Journal of Simulation (2015) 9, 121–128

www.palgrave-journals.com/jos/

Operating theatre modelling: integrating social measures

C Van Huele[1] and M Vanhoucke[1,2,3]*

[1]*Ghent University, Gent, Belgium;* [2]*Vlerick Leuven Gent Management School, Gent, Belgium; and* [3]*University College London, London, UK*

Hospital resource modelling literature is primarily focussed on productivity and efficiency measures. In this paper, our focus is on the alignment of the most valuable revenue factor, the operating room (OR) with the most valuable cost factor, the staff. When aligning these economic and social decisions, respectively, into one sustainable model, simulation results justify the integration of these factors. This research shows that integrating staff decisions and OR decisions results in better solutions for both entities. A discrete event simulation approach is used as a performance test to evaluate an integrated and an iterative model. Experimental analysis show how our integrated approach can benefit the alignment of the planning of the human resources as well as the planning of the capacity of the OR based on both economic related metrics (lead time, overtime, number of patients rejected) and social related metrics (personnel preferences, aversions, roster quality).

Journal of Simulation (2015) **9**(2), 121–128. doi:10.1057/jos.2014.32; published online 5 December 2014

Keywords: hospitals; optimization; discrete event simulation

1. Introduction

Implementing sustainability strategies in hospitals requires a holistic approach. An idealistic solution is to model and simulate a hospital as a whole. However, there has been remarkably little work done, despite the clear linkage between different entities within a hospital (Günal and Pidd, 2010). These links include the flows of patients between emergency departments, operating theatres and inpatient wards, governed by the hospital staff. The objectives of the different hospital's entities should be aligned in order to improve efficiency, quality of care and social factors. The contribution of this paper is twofold. First, we propose an integration of the scheduling of the surgeries and rostering of the surgeons in one phase. Second, we are able to prove through discrete event simulation (DES) that integrating the social measure into the classically economic measure for an operating theatre even improves the solution on both economic (operating theatre planning) and social (staff rostering) level.

In hospitals, 70% of all admissions are surgery related (Macario *et al* (1995)) and the operating theatre is seen as the core of the hospital with about 40% of all expenses (Litvak and Long, 2000). When implementing sustainability in hospitals, the operating theatre is consequently one of the most prominent candidates for improvement. The planning of the operating room (OR) is a very challenging task from a number of perspectives. On the one hand, there are the surgeries with their dependencies concerning arrival time and bed availability and on the other hand, the physicians require rosters that abide by work effort equalities, holiday applications and skills.

The majority of scheduling simulation studies in hospitals are directed at surgery scheduling in order to distribute patient demand for the physicians and the staff (Jun *et al*, 1999). A number of studies have tackled the problem from the reverse side, that is, the staff should be scheduled to meet the patient demand. Obviously, walk-in clinics and emergency departments are unable to change the arrival rate of patients and are obliged to roster their staff in accordance. The integration of staff preferences in the operating theatre schedule has also been discussed by Roland and Riane (2011). The main difference of this paper as opposed to similar papers in literature (Roland and Riane, 2011) that integrate both the staff rosters and the surgery schedule is the assignment of these physicians to the surgeries. In practice, these decisions are not yet integrated in the central planning system, but are part of the consultation phase. This paper differs from this similar work by implementing both problems in a single phase decision formulation. Our experimental analysis shows that this integration of decision stages offers better results on all levels, both economic and social.

Simulation techniques provide a variety of possibilities for health-care analysis. For example, performance analysis (eg, What is the capacity of the current health system?) and what-if analysis (eg, What is the effect of changes to resource scheduling to management policies?). Current literature includes several studies that analyse the performance of emergency departments, operating theatres and outpatient clinics, which use simulation and/or optimization techniques (Rohleder *et al*, 2007). As an example of performance analysis, VanBerkel and Blake (2007) propose a DES model to analyse the effect (patient throughput

*Correspondence: M Vanhoucke, Ghent University, Tweekerkenstraat 2, Gent, Oost Vlaanderen, 9000, Belgium; University College London, Gower Street, London WC1E 6BT, UK.
E-mail: mario.vanhoucke@ugent.be

and waiting times for elective patients) of redistributing beds and OR time between two sites of a general surgery division. Harper and Gamlin (2003) perform a what-if analysis by a simulation modelling approach the analyses patient flows under uncertainty in service times with respect to different proposal schedules in order to reduce patient waiting times.

