation examined by Stock and Watson (2007) can for instance be obtained by restricting β_t^π, ϱ and $\sigma^2_{\epsilon,\pi}$ to zero. The bivariate unobserved components specification for inflation and unemployment of Stella and Stock (2012) is nested when we replace the output gap by the unemployment gap, set ϱ and $\sigma^2_{\epsilon,\pi}$ to zero and restrict β_t^π to be constant.

A key question therefore is which model components are relevant and which can be excluded. However, model specification for state space models is a difficult task as this leads to non-regular testing problems. Consider for instance the question whether the slope of the Phillips curve should be modeled as constant or time-varying. This implies testing $\sigma^2_{\eta,\pi} = 0$ against $\sigma^2_{\eta,\pi} > 0$, which is a non-regular testing problem as the null hypothesis lies on the boundary of the parameter space. A similar problem arises when testing whether the temporary component ζ_t should be included in equation (3.14) or whether the stochastic volatilities are relevant.

In principle we could derive the reduced form VARMA representation of our model and apply standard structural break tests for mean and variances. However, by testing for time-variation in the UC model instead of the reduced form VARMA model, we can distinguish between changes in the volatilities to permanent versus transitory shocks. Moreover, deriving the reduced form VARMA representation requires assumptions regarding the order of integration of all variables. However, for output the order of integration is an outcome of our testing procedure as we allow potential output growth to be either constant or evolve as a random walk in which case output is I(2).

As an alternative, we use a Bayesian stochastic model specification search. The Bayesian approach is well-suited to deal with non-regular testing problems by computing posterior probabilities for each of the candidate models. In particular, Frühwirth-Schnatter and Wagner (2010) show how to extend Bayesian variable selection in standard regression models to state space models. Their approach relies on a non-centered parameterization of the state space model in which (i) binary stochastic indicators for each of the model components are sampled together with the parameters and (ii) the standard inverse Gamma prior for the variances of innovations to the components is replaced by a Gaussian prior centered at zero for the square root of these variances. The exact implementation applied to our state space model is outlined below.

Non-centered parameterization

Frühwirth-Schnatter and Wagner (2010) argue that a first piece of information on the hypothesis whether a variance parameter in a state space model is zero or not can be obtained by considering a non-centered parameterization. For the variances of the innovations to the slope of the Phillips curve and Okun's Law, i.e. $\sigma^2_{\eta,\pi}$ and $\sigma^2_{\eta,u}$, this implies rearranging equations (3.10) and (3.17) to

$$\beta^j_{t+1} = \beta^j_0 + \sigma_{\eta,j}\widetilde{\beta}^j_{t+1}, \tag{3.23}$$

$$\text{with} \qquad \widetilde{\beta}^j_{t+1} = \widetilde{\beta}^j_t + \widetilde{\eta}^j_t, \qquad \widetilde{\beta}^j_0 = 0, \qquad \widetilde{\eta}^j_t \sim i.i.d.\mathcal{N}(0,1), \tag{3.24}$$

for $j = \pi, u$ and where β^j_0 is the initial value of the level of β^j_t. A crucial aspect of the non-centered parameterization is that it is not identified, i.e. the signs of $\sigma_{\eta,j}$ and $\widetilde{\beta}^j_t$ can be changed by multiplying both with -1 without changing their product in equation (3.23). As a result of the non-identification, the likelihood function is symmetric around 0 along the $\sigma_{\eta,j}$ dimension and therefore multimodal. If the slope of the Phillips curve is time-varying, i.e. $\sigma^2_{\eta,j} > 0$, then the likelihood function will concentrate around the two modes $-\sigma_{\eta,j}$ and $\sigma_{\eta,j}$. For $\sigma^2_{\eta,j} = 0$ the likelihood function will become unimodal around zero. As such, allowing for non-identification of $\sigma_{\eta,j}$ provides useful information on whether $\sigma^2_{\eta,j} > 0$.

Likewise, the non-centered parameterization of the stochastic volatility terms in equation (3.18) is given by

$$h^k_{t+1} = h^k_0 + \sigma_{\gamma,k}\widetilde{h}^k_{t+1}, \tag{3.25}$$

$$\text{with} \quad \widetilde{h}^k_{t+1} = \widetilde{h}^k_t + \widetilde{\gamma}^k_t, \qquad \widetilde{h}^k_0 = 0, \qquad \widetilde{\gamma}^k_t \sim i.i.d.\mathcal{N}(0,1), \tag{3.26}$$

for $k = y, \pi, u, c, \zeta$ and where $h^k_0 = 0$ is the initial value of the level of h^k_t.

Finally, the non-centered parameterization of the time-varying drift in equation (3.3) is given by

$$\kappa_{t+1} = \kappa_0 + \sigma_\kappa \widetilde{\kappa}_{t+1}, \tag{3.27}$$

$$\text{with} \quad \widetilde{\kappa}_{t+1} = \widetilde{\kappa}_t + \widetilde{\psi}^\kappa_t, \qquad \widetilde{\kappa}_0 = 0, \qquad \widetilde{\psi}^\kappa_t \sim i.i.d.\mathcal{N}(0,1), \tag{3.28}$$

and where $\kappa_0 = 0$ is the initial value of the level of κ_t.

Parsimonious specification

A second advantage of the non-centered parameterization is that when e.g. $\sigma^2_{\eta,\pi} = 0$ the transformed component $\widetilde{\beta}^\pi_t$, in contrast to β_t, does not degenerate to the time-invariant slope of the Phillips curve as this is now represented by β^π_0. As such, the question whether the slopes of the Phillips curve and Okun's Law are time-varying or not can be expressed as a variable selection problem in equation (3.23). To this aim Frühwirth-Schnatter and Wagner (2010) introduce the parsimonious specification

$$\beta^j_t = \beta^j_0 + \delta_j \sigma_{\eta,j} \widetilde{\beta}^j_t, \tag{3.29}$$

for $j = \pi, u$ and where δ_j is a binary indicator which is either 0 or 1. If $\delta_j = 0$, the component $\widetilde{\beta}^j_t$ drops from the model such that β^j_0 represents the constant slope parameter. If $\delta_j = 1$ then $\widetilde{\beta}^j_t$ is included in the model and $\sigma_{\eta,j}$ is estimated from the data. In this case β^j_0 is the initial value of the slope parameter.

Likewise, the parsimonious non-centered parameterization of the stochastic volatility terms in equation (3.25) is given by

$$h^k_t = h^k_0 + \theta_k \sigma_{\gamma,k} \widetilde{h}^k_t, \tag{3.30}$$

for $k = y, \pi, u, c, \zeta$ and where θ_k is again a binary indicator that is either 0 or 1. If $\theta_k = 0$, the component $\widetilde{h}_t^k$ drops from the model such that $\left(\exp\{h_0^k\}\right)^2$ is the constant variance of ψ_t^k. If $\theta_k = 1$ then $\widetilde{h}_t^k$ is included in the model and $\sigma_{\gamma,j}$ is estimated from the data. In this case $\left(\exp\{h_0^k\}\right)^2$ is the initial value of the time-varying variance of ψ_t^k.

Finally, the parsimonious non-centered parameterization of the time-varying drift term in equation (3.27) is given by

$$\kappa_t = \kappa_0 + \lambda\sigma_\kappa\widetilde{\kappa}_t, \tag{3.31}$$

where λ is a binary indicator that is either 0 or 1. If $\lambda = 0$, the component $\widetilde{\kappa}_t$ drops from the model such that κ_0 is the constant drift in potential output. If $\lambda = 1$ then $\widetilde{\kappa}_t$ is included in the model and σ_κ is estimated from the data. In this case κ_0 is the initial value of the time-varying drift κ_t.

Collecting the binary indicators in the vector $\mathcal{M} = (\delta_\pi, \delta_u, \theta_y, \theta_\pi, \theta_u, \theta_c, \theta_\zeta, \lambda)$, each model is indicated by a value for $\mathcal{M}$, e.g. $\mathcal{M} = (0, 1, 0, 0, 0, 1, 0, 1)$ is a model with a constant Phillips curve slope, a time-varying Okun's Law coefficient, stochastic volatility in the innovations to the output gap component, a constant variance for the innovations to the trend components in output, inflation and unemployment as well as to the AR(1) inflation gap component and a time-varying drift in potential output.

Gaussian prior centered at zero

Our Bayesian estimation approach requires choosing prior distributions for the model parameters $\rho = (\rho_1, \rho_2)$, ϱ, $\beta_0 = (\beta_0^\pi, \beta_0^u)$ and $h_0 = (h_0^y, h_0^\pi, h_0^u, h_0^c, h_0^\zeta)$, for the binary indicators $\mathcal{M}$ and for the variances of the idiosyncratic factors $\sigma_\varepsilon^2 = \left(\sigma_{\varepsilon,y}^2, \sigma_{\varepsilon,\pi}^2, \sigma_{\varepsilon,u}^2\right)$, the innovations to the drift component σ_κ^2, the time-varying parameters $\sigma_\eta^2 = \left(\sigma_{\eta,\pi}^2, \sigma_{\eta,u}^2\right)$ and the stochastic volatility components $\sigma_\gamma^2 = \left(\sigma_{\gamma,y}^2, \sigma_{\gamma,\pi}^2, \sigma_{\gamma,u}^2, \sigma_{\gamma,c}^2, \sigma_{\gamma,\zeta}^2\right)$.

It is well-known that when using an inverse Gamma prior distribution for the variance parameters, the choice of the shape and scale hyperparameters that define this distribution have a strong influence on the posterior when the true value of the variance is close to zero. More specifically, as the inverse Gamma does not have probability mass at zero, using it as a prior distribution tends to push the posterior density away from zero. This is of particular importance when estimating the variances of the innovations to the time-varying parameters, to the drift in potential output and to the stochastic volatilities, as for these components we want to decide whether they are relevant or not. A further important advantage of the non-centered parameterization is therefore that it allows us to replace the standard inverse Gamma prior on a variance parameter σ^2 by a Gaussian prior centered at zero on σ. Centering the prior distribution at zero makes sense as for both $\sigma^2 = 0$ and $\sigma^2 > 0$, σ is symmetric around zero. Frühwirth-Schnatter and Wagner (2010) show that, compared to using an inverse Gamma prior for σ^2, the posterior density of σ is much less sensitive to the hyperparameters of the Gaussian distribution and is not pushed away from zero when $\sigma^2 = 0$.

As such we choose a Gaussian prior distribution centered at zero for σ_η, σ_κ and σ_γ, which are the standard deviations of the innovations to the time-varying parameters, to the drift in potential output and to the stochastic volatilities. For the variances of the idiosyncratic factors σ_ε^2, which are always included in the model, we choose the standard inverse Gamma prior distribution. For each of the model parameters in ρ, ϱ and β we assume a normal prior distribution. Details on the prior distributions are presented in Subsection 3.3.2 below. For the binary indicators $\mathcal{M}$ we choose a uniform prior distribution over all combinations of the indicators such that each model has the same prior probability, i.e. $p(\mathcal{M}) = 2^{-8}$, and each model component has a prior probability $p_0 = 0.5$ of being included in the model.

3.2.3 MCMC algorithm

In a standard linear Gaussian state space model, the Kalman filter can be used to filter the unobserved states from the data and to construct the likelihood function such that the unknown parameters can be estimated using maximum likelihood. However, the inclusion of the time-varying parameters β_t^π and β_t^u on the unobserved output gap y_t^c and the stochastic volatilities h_t^k in the state space model given in eq. (3.1) - (3.18) and the use of the stochastic model specification search outlined in Subsection 3.2.2 imply a highly non-linear estimation problem for which the standard approach via the Kalman filter and maximum likelihood is not feasible. Instead we use the Gibbs sampler which is a MCMC method to simulate draws from the intractable joint and marginal posterior distributions of the unknown parameters and the unobserved states using only tractable conditional distributions. Intuitively, this amounts to reducing the complex non-linear model into a sequence of blocks for subsets of parameters/states that are tractable conditional on the other blocks in the sequence.

For notational convenience, define a state vector $\alpha_t = (y_t^\tau, \pi_t^\tau, u_t^\tau, y_t^c, \zeta_t, \kappa_t)$, a time-varying parameter vector $\beta_t = (\beta_t^\pi, \beta_t^u)$, a stochastic volatilities vector $h_t = \left(h_t^y, h_t^\pi, h_t^u, h_t^c, h_t^\zeta\right)$ and an indicator vector $\iota_t = \left(\iota_t^y, \iota_t^\pi, \iota_t^u, \iota_t^c, \iota_t^\zeta\right)$. The unknown parameters are collected in the vector $\phi = (\rho, \varrho, \beta_0, \sigma, \sigma_\varepsilon^2)$, with $\sigma = (\sigma_\eta, \sigma_\kappa, \sigma_\gamma)$. Finally, let $x_t = (y_t, \pi_t, u_t)$ be the data vector. Stacking observations over time, we denote $x = \{x_t\}_{t=1}^T$ and similarly for α, β, h and ι. The posterior density of interest is then given by $f(\alpha, \beta, h, \iota, \phi, \mathcal{M}|x)$. Following Frühwirth-Schnatter and Wagner (2010) our MCMC scheme is as follows:

1. Sample the binary indicators in $\mathcal{M}$ from $f(\mathcal{M}|\alpha, \beta, h, x)$ marginalizing over the parame-

ters ϕ and sample the unrestricted parameters in ϕ from $f(\phi|\alpha, \beta, h, \mathcal{M}, x)$ while setting the restricted parameters, i.e. the elements in σ for which the corresponding component is not included in the model $\mathcal{M}$, equal to 0.

2. Sample the trend and temporary components α from $f(\alpha|\beta, h, \phi, \mathcal{M}, x)$, the time-varying parameters β from $f(\beta|\alpha, h, \phi, \mathcal{M}, x)$, the mixture indicators ι from $f(\iota|\alpha, \beta, h, \phi, \mathcal{M}, x)$ and the stochastic volatilities h from $f(h|\alpha, \beta, \iota, \phi, \mathcal{M}, x)$.

3. Perform a random sign switch for $\sigma_{\eta,j}$ and $\{\widetilde{\beta}_t^j\}_{t=1}^T$; for σ_κ and $\{\widetilde{\kappa}_t\}_{t=1}^T$ and for $\sigma_{\gamma,k}$ and $\{\widetilde{h}_t^k\}_{t=1}^T$, e.g. $\sigma_{\eta,\pi}$ and $\{\widetilde{\beta}_t^\pi\}_{t=1}^T$ are left unchanged with probability 0.5 while with the same probability they are replaced by $-\sigma_{\eta,\pi}$ and $\{-\widetilde{\beta}_t^\pi\}_{t=1}^T$.

Given an arbitrary set of starting values, sampling from these blocks is iterated J times and, after a sufficiently long burn-in period B, the sequence of draws $(B+1, ..., J)$ approximates a sample from the virtual posterior distribution $f(\alpha, \beta, h, \iota, \phi, \mathcal{M}|x)$. Details on the exact implementation of each of the blocks can be found in 3.A. The results reported below are based on 35,000 Gibbs sampler iterations, with the first 10,000 discarded as a burn-in period. We store every 5^{th} of the remaining 25,000 iterations, leaving 5,000 draws for inference.

3.3 Estimation results

3.3.1 Data

We estimate the model using quarterly U.S. data from 1959Q2 - 2014Q3. Inflation is measured by the annualized quarterly change in the core personal consumption expenditures (PCE) index. For unemployment we use the civilian unemployment rate as collected by the Bureau of Labor Statistics. Output is measured by the log of real GDP. All series are taken from St. Louis Federal Reserve Economic Data.

3.3.2 Prior choice

Table 3.1 reports summary information on our prior distributions for the unknown parameters. For the variance parameters of the idiosyncratic factors $\sigma_\varepsilon^2 = \left(\sigma_{\varepsilon,y}^2, \sigma_{\varepsilon,\pi}^2, \sigma_{\varepsilon,u}^2\right)$ we use the inverse Gamma prior $IG(c_0, C_0)$ where the shape $c_0 = \nu_0 T$ and scale $C_0 = s_0\sigma_0^2$ parameters are calculated from the *prior belief* σ_0^2 about the variance parameter and the *prior strength* ν_0 which is expressed as a fraction of the sample size T. Following the notation in

Since this prior is conjugate, $\nu_0 T$ can be interpreted as the number of fictitious observations used to construct the prior belief σ_0^2.

Frühwirth-Schnatter and Wagner (2010), for the remaining parameters we use a Gaussian prior $\mathcal{N}(a_0, A_0\sigma_e^2)$ in a regression with homoskedastic errors and $\mathcal{N}(a_0, A_0)$ when the errors exhibit stochastic volatility. Details on the notation are given in 3.A. Each of the prior choices is discussed below. Note that in Table 3.1 and in the text we report and discuss standard deviations rather than variances as the former are easier to interpret

- **Idiosyncratic components, $\sigma^2_{\varepsilon,g}$, $\sigma^2_{\varepsilon,g}$, $\sigma^2_{\varepsilon,\pi}$:** We set the prior beliefs to $\sigma_{\varepsilon,y} = 0.1$, $\sigma_{\varepsilon,\pi} = 1.0$, and $\sigma_{\varepsilon,u} = 0.5$. The strength of all three priors is 0.1. The larger value for $\sigma_{\varepsilon,\pi}$ is in line with the literature, which usually finds relative large measurement errors in inflation.

- **Volatility of trend components, $\exp\{h^y\}$, $\exp\{h^\pi\}$, $\exp\{h^u\}$:** The prior beliefs a_0 for the constant volatility part h_0 of the level shocks to potential output, trend inflation, and the NAIRU are set to $\ln(0.1)$, $\ln(0.2)$, and $\ln(0.05)$ respectively with the prior standard deviation $\sqrt{A_0}$ set to 0.1. Note that the prior belief $\ln(0.1)$ for potential output implies that, if there is no time-varying volatility, 95% of the innovations lie between -0.2 and $+0.2$ per quarter. For inflation and unemployment the 95% interval ranges from -0.4 to $+0.4$ and -0.1 to $+0.1$ respectively. These values are within the range of previous estimates and are of an economically reasonable value. The prior distribution on the time-varying part of the volatility of the trends is uninformative and centered at zero: $\sigma_{\gamma,k} \sim \mathcal{N}(0, 1)$, for $k = y, \pi, u$.

- **Potential output growth, κ:** In line with existing estimates, our prior belief about the time-invariant part of the output drift is given by $\kappa \sim \mathcal{N}(0.75, 0.1^2)$. For example, Morley, Nelson, and Zivot (2003), Sinclair (2009) and Mitra and Sinclair (2012) find values between 0.79 and 0.86 for quarterly U.S. postwar data. Similar to the volatilities of the trends, we set the time-varying part of potential output growth to $\sigma_\kappa \sim \mathcal{N}(0, 1)$.

- **Output gap, ρ, $\exp\{h^c\}$:** While the output gap is stationary by assumption, it is often found to be a very persistent process (see e.g. Morley, Nelson, and Zivot, 2003; Kim, Manopimoke, and Nelson, 2014). In order to ensure stationarity, we find it useful to impose prior information on the sum of the AR(2) parameters instead of restricting each parameter separately. Hence, we use an informative prior on the sum $(\rho_1 + \rho_2) \sim \mathcal{N}(0.9, 0.015^2)$ and a much less informative prior on the first lag $\rho_1 \sim \mathcal{N}(1.25, 0.5^2)$. The prior belief of 0.90 for $(\rho_1 + \rho_2)$ is an average of values typically found in the literature

See for instance Morley, Piger, and Rasche (2015); Kim, Manopimoke, and Nelson (2014); Stock and Watson (2007). Regarding the smoothness of the NAIRU, we are close to Fleischman and Roberts (2011) who estimate the NAIRU's standard deviation around 0.1.

Table 3.1: Prior distributions of model parameters

Inverse Gamma priors: $IG(c_0, C_0) = IG(\nu_0 T, \nu_0 T \sigma_0^2)$				Percentiles	
		σ_0	ν_0	2.5%	97.5%
idiosyncratic component output	$\sigma_{\varepsilon,y}$	0.10	0.10	0.080	0.141
idiosyncratic component inflation	$\sigma_{\varepsilon,\pi}$	1.00	0.10	0.777	1.408
idiosyncratic component unemployment	$\sigma_{\varepsilon,u}$	0.50	0.10	0.387	0.701
Gaussian priors homoskedastic errors: $\mathcal{N}(a_0, A_0\sigma_e^2)$				Percentiles	
Regression parameters		a_0	$\sqrt{A_0} \times \sigma_e$	2.5%	97.5%
const. Phillips curve slope	β_0^π	0.20	0.25×1.0	−0.290	0.690
const. Okun coefficient	β_0^u	−0.50	0.25×0.5	−0.745	−0.255
Non-centered components					
std. of time-varying Phillips curve	$\sigma_{\eta,\pi}$	0.00	1.00×1.0	−1.960	1.960
std. of time-varying Okun coefficient	$\sigma_{\eta,u}$	0.00	1.00×0.5	−0.980	0.980
Gaussian priors SV errors: $\mathcal{N}(a_0, A_0)$				Percentiles	
Regression parameters		a_0	$\sqrt{A_0}$	2.5%	97.5%
1st AR lag: output gap	ρ_1	1.25	0.50	0.270	2.230
sum of AR lags: output gap	$\rho_1 + \rho_2$	0.90	0.015	0.871	0.930
AR lag: AR(1) inflation component	ϱ	0.70	0.05	0.602	0.798
const. output drift	κ_0	0.75	0.10	0.554	0.946
Stochastic volatility parameters					
const. volatility of potential output	h_0^y	$\ln(0.10)$	0.10	$\ln(0.082)$	$\ln(0.122)$
const. volatility of trend inflation	h_0^π	$\ln(0.20)$	0.10	$\ln(0.164)$	$\ln(0.243)$
const. volatility of NAIRU	h_0^u	$\ln(0.05)$	0.10	$\ln(0.041)$	$\ln(0.061)$
const. volatility of output gap	h_0^c	$\ln(0.60)$	0.10	$\ln(0.493)$	$\ln(0.730)$
const. volatility of temporary inflation	h_0^ζ	$\ln(0.70)$	0.10	$\ln(0.575)$	$\ln(0.852)$
Non-centered components					
std. of SV: potential output	$\sigma_{\gamma,y}$	0.00	1.00	−1.960	1.960
std. of SV: trend inflation	$\sigma_{\gamma,\pi}$	0.00	1.00	−1.960	1.960
std. of SV: NAIRU	$\sigma_{\gamma,u}$	0.00	1.00	−1.960	1.960
std. of SV: output gap	$\sigma_{\gamma,c}$	0.00	1.00	−1.960	1.960
std. of SV: AR(1) inflation component	$\sigma_{\gamma,\zeta}$	0.00	1.00	−1.960	1.960
std. of time-varying output drift	σ_κ	0.00	1.00	−1.960	1.960

Notes: We set IG priors on the variance parameters σ^2 but in the top panel of this table we report details on the implied prior distribution for the standard deviations σ as these are easier to interpret. Likewise, in the bottom panel of the table we report $\sqrt{A_0}$ instead of A_0. For the stochastic volatility parameters h_0 we report a logarithm expression for the mean and percentiles as the arguments can then easily be interpreted as the mean and percentiles of $exp\{h_0\}$.

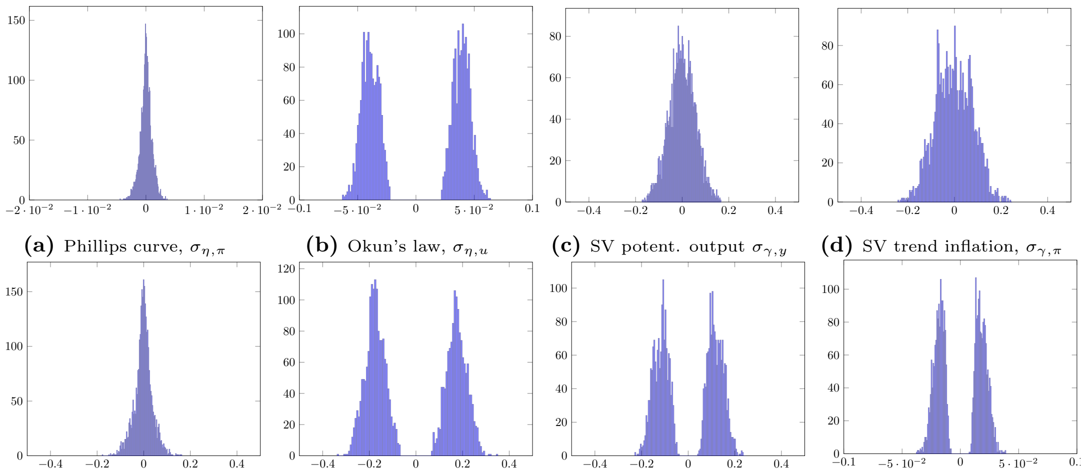
on trend-cycle decomposition of U.S. GDP (see e.g. Kuttner, 1994; Morley, Nelson, and Zivot, 2003; Luo and Startz, 2014). The small prior standard deviation of 0.015 is to ensure that the output gap is stationary. Setting a lower belief together with a higher standard deviations results in a similar posterior, though. The prior belief of 1.25 for ρ_1, which implies a prior belief of -0.35 for ρ_2, is in line with the typical hump-shaped pattern in response to cyclical shocks. With a prior standard deviation of 0.5 we are very uninformative on these individual parameters, though. The prior distribution for the time-invariant part of the cyclical volatility is given by $h_0^c \sim \mathcal{N}(\ln(0.6), 0.1^2)$, implying that 95% of the shocks lie between -1.2 and $+1.2$. Again, an uninformative prior for the time-varying volatility part is used, i.e. $\sigma_{\gamma,c} \sim \mathcal{N}(0,1)$.

- **AR(1) inflation component, ϱ, $\exp\left\{h^\zeta\right\}$:** We set the prior distribution for the autoregressive coefficient of the AR(1) inflation component to $\varphi \sim \mathcal{N}(0.7, 0.05^2)$. The relative small standard deviation ensures that ϱ lies within a region of medium persistence. With values too close to one, the AR(1) component becomes highly persistent and soaks up all variation in trend inflation. If ϱ becomes too small, ζ is indistinguishable from the white noise inflation component ε_t^π. The prior distribution of the time-invariant part of the volatility component is set to $h_0^\zeta \sim \mathcal{N}(\ln(0.7), 0.1^2)$. A loose prior is used for the standard deviation of the time-varying component: $\sigma_{\gamma,\zeta} \sim \mathcal{N}(0,1)$, allowing for a high degree of time variation in ζ_t as found in Kim, Manopimoke, and Nelson (2014).

- **Slope of Phillips curve, β^π:** Estimates for the slope of the Phillips curve in the literature differ depending on whether a forward or backward-looking curve is modeled. In forward-looking specifications, β^π is often found small and statistically insignificant (see e.g. Kim, Manopimoke, and Nelson, 2014). For the time-invariant part β_0^π we set a prior distribution of $\beta_0^\pi \sim \mathcal{N}(0.2, 0.25^2)$. Our prior belief about the degree of time variation in the Phillips curve is uninformative, i.e. $\sigma_{\eta,\pi} \sim \mathcal{N}(0,1)$.

- **Okun coefficient, β^u:** According to Lee (2000) and Reifschneider, Wascher, and Wilcox (2013) the impact of the unemployment gap on the output gap is close to -2 for the U.S., which would correspond to a value of -0.5 in our model as we express this relationship in reverse. Owyang and Sekhposyan (2012) estimate a rolling regression and find a very similar value on average. Thus, we set the prior distribution to $\beta_0^u \sim \mathcal{N}(-0.5, 0.125^2)$. The prior on the degree of time variation in Okun's Law is set to $\sigma_{\eta,u} \sim \mathcal{N}(0,1)$.