DES is one of the most heavily used analysis tools applicable for the analysis of health-care systems. Especially the modelling of inpatient facilities, such as the operating theatre was an actively discussed topic. Patient flows, bed occupancy levelling, length of stay (LOS) modelling, scheduling of surgeries are one of the most important issues. For example, in Persson and Persson (2010) a discrete event model shows how different management policies affect different performance metrics, such as patient waiting time, cancellations and the utilization of the OR. However the model does not allow social metrics. Patient-related or staff-related preferences that aim to ameliorate their social context are not present in most research papers concerning patient planning. In Virtue *et al* (2011) a DES approach is used to prove that average simulation times can be used to model average process times. Mallor and Azcrate (2011) combines optimization with simulation to study bed occupancy levels in an intensive care unit.

The rest of the paper is organized as follows. The next section discusses the modelling and integration of the surgery and physician assignment problem. The following section introduces the DES model in order to compare our proposed single phase decision stage with the two phase decision stage. We study the effects on both the economical and social metrics in the penultimate section. In the final section we discuss future work and give an overview of our proposed framework.

2. Hospital modelling: the OR

We focus on the modelling and scheduling of a patient flow in the operating theatre in order to maximize the throughput, while also maximizing the preferences of the staff. In order to achieve this combination, both surgery- and staff-related decisions must be integrated. When these decisions are taken sequentially or iteratively, the patient is first assigned to an eligible physician according to their preferences and afterwards the corresponding surgeries are assigned to the available OR days. This two-phase approach yields suboptimal results, because the search space is reduced because of the assignment of patients to physicians before the assignment of patients to OR days. In this paper, we propose to combine these problems into one decision problem. DES techniques are used to do performance analysis and what-if analysis.

In order to validate our assumption that the integrated decision implies better results than the iterative procedure for both economical and social factors, we define two models:

1. ITER: Iterative model
2. INT: Integrative model

These two models are both based on the same basic model as follows.

Let us consider a set of patients queued every week, eligible for surgery. These surgeries (defined by the set S) should then be assigned to one of the ORs of the set of ORs, R and on one of the days of the week of the set of days D. In that OR day, the surgery has to be scheduled in the time slots T according to the duration of the surgery d_s for the surgery $s \in S$. Let x_{stdrp} be equal to 1 if surgery s starts on time instant t, on day d and in OR r by physician p, and otherwise 0. Let f_{st} be the throughput coefficient for every surgery s and dependent on t. Then f_{st} is equal to a pre-defined throughput favouring constant K if $t > t_{rd}$ with t_{rd} the overtime instance and f_{st} equal to H if $t \leqslant t_{rd}$, with $K \gg H$. Let us define A_{sp} as a parameter specifying which surgery has to be scheduled by which physician. This is an important parameter of our model, since this parameter is used in the iterative (ITER) model and not in the integrative (INT) model. In the following, a basic model mixed integer programming model (MIP) containing only the surgery and physician assignment constraints is given.

Model:

$$\max \quad \sum_{s\in S}\sum_{p\in P}\sum_{t\in T}\sum_{r\in R}\sum_{d\in D} f_{st}x_{stdrp} \tag{1}$$

$$\sum_{p\in P}\sum_{s\in S}\sum_{\tau=t-d_s+1}^{t} x_{s\tau drp} \leqslant 1 \qquad \forall r \in R,\ \forall d \in D,\ \forall t \in T \tag{2}$$

$$\sum_{r\in R}\sum_{s\in S}\sum_{\tau=t-d_s+1}^{t} x_{s\tau drp} \leqslant 1 \qquad \forall t \in T,\ \forall d \in D,\ \forall p \in P \tag{3}$$

$$x_{stdrp} = 0 \qquad \forall s \in S,\ \forall r \in R,\ \forall p \in P,\ \forall d \in D,\ \forall t \in T \mid (t+d_s) > |T| \tag{4}$$

$$\sum_{d\in D}\sum_{t\in T} x_{stdrp} \leqslant A_{sp} \qquad \forall s \in S,\ \forall r \in R,\ \forall p \in P \tag{5}$$

$$x_{stdrp} \in \{0,1\} \quad \forall p \in P,\ \forall r \in R,\ \forall d \in D,\ \forall t \in T,\ \forall s \in S \tag{6}$$

The Objective (1) is to maximize the throughput of the surgeries by scheduling as many surgeries as possible on a weekly basis in order to optimize operating time. Because of the structure of the cost coefficient c_{st}, the overtime is minimized as well. Constraint (2) ensures that no surgery starts before the end of the previous surgery in that OR. Constraint (3) is quite similar by forcing that no surgery starts before the end of the previous surgery performed by the same physician. No surgery can start before the end of the day minus the duration of that surgery, therefore all x variables for those time instances are set to 0 in Constraint (4). Constraint (5) is only present in model ITER and mimics the pre-defined assignment of physicians to patients (surgeries). Finally, the integrality constraints on the x variable are presented in Equation (6).

Besides these basic assignment constraints, we implemented five soft constraint in order to optimize the roster of each physician. These soft constraints are (ctA), (ctN), (ctO), (ctW) and (ctQ).

- (*ctA*): Links the skills of the physician to the type of surgery which has to be performed.
- (*ctN*): Allows the physician to specify to work a certain number of hours in a week.
- (*ctO*): Allows the physician to take a holiday or to request unavailable time instances during the week.
- (*ctW*): Allows the physician to request a limit on the number of consecutive working days.
- (*ctQ*): Allows the physician to request a limit on the number of hours work per day.

These soft constraints are added as goal constraints in the MIP model described above. In the objective function, the deviation on these constraints per physician is then penalized in order to introduce fairness among the physicians. Every physician's roster is consequently optimized to satisfy the physician constraints.

Each patient has a LOS for which the patient remains in hospital. A patient can only be scheduled for surgery if there is an available bed, which has to be available during the LOS of that patient. The entire MIP model is formulated in Appendix.

3. DES hybridization with optimization

As a simulation model provides estimates of some characteristics of system performance under a set of given constraints, we suggest a discrete-event simulation to examine the efficiency of the operating theatre.

Figure 1 describes the proposed system that models the patient arrivals, surgery duration and surgery scheduling. Every simulation step, representing 5 scheduling days in the operating theatre, starts with the generation of instances of patients, sampled out of the historical arrival distribution. These surgeries are then planned in the operating theatre in the optimization phase by both the iterative model and the integrative model considering the surgery duration means. In the execution phase, these surgery schedules and physician rosters are evaluated through simulation. It is assumed that every patient arrives exactly at the scheduled surgery time. The model does not take into account no-shows nor cancellations. We have chosen not to integrate the cancellations nor no-shows into the simulation in order to focus the results of our simulation on the integrated *versus* iterative procedure. We do believe that no-shows will not influence the managerial result of this paper. Cancellation is a different story, minimizing the number of cancellations would benefit from better planning. Patients arrive in a first in first out queue at every OR. The planned surgeries' duration is then sampled from the historical log-normal distribution to simulate the surgery process. The economic performance parameters like lead time, number of processed, rejected surgeries and average overtime are measured. Social metrics regarding the physician preferences consist of the ratio of satisfied physician constraints ctA, ctQ, ctO, ctN and ctW. Each patient is instantiated with the throughput f_{st} parameter that doubles every week in order to prioritize non-scheduled patients. The unprocessed patients are then added to the list of patients eligible for scheduling in the next week. When a patient is not scheduled for 2 consecutive weeks (because of infeasibility), the patient is rejected and eliminated from the queue.

Both models, ITER and INT are based on the same MIP defined in the section 'Hospital modelling: the OR' and extensively defined in the appendix (Equations (A.1)–(A.19)).

The iterative model mimics a two-phased approach. In the first phase, the patient makes an appointment with one of the physicians eligible to do that surgery. In the second phase, the surgery is scheduled in the week, according to the physician's preferences and the patient is notified of the surgery date.