Figure 3.1: Posterior distributions of the standard deviations, unrestricted model (all binary indicators set to 1)



(a) Phillips curve, $\sigma_{\eta,\pi}$ **(b)** Okun's law, $\sigma_{\eta,u}$ **(c)** SV potent. output $\sigma_{\gamma,y}$ **(d)** SV trend inflation, $\sigma_{\gamma,\pi}$

(e) SV NAIRU, $\sigma_{\gamma,u}$ **(f)** SV output gap, $\sigma_{\gamma,c}$ **(g)** SV temp. inflation, $\sigma_{\gamma,\zeta}$ **(h)** Potent. output drift, σ_{κ}

3.3.3 Results stochastic model specification search

We first estimate an unrestricted model with all binary indicators set to one to generate posterior distributions for the standard deviations (σ) of the innovations to the 8 non-centered components of interest. If these distributions are bimodal, with low or no probability mass at zero, this can be taken as a first indication of time variation in the considered component. Results are shown in Figure 3.1. Clear-cut bimodality is found in the posterior distribution of the standard deviation of the innovations to the Okun's Law parameter ($\sigma_{\eta,u}$), to the volatility of the output gap ($\sigma_{\gamma,c}$) and the temporary inflation component ($\sigma_{\gamma,\zeta}$) and to the drift in potential output (σ_{κ}). For the stochastic volatility of trend inflation evidence is less clear. While the distribution $\sigma_{\gamma,\pi}$ appears to have two modes, it also has a considerable probability mass at zero. For the innovations to the Phillips curve parameter and to the stochastic volatility components in trend output and trend unemployment, the posterior distributions of $\sigma_{\eta,\pi}$, $\sigma_{\gamma,y}$ and $\sigma_{\gamma,u}$ are clearly unimodal at zero. This suggests that these components are stable over time.

As a more formal test for time variation, we next sample the stochastic binary indicators together with the other parameters in the model. Table 3.2 displays the individual posterior probabilities for the binary indicators being one. These probabilities are calculated as the average selection frequencies over all iterations of the Gibbs sampler. The second row shows results for our benchmark case $A_0 = 1$. This implies a relatively loose prior on the degree of time variation σ. To check robustness, the other rows show results over alternative values for A_0. The first row shows results for the case where $A_0 = 0.1$. This corresponds to a relatively stronger prior that allows for less time variation. The third and fourth row show results for diffuse prior distributions that allow for large variances on the time-varying components. The following conclusions can be drawn. First, the model selection rejects time variation in the slope of the Phillips curve. Over all four prior specifications, the posterior probability for a model with a time-varying Phillips curve slope is either far below or just above one percent. Second, the data clearly favor time variation in the Okun's Law parameter. Third, for the trend components in output, inflation and unemployment, a model with a constant volatility fits the data best. In our benchmark case ($A_0 = 1$) the posterior probabilities of a stochastic volatility component in the trend components varies between 8 and 18%, while the probabilities fall well below 5% when more diffuse priors are used. When the prior distribution allows for little time variation ($A_0 = 0.1$), the inclusion probabilities of the stochastic volatility components increase, but remain below 0.5.

Table 3.2: Posterior inclusion probabilities, different prior variances A_0

Prior		Posterior							
		Time-varying parameter			Stochastic volatility				
		Phillips curve	Okun's law	Output drift	Potential output	Trend inflation	NAIRU	Output gap	Temp. inflation
p_0	A_0	δ_π	δ_u	λ	θ_y	θ_π	θ_u	θ_c	θ_ζ
0.5	0.1	0.0110	1.0000	1.0000	0.2150	0.1704	0.3084	1.0000	1.0000
0.5	1	0.0026	1.0000	1.0000	0.1456	0.0828	0.1992	1.0000	1.0000
0.5	10	0.0007	1.0000	1.0000	0.0247	0.0263	0.0923	1.0000	1.0000
0.5	100	0.0000	1.0000	1.0000	0.0219	0.0206	0.0429	1.0000	1.0000

In the baseline specification, we assign a 0.5 prior probability to each of the binary indicators being one. As noted by Scott, Berger, et al. (2010), this prior choice does not provide multiplicity control for the Bayesian variable selection. When the number of possible variables is very large and each of the binary indicators has a prior probability of 0.5, the fraction of selected variables will very likely be around 0.5. Our findings appear to be unaffected by this issue, though. First, the number of variables to be selected is only 8 in this paper. Second, we re-estimate the (unrestricted) model with different priors. Specifically, the prior inclusion probability on each of the 8 components is set to 0.1 and 0.9 respectively. The resulting posterior probabilities are reported in Table 3.3. For all prior choices the same model is selected,

The increase in the posterior probability may appear counter intuitive, but is due to the fact that by restricting the amount of time variation the competing models become similar in their marginal likelihoods and thus the posterior probability shrinks towards the prior probability $p_0 = 0.5$.

i.e. the indicators $\delta_u, \theta_c, \theta_\zeta$ and λ have inclusion probabilities of ≥ 0.5, while the indicators $\delta_\pi, \theta_y, \theta_\pi$ and θ_u are excluded in the majority of all draws.

Table 3.3: Posterior inclusion probabilities, prior probabilities p_0

Prior		Posterior							
		Time-varying parameter			Stochastic volatility				
		Phillips curve	Okun's law	Output drift	Potential output	Trend inflation	NAIRU	Output gap	Temp. inflation
p_0	A_0	δ_π	δ_u	λ	θ_y	θ_π	θ_u	θ_c	θ_ζ
0.5	1	0.0026	1.0000	1.0000	0.1456	0.0828	0.1992	1.0000	1.0000
0.9	1	0.0192	1.0000	1.0000	0.4606	0.4478	0.4484	1.0000	1.0000
0.1	1	0.0004	1.0000	1.0000	0.0174	0.0302	0.0496	1.0000	1.0000

Besides inference on the importance of time variation in the individual components, the model selection search also allows to compute overall model probabilities. The introduction of 8 binary indicators leads to 2^8 possible models. As 4 out of the 8 binary indicators have low individual probabilities, most models have a probability of zero. As a result, in the benchmark case where $A_0 = 1$ only 7 models are selected in more than 1% of the Gibbs iterations. The posterior probabilities for these models are reported in Table 3.4. The favored model has a time-varying Okun's Law parameter and stochastic volatility in the output gap and the transitory inflation component, while the Phillips curve slope is constant and there is no stochastic volatility in the three trend components. This model choice is robust to different prior specification, i.e. the model in row one has the highest probability for each of the four considered values of A_0. In the case of strict priors ($A_0 = 0.1$), this model has a posterior probability of 45%, which rises to 92% when diffuse priors ($A_0 = 100$) are used. The three other models with notable probabilities larger than 10% include stochastic volatility in either potential output, trend inflation or in the NAIRU. As the variance of the prior A_0 increases, the probabilities of these models shrink towards zero.

For completeness, Figure 3.2 shows the evolution of the four components for which the time-variation does not show up as relevant using the model selection. These components will be restricted to be constant in the remainder of this paper. The evolution of the significant time-varying components is discussed more in detail in below.

Table 3.4: Posterior model probabilities over different prior variances A_0 (with $p_0 = 0.5$)

Model								Posterior probability			
δ_π	δ_u	λ	θ_y	θ_π	θ_u	θ_c	θ_ζ	$A_0 = 0.1$	$A_0 = 1$	$A_0 = 10$	$A_0 = 100$
0	1	1	0	0	0	1	1	0.4454	0.6415	0.8620	0.9180
0	1	1	0	0	1	1	1	0.1976	0.1334	0.0864	0.0395
0	1	1	0	1	0	1	1	0.0888	0.1080	0.0228	0.0183
0	1	1	1	0	0	1	1	0.1290	0.0555	0.0223	0.0208
0	1	1	0	1	1	1	1	0.0438	0.0342	0.0034	0.0023
0	1	1	1	1	0	1	1	0.0192	0.0121	0.0001	0.0000
0	1	1	1	0	1	1	1	0.0502	0.0111	0.0023	0.0011

3.3.4 Parameter estimates and unobserved components

In this section we present the results of the model that is favored by the stochastic model selection. We will refer to this as the parsimonious model. As a convergence check we plot the 20^{th}-order autocorrelations for all parameter and component draws in Figure 3.3. This diagnostic has been used before in Primiceri (2005) and Liu and Morley (2014). The majority of autocorrelations lie well below 0.1, while for a few parameters we find values between 0.2 and 0.3. Only one value is as high as 0.5. We take this as evidence for satisfactory convergence of the Markov-Chain.

The posterior distributions of the parsimonious model's time-invariant parameters are plotted in Figure 3.4. Descriptive statistics are given in Table 3.5. For the standard deviations of the non-centered variables the posterior distributions are bimodal. Thus, we report descriptive statistics on the unimodal posterior of the respective squared standard deviation parameters. The evolution of the unobserved components is shown in Figures 3.5-3.9 and discussed more in detail below.

Inflation

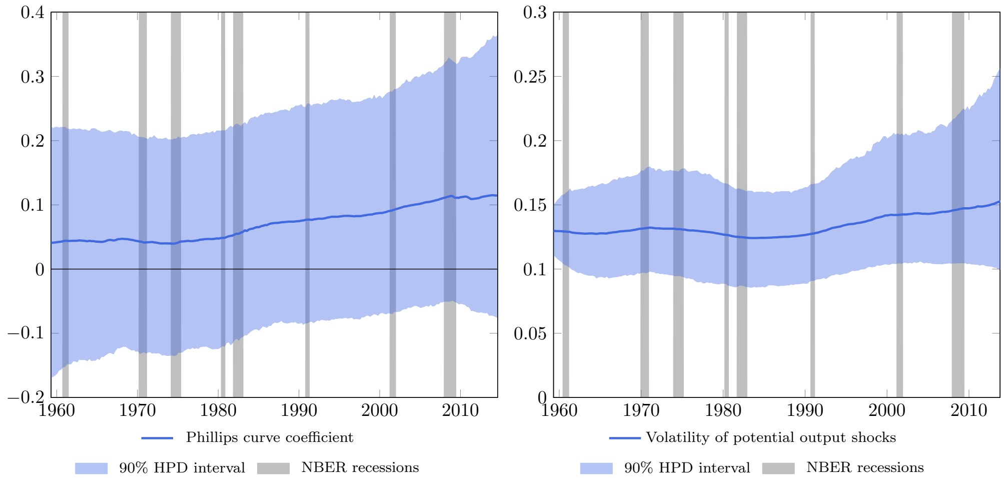
Figure 3.5 plots actual inflation against the median of the posterior distribution of trend inflation and its 90% highest posterior density (HPD) interval for the parsimonious model. Trend inflation evolves smoothly and tracks the low-frequency movements in observed inflation. It steadily rises over the Great Inflation period from the late 1960s until the late 1970s and then

Note that the posterior distributions of the standard deviations for the non-centered variables in the unrestricted model are reported in Figure 3.1. As there is no noticeable difference in these distributions in the parsimonious model, they are not included in Figure 3.4.

Table 3.5: Posterior distributions of model parameters (parsimonious model)

Inverse Gamma			Percentiles	
		median	2.5%	97.5%
idiosyncratic component output	$\sigma_{\varepsilon,y}$	0.122	0.091	0.168
idiosyncratic component inflation	$\sigma_{\varepsilon,\pi}$	0.544	0.492	0.607
idiosyncratic component unemployment	$\sigma_{\varepsilon,u}$	0.239	0.217	0.266
Gaussian			Percentiles	
Regression parameters		median	2.5%	97.5%
const. Phillips curve slope	β_0^π	0.067	−0.061	0.196
const. Okun coefficient	β_0^u	−0.438	−0.525	−0.350
1st AR lag: output gap	ρ_1	1.299	1.166	1.418
sum of AR lags: output gap	$\rho_1 + \rho_2$	0.942	0.922	0.963
AR lag: AR(1) inflation component	ϱ	0.759	0.676	0.846
const. output drift	κ_0	1.023	0.933	1.118
Stochastic volatility parameters				
const. volatility of potential output	$\exp\{h_0^y\}$	0.131	0.113	0.152
const. volatility of trend inflation	$\exp\{h_0^\pi\}$	0.208	0.181	0.239
const. volatility of NAIRU	$\exp\{h_0^u\}$	0.079	0.070	0.089
const. volatility of output gap	$\exp\{h_0^c\}$	0.582	0.519	0.646
const. volatility of temporary inflation	$\exp\left\{h_0^\zeta\right\}$	0.505	0.441	0.579
Non-centered components				
variance of time-varying Okun coefficient	$\sigma^2_{\eta,u}$	0.0016	0.0009	0.0029
variance of SV: output gap	$\sigma^2_{\gamma,c}$	0.0350	0.0131	0.0757
variance of SV: AR(1) inflation component	$\sigma^2_{\gamma,\zeta}$	0.0155	0.0053	0.0397
variance of time-varying output drift	σ^2_{κ}	0.0003	0.0001	0.0007

Figure 3.2: Evolution of the time-varying components not selected by the model search



(a) Phillips curve (b) SV potential output

(c) SV trend inflation (d) SV NAIRU

Figure 3.3: 20^{th}-order autocorrelations of all parameter and component draws

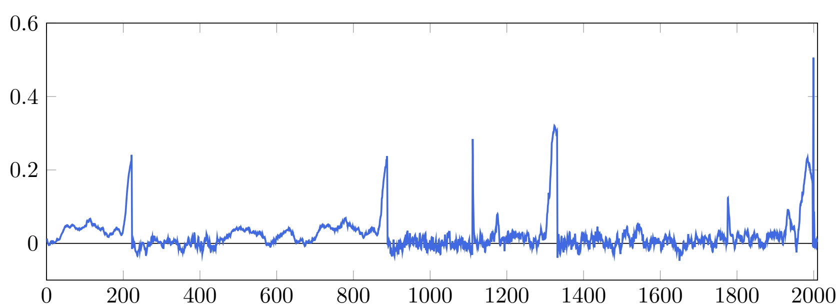
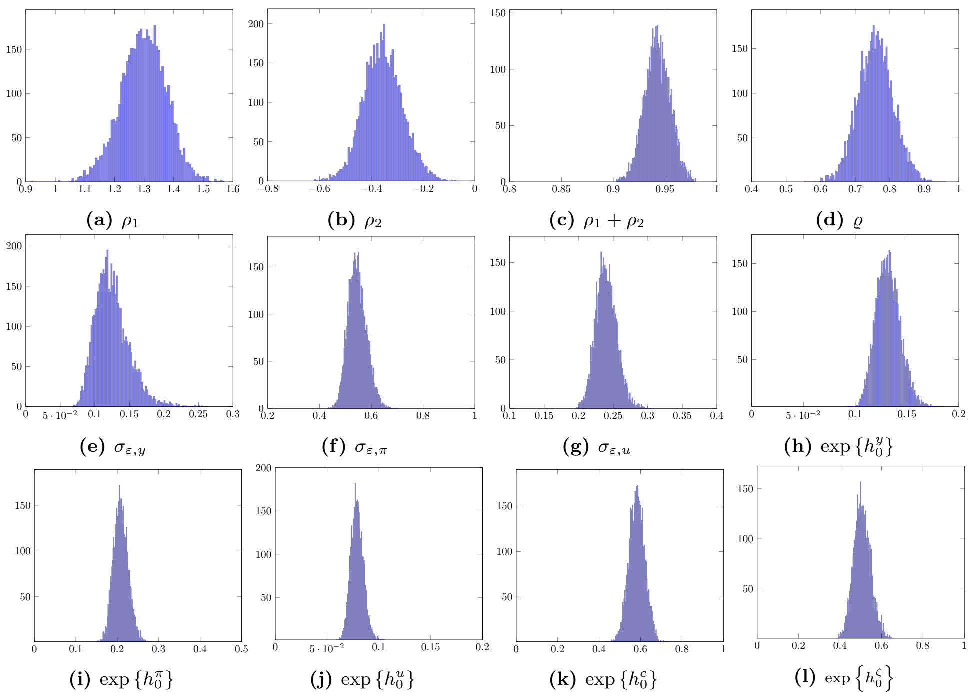


Figure 3.4: Posterior distributions of parameters (parsimonious model)



(a) ρ_1 **(b)** ρ_2 **(c)** $\rho_1 + \rho_2$ **(d)** ϱ

(e) $\sigma_{\varepsilon,y}$ **(f)** $\sigma_{\varepsilon,\pi}$ **(g)** $\sigma_{\varepsilon,u}$ **(h)** $\exp\{h_0^y\}$

(i) $\exp\{h_0^\pi\}$ **(j)** $\exp\{h_0^u\}$ **(k)** $\exp\{h_0^c\}$ **(l)** $\exp\{h_0^\zeta\}$

falls back during the disinflation period of the 1980s and 1990s. Since the late 1990s trend inflation remains low and stable at around 2%. Our estimated trend inflation series is very similar to those reported by Cogley and Sbordone (2008), Kim, Manopimoke, and Nelson (2014) and Stella and Stock (2012). The variance of innovations to trend inflation was found to be constant over time by the model selection procedure and is estimated with a posterior median of 0.21. This result is consistent with Kim, Manopimoke, and Nelson (2014) who find similar values over three distinct regimes and could not reject the null of constant volatility for trend inflation. It contrasts with Stock and Watson (2007) who find considerable variability in the variance of innovations to trend inflation. However, they do not allow for a persistent transitory component (ζ_t in our model) in the inflation gap such that trend inflation has to incorporate this component. Similar to the recent literature, we find that the inflation gap and idiosyncratic shocks are the most important driver of inflation. On average, they account for more than 90% of the variance of inflation changes at the one-quarter horizon. The inflation gap itself is driven by the output gap y_t^c but more importantly by the persistent AR(1) component ζ_t. As can be seen from Figure 3.6, ζ_t and its stochastic volatility peak in the 1970s. Inflation is not very sensitive to the output gap. As shown in panel (a) of Figure 3.7, we estimate the slope of the (time-invariant) Phillips curve to be very small with a posterior

In fact when we drop ζ_t, the model selection procedure selects a specification with stochastic volatility for trend inflation (results not reported).

median of 0.07 and a 90% HPD interval ranging from -0.06 to 0.20. This finding confirms Kim, Manopimoke, and Nelson (2014) but contrasts with Morley, Piger, and Rasche (2015) who find the real activity gap, as measured by the unemployment gap, to be an important driver of the inflation gap.

Our estimates shed light on a number of important episodes of U.S. monetary economic history. First, the Great Inflation of the late 1960s and the 1970s is reflected in a prolonged rise in trend inflation combined with an increase in the level and volatility of the temporary inflation component ζ_t. In our model, the latter captures the variation in inflation that is not explained by the conventional forward-looking Phillips curve. From our estimates, this component mainly seems to capture the extent to which the oil price shocks of 1973-74 and 1979-80 drove up inflation without increasing inflation expectations or being reflected in the output gap. Second, the aggressive disinflation strategy pursued by Paul Volcker when he became chairman of the Federal Reserve resulted in a steady but strong decline in trend inflation. Together with the sudden drop in the temporary inflation component, due to a drop in oil prices, this resulted in a sharp decline in realized inflation. The impact of the disinflation strategy on output depends on the credibility of monetary policy (see e.g. Ball, 1994). Imperfect credibility raises the output cost of reducing inflation. Our results point to a large output gap in the beginning of the disinflation period. In line with the small and stable slope of the Phillips curve, this is accompanied by only moderate negative deviations of realized inflation from its trend. This pattern changes during the second half of the disinflationary period, where the output gap decreases and realized inflation tracks trend inflation more closely. We take this as evidence that the credibility of the FED improved over time. This explanation is in line with the findings of Goodfriend and King (2005), who build a model with imperfect credibility, i.e. the FED acquires credibility over time as agents change their beliefs about whether the new policy regime is permanent. According to the authors, the initial real effects of the Volcker disinflation were mainly due to its imperfect credibility. Third, our results also contribute to the discussion on the missing deflation puzzle during the Great Recession. Specifically, this paper casts doubt on the existence of such a puzzle as the link between inflation and real activity is weak over the full sample. The fact that actual inflation does not deviate substantially from trend inflation is therefore consistent with a relatively large output gap.

Figure 3.5: Trend inflation (parsimonious model)

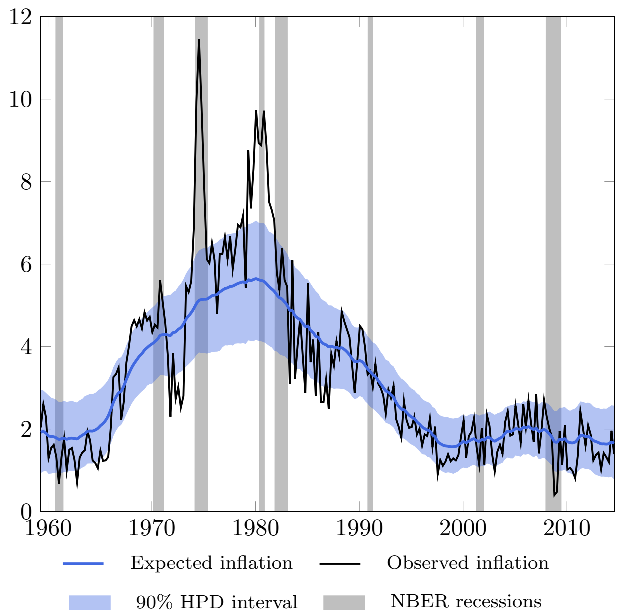
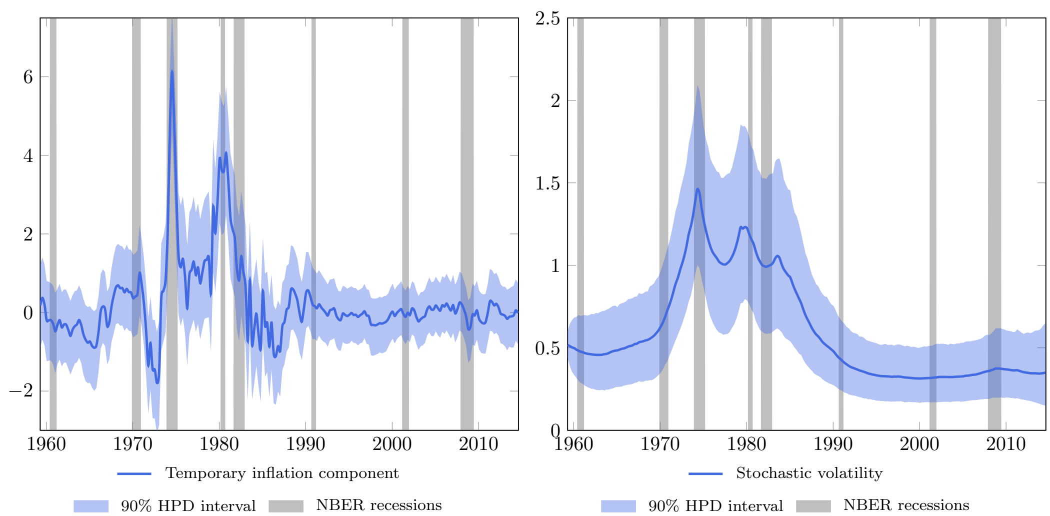


Figure 3.6: AR(1) inflation component (parsimonious model)



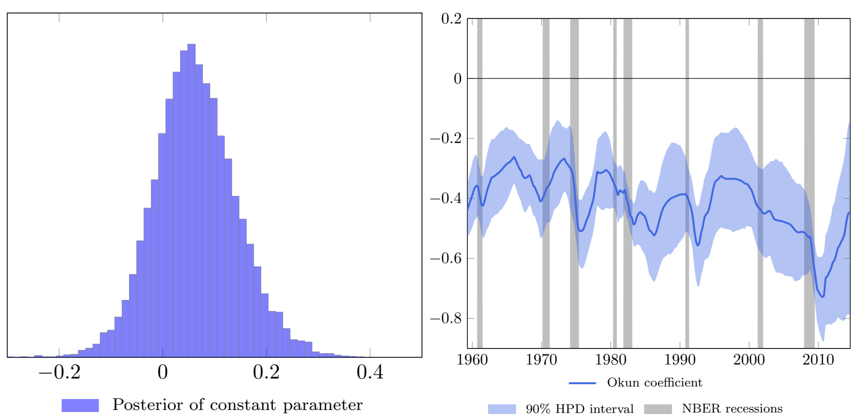
(a) AR(1) inflation component (ζ_t)

(b) Stochastic volatility of ζ_t

Output

Figure 3.8 plots the posterior results for the various components in output. Potential output, depicted in panel (a), is estimated as a smooth upward trend that tracks the low frequency movements in U.S. real GDP. The constant volatility of shocks to the level of potential output is found to be small with a posterior median of 0.13, while the drift in potential output, depicted in panel (b), exhibits substantial time variation. The downward trend in the drift term implies potential output growth to slow down from around 4% on an annual base in the early 1960s to about 1.6% at the end of the sample. This overall movement is highly consistent with the CBO's estimates for potential output growth, although this series is more volatile. The first sizable drop in potential output growth is in the early to mid 1970s. This is a well-known feature of the data generally referred to as the great productivity slowdown. From the late 1970s to the early 2000s potential output growth varies around an annual rate of 3%. The second sizable drop occurred during the 2000s with the most recent estimates pointing to a pessimistic scenario where slow growth is the 'new normal'. Our results support Perron and Wada (2009) who highlight the importance of accounting for breaks in potential output growth for UC models. By analyzing data from 1947 to 1998 they find one break in 1973.

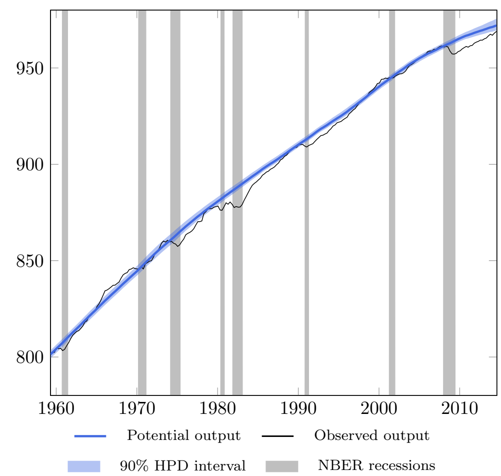
Figure 3.7: Phillips curve and Okun's Law (parsimonious model)



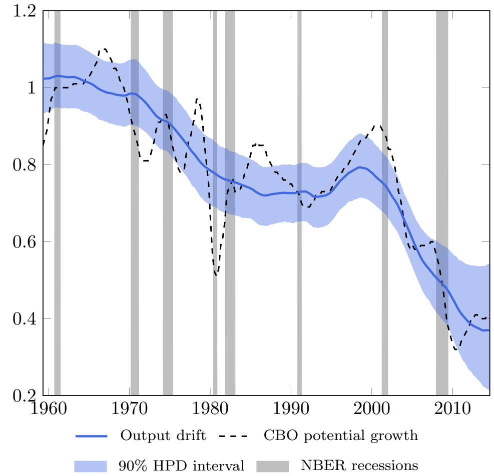
(a) Phillips curve β_t^π (b) Okun's Law β_t^u

Panel (c) of Figure 3.8 shows the estimated output gap together with the CBO gap. Both series evolve very similar and are able to identify the recession periods as dated by the NBER. A somewhat sizable difference in the level of the two series is observed during the 1980s. This is due to the fact that our model attributes most of the variation in real GDP during the early 1980s to cyclical shocks while the CBO assigns a larger fraction to potential output growth-related shocks as visualized by the sharp drop in the CBO potential growth series displayed in panel (b) in that period. The Great Moderation shows up in panel (d) as a considerable drop in the stochastic volatility of innovations to the output gap in the 1980s and a low volatility period that continued until 2007. During the Great Recession, volatility increases considerably but has not led to a permanent increase as it returns almost to its pre-crisis level in 2009.

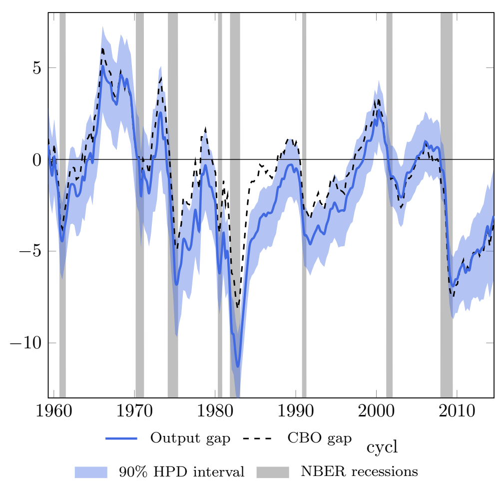
Figure 3.8: Output components (parsimonious model)



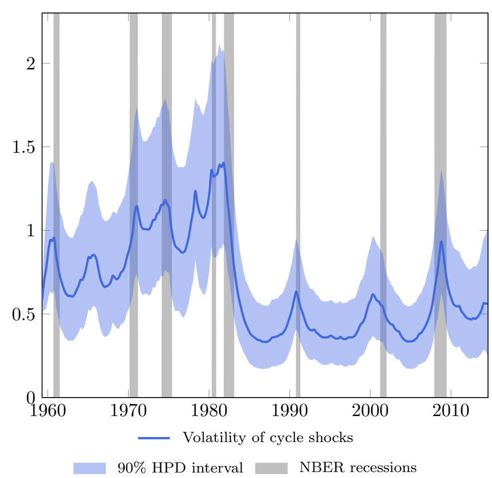
(a) Potential output



(b) Output growth



(c) Output gap



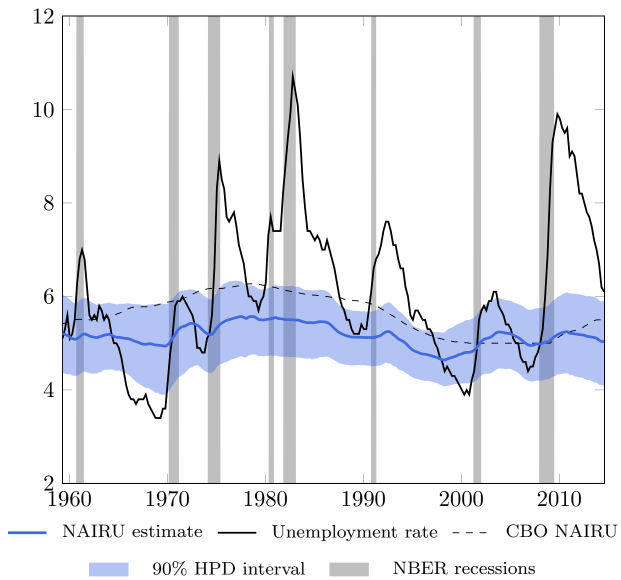
(d) Stochastic volatility of output gap

Unemployment

The NAIRU is shown in Figure 3.9 along with the CBO's NAIRU estimate and the actual unemployment rate. We find that the NAIRU evolves very smoothly over time which implies that most of the variations in unemployment are assigned to cyclical (demand-related) factors. However, the recent decline in the unemployment rate may also partially be driven by people exiting the labor force, possibly discouraged jobless workers. Our NAIRU estimate is consistent with Laubach (2001) and Basistha and Startz (2008). Similar to the latter study, our multivariate model results in a relatively narrow 90% HPD interval for the NAIRU.

Regarding the relation between the output gap and the unemployment gap, we find sub-

Figure 3.9: NAIRU (parsimonious model)



stantial time variation in Okun's Law parameter as displayed in Figure 3.7, panel (b). The time variation captures both changes at business cycle frequency and long-run changes, which is similar to the findings of Knotek (2007). We find that the sensitivity of the unemployment rate to cyclical output is higher during recessions than during recoveries. This asymmetric pattern holds for most of the postwar business cycles, except for the period after the 2001 recession, over which labor market sensitivity continues to increase. Before this turning point, the Okun coefficient fluctuates around a value of -0.4, suggesting that a positive output gap of 1% is associated with a negative deviation of the unemployment rate from the NAIRU of -0.4%. Since the 2001 recession, the Okun coefficient has decreased to roughly -0.7 in the Great Recession. Recently the Okun coefficient has quickly returned back to the historical average. According to the reasoning in Daly and Hobijn (2010), the spike in the Okun coefficient around the year 2009 can be explained by a surge in labor productivity, accompanied by a reduction in employment and hours worked which led to a break in the pattern between unemployment and output as observed over the past 60 years.

There exist several explanations for why the correlation between output and unemployment may depend on the business cycle stance. Starting from a microeconomic model, Campbell and Fisher (2000) explain how asymmetries in firms' adjustment costs can lead to asymmetric job creation and destruction rates at the macro level. Palley (1993) focuses on the aggregate labor market and explains the negative excess sensitivity of cyclical unemployment to cyclical output with sectoral shifts and changing behavior of female labor force participants.

Silvapulle, Moosa, and Silvapulle (2004) offer an explanation based on over-pessimistic firm behavior. If bad news is believed more quickly than good news, firms tend to adjust the workforce relatively quick in recessions, but are reluctant to hire during recoveries. The authors argue that such behavior leads to asymmetry in the Okun coefficient typically found in U.S. data. Moreover, our findings are in line with the literature on insider-outsider models pioneered by Lindbeck and Snower (1988) and Blanchard and Summers (1986). After a cyclical rise in unemployment, the remaining workers (so-called insiders) may demand higher wages during the following recovery due to labor turnover costs. Instead of creating new jobs for the unemployed workers (so-called outsiders), economic recovery translates into higher insider wages. Such behavior gives rise to asymmetry in the Okun coefficient, leading to persistent cyclical unemployment.

Our estimates also contribute to the discussion on jobless recoveries in the United States. We do not find that the business cycle sensitivity of the Okun coefficient has changed in the 1990s. Rather, our results suggest that recoveries have always been 'jobless' in the sense that the unemployment rate adjusts faster during recessions than during recoveries. The argument that the unemployment rate has become less sensitive to output growth over time is not supported by our model. However, the notion of 'jobless recoveries' is typically related to job growth and thus makes a statement about employment dynamics. The estimated Okun's Law coefficient reflects sensitivity of the unemployment rate and is sensitive to changes in labor force participation. Slower than average job growth could be counteracted by decreasing labor force participation, leaving the unemployment rate and therefore also the Okun's Law coefficient unchanged.

In sum, we find substantial time variation in various model's parameters. There is a sizable reduction in the volatility of output gap shocks and inflation gap shocks. We also find a significant decline in potential output growth in the 1970s and even more pronounced in the 2000s. Moreover, there is time variation in the Okun's Law parameter with unemployment being more sensitive to the output gap in recessions than in expansions.

3.4 Model extensions and robustness checks

In this section we check the robustness of our results along several dimensions. First, we replace the NKPC by a backward-looking Phillips curve in order to see if our findings regarding the inflation dynamics and the stability of β^{π} depend on the forward-looking specification of the Phillips curve. Second, we use the unemployment gap, instead of the output gap, as a measure of real activity in the Phillips curve.

3.4.1 Backward-looking Phillips curve

As described in Section 3.1, the literature provides mixed support for the NKPC in empirical applications. In contrast to the theoretical foundations, some studies find an important backward-looking component in inflation dynamics, i.e. inflation depends on its own lagged values. We therefore check for the robustness of our findings by replacing equation (3.14) by the following backward-looking Phillips curve specification

$$\pi_t = \sum_{p=1}^{4} b_p \pi_{t-p} + \beta_t^\pi \widetilde{y}_t^c + \varepsilon_t^\pi, \tag{3.32}$$

where the sum of the coefficients on lagged inflation is assumed to be one. The model is identical to the baseline model except for the absence of a stochastic trend in inflation and the temporary inflation component ζ. This specification matches standard backward-looking Phillips curve models in the literature. Table 3.7 gives the posterior probability of time variation in the slope of the Phillips curve for both the baseline and the backward-looking model. The Phillips curve is found to be stable in both specifications. Moreover, the finding is robust to different prior distributions for the degree of time variation. In all settings, the posterior probability of a time-varying Phillips curve are in the area of 1% or below. However, the estimated slope coefficients differ between the forward and the backward-looking model. Figure 3.10 plots the posterior distributions of the time-invariant slope parameter for both models. The probability mass of the coefficient in the backward-looking model is strictly positive and has a slightly higher median. This is in line with the literature that usually finds a bigger and more significant Phillips curve slope in backward compared to purely forward-looking models.

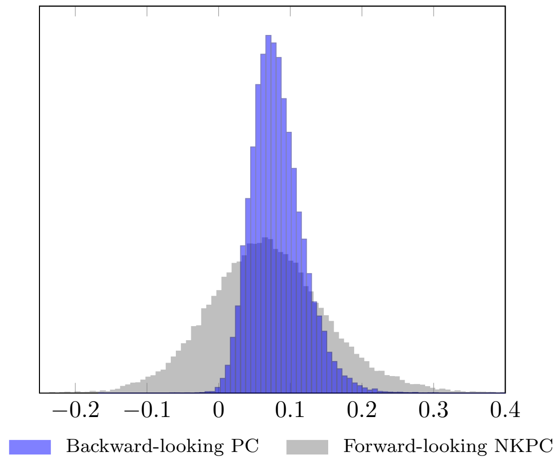
3.4.2 Alternative inflation measures

In the empirical literature on the Phillips curve, no single preferred inflation measure has emerged. This paper focuses on the core PCE inflation series, since it eliminates large outliers associated with energy price fluctuations as pointed out by Stock and Watson (2010). However, other inflation measures have been used repeatedly in the literature such as core and headline CPI inflation or the implicit GDP deflator. We check the robustness of our results by estimating the model for different inflation measures but leaving all prior distributions unchanged. Columns 2-4 in Table 3.6 report the estimated posterior distribution of the constant

Results are nearly identical when the unit-root assumption is relaxed. The sum of the coefficients in the unrestricted case is close to one.

Among many others see e.g. Rudd and Whelan (2005) and Ball and Mazumder (2011).

Figure 3.10: Posterior distributions of β_0^π: Backward and Forward-looking



Phillips curve slope. The median slope estimates range between 0.067 for the GDP deflator and 0.110 for headline CPI and headline PCE inflation. In all five specifications the 95% credible interval covers positive and negative values. Thus, the slope of the Phillips curve is found to be more or less flat regardless of the inflation measure used. Columns 5-8 report the posterior inclusion probabilities of a time-varying Phillips curve and the time-varying volatilities in trend and temporary inflation. Again, findings are robust to the different inflation measures. The posterior probability of time-variation in the Phillips curve is below 1% in all cases. Results differ more for the stochastic volatility component in trend inflation. However, probabilities remain below 5% except for the baseline measure and thus no evidence of time-varying shocks to trend inflation is found. Finally, stochastic volatility is always included in the temporary inflation component. We conclude that our findings are robust to alternative measures of price inflation.

3.4.3 Unemployment gap instead of output gap

The baseline model finds a constant Phillips curve slope and a time-varying Okun's Law parameter. Consequently, when we replace the output gap by the unemployment gap in the Phillips curve, the impact of unemployment on inflation is time-varying. However, when we estimate the model with the unemployment gap in the Phillips curve, the model selection procedure rejects a time-varying slope parameter. We believe that this is due to the fact that the real activity measure, independently on weather we proxy it by the output gap or the unemployment gap, has little impact on inflation. Thus, all conclusions drawn remain

Estimates for the output and unemployment components do not change notably, but are available on request.

Table 3.6: Robustness of parameter estimates to different inflation measures

	Posterior parameter distribution			Posterior inclusion probability		
	Slope coefficient β_0^π			Phillips curve	SV trend inflation	SV temp. inflation
	median	2.5%	97.5%	δ_π	θ_π	θ_ζ
CPI	0.110	-0.080	0.303	0.0070	0.0303	1.0000
CPI excl. F&E	0.094	-0.032	0.245	0.0080	0.0430	1.0000
PCE	0.110	-0.043	0.272	0.0026	0.0378	1.0000
PCE excl. F&E	0.067	-0.061	0.196	0.0026	0.0828	1.0000
GDP deflator	0.076	-0.072	0.235	0.0024	0.0476	1.0000

Priors are set to $p_0 = 0.5$ and $A_0 = 1$.

Table 3.7: Stability of Phillips curve for different models

Prior		Posterior probability of $\delta_\pi = 1$	
p_0	A_0	New-Keynesian	Backward-looking
0.5	0.1	0.0110	0.0040
0.5	1	0.0026	0.0012
0.5	10	0.0007	0.0008
0.5	100	0.0000	0.0004

unchanged when replacing the output gap by the unemployment gap.

3.5 Conclusion

We have investigated the degree of time variation in the parameters of a multivariate unobserved components model designed for the U.S. economy over the period 1959 to 2014. The empirical model decomposes real GDP, inflation and the unemployment rate into a common stochastic cyclical factor and their respective stochastic trends and idiosyncratic components. Key parameters such as the growth rate of potential output, the slope of the Phillips curve, Okun's Law coefficient as well as all variance parameters are allowed to vary over time. Importantly, while allowing for time variation the priors in the Bayesian estimation strategy are set to nest the case that the parameters are actually time-invariant. In a first estimation step, a stochastic model selection procedure is employed to test which parameters are time-varying. We find that potential output growth, Okun's Law coefficient, the variance of innovations to the output gap and to a persistent inflation gap component are time-varying while the slope of the Phillips curve and the variances of innovations to all trend components are time-invariant. Our estimation result show a clear decrease in potential output growth, which can be characterized by substantial drops in the 1970s and the 2000s. Okun's Law coefficient is found to be lower in recessions than in expansions, i.e. unemployment is more sensitive to the output gap in a downturn and reacts less sensitive in a recovery. With regard to the dynamics of inflation, we find that the inflation gap and idiosyncratic shocks are the major determinants of inflation changes. However, the inflation gap is not very sensitive to the output gap but is driven by a persistent AR(1) component. The latter component exhibits stochastic volatility and mainly captures the large and persistent swings in inflation during the inflationary period in the 1970s and 1980s.

Bibliography

ATKESON, A., AND L. E. OHANIAN (2001): "Are Phillips Curves Useful for Forecasting Inflation?," *Quarterly Review*, Winter, 2–11.

BALL, L. (1994): "Credible Disinflation with Staggered Price-Setting," *American Economic Review*, 84(1), 282–289.

BALL, L., AND S. MAZUMDER (2011): "Inflation Dynamics and the Great Recession," *Brookings Papers on Economic Activity*, 42(1), 337–405.

BALL, L. M., D. LEIGH, AND P. LOUNGANI (2013): "Okun's Law: Fit at Fifty?," NBER Working Papers 18668.

BASISTHA, A., AND R. STARTZ (2008): "Measuring the NAIRU with Reduced Uncertainty: A Multiple-Indicator Common-Cycle Approach," *The Review of Economics and Statistics*, 90(4), 805–811.

BEVERIDGE, S., AND C. R. NELSON (1981): "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'," *Journal of Monetary Economics*, 7(2), 151–174.

BLANCHARD, O. J., AND L. H. SUMMERS (1986): "Hysteresis And The European Unemployment Problem," in *NBER Macroeconomics Annual 1986, Volume 1*, NBER Chapters, pp. 15–90. National Bureau of Economic Research, Inc.

CALVO, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

CAMPBELL, J. R., AND J. D. M. FISHER (2000): "Aggregate Employment Fluctuations with Microeconomic Asymmetries," *The American Economic Review*, 90(5), pp. 1323–1345.

CARTER, C., AND R. KOHN (1994): "On Gibbs Sampling for State Space Models," *Biometrika*, 81(3), 541–53.

Chan, J. C. C., G. Koop, and S. M. Potter (2015): "A Bounded Model of Time Variation in Trend Inflation, Nairu and the Phillips Curve," *Journal of Applied Econometrics*.

Cogley, T., and A. M. Sbordone (2008): "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review*, 98(5), 2101–2126.

Daly, M., and B. Hobijn (2010): "Okun's Law and the Unemployment Surprise of 2009," *FRBSF Economic Letter*, (mar8).

Daly, M. C., B. Hobijn, A. Şahin, and R. G. Valletta (2012): "A Search and Matching Approach to Labor Markets: Did the Natural Rate of Unemployment Rise?," *Journal of Economic Perspectives*, 26(3), 3–26.

De Jong, P., and N. Shephard (1995): "The Simulation Smoother for Time Series Models," *Biometrika*, 82(2), 339–50.

Del Negro, M., and G. Primiceri (2014): "Time-Varying Structural Vector Autoregressions and Monetary Policy: A Corrigendum," Staff Reports 619, Federal Reserve Bank of New York.

Fernández-Villaverde, J., and J. Rubio-Ramírez (2010): "Macroeconomics and Volatility: Data, Models, and Estimation," NBER Working Papers 16618.

Fleischman, C. A., and J. M. Roberts (2011): "From Many Series, One Cycle: Improved Estimates of the Business Cycle from a Multivariate Unobserved Components Model," Discussion Paper 2011-46, Board of Governors of the Federal Reserve System (U.S.).

Frühwirth-Schnatter, S., and H. Wagner (2010): "Stochastic Model Specification Search for Gaussian and Partially Non-Gaussian State Space Models," *Journal of Econometrics*, 154(1), 85–100.

Fuhrer, J., and G. Moore (1995): "Inflation Persistence," *The Quarterly Journal of Economics*, 110(1), 127–159.

George, E., and R. McCulloch (1993): "Variable Selection via Gibbs Sampling," *Journal of the American Statistical Association*, 88(423), 881–889.

Goodfriend, M., and R. G. King (2005): "The Incredible Volcker Disinflation," *Journal of Monetary Economics*, 52(5), 981–1015.

——— (2012): "The Great Inflation Drift," in *The Great Inflation: The Rebirth of Modern Central Banking*, ed. by M. D. Bordo, and A. Orphanides, pp. 181–216. University of Chicago Press.

Hall, R. E. (2011): "The Long Slump," Discussion paper, National Bureau of Economic Research.

Hamilton, J. D. (2008): "Macroeconomics and ARCH," NBER Working Papers 14151.

Kim, C.-J., P. Manopimoke, and C. R. Nelson (2014): "Trend Inflation and the Nature of Structural Breaks in the New Keynesian Phillips Curve," *Journal of Money, Credit and Banking*, 46(2-3), 253–266.

Kim, S., N. Shephard, and S. Chib (1998): "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models," *Review of Economic Studies*, 65(3), 361–393.

Knotek, E. S. (2007): "How Useful is Okun's Law?," Discussion Paper Q IV, Federal Reserve Bank of Kansas City.

Kuttner, K. N. (1994): "Estimating Potential Output as a Latent Variable," *Journal of Business & Economic Statistics*, 12(3), 361–368.

Laubach, T. (2001): "Measuring the Nairu: Evidence from Seven Economies," *The Review of Economics and Statistics*, 83(2), 218–231.

Lee, J. (2000): "The Robustness of Okun's Law: Evidence from OECD Countries," *Journal of Macroeconomics*, 22(2), 331–356.

Lindbeck, A., and D. J. Snower (1988): "Cooperation, Harassment, and Involuntary Unemployment: An Insider-Outsider Approach," *American Economic Review*, 78(1), 167–88.

Liu, Y., and J. Morley (2014): "Structural Evolution of the Postwar U.S. Economy," *Journal of Economic Dynamics and Control*, 42(C), 50–68.

Luo, S., and R. Startz (2014): "Is It One Break or Ongoing Permanent Shocks That Explains U.S. Real GDP?," *Journal of Monetary Economics*, 66(C), 155–163.

Mankiw, N. G. (2001): "The Inexorable and Mysterious Tradeoff between Inflation and Unemployment," *The Economic Journal*, 111(471), 45–61.

Mavroeidis, S., M. Plagborg-Møller, and J. H. Stock (2014): "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature*, 52(1), 124–188.

Mishkin, F. S. (2007): "Inflation Dynamics," *International Finance*, 10(3), 317–334.

Mitra, S., and T. M. Sinclair (2012): "Output Fluctuations In The G-7: An Unobserved Components Approach," *Macroeconomic Dynamics*, 16(3), 396–422.

Morley, J., J. M. Piger, and R. H. Rasche (2015): "Inflation in the G7: Mind the Gap(s)?," *Macroeconomic Dynamics*, 19(04), 883–912.

Morley, J. C., C. R. Nelson, and E. Zivot (2003): "Why are the Beveridge-Nelson and Unobserved-Components Decompositions of GDP So Different?," *The Review of Economics and Statistics*, 85(2), 235–243.

Omori, Y., S. Chib, N. Shephard, and J. Nakajima (2007): "Stochastic volatility with leverage: Fast and efficient likelihood inference," *Journal of Econometrics*, 140(2), 425–449.

Owyang, M. T., and T. Sekhposyan (2012): "Okun's Law over the Business Cycle: Was the Great Recession All That Different?," *Federal Reserve Bank of St. Louis Review*, (September), 399–418.

Palley, T. I. (1993): "Okun's Law and the Asymmetric and Changing Cyclical Behaviour of the USA Economy.," *International Review of Applied Economics*, 7(2), 144–62.

Perron, P., and T. Wada (2009): "Let's Take a Break: Trends and Cycles in US Real GDP," *Journal of Monetary Economics*, 56(6), 749–765.

Primiceri, G. E. (2005): "Time Varying Structural Vector Autoregressions and Monetary Policy," *Review of Economic Studies*, 72(3), 821–852.

Reifschneider, D., W. Wascher, and D. Wilcox (2013): "Aggregate Supply in the United States: Recent Developments and Implications for the Conduct of Monetary Policy," Finance and Economics Discussion Series 2013-77, Board of Governors of the Federal Reserve System (U.S.).

Roberts, J. M. (2006): "Monetary Policy and Inflation Dynamics," *International Journal of Central Banking*, 2(3), 193–230.

Rudd, J., and K. Whelan (2005): "New Tests of the New-Keynesian Phillips Curve," *Journal of Monetary Economics*, 52(6), 1167–1181.

——— (2007): "Modeling Inflation Dynamics: A Critical Review of Recent Research," *Journal of Money, Credit and Banking*, 39(s1), 155–170.

Scott, J. G., J. O. Berger, et al. (2010): "Bayes and Empirical-Bayes Multiplicity Adjustment in the Variable-Selection Problem," *The Annals of Statistics*, 38(5), 2587–2619.

Silvapulle, P., I. Moosa, and M. Silvapulle (2004): "Asymmetry in Okun's law," *Canadian Journal of Economics*, 37(2), 353–374.

Sinclair, T. M. (2009): "The Relationships between Permanent and Transitory Movements in U.S. Output and the Unemployment Rate," *Journal of Money, Credit and Banking*, 41(2-3), 529–542.

Staiger, D., J. H. Stock, and M. W. Watson (1997): "The NAIRU, Unemployment and Monetary Policy," *Journal of Economic Perspectives*, 11(1), 33–49.

Stella, A., and J. H. Stock (2012): "A State-Dependent Model for Inflation Forecasting," International Finance Discussion Papers 2012-1062, Board of Governors of the Federal Reserve System (U.S.).

Stock, J. H., and M. W. Watson (2007): "Why Has U.S. Inflation Become Harder to Forecast?," *Journal of Money, Credit and banking*, 39(s1), 3–33.

Stock, J. H., and M. W. Watson (2010): "Modeling Inflation After the Crisis," NBER Working Papers 16488, National Bureau of Economic Research, Inc.

Woodford, M. (2008): "How Important Is Money in the Conduct of Monetary Policy?," *Journal of Money, Credit and Banking*, 40(8), 1561–1598.

Appendix

3.A Gibbs sampling algorithm

In this appendix we provide details on the Gibbs sampling algorithm used in Subsection 3.2.3 to jointly sample the binary indicators $\mathcal{M}$, the hyperparameters ϕ, the trend and temporary components α, the time-varying parameters β, the mixture indicators ι and the stochastic volatilities h. The structure of our Gibbs sampling approach is based on Frühwirth-Schnatter and Wagner (2010).

Block 1: Sampling the binary indicators $\mathcal{M}$ and the parameters ϕ

For notational convenience, let us define a general regression model

$$w = z^{\mathcal{M}} b^{\mathcal{M}} + e, \qquad e \sim \mathcal{N}(0, \Sigma), \tag{3.33}$$

with w a vector including observations on a dependent variable w_t and z an unrestricted predictor matrix with rows z_t that contain the state processes from the vectors α_t, β_t and h_t that are relevant for explaining w_t. The corresponding unrestricted parameter vector with the relevant elements from ϕ is denoted b. $z^{\mathcal{M}}$ and $b^{\mathcal{M}}$ are then the restricted predictor matrix and restricted parameter vector that exclude those elements in z and b for which the corresponding indicator in $\mathcal{M}$ is 0. Furthermore, Σ is a diagonal matrix with elements $\sigma^2_{e,t}$ that may vary over time to allow for heteroskedasticity of a known form.

A naive implementation of the Gibbs sampler would be to sample $\mathcal{M}$ from $f(\mathcal{M} | \alpha, \beta, h, \phi, w)$ and ϕ from $f(\phi | \alpha, \beta, h, \mathcal{M}, w)$. However, this approach does not result in an irreducible Markov chain as whenever an indicator in $\mathcal{M}$ equals zero, the corresponding coefficient in ϕ is also zero which implies that the chain has absorbing states. Therefore, as in Frühwirth-Schnatter and Wagner (2010) we marginalize over the parameters ϕ when sampling $\mathcal{M}$ and next draw the parameters ϕ conditional on the indicators $\mathcal{M}$. The posterior distribution

$f(\mathcal{M}\,|\alpha,\beta,h,w)$ can be obtained using Bayes' Theorem as

$$f(\mathcal{M}\,|\alpha,\beta,h,w) \propto f(w\,|\mathcal{M},\alpha,\beta,h)\, p(\mathcal{M}), \tag{3.34}$$

with $p(\mathcal{M})$ being the prior probability of $\mathcal{M}$ and $f(w\,|\mathcal{M},\alpha,\beta,h)$ being the marginal likelihood of the regression model (3.33) where the effect of the parameters $b^{\mathcal{M}}$ and σ_e^2 has been integrated out. The closed form solution of the marginal likelihood depends on whether the error term e_t is homoskedastic or heteroskedastic. More specifically:

- In the homoskedastic case $\Sigma = \sigma_e^2 I_T$, under the normal-inverse gamma conjugate prior

$$b^{\mathcal{M}} \sim \mathcal{N}\left(a_0^{\mathcal{M}}, A_0^{\mathcal{M}}\sigma_e^2\right), \qquad \sigma_e^2 \sim IG(c_0, C_0), \tag{3.35}$$

the closed form solution for $f(w\,|\mathcal{M},\alpha,\beta,h)$ is

$$f(w\,|\mathcal{M},\alpha,\beta,h) \propto \frac{\left|A_T^{\mathcal{M}}\right|^{0.5}}{\left|A_0^{\mathcal{M}}\right|^{0.5}} \frac{\Gamma(c_T)\, C_0^{c_0}}{\Gamma(c_0)\, (C_T^{\mathcal{M}})^{c_T}}, \tag{3.36}$$

and the posterior moments $a_T^{\mathcal{M}}$, $A_T^{\mathcal{M}}$, c_T and $C_T^{\mathcal{M}}$ of $b^{\mathcal{M}}$and σ_e^2 can be calculated as

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}}\left(\left(z^{\mathcal{M}}\right)' w + \left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}}\right), \tag{3.37}$$

$$A_T^{\mathcal{M}} = \left(\left(z^{\mathcal{M}}\right)' z^{\mathcal{M}} + \left(A_0^{\mathcal{M}}\right)^{-1}\right)^{-1}, \tag{3.38}$$

$$c_T = c_0 + T/2\,, \tag{3.39}$$

$$C_T^{\mathcal{M}} = C_0 + 0.5\left(w'w + \left(a_0^{\mathcal{M}}\right)'\left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}} - \left(a_T^{\mathcal{M}}\right)'\left(A_T^{\mathcal{M}}\right)^{-1} a_T^{\mathcal{M}}\right). \tag{3.40}$$

- In the heteroskedastic case $\Sigma = \mathrm{diag}\left(\sigma_{e,1}^2, ..., \sigma_{e,T}^2\right)$, under the normal conjugate prior $b^{\mathcal{M}} \sim \mathcal{N}\left(a_0^{\mathcal{M}}, A_0^{\mathcal{M}}\right)$ the closed form solution for the marginal likelihood $f(w\,|\mathcal{M},\alpha,\beta,h)$ is

$$f(w\,|\mathcal{M},\alpha,\beta,h) \propto \frac{|\Sigma|^{-0.5}\left|A_T^{\mathcal{M}}\right|^{0.5}}{\left|A_0^{\mathcal{M}}\right|^{0.5}} \exp\left(-\frac{1}{2}\left(w'\Sigma^{-1}w + \left(a_0^{\mathcal{M}}\right)'\left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}} - \left(a_T^{\mathcal{M}}\right)'\left(A_T^{\mathcal{M}}\right)^{-1} a_T^{\mathcal{M}}\right)\right), \tag{3.41}$$

with

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}} \left(\left(z^{\mathcal{M}} \right)' \Sigma^{-1} w + \left(A_0^{\mathcal{M}} \right)^{-1} a_0^{\mathcal{M}} \right), \tag{3.42}$$

$$A_T^{\mathcal{M}} = \left(\left(z^{\mathcal{M}} \right)' \Sigma^{-1} z^{\mathcal{M}} + \left(A_0^{\mathcal{M}} \right)^{-1} \right)^{-1}. \tag{3.43}$$

Following George and McCulloch (1993), instead of using a multi-move sampler in which all the elements in $\mathcal{M}$ are sampled simultaneously, we use a single-move sampler in which each of the binary indicators δ_j (for $j = \pi, u$), θ_k (for $k = y, \pi, u, c, \zeta$) and λ in $\mathcal{M}$ is sampled from $f\left(\delta_j | \delta_{\backslash j}, \theta, \lambda, \alpha, \beta, h, x\right)$, $f\left(\theta_k | \delta, \theta_{\backslash k}, \lambda, \alpha, \beta, h, x\right)$ and $f\left(\lambda | \delta, \theta, \alpha, \beta, h, x\right)$ respectively. Block 1 is therefore split up in the following subblocks:

Block 1(a): Sampling the binary indicators δ and the parameters β, σ_η and σ_ε^2

In this block we sample the binary indicators $\delta = (\delta_\pi, \delta_u)$ and the parameters $\beta = (\beta_0^\pi, \beta_0^u)$, $\sigma_\eta = (\sigma_{\eta,\pi}, \sigma_{\eta,u})$ and $\sigma_\varepsilon^2 = \left(\sigma_{\varepsilon,y}^2, \sigma_{\varepsilon,\pi}^2, \sigma_{\varepsilon,u}^2\right)$ conditional on the states α, β and h. First, as there is no binary indicator in equation (3.1), $\sigma_{\varepsilon,y}^2$ can be sampled directly from $IG\left(c_T, C_T\right)$ with c_T as in equation (3.39) and $C_T = C_0 + 0.5(\varepsilon^{y\prime}\varepsilon^y)$ with ε^y calculated from $\varepsilon_t^y = y_t - y_t^\tau - y_t^c$.