It is possible to simulate the pre-fixed surgeries to physicians in our model by setting the data parameter A_{sp} of Constraint (A.7) in such a way that it is subject to,

$$\sum_{p \in P} A_{sp} = 1, \qquad \forall s \in S, \forall r \in R \tag{7}$$

$$\sum_{p \in P} \sum_{r \in R} A_{sp} = \mid R \mid, \qquad \forall s \in S \tag{8}$$

In this way, all surgeries are pre-assigned to a specific randomly chosen physician. Equation (7) ensures that only one

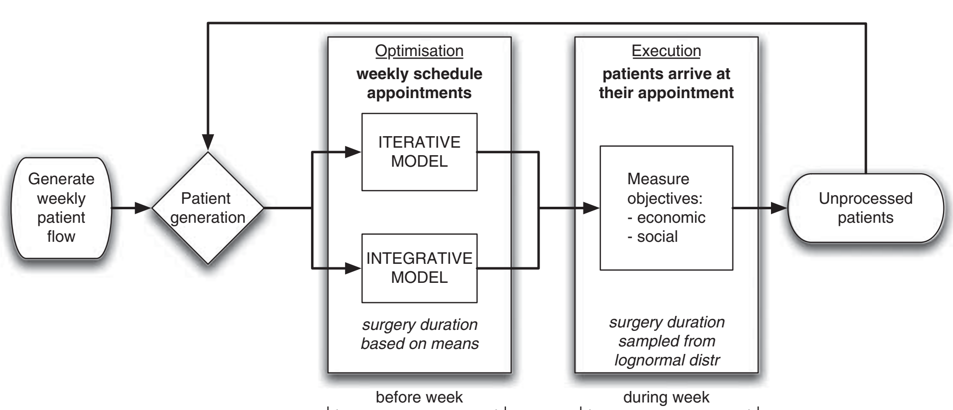

Figure 1 DES: Operating theatre.

physician is chosen for every surgery. However, this allows surgeries to be performed by multiple physicians in different ORs. Equation (8) forces every surgery to be eligible to be performed in every OR. This trick in parameter settings thus mimics an iterative approach and will be used to compare the results with the integrated approach.

We therefore propose an integration of the surgery and physician assignment to circumvent the suboptimal results by the iterative method. The data parameter A_{sp} does not impose any pre-defined assignment. Consequently, the search space is larger and physician preference infeasibilities can be avoided.

4. Results

The data regarding the surgeries in this study was based on data received from the General Hospital Maria Middelares in Sint-Niklaas, Belgium over a 3 months period. The data included surgery information of different surgical specialities: stomatology, orthopaedic surgery and neurological surgery. The information resulted in a list of surgery records from the $|R|=3$ ORs in the hospital, not including emergency cases. The horizon consists of $|D|=5$ opening days and a day consists of 10 hours surgery (ie, from 8:00 until 18:00) with a time division denoted by $|T|=20$ resulting in half-hour time blocks.

Table 1 Surgery specialties data

Specialty	*Distribution fit (location, scale)*	*Mean*	*Number of surgeries*
Neurological	ln $\mathcal{N}(0.387, 0.777)$	1.83	207
Stomatology	ln $\mathcal{N}(0.258, 0.633)$	1.61	315
Orthopaedic	ln $\mathcal{N}(0.237, 0.670)$	1.53	852

Every week a set of patients are generated, sampled out of the fitted distribution of weekly patient arrival from the data. The hospital Maria Middelares patient quantity arrival pattern is normally distributed with $\mathcal{N}(40.8, 3.6)$ and the surgery duration was sampled from the data set of three surgery types: neurological, stomatological and orthopaedic surgery, presented in Table 1.

All experiments were performed on the Stevin Supercomputer Infrastructure (Gengar) provided by Ghent University. The cluster contains 94 computing nodes (IBM HS 21 XM blade), each of which contains a dual-socket quad-core Intel Xeon L5420 (Intel Core microarchitecture, 2.5 GHz, 6 MB L2 cache per quad-core chip), thus 8 cores/node with 16 GB RAM. The model was written in C++ and linked with the CPLEX 12.5 optimization library. In the optimization phase the MIP solver is aborted after 3600 seconds when no solution is found with a MIP gap of 1%. The simulation phase is written in the R statistical programming language and calls the optimization phase in every simulation step.