Next, using equation (3.29), equations (3.14) and (3.15) can be rewritten in the general linear regression format of (3.33) as

$$\overbrace{\pi_t - \pi_t^\tau - \zeta_t}^{w_t} = \overbrace{\left[\begin{array}{cc} \widetilde{y}_t^c & \delta_\pi \widetilde{\beta}_t^\pi \widetilde{y}_t^c \end{array}\right]}^{z_t^{\mathcal{M}}} \overbrace{\left[\begin{array}{c} \beta_0^\pi \\ \sigma_{\eta,\pi} \end{array}\right]}^{b^{\mathcal{M}}} + \overbrace{\varepsilon_t^\pi}^{e_t}, \tag{3.44}$$

$$\underbrace{u_t - u_t^\tau}_{w_t} = \underbrace{\left[\begin{array}{cc} y_t^c & \delta_u \widetilde{\beta}_t^u y_t^c \end{array}\right]}_{z_t^{\mathcal{M}}} \underbrace{\left[\begin{array}{c} \beta_0^u \\ \sigma_{\eta,u} \end{array}\right]}_{b^{\mathcal{M}}} + \underbrace{\varepsilon_t^u}_{e_t}, \tag{3.45}$$

where in both the restricted vector $z_t^{\mathcal{M}}$ and the restricted parameter vector $b^{\mathcal{M}}$ the second term is excluded when $\delta_j = 0$ (for $j = \pi, u$). Note that, next to the parameters in $b^{\mathcal{M}}$ and σ_e^2, each of the specifications (3.44) and (3.45) depends only on the data w_t, on some of the states in α_t and β_t and on δ_j. As such, we can simplify the specification of the posterior from $f\left(\delta_j | \delta_{\backslash j}, \theta, \lambda, \alpha, \beta, h, x\right)$ to $f\left(\delta_j \,| \alpha, \beta, w\,\right)$ for which we have $f\left(\delta_j \,| \alpha, \beta, w\,\right) \propto f\left(w\,| \delta_j, \alpha, \beta\,\right) p\left(\delta_j\right)$. As the error terms ε_t^π in the inflation equation and ε_t^u in the employment equation are homoskedastic, we have $\Sigma = \sigma_e^2 I_T$ in the general notation of equation (3.33) such that the marginal likelihood $f(w\,|\delta_j,\alpha,\beta)$ can be calculated as in equation (3.36). The binary indicator δ_j can then be sampled from the Bernoulli distribution with probability

$$p(\delta_j = 1\,|\alpha,\beta,w) = \frac{f(\delta_j = 1\,|\alpha,\beta,w)}{f(\delta_j = 0\,|\alpha,\beta,w) + f(\delta_j = 1\,|\alpha,\beta,w)}, \tag{3.46}$$

while $\sigma^2_{\varepsilon,j}$ can be sampled from $IG\left(c_T, C_T^{\mathcal{M}}\right)$ and, conditionally on $\sigma^2_{\varepsilon,j}$, $b^{\mathcal{M}}$ from $\mathcal{N}\left(a_T^{\mathcal{M}}, A_T^{\mathcal{M}}\sigma^2_{\varepsilon,j}\right)$, for $j = \pi, u$ and with $a_T^{\mathcal{M}}$, $A_T^{\mathcal{M}}$, c_T and $C_T^{\mathcal{M}}$ as defined in equations (3.37)-(3.40). Note that $b^{\mathcal{M}} = \left(\beta_0^j, \sigma_{\eta,j}\right)'$ when $\delta_j = 1$ and $b^{\mathcal{M}} = \beta_0^j$ when $\delta_j = 0$. In the former case $\sigma_{\eta,j}$ is sampled from the posterior while in the latter case we set $\sigma_{\eta,j} = 0$.

Block 1(b): Sampling the binary indicators θ and the parameters h_0 and σ_γ

In this block we sample the binary indicators $\theta = (\theta_y, \theta_\pi, \theta_u, \theta_c, \theta_\zeta)$ and the parameters $h_0 = \left(h_0^y, h_0^\pi, h_0^u, h_0^c, h_0^\zeta\right)$ and $\sigma_\gamma = (\sigma_{\gamma,y}, \sigma_{\gamma,\pi}, \sigma_{\gamma,u}, \sigma_{\gamma,c}, \sigma_{\gamma,\zeta})$ conditional on the states α, β and h. Using equation (3.30), equation (3.20) can be rewritten in the general linear regression format of (3.33) as

$$\overbrace{g_t^k - \left(m_{\iota_t^k} - 1.2704\right)}^{w_t} = \overbrace{2\begin{bmatrix} 1 & \theta_k \tilde{h}_t^k \end{bmatrix}}^{z_t^{\mathcal{M}}} \overbrace{\begin{bmatrix} h_0^k \\ \sigma_{\gamma,k} \end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\tilde{\epsilon}_t^k}^{e_t}, \tag{3.47}$$

for $k = y, \pi, u, c, \zeta$, with $\tilde{\epsilon}_t^k = \epsilon_t^k - \left(m_{\iota_t^k} - 1.2704\right)$ is ϵ_t^k recentered around zero and where using equations (3.2), (3.9), (3.16), (3.4) and (3.11), $g_t^k = \ln\left(\left(\exp\{h_t^k\}\psi_t^k\right)^2 + .001\right)$ can be calculated as

$$g_t^y = \ln\left(\left(y_t^\tau - y_{t-1}^\tau - \kappa_t\right)^2 + .001\right), \tag{3.48}$$

$$g_t^\pi = \ln\left(\left(\pi_t^\tau - \pi_{t-1}^\tau\right)^2 + .001\right), \tag{3.49}$$

$$g_t^u = \ln\left(\left(u_t^\tau - u_{t-1}^\tau\right)^2 + .001\right), \tag{3.50}$$

$$g_t^c = \ln\left(\left(y_t^c - \rho_1 y_{t-1}^c - \rho_2 y_{t-2}^c\right)^2 + .001\right), \tag{3.51}$$

$$g_t^\zeta = \ln\left(\left(\zeta_t - \varrho\zeta_{t-1}\right)^2 + .001\right). \tag{3.52}$$

As specification (3.47) depends only on the data w_t, on the stochastic volatility term h_t^k and on θ_k, we can simplify the specification of the posterior from $f\left(\theta_k|\delta, \theta_{\backslash k}, \lambda, \alpha, \beta, h, x\right)$ to $f(\theta_k\,|h,w)$. Using Bayes' Theorem, we have $f(\theta_k\,|h,w) \propto f(w\,|\theta_k,h)\,p(\theta_k)$. Given the mixture distribution of ϵ_t^k defined in equation (3.22), the error term $\tilde{\epsilon}_t^k$ in equation (3.47) has a heteroskedastic variance $v^2_{\iota^k_t}$ such that $\Sigma = \text{diag}\left(v^2_{\iota^k_1}, ..., v^2_{\iota^k_T}\right)$ in the general notation of equation (3.33). In this case, the marginal likelihood $f\left(w\,|\theta_k, h\right)$ can be calculated as in equation (3.41). The binary indicator θ_k can then be sampled from the Bernoulli distribution with probability $p\left(\theta_k = 1\,|h, w\right)$ calculated from an equation similar to (3.46). Next, $b^{\mathcal{M}}$ can be sampled from $\mathcal{N}\left(a^{\mathcal{M}}_T, A^{\mathcal{M}}_T\right)$ for $k = y, \pi, u, c, \zeta$ and with $a^{\mathcal{M}}_T$ and $A^{\mathcal{M}}_T$ as defined in equations (3.42) and (3.43). Note that $b^{\mathcal{M}} = \left(h^k_0, \sigma_{\gamma,k}\right)'$ when $\theta_k = 1$ and $b^{\mathcal{M}} = h^k_0$ when $\theta_k = 0$. In the latter case, we set $\sigma_{\gamma,k} = 0$.

Block 1(c): Sampling the binary indicator λ and the parameters κ_0 and σ_κ

In this block we sample the binary indicator λ and the parameters κ_0 and σ_κ conditional on the states α, β and h. Using equation (3.31), equation (3.2) can be rewritten in the general linear regression format of (3.33) as

$$\overbrace{y^\tau_t - y^\tau_{t-1}}^{w_t} = \overbrace{\begin{bmatrix} 1 & \lambda\tilde{\kappa}_t \end{bmatrix}}^{z^{\mathcal{M}}_t} \overbrace{\begin{bmatrix} \kappa_0 \\ \sigma_\kappa \end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\exp\{h^y_t\}\psi^y_t}^{e_t}, \tag{3.53}$$

with $\Sigma = \text{diag}\left(\exp\{h^y_1\}^2, \ldots, \exp\{h^y_T\}^2\right)$. The indicator λ can then be sampled from the posterior distribution $f\left(\lambda\,|\alpha, w\right) \propto f\left(w\,|\lambda, \alpha\right) p\left(\lambda\right)$ with the marginal likelihood $f\left(w\,|\lambda, \alpha\right)$ calculated from equation (3.41). Next, $b^{\mathcal{M}}$ can be sampled from $\mathcal{N}\left(a^{\mathcal{M}}_T, A^{\mathcal{M}}_T\right)$ with $a^{\mathcal{M}}_T$ and $A^{\mathcal{M}}_T$ as defined in equations (3.42) and (3.43). Note that $b^{\mathcal{M}} = (\kappa_0, \sigma_\kappa)'$ when $\lambda = 1$ and $b^{\mathcal{M}} = \kappa_0$ when $\lambda = 0$. In the latter case, we set $\sigma_\lambda = 0$.

Block 1(d): Sampling the parameters ρ and ϱ

For sampling $\rho = (\rho_1, \rho_2)$ conditional on the states α, β and h, equation (3.4) can be written in the general notation of equation (3.33) as: $w_t = y^c_t$, $z_t = (y^c_{t-1}, y^c_{t-2})$, $b = (\rho_1, \rho_2)'$ and $e_t = \exp\{h^c_t\}\psi^c_t$, such that $\Sigma = \text{diag}\left(\exp\{h^c_1\}^2, \ldots, \exp\{h^c_T\}^2\right)$. Under the normal prior distribution $\mathcal{N}\left(a_0, A_0\right)$, ρ can then be sampled from the posterior $\mathcal{N}\left(a_T, A_T\right)$ with a_T and A_T as in equations (3.42) and (3.43).

Likewise, for filtering ϱ conditional on the states α, β and h, equation (3.11) can be written in the general notation of equation (3.33) as: $w_t = \zeta_t$, $z_t = \zeta_{t-1}$, $b = \varrho$ and $e_t = \exp\{h^\zeta_t\}\psi^\zeta_t$, such that $\Sigma = \text{diag}\left(\exp\{h^\zeta_1\}^2, \ldots, \exp\{h^\zeta_T\}^2\right)$. Under the normal prior distribution $\mathcal{N}\left(a_0, A_0\right)$, ϱ can again be sampled from the posterior $\mathcal{N}\left(a_T, A_T\right)$ with a_T and A_T as in equations (3.42) and (3.43).

Block 2: Sampling the state vectors α, β and h and mixture indicators ι

In this block we use a forward-filtering and backward-sampling approach of Carter and Kohn (1994) and De Jong and Shephard (1995) to sample the states α, β and h based on a general state space model of the form

$$w_t = Z_t^{\mathcal{M}} s_t^{\mathcal{M}} + e_t, \qquad e_t \sim iid\mathcal{N}(0, H_t), \tag{3.54}$$

$$s_{t+1} = R_0 + R_1 s_t + K_t v_t, \qquad v_t \sim iid\mathcal{N}(0, Q_t), \qquad s_1 \sim iid\mathcal{N}(a_1, A_1), \tag{3.55}$$

where w_t is now a vector of observations and s_t an unobserved state vector. The matrices Z_t, R_0, R_1, K_t, H_t, Q_t and the expected value a_1 and variance A_1 of the initial state vector s_1 are assumed to be known (conditioned upon). The vector $s_t^{\mathcal{M}}$ and the matrix $Z_t^{\mathcal{M}}$ are again restricted versions of s_t and Z_t with the elements excluded depending on the model indicators $\mathcal{M}$. The error terms e_t and v_t are assumed to be serially uncorrelated and independent of each other at all points in time. As equations (3.54)-(3.55) constitute a linear Gaussian state space model, the unknown state variables in s_t can be filtered using the standard Kalman filter. Sampling $s = [s_1, \ldots, s_T]$ from its conditional distribution can then be done using the multimove simulation smoother of Carter and Kohn (1994) and De Jong and Shephard (1995).

Block 2(a) Sampling the trend and temporary components α

We first filter and draw the state vector $\alpha = (y^\tau, \pi^\tau, u^\tau, \kappa, y^c, \zeta)$ conditionally on the time-varying parameters β, the stochastic volatilities h and the hyperparameters ϕ. More specifically, using the general notation in equations (3.54)-(3.55), the unrestricted (i.e. $\lambda = 1$) conditional state space representation is given by

$$\underbrace{\begin{bmatrix} y_t \\ \pi_t \\ u_t \end{bmatrix}}_{w_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & a\beta_t^\pi & b\beta_t^\pi \\ 0 & 0 & 1 & 0 & 0 & \beta_t^u & 0 \end{bmatrix}}_{Z_t^{\mathcal{M}}} \underbrace{\begin{bmatrix} y_t^\tau \\ \pi_t^\tau \\ u_t^\tau \\ \widetilde{\kappa}_t \\ \zeta_t \\ y_t^c \\ y_{t-1}^c \end{bmatrix}}_{s_t^{\mathcal{M}}} + \underbrace{\begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^\pi \\ \varepsilon_t^u \end{bmatrix}}_{e_t}, \tag{3.56}$$

$$\underbrace{\begin{bmatrix} y^\tau_{t+1} \\ \pi^\tau_{t+1} \\ u^\tau_{t+1} \\ \widetilde{\kappa}_{t+1} \\ \zeta_{t+1} \\ y^c_{t+1} \\ y^c_t \end{bmatrix}}_{s_{t+1}} = \underbrace{\begin{bmatrix} \kappa_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{R_0} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & \sigma_\kappa & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varrho & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_1 & \rho_2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{R_1} \underbrace{\begin{bmatrix} y^\tau_t \\ \pi^\tau_t \\ u^\tau_t \\ \widetilde{\kappa}_t \\ \zeta_t \\ y^c_t \\ y^c_{t-1} \end{bmatrix}}_{s_t} + K_t \underbrace{\begin{bmatrix} \psi^y_t \\ \psi^\pi_t \\ \psi^u_t \\ \psi^\kappa_t \\ \psi^\zeta_t \\ \psi^c_t \end{bmatrix}}_{\nu_t}, \tag{3.57}$$

$$\text{with} \quad K_t = \begin{bmatrix} \exp\{h^y_t\} & 0 & 0 & 0 & 0 & 0 \\ 0 & \exp\{h^\pi_t\} & 0 & 0 & 0 & 0 \\ 0 & 0 & \exp\{h^u_t\} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\psi,\kappa} & 0 & 0 \\ 0 & 0 & 0 & 0 & \exp\{h^\zeta_t\} & 0 \\ 0 & 0 & 0 & 0 & 0 & \exp\{h^c_t\} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and where $a = \left(1 - \omega\rho_1 - \omega^2\rho_2\right)^{-1}$, $b = a\omega\rho_2$, $H_t = diag\left(\sigma^2_{\varepsilon,y}, \sigma^2_{\varepsilon,\pi}, \sigma^2_{\varepsilon,u}\right)$ and $Q_t = I_6$. The random walk components y^τ_t, π^τ_t, u^τ_t and $\widetilde{\kappa}_t$ are initialized by setting $a_1 = 0$ and $A_1 = 1000$ for each of these components while the stationary components ζ_t and y^c_t are initialized from their unconditional distributions. Note that using κ_0, σ_κ and $\widetilde{\kappa}_t$, κ_t can easily be reconstructed from equation (3.27).

In the restricted model (i.e. $\lambda = 0$) $\widetilde{\kappa}_t$ is excluded from $s^{\mathcal{M}}_t$, with appropriate adjustment of the other matrices. In this case, no forward-filtering and backward-sampling is needed and $\widetilde{\kappa}_t$ can be sampled directly from its prior using equation (3.28).

Block 2(b): Sampling the time-varying parameters β

We next filter and draw the time-varying parameters $\beta = (\beta^\pi, \beta^u)$ conditionally on the state vector α, the stochastic volatilities h, the hyperparameters ϕ and the binary indicators $\mathcal{M}$. More specifically, using equation (3.29) in (3.14) and (3.15), the unrestricted (i.e. $\delta_j = 1$) conditional state space representations for the time-varying parameters $\widetilde{\beta}^\pi_t$ and $\widetilde{\beta}^u_t$ are given by

$$\overbrace{\left[\ \pi_t - \pi_t^\tau - \zeta_t - \beta_0^\pi \widetilde{y}_t^c\ \right]}^{w_t} = \overbrace{\left[\ \sigma_{\eta,\pi}\widetilde{y}_t^c\ \right]}^{Z_t^{\mathcal{M}}}\overbrace{\left[\ \widetilde{\beta}_t^\pi\ \right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\ \varepsilon_t^\pi\ \right]}^{e_t}, \tag{3.58}$$

$$\underbrace{\left[\ \widetilde{\beta}_{t+1}^\pi\ \right]}_{s_{t+1}} = \underbrace{\left[\ 1\ \right]}_{R_1}\underbrace{\left[\ \widetilde{\beta}_t^\pi\ \right]}_{s_t} + \underbrace{\left[\ 1\ \right]}_{K_t}\underbrace{\left[\ \widetilde{\eta}_t^\pi\ \right]}_{\nu_t}, \tag{3.59}$$

with $H_t = \sigma^2_{\varepsilon,\pi}$ and $Q_t = 1$, and

$$\overbrace{\left[\ u_t - u_t^\tau - \beta_0^u y_t^c\ \right]}^{w_t} = \overbrace{\left[\ \sigma_{\eta,u} y_t^c\ \right]}^{Z_t^{\mathcal{M}}}\overbrace{\left[\ \widetilde{\beta}_t^u\ \right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\ \varepsilon_t^u\ \right]}^{e_t}, \tag{3.60}$$

$$\underbrace{\left[\ \widetilde{\beta}_{t+1}^u\ \right]}_{s_{t+1}} = \underbrace{\left[\ 1\ \right]}_{R_1}\underbrace{\left[\ \widetilde{\beta}_t^u\ \right]}_{s_t} + \underbrace{\left[\ 1\ \right]}_{K_t}\underbrace{\left[\ \widetilde{\eta}_t^u\ \right]}_{\nu_t}, \tag{3.61}$$

with $H_t = \sigma^2_{\varepsilon,u}$ and $Q = 1$. Both random walk components $\widetilde{\beta}_t^\pi$ and $\widetilde{\beta}_t^u$ are initialized by setting $a_1 = 0$ and $A_1 = 1000$.

In the restricted model (i.e. $\delta_j = 0$), $Z^{\mathcal{M}}$ and $s^{\mathcal{M}}$ are empty. In this case, no forward-filtering and backward-sampling is needed and $\widetilde{\beta}_t^j$ can be sampled directly from its prior using equation (3.24). Note that the sampling of the state vector α in block 2(a) depends on β_t^j rather than on $\widetilde{\beta}_t^j$. Using β_0^j, $\sigma_{\eta,j}$ and $\widetilde{\beta}_t^j$, β_t^j can easily be reconstructed from equation (3.23).

Block 2(c): Sampling the mixture indicators ι and the stochastic volatilities h

In this block we draw the mixture indicators $\iota = \left(\iota^y, \iota^\pi, \iota^u, \iota^c, \iota^\zeta\right)$ and the stochastic volatilities $h = \left(h^y, h^\pi, h^u, h^c, h^\zeta\right)$ conditionally on the state vector α, the time-varying parameters β, the hyperparameters ϕ and the binary indicators $\mathcal{M}$. Following Del Negro and Primiceri (2014), in this block we first sample the mixture indicator ι_t^k (for $k = y, \pi, u, c, \zeta$) from its conditional probability mass

$$p\left(\iota_t^k = i | h_t^k, \epsilon_t^k\right) \propto q_i f_{\mathcal{N}}\left(\epsilon_t^k | 2h_t^k + m_i - 1.2704, \nu_i^2\right), \tag{3.62}$$

with values for $\{q_i, m_i, \nu_i^2\}$ taken from Table 1 in Omori, Chib, Shephard, and Nakajima (2007).

Next, we filter and sample the stochastic volatility terms $\widetilde{h}_t^k$ (for $k = y, \pi, u, c, \zeta$) conditioning on the transformed states g_t^k defined in equations (3.48)-(3.52), on the mixture indicators

ι_t^k and on the parameters ϕ. More specifically, the unrestricted (i.e. $\theta_k = 1$) conditional state space representation is given by

$$\overbrace{\left[\; g_t^k - \left(m_{\iota_t^k} - 1.2704\right) - 2h_0^k \;\right]}^{w_t} = \overbrace{\left[\; 2\theta^k \sigma_{\gamma,k} \;\right]}^{Z_t^{\mathcal{M}}} \overbrace{\left[\; \widetilde{h}_t^k \;\right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\; \widetilde{\epsilon}_t^k \;\right]}^{e_t}, \tag{3.63}$$

$$\underbrace{\left[\; \widetilde{h}_{t+1}^k \;\right]}_{s_{t+1}} = \underbrace{[1]}_{R_1} \underbrace{\left[\; \widetilde{h}_t^k \;\right]}_{s_t} + \underbrace{\left[\; 1 \;\right]}_{K_t} \underbrace{\left[\; \widetilde{\gamma}_t^k \;\right]}_{\nu_t}, \tag{3.64}$$

with $H_t = v_{\iota_t^k}^2$, $Q_t = 1$ and where $\widetilde{\epsilon}_t^k = \epsilon_t^k - \left(m_{\iota_t^k} - 1.2704\right)$ is ϵ_t^k recentered around zero. The random walk components $\widetilde{h}_t^k$ are initialized by setting $a_1 = 0$ and $A_1 = 1000$.

In the restricted model (i.e. $\theta_k = 0$), $Z^{\mathcal{M}}$ and $s^{\mathcal{M}}$ are empty. In this case, no forward-filtering and backward-sampling is needed and $\widetilde{h}_t^k$ can be sampled directly from its prior using equation (3.26). Note that the sampling of the state vector α in block 2(a) depends on h_t^k rather than on $\widetilde{h}_t^k$. Using h_0^k, $\sigma_{\gamma,k}$ and $\widetilde{h}_t^k$, h_t^k can easily be reconstructed from equation (3.25).

4 | Demographics and Business Cycle Volatility: A Spurious Relationship?

with Gerdie Everaert

Abstract: This paper replicates the estimation results of three studies about the impact of the age composition of the labor force on business cycle volatility and investigates whether they signal a meaningful long-run relationship. We show that both the volatile-age labor force share variable and the business cycle volatility measure are non-stationary but find no robust evidence of a cointegrating relation. This suggests that the estimation results reported in the literature are in fact spurious. This conclusion is further supported by the finding that the estimation results are very sensitive to small changes in the data and/or to the specification of the volatile-age labor force share.
JEL classification: E32, J11

Hauke Vierke acknowledges financial support from Ghent University's Special Research Fund (BOF).

4.1 Introduction

In a well-cited paper Jaimovich and Siu (2009), hereafter Ja&Si, argue that the run-up of U.S. volatility in the mid-1960s and the marked decline since the mid-1980s, known as the Great Moderation, are the consequence of long swings in the age composition of the American population induced by the baby boom and subsequent baby bust. Ja&Si start from well-documented differences in the responsiveness of labor market activity to the business cycle over individuals of different ages. Both 'the young' and 'the old' tend to experience greater sensitivity of employment and hours worked than the prime-aged. Given this U-shaped pattern Ja&Si define the *volatile-age* labor force share s_{it} as the fraction of the 15-64 year old labor force accounted for by those aged 15-29 and 60-64. Using an unbalanced panel for the G7 countries covering the period 1963-1999, s_{it} is then linked to the time-varying standard deviation of output σ_{it} in the following benchmark regression

$$\sigma_{it} = \alpha_i + \beta_t + \gamma s_{it} + \varepsilon_{it}, \tag{4.1}$$

with α_i a country fixed effect and β_t a time fixed effect. Using a variety of alternative measures for σ_{it}, Ja&Si show that shifts in the volatile-age share variable s_{it} account for a significant fraction of the observed variation in cyclical volatility in the G7 countries. Similar results are obtained by Lugauer (2012) for a panel of 50 U.S. states and by Lugauer and Redmond (2012) for a panel of 51 advanced and developing countries.

A theoretical explanation for the effect of demographics in output volatility is given by Jaimovich, Pruitt, and Siu (2013), who modify a standard neoclassical model to include capital-experience complementarity in the production function. The model distinguishes between young and old workers, where the latter have greater labor market experience and, thus, higher capital complementarity. As the elasticity of substitution between capital and labor differs among the two age groups, technology shocks induce an asymmetric change in labor demand, where labor demand for the young is more volatility than for the prime-aged workers.

Although the exact timing and specific evolution has been different, most developed countries have experienced a similar shift in the age distribution of the labor force over the postwar period which seems to coincide with a decline in macroeconomic volatility. The aim of this paper is to investigate whether this alleged long-run relationship is meaningful or is in fact spurious. First, we look at the properties of the G7 data used by Ja&Si for the volatility measures and the labor share variable. We show that both are non-stationary and exhibit significant cross-sectional dependence. The latter implies that the long swings in the data that are, to a certain extent, common over the G7 countries. Second, we replicate the results of Ja&Si and show that (4.1) is not a cointegrating relation. This does not automatically imply that the results are not meaningful as Phillips and Moon (1999) have shown that a spurious regression can be avoided in a panel with a large number of independent cross-sections. However, the small cross-sectional dimension of the G7 panel dataset together with the presence of significant cross-sectional dependence raise questions on whether the Phillips and Moon result applies. Third, to shed further light on whether there is a stable long-run relation between business cycle volatility and demographics, we use the richer panel datasets of Lugauer (2012) and Lugauer and Redmond (2012). The larger number of cross-sections allows us to use the common correlated effects (CCE) approach of Pesaran (2006) to account for cross-sectional dependence. We still find no evidence of cointegration and the estimation results are found to be very sensitive to small changes in the data and/or to the specification of the volatile-age labor force share.

4.2 Time series properties and cross-sectional dependence

As a first step in the empirical analysis, Table 4.1 reports unit root and cross-sectional dependence tests on the cyclical volatility σ_{it} and the volatile-age labor share variable s_{it} taken from the Ja&Si dataset. As a measure for σ_{it}, we consider both the standard deviation of HP filtered real GDP during a 10-year window, hereafter HP, and the instantaneous standard deviation of real output growth calculated from the stochastic volatility model of Stock and Watson (2003, 2005), hereafter SW. More details on the construction of these variables can be found in Ja&Si.

The top left panel of Table 4.1 reports country-specific Augmented Dickey and Fuller (1979) (ADF) and Maddala and Wu (1999) (MW) panel unit root tests for a specification with a constant. The null hypothesis of a unit root in the HP and SW volatility measures cannot be rejected at the 5% level of significance for any of the individual countries nor for the panel as a whole. For the labor share variable s_{it} the unit root hypothesis is rejected for only 2 out of 7 countries, i.e. for Canada and Japan, while not rejected using the panel MW test.

The bottom left panel of Table 4.1 reports the average cross-sectional correlation $\bar{\hat{\rho}}$ in the data together with the Pesaran (2004) cross-sectional dependence (CD) test. To avoid spurious non-zero correlations due to non-stationarity of the data in levels, we also report results for the first-differenced data and for the error terms of the ADF regressions. For

The data are available at the AER website: *https://www.aeaweb.org/aer/data/june09/20070168_data.zip.*

Table 4.1: Properties of the Ja&Si data: unit root and cross-sectional dependence tests

	Sample period: 1963-1999, unbalanced panel of G7 countries					
Model:	constant			constant + time dummies		
	σ_{it}		s_{it}	σ_{it}		s_{it}
	HP	SW		HP	SW	
Country-specific ADF unit root tests						
Canada	−0.88	−2.44	−3.87	−1.28	−2.33	−2.82
	[0.79]	[0.14]	[0.01]	[0.63]	[0.17]	[0.07]
France	−2.76	−2.38	1.40	−0.31	−0.56	−0.01
	[0.08]	[0.16]	[0.99]	[0.91]	[0.87]	[0.95]
Germany	−0.17	−1.45	0.41	−2.14	−1.27	−3.40
	[0.94]	[0.55]	[0.98]	[0.23]	[0.63]	[0.02]
Italy	−0.48	−1.28	−0.60	−0.54	−0.73	−1.80
	[0.88]	[0.63]	[0.84]	[0.85]	[0.81]	[0.37]
Japan	−1.26	−1.60	−4.64	−1.37	−1.43	−2.88
	[0.64]	[0.48]	[0.00]	[0.59]	[0.56]	[0.06]
U.K.	−0.34	−0.99	2.34	1.17	−1.57	−1.85
	[0.91]	[0.75]	[0.99]	[0.99]	[0.48]	[0.35]
U.S.	−0.42	−0.92	−1.63	−1.69	−2.02	−2.30
	[0.90]	[0.77]	[0.45]	[0.44]	[0.28]	[0.18]
Panel unit root tests						
MW	7.36	12.40	20.45	7.13	10.37	26.45
	[0.92]	[0.57]	[0.12]	[0.93]	[0.74]	[0.02]
Cross-sectional dependence tests						
Data in levels						
$\overline{\widehat{\rho}}$	0.21	0.27	0.59	−0.15	−0.14	−0.22
CD	5.74	7.22	13.18	−3.71	−3.61	−4.68
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Data in first-differences						
$\overline{\widehat{\rho}}$	0.18	0.06	0.13	−0.16	−0.04	−0.05
CD	4.73	1.64	2.49	−3.53	−1.19	−1.08
	[0.00]	[0.10]	[0.01]	[0.00]	[0.23]	[0.28]
Error terms of the ADF regressions						
$\overline{\widehat{\rho}}$	0.11	0.13	0.21	−0.11	0.03	0.20
CD	2.92	3.28	4.94	−2.63	0.50	4.24
	[0.00]	[0.00]	[0.00]	[0.00]	[0.61]	[0.00]

Notes: The lag length of the country-specific ADF unit root tests is selected using the Akaike information criterion with a maximum lag length of $\text{int}\{12\,(T\,/100\,)^{0.25}\}$. The ADF tests for the model with constant include a constant while for the model with constant and time dummies the data are first regressed on fixed and time effects and the residuals are next used to calculate the ADF statistics. The MW panel unit root test is defined as $-2\sum_{i=1}^{N}\ln(p_i)$ where p_i is the p-value corresponding to the unit root test of the ith country. The average cross-correlation coefficient $\overline{\widehat{\rho}} = (2\,/N\,(N-1)\,)\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\widehat{\rho}_{ij}$ is the average of the country-by-country cross-correlation coefficients $\widehat{\rho}_{ij}$ (for $i \neq j$). The cross-sectional dependence CD test is defined as $\sqrt{2T\,/N\,(N-1)}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\widehat{\rho}_{ij}$, which is asymptotically standard normal under the null of cross-sectional independence. p-values are in square brackets.

each of the variables and error terms significant positive cross-sectional dependence is found. This is in line with the graphs in Ja&Si which show a similar long-run pattern in σ_{it} and s_{it} over countries. The only clear exception is Japan, which experienced an overall postwar moderation in output volatility and aging of the workforce similar to the other countries in the sample, but the evolution over time was quite different.

To control for cross-sectional dependence, Ja&Si include a full set of time dummies. The top right panel of Table 4.1 therefore also reports unit root tests after projecting out time effects (model = constant + time dummies). This does not change the main conclusion with respect to the non-stationarity of the data. The null hypothesis of a unit root can still not be rejected for the HP and SW volatility measures using either the country-specific ADF tests or the MW panel test. Especially for the SW measure this should not come as a surprise as (the log of) volatility is explicitly generated as a random walk process. For the labor share s_{it} the unit root hypothesis is again rejected for Canada and Japan, although here only at the 10% level, while the unit root is now also rejected for Germany. As a result the MW panel test rejects the unit root hypothesis at the 2% level of significance. The bottom right part of Table 4.1 shows that the time dummies are able to remove most of the positive cross-sectional dependence in the data. The small number of countries available to estimate the time effects even induces significant negative cross-sectional dependence. Only for the error terms of the ADF specification for the labor share variable the positive cross-sectional dependence does not disappear.

Taking stock, the unit root tests show that the data, especially the volatility measures, exhibit non-stationary behavior such that one should be careful when using them in a standard regression context. In the next section we analyse the consequences of this finding for the estimation results in Ja&Si.

4.3 The Jaimovich and Siu (AER, 2009) regression

The top left panel of Table 4.2 replicates the baseline coefficient estimates of 4.02 when using the HP volatility measure and 3.34 when using the SW measure reported by Ja&Si in their Table 4. These results show that the volatile-age workforce participants have a strong and significant positive impact on business cycle volatility. Next to results for a model with both individual and time fixed effects (FETE), as used by Ja&Si, we also report fixed effects (FE)

The finding of significant cross-sectional dependence implies that the MW panel unit root test, which ignores cross-sectional dependence, is inappropriate.

Ja&Si also report results using an IV instead of a LS estimator to account for the possible endogeneity of the labor share variable. Although in a non-stationary panel endogeneity is typically dealt with in a different way, we experimented with the IV approach but this did not change the qualitative results.

estimation results to signify the difference. For the full G7 sample, excluding the time effects does not result in fundamentally different results. Not removing the common trend in the data by not including time dummies results in point estimates that are a bit higher and standard errors that are a bit lower. Given the non-stationarity of the data detected in the previous section, Table 4.2 also includes country-specific ADF and MW panel cointegration tests using the estimated error terms $\widehat{\varepsilon}_{it}$. These results show that equation (4.1) is not a cointegration regression as the null hypothesis of a unit root in ε_{it} cannot be rejected. This raises the question whether the correlation between demographics and volatility is spurious. Ja&Si do suggest themselves that the observed strong positive correlation could be spurious because of omitted non-stationary factors such as unstable oil prices and monetary policy in the 1970s. They argue however that the different evolution in demographics and volatility over countries, most markedly in Japan compared to the other countries, should avoid spurious correlation. Phillips and Moon (1999) indeed show that pooling over a large number of cross-sections attenuates the strong effect of the non-stationary residuals while retaining the strength of the signal. In this case, a consistent estimate of the long-run relation can be obtained even if there is no cointegration. An important condition for this result to hold is that the error terms are cross-sectionally independent. Urbain and Westerlund (2011) for instance show that the Phillips and Moon result does not longer hold when the non-stationary in the error term is induced by a common factor structure. The bottom left panel of Table 4.2 shows that although there are signs of cross-sectional dependence in the error terms this is not overwhelming especially when using the SW volatility measure. However, given the small cross-sectional dimension of the G7 panel dataset and some cross-sectional dependence the question remains whether the Phillips and Moon result holds or whether the results are in fact spurious. In this respect, it is informative to see that the positive correlation disappears, and even becomes negative, when excluding Japan from the panel in a model including both individual and time fixed effects. When including only individual fixed effects the strong positive correlation remains, though. This shows that when controlling for the common evolution in volatility in the remaining 6 G7 countries, there is no longer a role for the country-specific evolution in demographics. The different evolution in the variables for Japan together with the small cross-sectional dimension in the data seems to undermine the effectiveness of the time dummies to adequately capture the common factors in the full G7 dataset.

Table 4.2: The Jaimovich and Siu (AER, 2009) regression

	Sample period: 1963-1999, unbalanced panel of G7 countries							
	Full G7 sample				G7 sample excl. Japan			
	HP		SW		HP		SW	
	FE	FETE	FE	FETE	FE	FETE	FE	FETE
Estimation results								
γ	4.70	4.02	4.89	3.34	4.39	−0.34	4.24	−3.09
	(0.79)	(1.13)	(0.70)	(1.17)	(0.95)	(1.64)	(0.82)	(1.78)
Country-specific ADF cointegration tests								
Canada	−1.22	−1.15	−2.78	−2.35	−1.25	−1.31	−2.92	−2.35
	[0.65]	[0.69]	[0.08]	[0.17]	[0.64]	[0.61]	[0.06]	[0.17]
France	−1.01	0.11	1.40	−0.40	−1.11	−0.44	1.21	−0.72
	[0.74]	[0.96]	[0.99]	[0.90]	[0.70]	[0.89]	[0.99]	[0.83]
Germany	−1.62	−1.67	−0.92	−1.37	−1.60	−2.69	−0.77	−1.93
	[0.46]	[0.44]	[0.77]	[0.58]	[0.47]	[0.09]	[0.81]	[0.31]
Italy	−1.62	−2.21	−0.48	−1.82	−1.59	−0.50	−0.66	−1.33
	[0.45]	[0.21]	[0.87]	[0.36]	[0.46]	[0.87]	[0.83]	[0.59]
Japan	−2.15	−1.93	−2.11	−1.82				
	[0.23]	[0.32]	[0.24]	[0.37]				
U.K.	1.55	0.88	−2.68	−2.34	−0.17	0.40	−2.45	−1.90
	[0.99]	[0.99]	[0.10]	[0.17]	[0.93]	[0.98]	[0.14]	[0.33]
U.S.	−1.05	−1.48	−1.54	−1.84	−1.00	−1.52	−1.42	−1.82
	[0.73]	[0.54]	[0.50]	[0.36]	[0.75]	[0.52]	[0.56]	[0.37]
Panel cointegration test								
MW	8.30	9.20	14.96	14.55	5.43	7.71	11.71	11.65
	[0.87]	[0.82]	[0.38]	[0.41]	[0.94]	[0.81]	[0.47]	[0.47]
Cross-sectional dependence in the first-differenced error terms								
$\overline{\widehat{\rho}}$	0.10	−0.15	0.06	−0.03	0.14	−0.17	0.08	−0.04
CD	2.35	−3.42	1.16	−0.93	2.65	−3.08	1.32	−1.12
	[0.02]	[0.00]	[0.25]	[0.35]	[0.01]	[0.00]	[0.19]	[0.26]

Notes: Newey-West standard errors in parentheses, p-values in square brackets.
The cointegration tests are unit root tests on the estimated error terms $\widehat{\varepsilon}_{it}$.
See Table 4.1 for further notes.

4.4 Two datasets with a richer cross-sectional dimension

The Phillips and Moon (1999) result that a consistent estimate of the long-run relation between non-stationary variables can be obtained even if there is no cointegration only holds when pooling over a large number of independent cross-sections. Banerjee and Carrion–i–Silvestre (2015) show that consistent estimation is possible once cross-sectional dependence is controlled for using the pooled common correlated effects (CCEP) estimator of Pesaran (2006) and Kapetanios, Pesaran, and Yamagata (2011). By including cross-sectional averages of the data to proxy for the unobserved common factors, the CCEP approach is able to account for a more general heterogeneous cross-sectional dependence compared to the homogeneous pattern when using time fixed effects. Moreover, CCEP allows for consistent estimation under very general integration properties of both the data and the unobserved common factors that generate the cross-sectional dependence. As these results requires the number of cross-sections to grow to infinity, in the next section we look at two panels with a larger cross-sectional dimension than the G7 panel considered by Ja&Si.

4.4.1 Lugauer and Redmond (EconLet, 2012)

Lugauer and Redmond (2012) collect a balanced panel dataset for 51 countries over the period 1957-2000. GDP data from the Penn World Table (2009, version 6.3) are used to calculate output volatility, which is defined as the 9-year rolling standard deviation of log annual GDP, detrended using the HP filter. Demographic data are taken from the United Nations World Population Prospects (2008). Following Ja&Si, the volatile-age labor share variable is defined as the fraction of the working age population aged 15 to 29 plus those aged 60 to 64. Country-specific ADF unit root tests (not reported) after projecting out time fixed effects show that the null hypothesis of a unit root is rejected at the 5% level of significance in 3 countries for the volatility measure and in 14 countries for the age share variable. A MW panel unit root test rejects the unit root hypothesis for the age share variable but not for the volatility measure. This marked difference in time series behavior already suggests that demographics can probably not explain the stochastic trend in volatility in a lot of countries.

The FETE column in the upper left panel of Table 4.3 replicates the result in LR that the age distribution has an economically large impact on volatility, i.e. a 10% points increase in the share variable raises the standard deviation of cyclical output by 0.38, although only being significant at the 10% level of significance. The CCEP estimator yields an even larger estimate which is moreover significant at the 5% level. The cointegration test shows that only in a

Although these data are publicly available, we thank Steven Lugauer for kindly providing the original data.

Table 4.3: The Lugauer and Redmond (EconLet, 2012) regression

	Original sample, all ($N = 51$)			Original sample, OECD ($N = 22$)			Revised sample, all ($N = 51$)		
	FE	FETE	CCEP	FE	FETE	CCEP	FE	FETE	CCEP
Estimation results									
γ	7.71	3.84	5.99	4.47	2.19	0.01	5.77	−0.62	0.92
	(1.46)	(1.99)	(2.49)	(1.04)	(1.89)	(2.34)	(1.40)	(1.74)	(2.11)
Cointegration tests									
ADF: # $p_i < 5\%$	5	2	3	3	2	1	4	4	1
Panel MW	116.83	104.78	96.71	70.81	70.09	48.85	115.41	126.37	95.30
	[0.15]	[0.41]	[0.63]	[0.01]	[0.01]	[0.28]	[0.17]	[0.05]	[0.67]
r	–	–	4	–	–	2	–	–	4
Cross-sectional dependence in the first-differenced error terms									
$\overline{\widehat{\rho}}$	0.04	0.01	−0.00	0.12	−0.03	−0.04	0.04	−0.00	−0.01
CD	8.72	1.54	−0.34	11.51	−3.43	−3.62	8.66	−0.09	−1.43
	[0.00]	[0.12]	[0.74]	[0.00]	[0.00]	[0.00]	[0.00]	[0.93]	[0.15]

Notes: For the country-specific ADF cointegration test we report the number of countries for which the unit root hypothesis is rejected, i.e. for which the p-value p_i is smaller than 5%.

For the CCEP estimator, the cointegration test is a unit root test on the idiosyncratic part of the error terms, which is obtained by removing r common factors from the estimated error terms using the principal components approach suggested by Bai and Ng (2004). The value of r is chosen to make sure that the idiosyncratic error terms are cross-sectionally independent. See Everaert (2014) for more details.

See Tables 4.1 and 4.2 for further notes.

small number of countries this constitutes a cointegration relation while the panel MW test does not reject the null hypothesis of no cointegration. Despite the finding of no cointegration but given that the error terms of the FETE and CCEP regressions are independent over cross-sections, the result in Banerjee and Carrion–i–Silvestre (2015) implies that these estimators should still yield consistent estimates. In this respect, only the FE estimate is not trustworthy as there is significant cross-sectional dependence left in the error terms.

However, having a closer look at the data reveals that the estimates are driven by only a small number of countries. After removing 5 countries (the Democratic Republic of the Congo, Nicaragua, Ethiopia, Nigeria and Mauritius) which exhibit very large swings in volatility, the relation between demographics and volatility completely disappears, i.e. the FETE and CCEP estimates drop to 0.25 and 0.28 respectively and are no longer significant (more detailed results are available on request). As dropping specific countries is a somewhat ad hoc choice, Table 4.3 reports results of two alternative robustness checks. First, the middle panel shows that demographics also have no significant impact on volatility when using only the more homogeneous panel of OECD countries. As a further robustness check, we use a more

Note that the MW test on the error terms of the FE and FETE regressions rejects the null of no cointerecent version of the Penn World Table (2012, version 7.1) and the 2012 revision of the World Population Prospects, but leave the sample period unchanged (1957 - 2000). The estimation results in the right panel of Table 4.3 show that data revisions apparently also remove the alleged strong relation between demographics and volatility.

4.4.2 Lugauer (ReStat, 2012)

Lugauer (2012) uses a panel including data on 50 U.S. states over the period 1981-2004. Output volatility is defined as in Lugauer and Redmond (2012), with state-level GDP data taken from the Bureau of Economic Analysis. Demographic variables are based on U.S. Census data. The age share variable used in the baseline regression is the *youth share*, defined as the fraction of the population aged 20 to 54 under the age of 35. State-specific ADF unit root tests (not reported) after projecting out time fixed effects show that the null hypothesis of a unit root is rejected at the 5% level of significance in 1 state for the HP volatility measure and in 3 states for the age share variable. The null hypothesis of a unit root is also not rejected using a MW panel unit root test.

Table 4.4 replicates the FETE estimate of 3.13 reported by Lugauer. Again this is not a cointegrating relation, i.e. the null hypothesis of a unit root is rejected in only 1 out of the 50 states while the MW panel test does not reject the null hypothesis. Also note that the time dummies are now no longer able to remove all of the cross-sectional dependence. The CCEP estimate of 1.20 is much smaller but still significant at the 5% level. Interestingly, this estimator is able to remove all of the cross-sectional dependence with the MW test on the idiosyncratic part of the error term rejecting the null of no cointegration.

Important to note is that the youth share used by Lugauer (2012) as demographic variable is different from the volatile-age share variable suggested by Ja&Si. As such, this variable does not capture the alleged U-shaped pattern of employment volatility as it excludes the youngest (15-19) and oldest (60-64) workers. In column 5-7 of Table 4.4 we therefore present regression results when the demographic variable is defined as in Ja&Si. The coefficient turns negative and, in the case of the FETE estimator, is even statistically significant. The CCEP regression is now no longer a cointegrating relation.

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gration. This is due to the fact that for the OECD sample the MW test also rejects the null of a unit root in the HP volatility measure.

The data are available at the Review of Economics and Statistics Database: *http://hdl.handle.net/1902.1/19663*.

To construct the volatile-age share variable we had to make us of the most recent Census revision. Replacing the original with the revised data but keeping the youth share definition did not notably change the results reported in columns 2-4 of Table 4.4.

Table 4.4: The Lugauer (ReStat, 2012) regression

	s_{it} = youth share ($N = 50$)			s_{it} = volatile-age share ($N = 50$)		
	FE	FETE	CCEP	FE	FETE	CCEP
Estimation results						
γ	4.64	3.13	1.20	6.50	-2.92	-1.01
	(0.24)	(1.24)	(0.51)	(0.31)	(1.11)	(0.83)
Cointegration tests						
ADF: # $p_i < 5\%$	1	1	4	1	0	4
Panel MW	82.73	107.27	124.07	107.51	94.47	103.08
	[0.89]	[0.29]	[0.05]	[0.29]	[0.64]	[0.40]
r	-	-	2	-	-	2
Cross-sectional dependence in the first-differenced error terms						
$\bar{\hat{\rho}}$	0.44	0.06	0.00	0.39	0.06	-0.01
CD	73.32	9.33	0.80	66.04	9.80	-1.30
	[0.00]	[0.00]	[0.42]	[0.00]	[0.00]	[0.19]

Notes: see Tables 4.1, 4.2 and 4.3.

4.5 Conclusion

This paper has replicated several important studies estimating the impact of the age distribution of the work force on cyclical output volatility. Using the original datasets we first replicate the estimation results. However, the existing literature ignores the non-stationary behavior and the cross-sectional dependence in the error terms. We therefore go on to show that there is clear evidence of a unit root in the error terms of the regressions. This suggest that the strong correlation between demographics and business cycle volatility reported in the literature may be spurious rather than signaling a meaningful long-run relationship. This suggestion is supported by the finding that the estimation results are very sensitive to small changes in the sample and/or in the definition of the labor share variable. Accounting for unobserved common factors using the CCEP estimator does not change this conclusion. Our results show that the current setup of using only a composite demographic change variable is probably too restrictive and more work is needed to come up with a satisfactory explanation for the great moderation and for shifts in volatility more in general.

Bibliography

BAI, J. S., AND S. NG (2004): "A PANIC Attack on Unit Roots and Cointegration," *Econometrica*, 72(4), 1127–1177.

BANERJEE, A., AND J. L. CARRION–I–SILVESTRE (2015): "Testing for Panel Cointegration Using Common Correlated Effects Estimators," Discussion Paper 15-02, Department of Economics, University of Birmingham.

DICKEY, D., AND W. FULLER (1979): "Distribution of the Estimators for Autoregressive Time-Series with a Unit Root," *Journal of the American Statistical Association*, 74(366), 427–431.

EVERAERT, G. (2014): "A Panel Analysis of the Fisher Effect with an Unobserved I(1) World Real Interest Rate," *Economic Modelling*, 41, 198–210.

JAIMOVICH, N., S. PRUITT, AND H. E. SIU (2013): "The Demand for Youth: Explaining Age Differences in the Volatility of Hours," *American Economic Review*, 103(7), 3022–3044.

JAIMOVICH, N., AND H. E. SIU (2009): "The Young, the Old, and the Restless: Demographics and Business Cycle Volatility," *American Economic Review*, 99(3), 804–826.

KAPETANIOS, G., M. H. PESARAN, AND T. YAMAGATA (2011): "Panels with Non-Stationary Multifactor Error Structures," *Journal of Econometrics*, 160(2), 326–348.

LUGAUER, S. (2012): "Estimating the Effect of the Age Distribution on Cyclical Output Volatility Across the United States," *The Review of Economics and Statistics*, 94(4), 896–902.

LUGAUER, S., AND M. REDMOND (2012): "The Age Distribution and Business Cycle Volatility: International Evidence," *Economics Letters*, 117(3), 694–696.

MADDALA, G. S., AND S. W. WU (1999): "A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test," *Oxford Bulletin of Economics and Statistics*, 61, 631–652.

Pesaran, M. (2004): "General Diagnostic Tests for Cross Section Dependence in Panels," Cambridge Working Papers in Economics 0435, Faculty of Economics, University of Cambridge.

Pesaran, M. (2006): "Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure," *Econometrica*, 74(4), 967–1012.

Phillips, P. C. B., and H. R. Moon (1999): "Linear Regression Limit Theory for Nonstationary Panel Data," *Econometrica*, 67(5), 1057–1112.

Stock, J. H., and M. W. Watson (2003): "Has the Business Cycle Changed and Why?," in *NBER Macroeconomics Annual 2002, Volume 17*, ed. by M. Gertler, and K. Rogoff, pp. 159–230. Cambridge, MA: MIT Press.

——— (2005): "Understanding Changes in International Business Cycle Dynamics," *Journal of the European Economic Association*, 3(5), 968–1006.

Urbain, J.-P., and J. Westerlund (2011): "Least Squares Asymptotics in Spurious and Cointegrated Panel Regressions with Common and Idiosyncratic Stochastic Trends," *Oxford Bulletin of Economics and Statistics*, 73(1), 119–139.

5 | What Drives Output Volatility? The Role of Demographics and Government Size Revisited

Abstract: This paper investigates the determinants of output volatility in a panel of 22 OECD countries. In contrast to the existing literature, we avoid ad hoc volatility measures based on rolling windows and account for the possible non-stationarity of the data. Specifically, we estimate output volatility from an unobserved components model where the volatility series is the outcome of both macroeconomic determinants and a latent integrated process. A Bayesian model selection is performed to test for the presence of the non-stationary component, similar to a cointegration test. Results suggest that output volatility is driven by demographics and openness of the economy, while the effects of government size and taxation are ambiguous.

JEL classification: C32, E32, E62, J11

The author would like to thank Gerdie Everaert, Tino Berger, Gert Peersman, Freddy Heylen, Helmut Herwartz, Joris Wauters, and participants of the Ghent Macro Research Group meeting for helpful comments, and Olaf Posch for sharing his dataset on taxation. Hauke Vierke acknowledges financial support from the National Bank of Belgium.

5.1 Introduction

Since its first documentation by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), the persistent reduction in United States business cycle volatility during the 1980s has inspired a large body of literature. Stock and Watson (2003) coined the term "Great Moderation" to describe the puzzling fall in volatility. Since then, many studies have tried to explain the time variation in business cycle volatility, mainly through advances in economic policy or changes in deep structural factors of the economy. The latter explanation is supported by recent studies on the role of demographics. While it was first documented in the United States, Del Negro and Otrok (2008) showed that the decline in volatility was a global phenomenon with important differences across countries regarding the magnitude, timing, and sources. International differences in output fluctuations have also been investigated in an earlier strand of literature that mainly focused on cross-country regressions to explain volatility among OECD countries. Results suggest an important role of government size for stabilizing the economy. However, the literature suffers from two important shortcomings: The use of ad hoc measures for output volatility and possible spuriousness of the regression result. The latter point was investigated extensively by Everaert and Vierke (2015) for the relationship between demographics and output volatility.

In this paper, a novel approach for estimating the determinants of output volatility is proposed. The contribution is threefold. First, instead of relying on rolling standard deviation, the volatility series is obtained from an unobserved components model with a latent volatility process, which is partly driven by macroeconomic covariates. The second key feature is the inclusion of an unobserved I(1) component in the volatility equation to account for possible non-stationary factors driving volatility. Third, in order to obtain the most parsimonious model, a Bayesian variable selection is performed on the non-stationary factor, similar to a cointegration test. The model is estimated for an unbalanced panel of 22 OECD countries with data from 1960 to 2007. We merge a recent literature strand on the effect of demographics with the literature on the role of government stabilization. Specifically, we try to explain output volatility over time and across countries with the demographic composition of the labor force as well as the size of the government, while controlling for tax composition and openness of the economy. The regression results suggest important effects of demographics, openness, and labor taxation.

The remainder of the paper is structured as follows: Section 5.2 reviews the literature on the relationship between government size, taxation, demographics, and output volatility.

We use the terms *output volatility* and *business cycle volatility* interchangeably in this paper.

Section 5.3 presents the unobserved components model and gives a short introduction to the testing problem involved with this approach. In section 5.4 the Gibbs sampling algorithm and the prior distributions are presented. Section 5.5 discusses the sources, transformations, and time-series properties of the data. Estimation results are presented in section 5.6, and section 5.7 concludes.

5.2 Literature review

This paper focuses on the effect of fiscal policy, mainly government size and taxation, and demographics on output volatility. From a theoretical point of view, there exist different perspectives on the role of the government in stabilizing output. While the Keynesian school argued that the economy is inherently unstable and cyclical fluctuations are costly, Lucas (1987) estimated the welfare cost of the business cycle, based on a micro-founded model, to be negligible. Abstracting from normative statements about the cost of the business cycle, the question arises whether governments are able to alter output fluctuations in the first place. A seminal contribution on the role of government size for output stabilization was given by Gali (1994), who investigates the effect of income taxation and government purchases on output volatility in a real business cycle (RBC) model with technology-driven shocks. The model predicts a destabilizing effect of tax increases as they reduce (after tax) productivity and thus lower steady-state employment. The resulting higher labor elasticity leads to a greater labor supply sensitivity to technology shocks. Higher government purchases, on the other hand, have an opposing effect and lead to lower output volatility. For different calibrations the destabilizing effect of higher labor taxes dominates. However, when taken to data from 22 OECD countries the estimated effects do not match the theoretical predictions. A cross-country regression reveals that income taxes and government purchases are "automatic stabilizers" in the Keynesian sense. Tax progressivity and transfer payments could therefore help to cushion the impact of shocks to aggregate production on disposable income. Gali (1994) argues that the model might be missing important features such as productivity-improving government spending.

Since the seminal work by Gali, many papers have investigated the effect of government size and/or taxation on output volatility. For example, Fatás and Mihov (2001) estimate the effect of government spending on output volatility for 20 OECD countries (1960-1997). They report that regardless of the volatility or government size measure, the effect on output is always stabilizing. In a related study, Martinez-Mongay and Sekkat (2005) test whether the negative relation between government size and output volatility depends on the tax mix.

Specifically, the authors are interested in the effect of distortionary taxes. Using data for 25 OECD countries (1960-2000), they find that labor and capital taxes have stabilizing effects, although the evidence is weak. Pisani-Ferry, Debrun, and Sapir (2008) extend the literature by exploiting the time series dimension of the data and perform a panel regression for 20 OECD countries (1960-2006). They confirm the link between government size and macroeconomic volatility for the beginning of the sample, but show that the relation disappeared during the 1990s. Carmignani, Colombo, and Tirelli (2011) pay special attention to the endogeneity of government size and estimate simultaneous equations. They find that greater volatility causes larger governments. In contrast to previous studies, their estimates imply a destabilizing effect of government expenditure on output volatility. Crespo Cuaresma, Reitschuler, and Silgoner (2011) estimate a dampening and non-linear effect of government size on output volatility for a sample of EU countries, which reverts at very high levels of government expenditure. In a recent contribution Posch (2011) derives the effect of different taxes on the volatility of output growth within a stochastic neoclassical growth model and tests the theoretical predictions within a panel regression of 20 OECD countries. Special attention is paid to the nature of the unobserved variance process. Different tax ratios are found to have ambiguous effects: Taxes on labor and corporate income are stabilizing, while capital taxes increase volatility. Overall, the existing literature suggests a dampening effect of government expenditure on output volatility, in line with the notion of automatic stabilizers. However, when some of the channels are investigated individually, the picture is less clear-cut and opposing effects arise.

Next to the role of the government for output stabilization, we take into account recent studies that attribute the Great Moderation to structural change in the form of demographic shifts. As the decline in United States' output volatility coincides with a decrease in the number of young workers relative to prime-age workers, the demographic composition of the labor force arises as a possible explanation for volatility shifts. A theoretical explanation was given by Jaimovich, Pruitt, and Siu (2013) within a standard neoclassical model. When the production function includes capital-experience complementarity, the elasticity of substitution between capital and labor differs among young and old workers, which leads to higher volatility in the wages and working hours of the younger workers. Jaimovich and Siu (2009) were the first to empirically test this relationship for a small sample of countries. They start from documenting differences in the volatility of employment and hours worked across different age groups. The youngest and oldest among the workforce experience larger employment fluctuations than prime-age workers. To reflect this U-shaped pattern the authors define a volatile-age share variable, constructed as the ratio of the 15-29 and 60-64 old workers over the entire workforce. Estimating a panel regression for the G7 countries they find a large and significant effect of the age share on output volatility, defined as the rolling standard deviation of the HP-filtered output gap. The estimated effect is large enough to account for up to one-third of the decline in output volatility in the United States. Lugauer (2012) confirms this significant effect of demographics for a large panel of the 50 federal states. Similar results are also found in Lugauer and Redmond (2012) for 51 advanced and developing countries.

Nevertheless, we identify two important shortcomings in the literature. First, the majority of the studies using panel methods do not discuss the time series properties of the variables considered, although many are potentially non-stationary. Notable exceptions, which pay attention to the problem of spurious regression results, are Posch (2011), who performs a cointegration analysis and confirms a long-run relation between taxes and volatility, and Kent, Smith, and Holloway (2005), who discuss the limits of a linear time trend or common time effects for capturing common volatility trends. Jaimovich and Siu (2009) mention that their findings could be spurious because of omitted non-stationary factors, but argue that cross-country differences in volatility and demographics ensure identification. Everaert and Vierke (2015) replicate three studies on the relationship between demographics and business cycle volatility and show that common time-dummies are insufficient to account for non-stationarity. As the majority of studies on the determinants of output volatility rely only on time dummies to cope with non-stationary common factors, this casts doubt on the reliability of their results.

Second, the latent volatility series is usually estimated via rolling standard deviations. This generates a high degree of persistence in the endogenous variable due to the overlapping windows, so that the error terms of the subsequent regression exhibit serial correlation. Moreover, complex dynamics in output volatility are averaged out through this procedure. The problem is partly acknowledged by Jaimovich and Siu (2009) and Lugauer (2012), who obtain an alternative volatility series from the stochastic volatility model of Stock and Watson (2003). However, volatility is explicitly modeled as a non-stationary process, which emphasizes the problem of spurious results when regressing the obtained series on a set of non-stationary explanatory variables. Jaimovich and Siu (2009) and Posch (2011) also address the unobservability of output volatility and estimate a GARCH model borrowed from Ramey and Ramey (1995), which relies on common time dummies to capture time-variation not explained by the covariates. Posch (2011) touches on the limits of this approach in the context of cointegration. We conclude that the time series treatment in the literature is insufficient. The different approaches are conflicting, as the same variable is assumed to be stationary in one application

While a large number of cross sections can eliminate the problem of spurious results, this only holds for the case of cross-sectionally independent error terms.

while explicitly modeled as I(1) in another one.

5.3 Empirical model

5.3.1 An unobserved components model with stochastic volatility

The empirical model is based on the following output growth decomposition

$$y_{i,t} = \mu_{i,t} + \rho_{1,i}(y_{i,t-1} - \mu_{i,t-1}) + \rho_{2,i}(y_{i,t-2} - \mu_{i,t-2}) + \exp\{h^*_{i,t}\}\varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim i.i.d.\mathcal{N}(0,1), \tag{5.1}$$

where $y_{i,t}$ is annual output growth in country $i = 1, ..., N$ at time $t = 1, ..., T$. $\mu_{i,t}$ is the time-varying mean growth rate, and $\exp\{h^*_{i,t}\}$ is the time-varying standard deviation of output shocks. Deviations from the mean growth rate are assumed to be transitory, i.e. the characteristic roots of the AR(2) process lie strictly outside the unit circle. The mean growth rate is assumed to evolve over time according to a random walk:

$$\mu_{i,t} = \mu_{i,t-1} + \eta^{\mu}_{i,t}, \qquad \eta^{\mu}_{i,t} \sim i.i.d.\mathcal{N}(0, \sigma^2_{\mu,i}). \tag{5.2}$$

Although a time-varying mean growth rate is not a stylized fact in empirical macroeconomics, we follow a recent finding by Berger, Everaert, and Vierke (2015) of a persistent decline in potential output growth. This is in line with the literature on the United States productivity slowdown in the early 1970s. Perron and Wada (2009), Kim, Piger, and Startz (2007), and Morley and Piger (2012) take this slowdown into account by allowing for structural breaks in the growth rate of potential output. The random walk specification can mirror a similar pattern, but allows for more complex dynamics as well. The volatility process $h^*_{i,t}$ is assumed to be driven by a vector of macroeconomic covariates, $x_{i,t}$, where the country-specific slope coefficients are collected in the vector β_i. These covariates or a subset of them may exhibit non-stationarity. Thus, the question of cointegration between output volatility and the explanatory variables arises. As shown by Everaert (2011), standard cointegration analysis yields spurious results when relevant integrated variables are omitted from the model. One solution is to model the omitted variables as a random walk component within an unobserved components framework, so that the long-run relation between the non-cointegrated variables can be estimated consistently via the Kalman filter. Hence, an unobserved non-stationary error component, $h_{i,t}$, is introduced to the model. In line with the literature on stochastic volatility, this component is modeled as a driftless random walk:

$$h^*_{i,t} = \beta_i' x_{i,t} + h_{i,t}, \tag{5.3}$$

$$h_{i,t} = h_{i,t-1} + \eta^h_{i,t}, \qquad \text{where } \eta^h_{i,t} \sim i.i.d.\mathcal{N}(0, \sigma^2_{h,i}). \tag{5.4}$$

Equations (5.1)-(5.4) constitute a state-space model, where (5.1) is the observation equation, linking the latent variables to the observed output series, and (5.2) - (5.4) are the state equations, describing the dynamics of the latent variables. Given estimates for the model parameters, the latent variables of this state space system can be estimated via the standard Kalman filter.

5.3.2 Model selection in the UC framework

A key feature of this paper is that we test for the presence of a non-stationary country-specific error component $h_{i,t}$. This testing problem is equivalent to testing the hypothesis $\sigma^2_{h,i} = 0$ against $\sigma^2_{h,i} > 0$. Note that in the first case, $h_{i,t}$ becomes a constant and takes on the role of a country-specific intercept in the volatility equation, i.e. it captures differences over countries that are not explained by the set of covariates and do not change over time. In the latter case, $h_{i,t}$ will capture omitted and/or unobservable non-stationary factors which permanently shift volatility. The proposed test is non-regular, since the null hypothesis lies at the boundary of the parameter space. In principle, this could be dealt with by a Lagrange multiplier test. This paper follows a different approach and applies a Bayesian procedure, mainly for two reasons. First, the Bayesian approach allows to make intuitive statements about the inclusion probability of the non-stationary factors. Second, the estimation of the proposed SV model becomes intractable for the classical approach. A Gibbs sampling approach, which relies on splitting the complex nonlinear model into conditionally normal blocks, makes estimation feasible and leads to a Bayesian testing procedure without many additional modifications.

Specifically, the stochastic model specification search (SMSS), introduced by Frühwirth-Schnatter and Wagner (2010), is applied. This approach requires the model to be re-written

Ex ante, no correlation structure is imposed on the factors across countries. As explained in the Appendix, all country-specific factors are filtered and sampled separately. This approach keeps the size of the covariance matrices manageable, but still allows for *ex post* correlation among the country-specific factors.

See Morley, Panovska, and Sinclair (2015) for an evaluation of the LM test for the presence of a non-stationary component and a discussion of its small sample properties.

in a non-centered parameterization. Equation (5.4) is rearranged in the following way:

$$h_{i,t} = h_{i,0} + \sigma_{h,i}\,\widetilde{h}_{i,t}, \tag{5.5}$$

$$\widetilde{h}_{i,t} = \widetilde{h}_{i,t-1} + \widetilde{\eta}^{h}_{i,t}, \qquad \text{where } \widetilde{h}_{i,0} = 0 \text{ and } \widetilde{\eta}^{h}_{i,t} \sim i.i.d.\mathcal{N}(0,1). \tag{5.6}$$

The non-centered parameterization splits the factor, $h_{i,t}$, into a constant part, $h_{i,0}$, and a time-varying part, $\widetilde{h}_{i,t}$. The standard deviation of the innovations to the random walk component now enters multiplicatively in equation (5.5). As such, the model is non-identified. To see that, note that the signs of $\sigma_{h,i}$ and $\widetilde{h}_{i,t}$ can be interchanged without affecting the sign of their product. In the non-centered parameterization, $\sigma_{h,i}$ holds valuable information about the presence of a time-varying component. If the posterior distribution is unimodal, the model favors a constant country-specific volatility factor. If the the distribution is bimodal around zero, a time-varying factor is favored. The non-centered parameterization leads to a more formal test for time-variation without much further modification. Frühwirth-Schnatter and Wagner (2010) introduce a binary indicator to select the most parsimonious specification. Here, the indicator is labeled δ_i, which takes on the value $\delta_i = 1$ for a model with time-varying volatility factor in country i and $\delta_i = 0$ if the country factor is constant over time. Hence, the final volatility equation is given by

$$h^{*}_{i,t} = \beta_i' x_{i,t} + h_{i,0} + \delta_i \sigma_{h,i}\,\widetilde{h}_{i,t}. \tag{5.7}$$

All country-specific indicators are collected in a vector $\mathcal{M} = (\delta_1, \delta_2, ..., \delta_N)$, where every possible combination of the binary indicators constitutes one specific model. We impose a uniform prior on the model probabilities, so that each model has the same prior probability. The models are then sampled according to their Bayesian model probability. Details on how to estimate the indicators are given in the Appendix of this paper. The SMSS in combination with the non-centered parameterization holds important advantages: It avoids inverse-Gamma prior distributions, which can lead to a substantial bias for the variance of the non-stationary factor when the true value is close to zero. Instead, the square root of the variance of the non-stationary component now enters as a regression coefficient in equation 5.5, for which a normal prior distribution can be used. This prior does not impose the same bias as the inverse-Gamma distribution. Moreover, the SMSS relies on binary model indicators that can easily be sampled during the Gibbs sampling procedure along with the remaining model parameters. These indicators can be used to test for non-stationary factors in the volatility equation.

5.4 Bayesian estimation

5.4.1 Gibbs sampling

This paper relies on Markov Chain Monte Carlo (MCMC) methods for estimation. The proposed model is highly nonlinear because of the stochastic volatility components, which enter exponentially into equation 5.1. Thus, the standard application of the Kalman Filter and Maximum likelihood technique is not feasible. Instead a Gibbs Sampling procedure is applied. Specifically, the complex model is split into blocks of parameters and components that are linear conditional on the other blocks. To linearize the nonlinear volatility process, we follow Kim, Shephard, and Chib (1998) and take the logarithm of the square of the process. The resulting linear model, which has non-Gaussian error terms, is then approximated by an offset mixture model. This section only holds a brief description. A detailed explanation of the algorithm is given in the Appendix. For notational convenience, the country-specific components are collected in the vectors $\mu_t = (\mu_{t,1}, ..., \mu_{t,N})$ and $\widetilde{h}_t = (\widetilde{h}_{t,1}, ..., \widetilde{h}_{t,N})$. Observations are stacked over time, i.e. $x = \{x_t\}_{t=1}^T$, $y = \{y_t\}_{t=1}^T$, $\mu = \{\mu_t\}_{t=1}^T$ and $\widetilde{h} = \left\{\widetilde{h}_t\right\}_{t=1}^T$. Country-specific parameters are collected in the vectors $h_0 = (h_{0,1}, ..., h_{0,N})$, $\sigma_h = (\sigma_{h,1}, ..., \sigma_{h,N})$, and $\beta = (\beta_1, ..., \beta_N)$. Moreover, denote $\rho_i = (\rho_{1,i}, \rho_{2,i})'$ and $\rho = (\rho_1, ..., \rho_N)$. All parameters are collected in $\psi = (\beta, \rho, h_0, \sigma_h, \sigma_\mu)$. The posterior density of interest is then given by $f(\widetilde{h}, \mu, \psi, \mathcal{M}|y, x)$. In short, the algorithm consists of the following Gibbs Sampling steps:

1. Sample the binary indicators $\mathcal{M}$ from $f(\mathcal{M}|\widetilde{h}, \mu, y, x)$ marginalizing over the parameters ψ and sample the unrestricted parameters in ψ from $f(\psi|\widetilde{h}, \mu, \mathcal{M}, y, x)$, while setting $\sigma_{h,i}$ equal to zero for countries for which $\delta_i = 0$ from $\mathcal{M}$.

2. Sample the latent components $\widetilde{h}$ from $f(\widetilde{h}|\mu, \psi, \mathcal{M}, y, x)$ and μ from $f(\mu|\widetilde{h}, \psi, \mathcal{M}, y, x)$.

3. Perform a random sign switch for σ_h and $\widetilde{h}$ with probability 0.5.

By sampling repeatedly from these blocks, we obtain parameters draws from the joint posterior after a sufficiently large number of "burn-in" iterations

5.4.2 Prior choice

This paper follows a Bayesian approach and therefore the choice of prior distributions deserves some explanations. Prior distributions are given in Table 5.1. The main parameters of interest are the β coefficients, which represent the effect of the explanatory variables on output volatility. For these parameters a diffuse prior is chosen. The prior is normal with mean zero and large standard deviation. Because of the non-centered parameterization, normal priors

Table 5.1: Prior distributions

Name	Description	Density	Specification	
			Mean a_0	Std $\sqrt{A_0}$
$\beta_{i,k}$	Correlation coefficients	$\mathcal{N}(a_0, A_0)$	0.0	10.0
$h_{i,0}$	Constant country-specific volatility	$\mathcal{N}(a_0, A_0)$	0.0	1.0
$\sigma_{h,i}$	Std of country-specific tv-volatility	$\mathcal{N}(a_0, A_0)$	0.0	1.0
$\rho_{1,i}$	AR coefficient output gap	$\mathcal{N}(a_0, A_0)$	0.6	0.05
$\rho_{2,i}$	AR coefficient output gap	$\mathcal{N}(a_0, A_0)$	-0.2	0.05
			Belief σ_0^2	Strength ν_0
$\sigma^2_{\mu,i}$	Variance of growth trend shocks	$IG(v_0 T, v_0 T \sigma_0^2)$	0.1	$T \times 0.1$
			Success rate p	
$p(\delta_i = 1)$	Inclusion probability of country-specific tv-volatility	Bern(p)	0.5	

can also be applied to the constant output volatility, $h_{0,i}$, and the standard deviation of the country-specific time-varying volatility, $\sigma_{h,i}$, which take on the role of regression coefficients. For both parameters the prior distributions are relatively uninformative with a zero-mean and unit standard deviation. The normal priors on the autoregressive coefficients ρ imply a low degree of persistence, which is in line with fitting a simple univariate AR model with constant volatility to annual output growth. For the standard deviation of the time-varying output growth rate, the prior distribution is the usual inverse-Gamma distribution $IG(c_0, C_0)$ where the shape c_0 and scale C_0 are expressed in terms of a *prior belief* σ_0^2 and a *prior strength* v_0. Specifically, we set $c_0 = v_0 T$ and $C_0 = s_0 \sigma_0^2$, so that the strength can be expressed as a fraction of the sample size. Our belief implies that roughly 95% of shocks to the mean growth rate at annual frequency lie between +/- 0.62%-points. The strength of this belief is relatively weak, as it implies a number of "fictitious" observations equal to 10% of the sample size. Finally, the prior probability for time-variation in country-specific output volatility, which is needed for the SMSS procedure, is given by a Bernoulli distribution with $p = 0.5$, which assigns equal weight to the null and alternative hypothesis.

5.5 Data and time series properties

This paper studies output volatility dynamics in an unbalanced panel of 22 OECD countries at annual frequency for the years 1960 to 2007. The explanatory variables include 7 indicators for fiscal policy, demographics, and openness. Details on variable construction and sources are given in Table 5.2. Output growth is measured by the first difference of the log of real GDP, which is standard in the literature. In line with the recent studies on the effects of demographics on volatility, we add the share of the working age population aged 15-29 and 60-64 as an explanatory variable. Empirical estimates by Jaimovich and Siu (2009) suggest a positive effect of the share variable. Regarding the impact of fiscal policy on output volatility, we consider both the revenue and expenditure side. On the expenditure side, we measure the size of the government by the government's final consumption expenditure expressed as a share of GDP. However, as first noted by Rodrik (1998) and then discussed by Fatás and Mihov (2001) and others, the effect of government size might suffer from a simultaneity bias. More open economies might experience greater volatility because of their exposure to international business cycle shocks. If governments can indeed reduce volatility, they will likely be bigger in more open economies, which would induce a downward bias of the estimated effect of government size. Hence, we include trade openness, defined as the sum of exports and imports as a share of GDP, into our regression analysis. Despite the argument of Rodrik (1998), the sign of the effect of openness is not clear-cut, as higher openness can also represent a form of risk diversification that can counteract shocks at the national level. On the revenue side, we measure the tax mix by three different tax variables: Consumption, capital, and labor tax ratios. We combine tax data from two different sources, McDaniel (2007) and Posch (2011), to achieve a larger sample. Specifically, Posch (2011) measures average taxes following the methodology of Mendoza, Razin, and Tesar (1994) using OECD data. This data set covers 22 countries, but is only available from 1970 onwards. The data provided by McDaniel (2007) are based on national accounts and thus start as early as 1950. Unfortunately, the sample covers only 15 out of the 22 countries. Thus, we merge the two sources and extend the data from Posch (2011) with the earlier data from McDaniel (2007) for these 15 countries. This leaves us with an unbalanced panel consisting of 22 countries and a total of 984 observations. Posch (2011) derives marginal tax effects in a stochastic neoclassical growth model and finds that theoretical effects are ambiguous. Labor taxes have negative effects on output volatility

The countries considered are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

See McDaniel (2007) for more details on the comparison with the Mendoza, Razin, and Tesar (1994) methodology.

Table 5.2: Data description and sources

Variable	Description	Source
y	First difference of log real GDP	Penn World Tables 8.0
age	Share of the the working age population (15-64) aged 15-29 and 60-64, see Jaimovich and Siu (2009)	World Population Prospects: The 2012 Revision
openness	Sum of exports and imports as a share of GDP	World Bank national accounts data
govsize	General government final consumption expenditure as a share of GDP	World Bank national accounts data
captax	Average tax rate on capital income	McDaniel (2007) & Posch (2011)
contax	Average tax rate on consumption expenditures	McDaniel (2007) & Posch (2011)
labtax	Average tax rate on labor income (incl. payroll tax and tax on household income)	McDaniel (2007) & Posch (2011)

in the model, while consumption taxes are volatility neutral. Capital taxes are a form of income taxation but also on investment and wealth. Thus, they have no clear-cut theoretical effect. Before taking the model to the data, the time series properties of the variables are discussed. As argued in the motivation of this paper, some of the explanatory variables may exhibit non-stationarity so that correlations are potentially spurious. Table 5.3 reports the results of country-specific Augmented Dickey and Fuller (1979) (ADF) and Maddala and Wu (1999) (MW) panel unit root test. In case of the ADF test, the number of cross-sections with an individual p-value lower than 5% is reported. For the MW test, the Fisher test statistic is given along with the corresponding p-value in brackets. Columns 2-3 refer to tests with a country-specific constant, while in columns 4-5 a time trend is added. Only looking at the individual ADF statistics suggests that most of the series considered are in fact unit root processes. For output growth the null of a single unit root can be rejected for almost all countries considered when individual tests are performed. Looking at the explanatory variables, the null of a single unit root in country-wise ADF tests cannot be rejected for the majority of countries. Results from the Madalla-Wu panel unit root test are less clear-cut. For a model with only country-specific intercepts, the data series for openness, capital taxes, and consumption taxes are found to have unit roots. If a time trend is added to the testing equation, unit roots cannot be rejected for openness, government size, consumption, and labor taxes. Taking together the evidence from country-wise and panel unit root tests, it appears

Table 5.3: Country-specific and panel unit root tests

Variable	constant		constant & time trend	
	ADF: $\#(p_i < 5\%)$	Panel MW	ADF: $\#(p_i < 5\%)$	Panel MW
y	20	331.26 [0.00]	20	332.83 [0.00]
age	4	100.37 [0.00]	3	122.73 [0.00]
openness	1	19.85 [0.99]	2	54.02 [0.14]
govsize	2	78.11 [0.00]	2	42.49 [0.54]
captax	2	50.42 [0.23]	4	71.15 [0.01]
contax	1	49.27 [0.27]	2	55.89 [0.11]
labtax	7	78.20 [0.00]	1	25.05 [0.99]

Optimal lag length was determined by Schwartz criterion with maximum number set to 4.

important to account for the time series properties of the data when estimating the model in order to avoid spurious results.

5.6 Estimation results

We iterate the Gibbs steps 10,000 times and drop the first 3,000 as a "burn in" period. For the first 1,500 of these "burn in" iterations, we restrict the binary indicators to be equal to one, after which they are sampled according to the SMSS. This ensures convergence to the ergodic distribution. The results presented in this section are thus based on the remaining 7,000 iterations. Before turning to the estimated effects of the macroeconomic covariates, this section presents results on the time-varying and latent output components: The unobserved trend growth rate, the country-specific latent I(1) component, and the unobserved overall volatility. Then, results on the explanatory variables are given when the full vector or only a subset of covariates is used.

The empirical model arises from the decomposition of annual output growth into a (stochastic) trend and innovation term. The estimated country-specific trend components from the baseline model are given in Figure 5.1. This model includes the full set of explanatory variables. Moreover, all binary indicators are sampled according to the SMSS procedure. Over all countries, the trend rate captures low-frequency movements, so that observed annual growth is mainly driven by the transitory component. The shorter series for some countries are due to limited data availability of the covariates.

One key feature of the approach presented in this paper is the inclusion of a latent non-

We check and reject that the unit roots in the tax rates are the result of the procedure for merging the data sets.

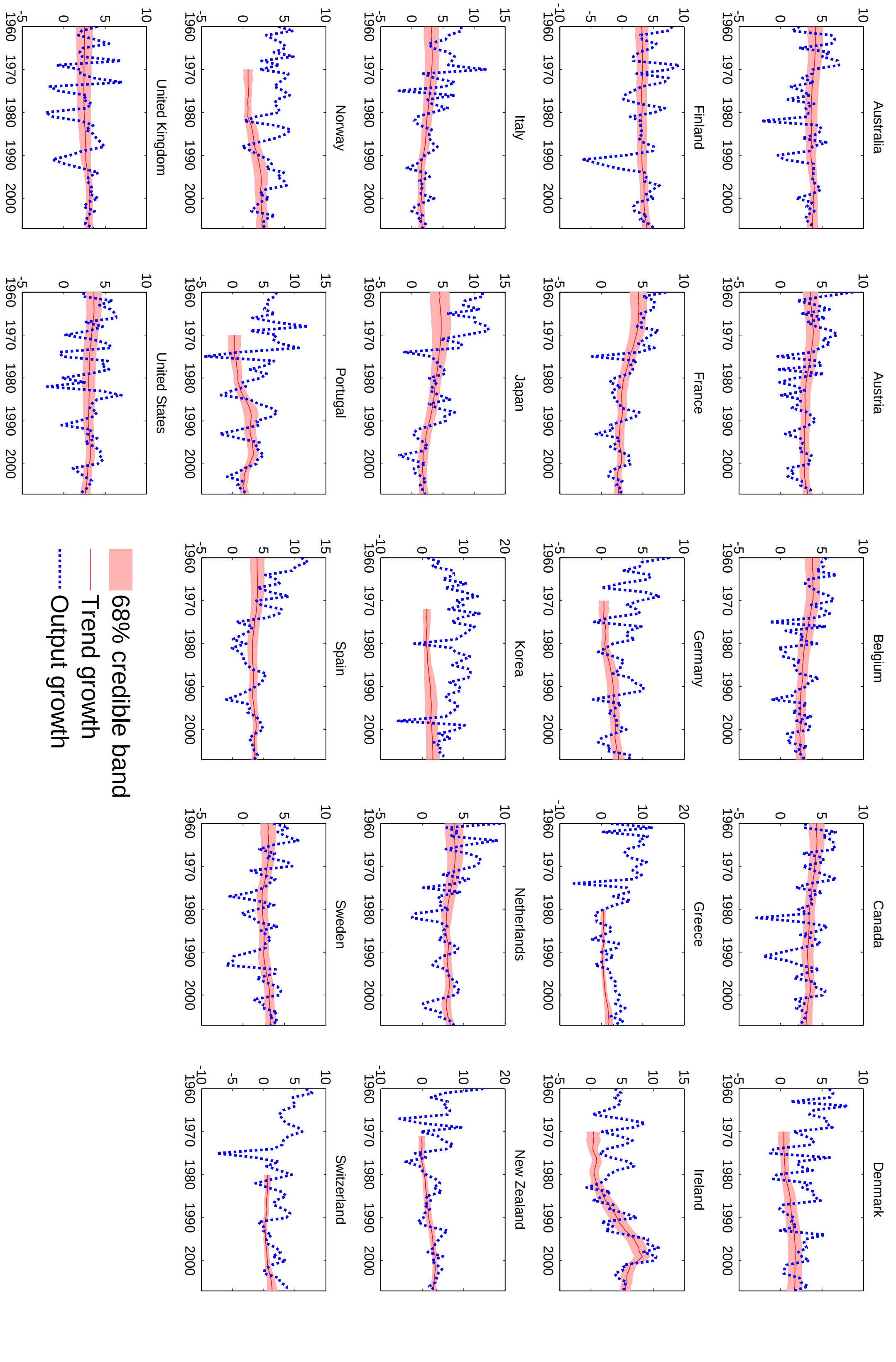
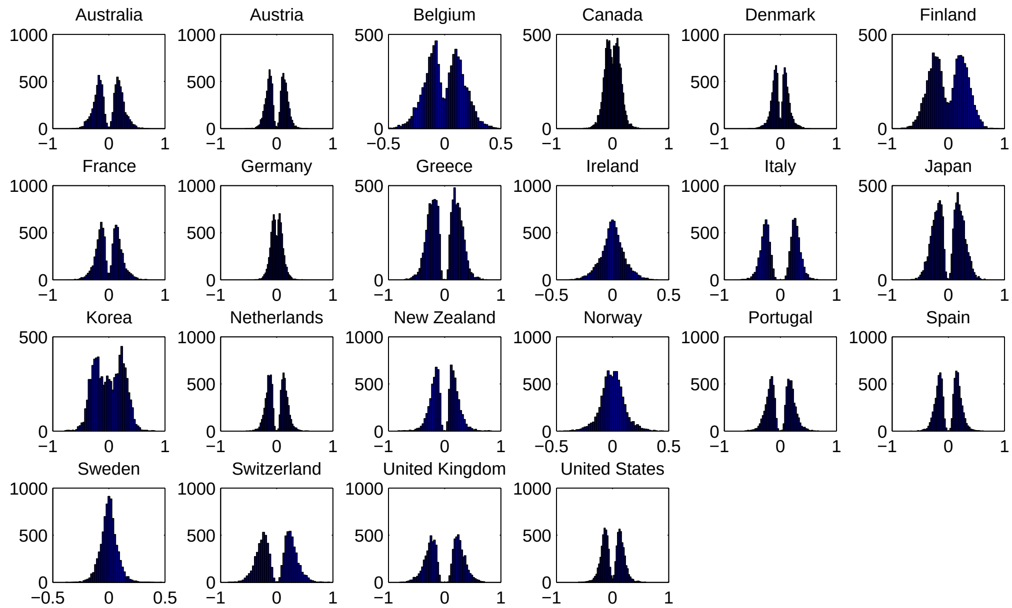


Figure 5.1: Median estimate of trend growth rate

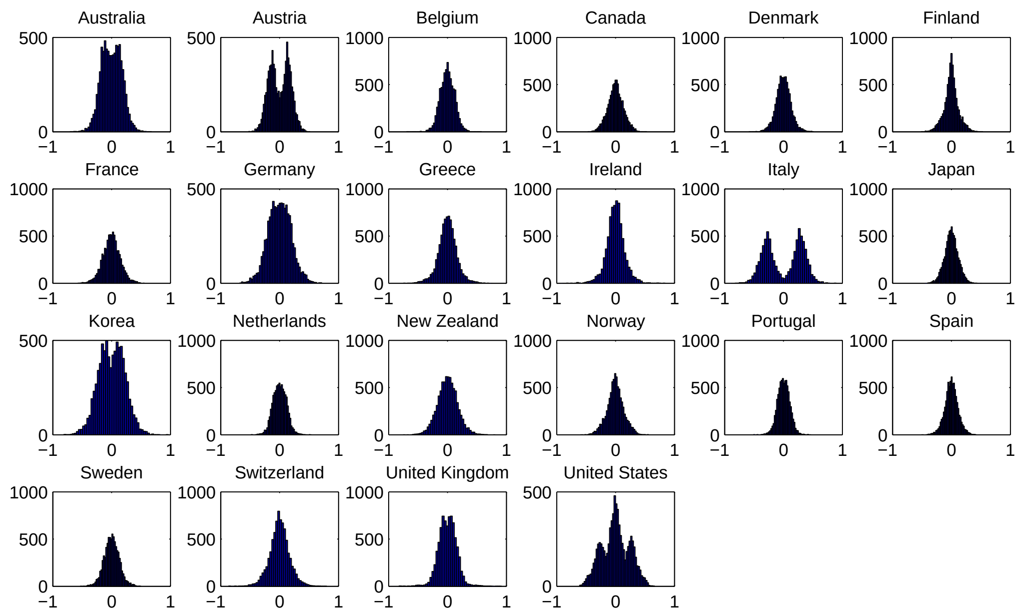
Figure 5.2: Posterior distributions of standard deviation for SV component, excl. covariates



stationary volatility component, which is selected according to Bayesian model probabilities. As a preliminary test, prior to the stochastic model specification search, one can investigate the importance of this component by looking at the posterior distribution of the country-specific standard deviation of the stochastic component. In Figures 5.2 and 5.3 the empirical distributions of $\sigma_{h,i}$ are plotted for both a model without any covariates and the full set of covariates. As such, Figure 5.2 gives preliminary evidence on the degree of time-variation in output volatility when volatility is purely stochastic, i.e. all $\beta_{i,k}$ are set to zero. Comparing Figures 5.2 and 5.3 shows how much of the time-variation can be accounted for by the explanatory variables. In order to obtain the full empirical distribution of the parameter, the binary indicators on the I(1) component are restricted to be $\delta_i = 1$.

Most of the distributions in Figure 5.2 are bimodal, giving support for a model with time-varying volatility in the majority of the 22 countries. However, once the explanatory variables are added to the equations, there is less evidence for the presence of the latent component. The distribution switches to a unimodal distribution in most cases. As the graphical evidence is mixed for some countries, e.g. Australia, the binary indicators and their respective inclusion probabilities can give quantitative evidence for the presence of a non-stationary component. Table 5.4 gives the posterior probability for the country-specific indicators being equal to one according to the SMSS. Columns 2 and 6 refer to a model without covariates, i.e. time-variation can only arise due to the the unobserved component. To make the effect of the number of covariates more visible, columns 3 and 7 report results for when only the age share

Figure 5.3: Posterior distributions of standard deviation for SV component, baseline model



is included. Columns 4 and 8 refer to the full model, i.e. all 7 covariates are included. Not surprisingly, the inclusion probabilities for a model without covariates are relatively high. This is in line with previous results from the literature that permanent volatility changes occurred in many countries since the 1980s. For the demographic model, 7 out of the 22 countries show an important non-stationary component. This confirms the findings of Everaert and Vierke (2015) on panel regressions when demographics is the only explanatory variable. However, the inclusion probabilities drop way below 0.5 in all countries for the full model specification. This is a first key result: A larger vector of covariates that demographics, openness, and various fiscal indicators does not produce a non-stationary error component, i.e. results are not spurious.

Furthermore, one can look at the estimated paths of the country-specific non-stationary factors, as given in Figure 5.4. For this exercise, binary indicators are set to $\delta_i = 1$ for all countries in order to obtain the full empirical distribution for the non-stationary components. As the non-stationary components are found to be important especially for the demographic model, we plot the components for this specific model. Most importantly, the individual dynamics are quite different across countries and common time dummies will not be able to capture the heterogeneity in non-stationary factors. Countries like Australia, Ireland, Italy, or Portugal show a hump-shaped pattern, while countries like Finland or Korea exhibit a larger spike in the second half of the sample. As already indicated by the estimated posterior standard deviation of the non-stationary components, the variation over time in other countries

Table 5.4: Posterior inclusion probabilities of I(1) factors

	Probability $\delta_i = 1$				Probability $\delta_i = 1$		
Country	Excl. x	Age	Full model	Country	Excl. x	Age	Full model
Australia	**0.55**	0.12	0.06	Japan	**0.88**	0.07	0.06
Austria	**0.56**	**0.97**	0.17	Korea	0.27	**0.69**	0.16
Belgium	0.24	0.08	0.05	Netherlands	**0.78**	0.25	0.08
Canada	0.12	0.07	0.08	New Zealand	**0.99**	0.34	0.09
Denmark	0.17	0.08	0.08	Norway	0.07	0.08	0.08
Finland	0.3	0.06	0.06	Portugal	**0.96**	**1.00**	0.07
France	0.34	0.07	0.06	Spain	**0.94**	**0.93**	0.08
Germany	0.08	0.10	0.13	Sweden	0.05	0.07	0.06
Greece	**1.00**	**1.00**	0.10	Switzerland	**0.94**	0.06	0.09
Ireland	0.07	0.16	0.10	United Kingdom	**1.00**	**0.81**	0.06
Italy	**1.00**	**1.00**	0.07	United States	**0.64**	0.18	0.07

Age: Only *age* variable is included.
Bold numbers indicate probabilities > 0.5.

Figure 5.4: Median estimates of non-stationary components (demographic model, $\delta_i = 1$)

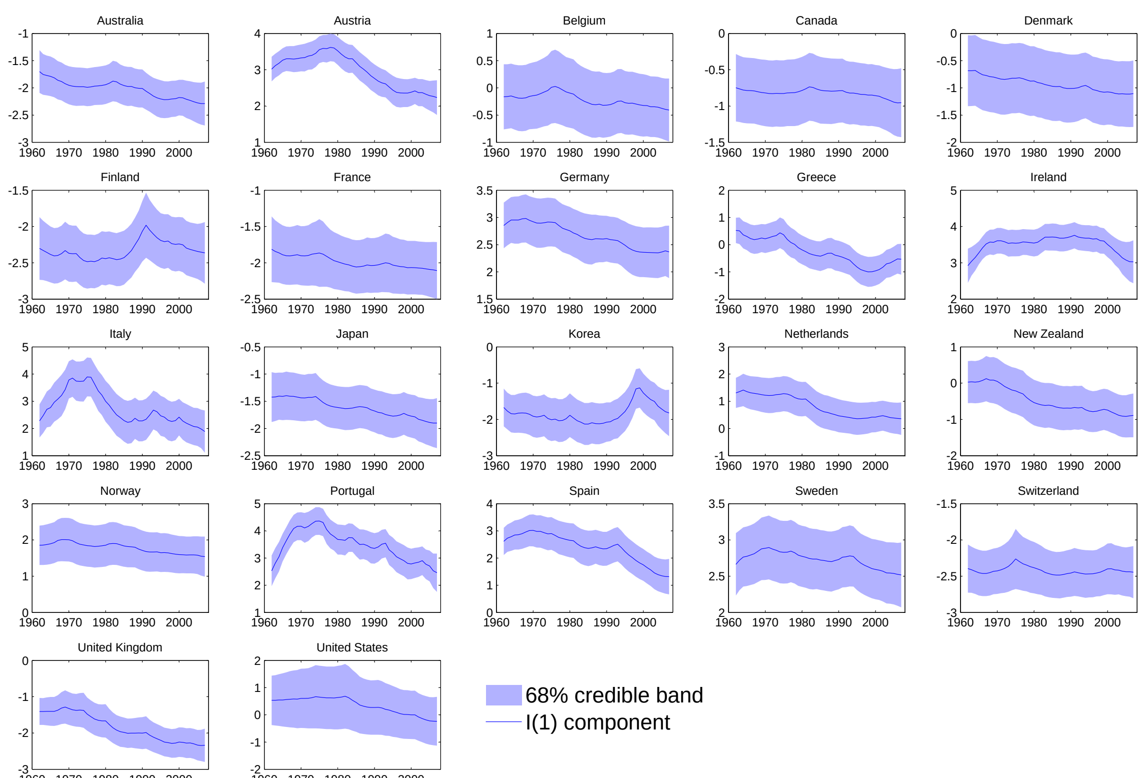
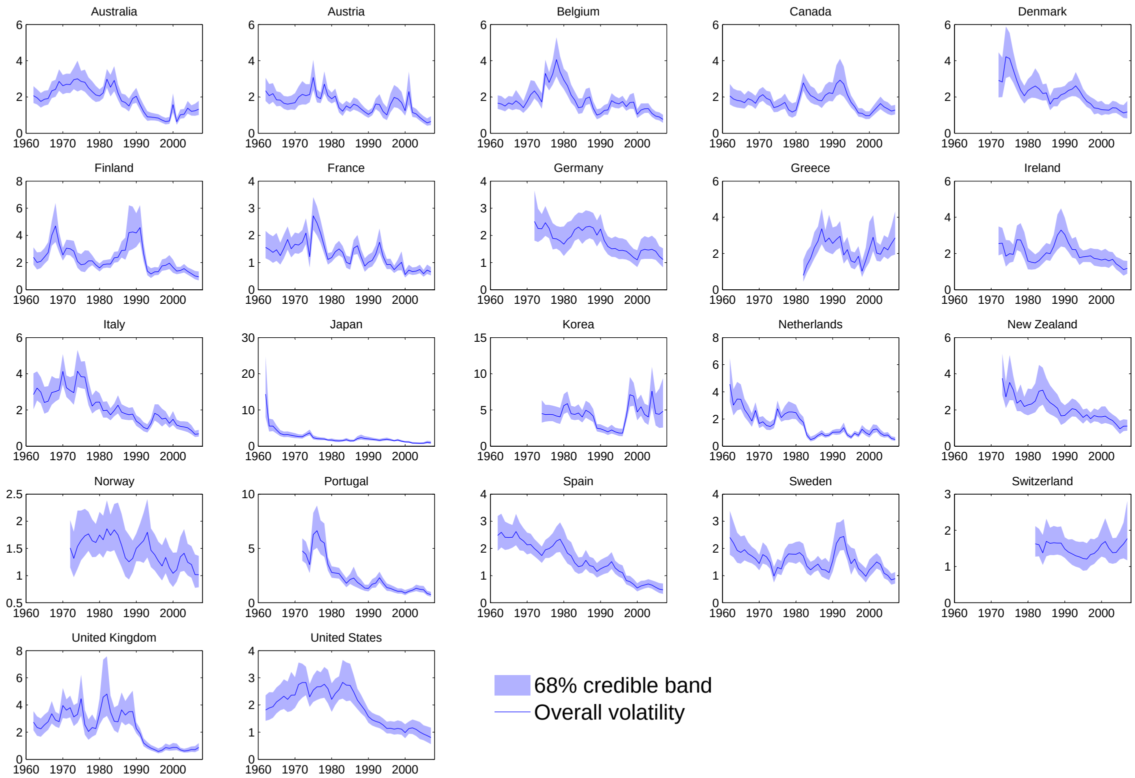


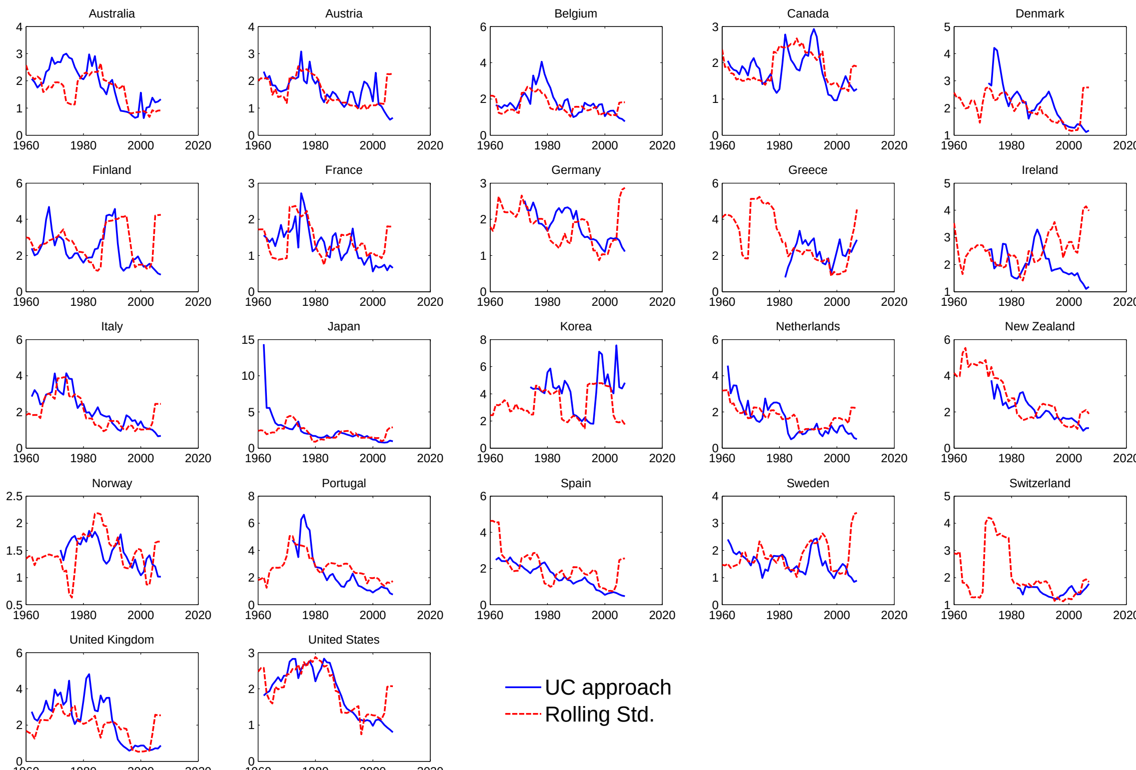
Figure 5.5: Median estimate of overall volatility (full model)



is limited.

The estimated overall country-specific volatilities, given by the sum of the explained and unexplained component, are plotted in Figure 5.5. There is clear evidence of a substantial decline in output volatility in the second half of the sample in the United Kingdom or the United States. Many other countries experience a similar decline, but the reduction is much less pronounced and the timing is quite different. Some economies even exhibit an increase in output volatility, such as Korea or Greece. This leads to the conclusion that the Great Moderation has been an international phenomenon, but with important differences across countries. The volatility measure in this paper differs from the ones commonly used in the literature. Therefore, Figure 5.6 compares the UC approach to an instantaneous measure with the standard deviation of output growth from a centered 9-year rolling window. We report only the median posterior estimates. For many countries, differences between the two series appear rather small. In case of the United States, for example, the rolling window-based standard deviation evolves quite closely to our instantaneous stochastic volatility series. This could suggest that an endogenous measure of volatility is less important, and a rolling standard deviation is a reasonable approximation. Moreover, an advantage of the rolling

Figure 5.6: Comparison between UC and rolling window approach



window approach is that no explicit assumption needs to be made about the volatility process. However, the estimated correlation coefficients can be quite different, especially if one does not account for the non-stationarity of the series in the regression analysis. Moreover, there exist important differences between the two volatility measures for other countries, such as Australia, Denmark, Japan, Korea or the United Kingdom. In addition, the existing literature usually uses centered windows for filtering the rolling standard deviation, which means that the volatility measure inhibits future realized volatility. As a consequence the centered window "predates" the Great Recession as can be seen by the sharp increase in the rolling measure at the very end of the sample. The instantaneous volatility measure is robust to this problem. The estimated effects of the explanatory variables are given in Table 5.5. We report the mean group estimator, given by the arithmetic average of the country-specific slope coefficients. Columns 2-5 refer to a regression based on the full tax data set. To demonstrate robustness we also report results when the original sources are used separately. This gives an indication on whether tax effects are mostly identified though cross-sectional or over-time variation. The effect of demographics on output volatility is strong and positive. The median estimate of 3.5 on the *age* variable implies an increase in the standard deviation of output growth of 0.35 for a 10%-point increase in the age share. This value is broadly in line with the finding from Jaimovich and Siu (2009) when an instantaneous volatility measure instead of a rolling window is used. Moreover, this effect is statistically different from zero, as the 95% credible band covers only positive parameter values. The effect is also robust over the different samples. For economic openness, we find a strong negative effect on output volatility. The median effect is -1.7, so that a 10%-point increase in the share of imports and exports to total output decreases output volatility by 0.17. It implies that trade openness as a form of risk diversification played an important role for the decrease in output volatility as on average the country-specific share increased by 35%-points. Our findings are thus in line with Haddad, Lim, Pancaro, and Saborowski (2013), who argue that trade openness reduces output growth volatility for well-diversified economies. This criterion certainly applies to the majority of our 22 country sample. We do not find a meaningful effect of government size on output volatility, when controlling for openness and the different tax channels. The standard error is very large, so that a zero effect of government size on output volatility cannot be ruled out across all three data sets. Interestingly, a large negative effect is obtained when output volatility is regressed only on demographics and government size. This reinforces the need to control for openness and the tax mix when evaluating the role of government size. However, we still find a meaningful effect of fiscal policy, mainly through labor taxes, although the estimate contains a moderate amount of uncertainty. We can rule out meaningful effects of capital or consumption taxes on output volatility. Labor taxes are found to have a stabilizing effect on the economy. The median estimate and the corresponding standard error differ across the three data sets. In the baseline specification (columns 2-5) we find that an increase of the labor tax ration of 10%$-$points reduces the standard deviation of output by 0.2. A zero effect can be ruled out at the 90%, but not the 95% credible level. Using only the data provided by Posch (2011), the effect is stronger and statistically different from zero. This finding is in line with previous results from Posch (2011) and suggests that the stabilizing effect of fiscal policy works mainly through labor taxation as an automatic stabilizer. Earlier results on a negative effect of government size are possibly due to missing control for the tax mix. However, we note that there might exist a trade-off between stabilization and efficiency, as discussed by Martinez-Mongay and Sekkat (2005) and Posch (2011) among others. Specifically, while higher distortionary taxes can work in favor of output stabilization, there exist well-described arguments on their negative effect of long-run growth.

Results are not reported here, but available on request.

Restricting the Posch data set to the 15 countries available in the McDaniel sample does not change the results substantially. However, the estimated effects of openness and labor taxes are even stronger.

Evaluating the effects on first and second moments on output growth simultaneously is beyond the scope of this paper, but opens up an interesting path for future research.

Table 5.5: Posterior distribution of mean group estimator

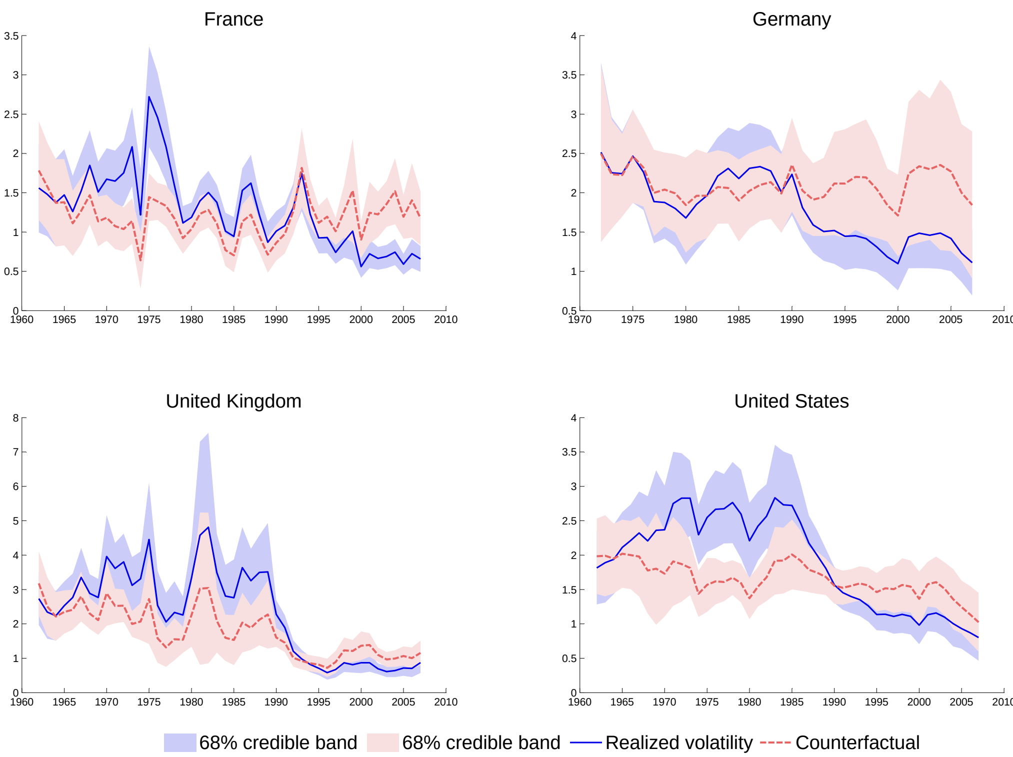
	Full set 22 countries, 1960-2007				McDaniel 15, 1960-2007		Posch 22, 1970-2007	
Variable	Median	Std	$P_{2.5}$	$P_{97.5}$	Median	Std	Median	Std
age	**3.507**	[0.647]	2.243	4.755	**3.514**	[0.795]	**3.477**	[0.742]
openness	**-1.679**	[0.485]	-2.587	-0.712	**-2.193**	[0.704]	**-2.066**	[0.571]
govsize	0.809	[1.871]	-2.861	4.538	2.654	[2.352]	1.837	[2.023]
captax	0.682	[0.878]	-1.053	2.385	-0.724	[1.651]	1.670	[0.992]
contax	0.299	[1.647]	-2.882	3.512	0.102	[1.949]	1.741	[1.725]
labtax	-1.992	[1.077]	-4.103	0.184	-1.468	[1.646]	**-3.575**	[1.231]

Bold numbers: 95% credible interval does not contain zero.

In order to demonstrate the contribution of the explanatory variables to the evolution of output volatility over time, we construct a selection of counterfactual volatility series by holding constant one ore more of the explanatory variables. Specifically, we take the estimated effects from the baseline model as given, but assume that the respective variable stays constant at a value equal to the 5-year average at the beginning of the sample. Counterfactual scenarios are not based on the mean group estimators, but on country-specific slope coefficients. Moreover, we construct a counterfactual series at every iteration of the Gibbs sampling algorithm to obtain the full posterior distribution. Thus, the credible bands around the counterfactuals take into account all sources of filtering and parameter uncertainty within the model. We focus on the effects of demographics and the fiscal policy variables. Here, we present only a selection of countries in order to keep the analysis brief. Figure 5.7 plots the counterfactual series for France, Germany, the United Kingdom, and the United States. The counterfactual series reflect a working age population with no demographic shifts, i.e. the share of young and very old workers is constant over the full sample. In all four countries considered, demographics explain most of the long-run swings in output volatility. When the age share variable is constant, the series still exhibit some distinct short-run fluctuations. The common long-run decline in volatility, however, is completely absent or less pronounced in the counterfactual. Overall, the graphical analysis is in line with the notion that a higher share of prime-age workers reduced output volatility. As most industrialized countries experience important shifts towards an aging society, actual volatility in the last decade lies below the counterfactual series that is based on the demographic composition from 40-50 years ago. In Germany, for example, the counterfactual volatility is twice as large as the actual series in

We use averages to account for possible outliers at the start of the sample.

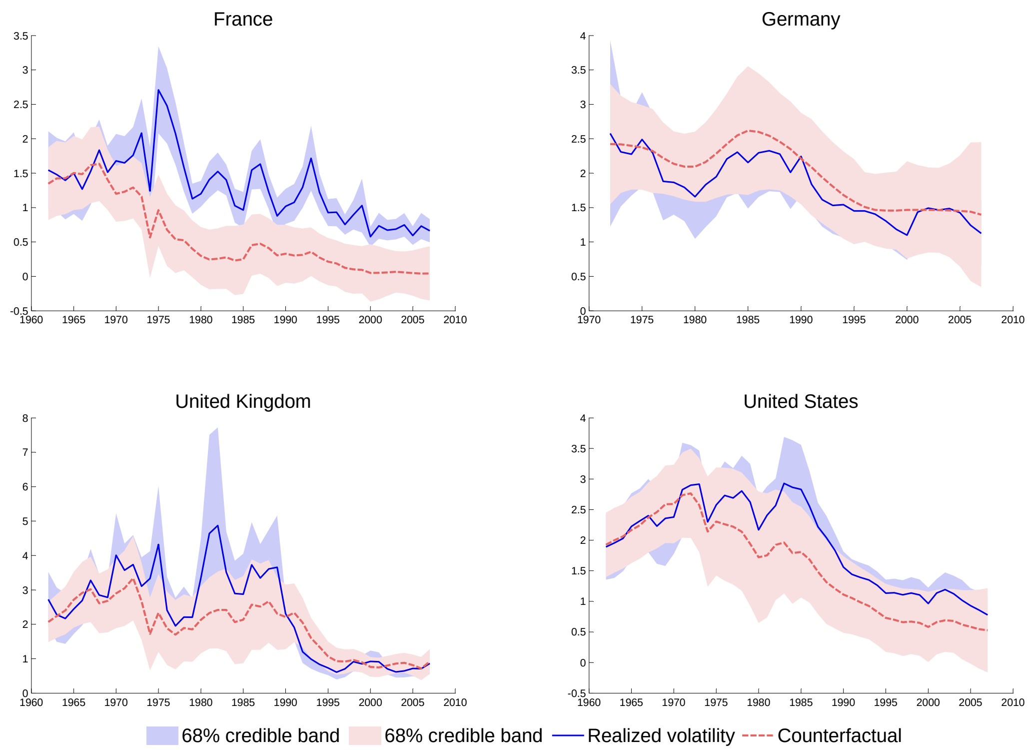
Figure 5.7: Counterfactual volatility series: Constant demographics



the second half. However, there is large uncertainty around the median volatility estimates. The large difference comes as no surprise as Germany is one of the fastest aging countries. Turning to the United States, we find supporting evidence that output volatility in the last 5 decades was in large parts driven by demographics. As argued by Jaimovich and Siu (2009) the "baby boom" in the United States lead to a large inflow of young workers during the 1970s and a subsequent large share of prime-aged workers since the 1990s. Note that our estimated volatility series indeed shows a large upward deviation from the counterfactual during the 1970s and 1980s, before falling below the counterfactual in the 1990s. However, there remains a persistent long-run decline in output volatility even after removing the influence of demographic. The median of the counterfactual volatility series drops by almost one standard deviation over the full sample.

When we hold constant all four fiscal variables, i.e. government size, consumption, capital, and labor taxes, we obtain the series plotted in Figure 5.8. In contrast to the effect of demographics, the fiscal variables account mainly for the short-run fluctuations in volatility. See for example the case of Germany, where the counterfactual exercise leads to a very smooth series that still exhibits a hump-shaped pattern. However, the very large credible bands around the German counterfactual indicate that the effect of fiscal variables is not well

Figure 5.8: Counterfactual volatility series: Constant fiscal policy



identified. In France and the United States, the counterfactual series lie below the realized series for large parts of the sample period, although the bands overlap in case of the United States. This indicates that fiscal policy had an overall de-stabilizing effect on the economies. However, we emphasize that results from this exercise should be handled carefully, as we assume policy-invariance of the parameters.

5.7 Conclusion

In this paper we have estimated the determinants of output volatility in an unbalanced panel of 22 OECD countries. In contrast to the existing literature, the volatility series arises endogenously from an unobserved components model, that treats output volatility as a latent and possibly non-stationary process. Thus, we avoid using ad hoc volatility measures based on rolling standard deviations. Moreover, we explicitly account for the possible of non-stationary factors driving output volatility. A Bayesian model selection procedure is used to test for the presence of non-stationary error components in the volatility equation, similar to a cointegration test. We merge the literature on the role of government size in output stabilization

In other words, the usual Lucas critique applies.

with new advances on the effect of demographics on business cycle volatility. We control for international openness and the tax mix when investigating the role of fiscal policy. We find that demographics played an important role for the evolution of output volatility during the last decades in many advanced economies. The evidence on the role of fiscal policy is mixed. When controlling for trade openness, the main channel for fiscal policy to stabilize output is through labor taxation. This finding is in line with the concept of taxes as automatic stabilizers. Finally, we construct counterfactual series for a selection of countries in order to demonstrate the contribution of individual determinants to output volatility over time.

Bibliography

Berger, T., G. Everaert, and H. Vierke (2015): "Testing for Time Variation in an Unobserved Components Model for the U.S. Economy," Working Papers of Faculty of Economics and Business Administration, Ghent University, Belgium 15/903.

Carmignani, F., E. Colombo, and P. Tirelli (2011): "Macroeconomic Risk and the (De)Stabilising Role of Government Size," *European Journal of Political Economy*, 27(4), 781–790.

Carter, C., and R. Kohn (1994): "On Gibbs Sampling for State Space Models," *Biometrika*, 81(3), 541–553.

Crespo Cuaresma, J., G. Reitschuler, and M. A. Silgoner (2011): "On the Effectiveness and Limits of Fiscal Stabilizers," *Applied Economics*, 43(9), 1079–1086.

De Jong, P., and N. Shephard (1995): "The Simulation Smoother for Time Series Models," *Biometrika*, 82(2), 339–50.

Del Negro, M., and C. Otrok (2008): "Dynamic Factor Models with Time-Varying Parameters: Measuring Changes in International Business Cycles," Staff Reports 326, Federal Reserve Bank of New York.

Del Negro, M., and G. Primiceri (2014): "Time-Varying Structural Vector Autoregressions and Monetary Policy: A Corrigendum," Staff Reports 619, Federal Reserve Bank of New York.

Dickey, D., and W. Fuller (1979): "Distribution of the Estimators for Autoregressive Time-Series with a Unit Root," *Journal of the American Statistical Association*, 74(366), 427–431.

Everaert, G. (2011): "Estimation and Inference in Time Series with Omitted I(1) Variables," *Journal of Time Series Econometrics*, 2(2), 1–28.

Everaert, G., and H. Vierke (2015): "Demographics and Business Cycle Volatility: A Spurious Relationship?," Unpublished Manuscript.

Fatás, A., and I. Mihov (2001): "Government Size and Automatic Stabilizers: International and Intranational Evidence," *Journal of International Economics*, 55(1), 3–28.

Frühwirth-Schnatter, S., and H. Wagner (2010): "Stochastic Model Specification Search for Gaussian and Partial Non-Gaussian State Space Models," *Journal of Econometrics*, 154(1), 85–100.

Gali, J. (1994): "Government Size and Macroeconomic Stability," *European Economic Review*, 38(1), 117–132.

George, E., and R. McCulloch (1993): "Variable Selection via Gibbs Sampling," *Journal of the American Statistical Association*, 88(423), 881–889.

Haddad, M., J. J. Lim, C. Pancaro, and C. Saborowski (2013): "Trade Openness Reduces Growth Volatility When Countries Are Well Diversified," *Canadian Journal of Economics*, 46(2), 765–790.

Jaimovich, N., S. Pruitt, and H. E. Siu (2013): "The Demand for Youth: Explaining Age Differences in the Volatility of Hours," *American Economic Review*, 103(7), 3022–44.

Jaimovich, N., and H. E. Siu (2009): "The Young, the Old, and the Restless: Demographics and Business Cycle Volatility," *American Economic Review*, 99(3), 804–26.

Kent, C., K. Smith, and J. Holloway (2005): "Declining Output Volatility: What Role for Structural Change?," in *The Changing Nature of the Business Cycle*, ed. by C. Kent, and D. Norman, RBA Annual Conference Volume. Reserve Bank of Australia.

Kim, C.-J., and C. R. Nelson (1999): "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle," *The Review of Economics and Statistics*, 81(4), 608–616.

Kim, C.-J., J. M. Piger, and R. Startz (2007): "The Dynamic Relationship between Permanent and Transitory Components of U.S. Business Cycles," *Journal of Money, Credit and Banking*, 39(1), 187–204.

Kim, S., N. Shephard, and S. Chib (1998): "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models," *Review of Economic Studies*, 65(3), 361–393.

Lucas, R. (1987): *Models of Business Cycles*. Cambridge: MIT Press.

Lugauer, S. (2012): "Estimating the Effect of the Age Distribution on Cyclical Output Volatility Across the United States," *The Review of Economics and Statistics*, 94(4), 896–902.

Lugauer, S., and M. Redmond (2012): "The Age Distribution and Business Cycle Volatility: International Evidence," *Economics Letters*, 117(3), 694–696.

Maddala, G. S., and S. W. Wu (1999): "A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test," *Oxford Bulletin of Economics and Statistics*, 61(S1), 631–652.

Martinez-Mongay, C., and K. Sekkat (2005): "Progressive Taxation, Macroeconomic Stabilization and Efficiency in Europe," European Economy - Economic Papers 233, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

McConnell, M. M., and G. Perez-Quiros (2000): "Output Fluctuations in the United States: What Has Changed since the Early 1980's?," *American Economic Review*, 90(5), 1464–1476.

McDaniel, C. (2007): "Average Tax Rates on Consumption, Investment, Labor and Capital in the OECD 1950-2003," Unpublished Manuscript.

Mendoza, E. G., A. Razin, and L. L. Tesar (1994): "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption," *Journal of Monetary Economics*, 34(3), 297–323.

Morley, J., I. B. Panovska, and T. M. Sinclair (2015): "Testing Stationarity with Unobserved Components Models," *Macroeconomic Dynamics (forthcoming)*.

Morley, J., and J. Piger (2012): "The Asymmetric Business Cycle," *The Review of Economics and Statistics*, 94(1), 208–221.

Omori, Y., S. Chib, N. Shephard, and J. Nakajima (2007): "Stochastic Volatility with Leverage: Fast and Efficient Likelihood Inference," *Journal of Econometrics*, 140(2), 425–449.

Perron, P., and T. Wada (2009): "Let's Take a Break: Trends and Cycles in US Real GDP," *Journal of Monetary Economics*, 56(6), 749–765.

Pisani-Ferry, J., X. Debrun, and A. Sapir (2008): "Government Size and Output Volatility: Should We Forsake Automatic Stabilization?," IMF Working Papers 08/122, International Monetary Fund.

POSCH, O. (2011): "Explaining Output Volatility: The Case of Taxation," *Journal of Public Economics*, 95(11), 1589–1606.

RAMEY, G., AND V. A. RAMEY (1995): "Cross-Country Evidence on the Link between Volatility and Growth," *American Economic Review*, 85(5), 1138–51.

RODRIK, D. (1998): "Why Do More Open Economies Have Bigger Governments?," *Journal of Political Economy*, 106(5), 997–1032.

STOCK, J. H., AND M. W. WATSON (2003): "Has the Business Cycle Changed and Why?," in *NBER Macroeconomics Annual 2002, Volume 17*, NBER Chapters, pp. 159–230. National Bureau of Economic Research, Inc.

Appendix

5.A Gibbs sampling algorithm

The structure of the Gibbs sampling algorithm is based on Frühwirth-Schnatter and Wagner (2010) and the description draws heavily from Berger, Everaert, and Vierke (2015). To linearize the nonlinear volatility specification, we follow the procedure by Kim, Shephard, and Chib (1998) and approximate the non-Gaussian distribution in the volatility equation by a mixture distribution. Specifically, we re-arrange and then linearize the growth equation in the following way:

$$y_{i,t} = \mu_{i,t} + \rho_{1,i}(y_{i,t-1} - \mu_{i,t-1}) + \rho_{2,i}(y_{i,t-2} - \mu_{i,t-2}) + \exp\{\beta_i' x_{i,t}\} \exp\{h_{i,t}\}\varepsilon_{i,t}, \tag{5.8}$$

where $\varepsilon_{i,t} \sim i.i.d.\mathcal{N}(0,1)$ and $h_{i,t}$ is defined as in (5.5). Both the explained and unexplained component of volatility enter multiplicatively. The Gibbs sampling approach allows for splitting and transforming this model into blocks that are conditionally linear. Thus, given β_i and $x_{i,t}$, estimates for the latent non-stationary component can be obtained from the following model:

$$\widetilde{y}_{i,t} = \exp\{h_{i,t}\}\varepsilon_{i,t}, \tag{5.9}$$

where $\widetilde{y}_{i,t} = (y_{i,t} - \mu_{i,t} - \rho_{1,i}(y_{i,t-1} - \mu_{i,t-1}) - \rho_{2,i}(y_{i,t-2} - \mu_{i,t-2}))(\exp\{\beta_i' x_{i,t}\})^{-1}$. This expression can be linearized by taking the natural-log of the squares:

$$\ln(\widetilde{y}_{i,t}^2 + c) = 2h_{i,t} + \epsilon_{i,t}, \tag{5.10}$$

where $c = 0.001$ is an offset constant and $\epsilon_{i,t} = \ln(\varepsilon_{i,t}^2)$. The last term follows a log-chi-square, which can be approximated by the following mixture of normal distributions:

$$f(\epsilon_{i,t}) = \sum_{j=1}^{M} q_j f_{\mathcal{N}}(\epsilon_{i,t} | m_j - 1.2704, \nu_j^2), \tag{5.11}$$

where q_j is the component probability of a specific normal distribution with mean $m_j - 1.2704$ and variance ν_j^2. This mixture can equivalently be expressed in terms of component probabilities:

$$\epsilon_{i,t}|(\iota_{i,t} = j) \sim \mathcal{N}(m_j - 1.2704, \nu_j^2) \quad \text{with } Pr(\iota_{i,t} = j) = q_j. \tag{5.12}$$

We follow Omori, Chib, Shephard, and Nakajima (2007) and use a mixture of $M = 10$ normal distributions to proxy the log-chi-square distribution. With this linearization at hand, the Gibbs sampling algorithm consists of the following steps:

Block 1: Sampling the binary indicators $\mathcal{M}$ and the hyperparameters ϕ

Block 1(a): Sampling $\mathcal{M}, h_0$ and σ_h

For notational convenience, let us define a general regression model:

$$w = z^{\mathcal{M}} b^{\mathcal{M}} + e, \qquad e \sim \mathcal{N}(0, \Sigma), \tag{5.13}$$

with w a vector including observations on a dependent variable w_t and z an unrestricted predictor matrix containing the state processes $\widetilde{h}_t$ that is relevant for explaining w_t. The corresponding unrestricted parameter vector with the relevant elements from ϕ is denoted b. $z^{\mathcal{M}}$ and $b^{\mathcal{M}}$ are then the restricted predictor matrix and restricted parameter vector that exclude those elements in z and b for which the corresponding indicator in $\mathcal{M}$ is 0. Furthermore, Σ is a diagonal matrix with elements $\sigma_{e,t}^2$ that may vary over time to allow for heteroskedasticity of a known form.

A naive implementation of the Gibbs sampler would be to sample $\mathcal{M}$ from $f(\mathcal{M}|\widetilde{h}, w, \phi)$ and ϕ from $f(\phi|\widetilde{h}, \mathcal{M}, w)$. However, this approach does not result in an irreducible Markov chain as whenever an indicator in $\mathcal{M}$ equals zero, the corresponding coefficient in ϕ is also zero which implies that the chain has absorbing states. Therefore, as in Frühwirth-Schnatter and Wagner (2010) we marginalize over the parameters ϕ when sampling $\mathcal{M}$ and next draw the parameters ϕ conditional on the indicators $\mathcal{M}$. The posterior distribution $f(\mathcal{M}\ |\widetilde{h}, w)$ can be obtained using Bayes' Theorem as

$$f(\mathcal{M}|\widetilde{h}, w) \propto f(w|\mathcal{M}, \widetilde{h}) p(\mathcal{M}), \tag{5.14}$$

with $p(\mathcal{M})$ being the prior probability of $\mathcal{M}$ and $f(w|\mathcal{M}, \widetilde{h})$ being the marginal likelihood of the regression model (5.13) where the effect of $b^{\mathcal{M}}$ and σ_e^2 is integrated out. The closed form solution of the marginal likelihood in the case of heteroskedasticity $\Sigma = \text{diag}\left(\sigma_{e,1}^2, ..., \sigma_{e,T}^2\right)$, under the normal conjugate prior $b^{\mathcal{M}} \sim \mathcal{N}\left(a_0^{\mathcal{M}}, A_0^{\mathcal{M}}\right)$, is given by

$$f\left(w\left|\mathcal{M},\widetilde{h}\right.\right) \propto \frac{|\Sigma|^{-0.5}\left|A_T^{\mathcal{M}}\right|^{0.5}}{\left|A_0^{\mathcal{M}}\right|^{0.5}}\exp\left(-\frac{1}{2}\left(w'\Sigma^{-1}w+\left(a_0^{\mathcal{M}}\right)'\left(A_0^{\mathcal{M}}\right)^{-1}a_0^{\mathcal{M}}\right.\right.$$
$$\left.\left.-\left(a_T^{\mathcal{M}}\right)'\left(A_T^{\mathcal{M}}\right)^{-1}a_T^{\mathcal{M}}\right)\right), \tag{5.15}$$

with

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}}\left(\left(z^{\mathcal{M}}\right)'\Sigma^{-1}w+\left(A_0^{\mathcal{M}}\right)^{-1}a_0^{\mathcal{M}}\right), \tag{5.16}$$

$$A_T^{\mathcal{M}} = \left(\left(z^{\mathcal{M}}\right)'\Sigma^{-1}z^{\mathcal{M}}+\left(A_0^{\mathcal{M}}\right)^{-1}\right)^{-1}. \tag{5.17}$$

Following George and McCulloch (1993), instead of using a multi-move sampler in which all the elements in $\mathcal{M}$ are sampled simultaneously, we use a single-move sampler in which each of the binary indicators δ_i (for $i = 1, ..., N$) is sampled separately.

Using equation (5.5), equation (5.10) can be rewritten in the general linear regression format of (5.13) as

$$\overbrace{g_{i,t}-\left(m_{\iota_{i,t}}-1,2704\right)}^{w_t} = \overbrace{2\begin{bmatrix} 1 & \delta_i\widetilde{h}_{i,t}\end{bmatrix}}^{z_t^{\mathcal{M}}}\overbrace{\begin{bmatrix} h_{i,0} \\ \sigma_{h,i}\end{bmatrix}}^{b^{\mathcal{M}}}+\overbrace{\widetilde{\epsilon}_{i,t}}^{e_t}, \tag{5.18}$$

for $i = 1, ..., N$, with $\widetilde{\epsilon}_{i,t} = \epsilon_{i,t} - \left(m_{\iota_{i,t}} - 1,2704\right)$ is $\epsilon_{i,t}$ recentered around zero and where $g_{i,t} = \ln(\widetilde{y}_{i,t}^2 + c)$ The marginal likelihood $f\left(w\left|\delta_i, h\right.\right)$ can be calculated as in equation (5.15). The binary indicator δ_i can then be sampled from the Bernoulli distribution with probability $p\left(\delta_i = 1\left|\widetilde{h}_i, w\right.\right)$ calculated from

$$p\left(\delta_i = 1\left|\widetilde{h}_i, w\right.\right) = \frac{f\left(\delta_i = 1\left|\widetilde{h}_i, w\right.\right)}{f\left(\delta_i = 0\left|\widetilde{h}_i, w\right.\right)+f\left(\delta_i = 1\left|\widetilde{h}_i, w\right.\right)}, \tag{5.19}$$

Next, $b^{\mathcal{M}}$ can be sampled from $\mathcal{N}\left(a_T^{\mathcal{M}}, A_T^{\mathcal{M}}\right)$ with $a_T^{\mathcal{M}}$ and $A_T^{\mathcal{M}}$ as defined in (5.16) and (5.17). Note that $b^{\mathcal{M}} = (h_{h,0}, \sigma_{h,i})'$ when $\delta_i = 1$ and $b^{\mathcal{M}} = h_{h,0}$ when $\delta_i = 0$. In the latter case, we set $\sigma_{h,i} = 0$.

Block 1(b): Sampling ρ

Using the general notation from (5.13), define

$$\overbrace{\begin{bmatrix} y_{i,t}-\mu_{i,t} \end{bmatrix}}^{w_t} = \overbrace{\begin{bmatrix} y_{i,t-1}-\mu_{i,t-1} & y_{i,t-2}-\mu_{i,t-2} \end{bmatrix}}^{Z_t} \overbrace{\begin{bmatrix} \rho_{1,i} \\ \rho_{2,i} \end{bmatrix}}^{b_t} + \overbrace{\begin{bmatrix} \exp\{h_{i,t}+\beta_i' x_{i,t}\} \end{bmatrix}}^{e_t} \tag{5.20}$$

with $\Sigma = \text{diag}(\exp\{h_{i,1}+\beta_i' x_{i,1}\}^2, ..., \exp\{h_{i,T}+\beta_i' x_{i,T}\}^2)$. The parameters can be sampled according to (5.16) and (5.17).

Block 1(c): Sampling $\sigma^2_{\mu,i}$

Again, we use the more general notation from (5.13) and sample the variance conditioning on $\mu_{i,t}$ according to:

$$\overbrace{\begin{bmatrix} \mu_{i,t}-\mu_{i,t-1} \end{bmatrix}}^{w_t} = \overbrace{\begin{bmatrix} \eta^{\mu}_{i,t} \end{bmatrix}}^{e_t}, \tag{5.21}$$

where $\sigma^2_{\mu,i}$ can be sampled from $\mathcal{IG}(c_T, C_T)$ with $C_T = C_0 + 0.5(\eta^{\mu\prime}\eta^{\mu})$ where η^{μ} is calculated from $\eta^{\mu}_t = \mu_{i,t} - \mu_{i,t-1}$, and where $c_T = c_0 + T/2$.

Block 1(d): Sampling the β_i

Starting from (5.8), we condition on $h_{i,t}$ and linearize again by taking the log of the squares:

$$\ln(\widetilde{y}^2_{i,t} + c) = 2\beta_i' x_{i,t} + \epsilon_{i,t}, \tag{5.22}$$

where $\widetilde{y}_{i,t} = (y_{i,t} - \mu_{i,t} - \rho_{1,i}(y_{i,t-1} - \mu_{i,t-1}) - \rho_{2,i}(y_{i,t-2} - \mu_{i,t-2}))\exp\{h_{i,t}\}^{-1}$ and $\epsilon_{i,t}$ follows a log-chi-square distribution. Similar to (5.18), we use the more general notation:

$$\overbrace{g_{i,t} - \left(m_{\iota_{i,t}} - 1,2704\right)}^{w_t} = \overbrace{2\beta_i}^{z_t}\overbrace{x_{i,t}}^{b} + \overbrace{\widetilde{\epsilon}_{i,t}}^{e_t},, \tag{5.23}$$

where the predictor matrix and corresponding parameter vector are always unrestricted and $g_{i,t} = \ln(\widetilde{y}^2_{i,t} + c)$. β_i can then be sampled from $\mathcal{N}(a_T, A_T)$ similar to (5.16) and (5.17), where all elements are unrestricted.

Block 2: Sampling mixture indicators ι and latent components μ and $\widetilde{h}_i$

Block 2(a): Sampling ι

Following Del Negro and Primiceri (2014), the mixture indicators are sampled before the stochastic non-stationary volatility component. Specifically, we use equation (5.10) and sample the indicator from its conditional probability mass

$$p(\iota_{i,t} = j|h_{i,t}, \epsilon_{i,t}) \propto q_j f_{\mathcal{N}}(\epsilon_{i,t}|2h_{i,t} + m_j - 1.2704, \nu_j^2), \tag{5.24}$$

with the values for q_j, m_j, and ν_j^2 taken from Omori, Chib, Shephard, and Nakajima (2007).

Block 2(b): Sampling $\widetilde{h}_i$

In this block we use a forward-filtering and backward-sampling approach of Carter and Kohn (1994) and De Jong and Shephard (1995) to sample the unobserved state $\widetilde{h}_i$ based on a general state space model of the form

$$w_t = Z_t^{\mathcal{M}} s_t^{\mathcal{M}} + e_t, \qquad e_t \sim iid\mathcal{N}(0, H_t), \tag{5.25}$$

$$s_{t+1} = R_0 + R_1 s_t + K_t v_t, \qquad v_t \sim iid\mathcal{N}(0, Q_t), \qquad s_1 \sim iid\mathcal{N}(a_1, A_1), \tag{5.26}$$

where w_t is now a vector of observations and s_t an unobserved state vector. The matrices Z_t, R_0, R_1, K_t, H_t, Q_t and the expected value a_1 and variance A_1 of the initial state vector s_1 are assumed to be known (conditioned upon). The vector $s_t^{\mathcal{M}}$ and the matrix $Z_t^{\mathcal{M}}$ are again restricted versions of s_t and Z_t with the elements excluded depending on the model indicators $\mathcal{M}$. The error terms e_t and v_t are assumed to be serially uncorrelated and independent of each other at all points in time. As equations (5.25)-(5.26) constitute a linear Gaussian state space model, the unknown state variables in s_t can be filtered using the standard Kalman filter. Sampling $s = [s_1, \ldots, s_T]$ from its conditional distribution can then be done using the multimove simulation smoother of Carter and Kohn (1994) and De Jong and Shephard (1995).

We filter and sample the stochastic volatility terms $\widetilde{h}_{i,t}$ (for $i = 1, ..., N$) conditioning on the transformed series $g_{i,t} = \ln(\widetilde{y}_{i,t}^2 + c)$, on the mixture indicators $\iota_{i,t}$ and on the parameters ϕ. More specifically, the unrestricted (i.e. $\delta_i = 1$) conditional state space representation is given by

$$\overbrace{\left[\ g_t^k - \left(m_{\iota_{i,t}} - 1,2704\right) - 2h_{i,0}\ \right]}^{w_t} = \overbrace{\left[\ 2\delta_i\sigma_{h,i}\ \right]}^{Z_t^{\mathcal{M}}}\overbrace{\left[\ \widetilde{h}_{i,t}\ \right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\ \widetilde{\epsilon}_t^k\ \right]}^{e_t}, \tag{5.27}$$

$$\underbrace{\left[\ \widetilde{h}_{i,t+1}\ \right]}_{s_{t+1}} = \underbrace{[1]}_{R_1}\underbrace{\left[\ \widetilde{h}_{i,t}\ \right]}_{s_t} + \underbrace{\left[\ 1\ \right]}_{K_t}\underbrace{\left[\ \widetilde{\eta}_{i,t}^h\ \right]}_{\nu_t}, \tag{5.28}$$

with $H_t = v_{\iota_t^k}^2$, $Q_t = 1$ and where $\widetilde{\epsilon}_t^k = \epsilon_t^k - \left(m_{\iota_t^k} - 1,2704\right)$ is ϵ_t^k recentered around zero. The random walk components $\widetilde{h}_t^k$ are initialized by setting $a_1 = 0$ and $A_1 = 0.0001$.

In the restricted model (i.e. $\delta_i = 0$), $Z^{\mathcal{M}}$ and $s^{\mathcal{M}}$ are empty. In this case, no forward-filtering and backward-sampling is needed and $\widetilde{h}_t^k$ can be sampled directly from its prior using equation (5.5). Using draws for $h_{i,0}$, $\sigma_{h,i}$ and $\widetilde{h}_{i,t}$, $h_{i,t}$ can easily be reconstructed from equation (5.6).

Block 2(c): Sampling μ_i

We sample the unobserved state μ_i based on the general state space model from (5.25) and (5.26), where all elements are unrestricted. Conditioning on the observed series $y_{i,t}$, the time-varying volatility $h_{i,t}^* = \exp\{h_{i,t} + \beta_i' x_{i,t}\}$, and the parameter $\sigma_{\mu,i}^2$ and ρ_i, the state space representation is given by

$$\overbrace{\left[\ y_{i,t} - \rho_{1,i}y_{i,t-1} - \rho_{2,i}y_{i,t-2}\ \right]}^{w_t} = \overbrace{\left[\ 1 \quad -\rho_{1,i} \quad -\rho_{2,i}\ \right]}^{Z_t}\overbrace{\begin{bmatrix} \mu_{i,t} \\ \mu_{i,t-1} \\ \mu_{i,t-2} \end{bmatrix}}^{s_t} + \overbrace{\left[\ \varepsilon_{i,t}\ \right]}^{e_t}, \tag{5.29}$$

$$\underbrace{\begin{bmatrix} \mu_{i,t+1} \\ \mu_{i,t} \\ \mu_{i,t-1} \end{bmatrix}}_{s_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{R_1}\underbrace{\begin{bmatrix} \mu_{i,t} \\ \mu_{i,t-1} \\ \mu_{i,t-2} \end{bmatrix}}_{s_t} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{K_t}\underbrace{\left[\ \eta_{i,t}^{\mu}\ \right]}_{\nu_t}, \tag{5.30}$$

with H_t a matrix with diagonal elements $\exp\{h_{i,t}^*\}$, $Q_t = \sigma_{\mu,i}^2$, and where $\mu_{i,t}$ is diffusely initialized by setting $a_1 = 0$ and $A_1 = 1000$.

Block 3: Random sign switch on σ_h and $\widetilde{h}$

For each cross section, a random sign switch is performed on $\sigma_{h,i}$ and $\left\{\widetilde{h}_i\right\}_{t=1}^T$ with probability 0.5.