4.1. Sustainability measures

In Figure 2 the results of the DES are visualized per metric; economic and social. For every of the 52 weeks, both the

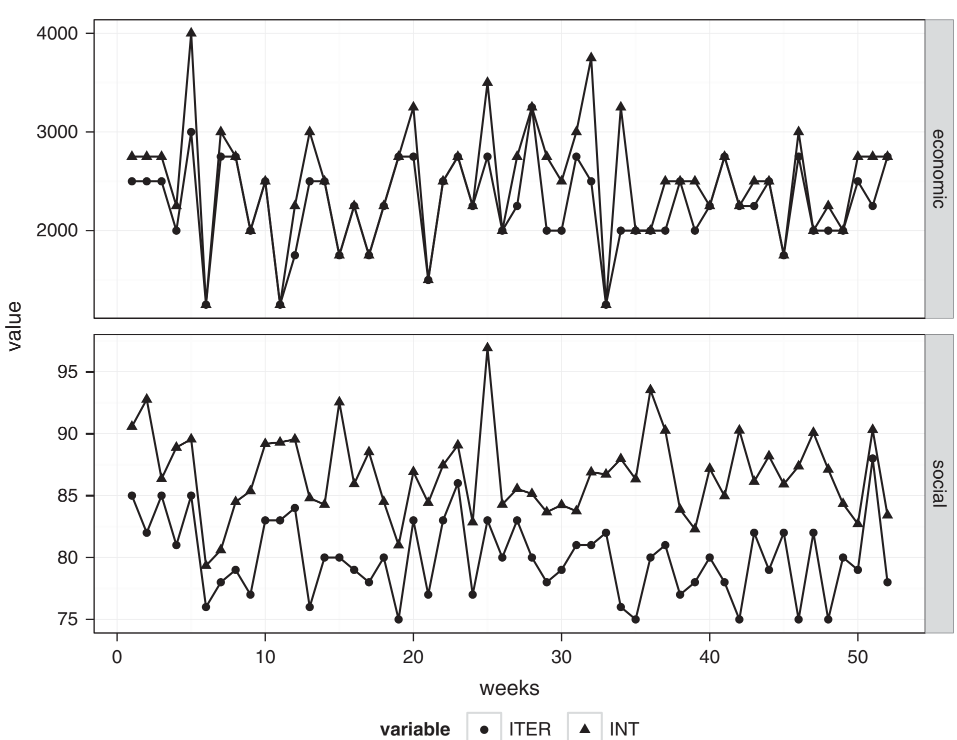

Figure 2 DES: Integrative *versus* iterative methods.

Table 2 Qualitative results for the integrated (IN) and iterative (IT) approach over yearly period

Approach	*Economic*				*Social*					*Computation time (s)*
	LT	*#P*	*#R*	*OT*	*ctA (%)*	*ctN (%)*	*ctO (%)*	*ctW (%)*	*ctQ (%)*	
Integrative	1.01	2003	40	€852	89	91	82	84	79	2072
Iterative	1.06	1927	116	€971	87	91	80	84	77	903

Table 3 DES: Influence of rejection policy on integrative approach

Policy	*Economic*				*Social*				
	LT	*#P*	*#R*	*OT*	*ctA(%)*	*ctN(%)*	*ctO(%)*	*ctW(%)*	*ctQ(%)*
$Rej = 2$	1.01	2003	40	€852	89	91	82	84	79
$Rej = 4$	1.01	2031	11	€858	88	89	81	84	78
$Rej = 6$	1.01	2045	0	€863	87	89	81	83	77

performance measures of the integrative model and the iterative model is shown. It can be observed that the integrative approach is equal or better than the iterative approach for every week on economic aggregate. The economic aggregate is defined by the throughput maximization and overtime minimization as defined by the objective function in the base model. The social aggregate is defined by the weighted sum of all five different measures (ctA, ctN, ctO, ctW, ctQ) with equal weight.

In Table 2 a summary of the qualitative results for the DES simulation are presented for both methods, the iterative model and the integrative model. Economic factors are shown in the first three columns, while the social factors are shown in the last five columns. Economic factors include the average number of days from scheduling day until the leave date (lead time (*LT*)), the number of patients processed (*#P*), the number of patients rejected (*#R*) and the average weekly overtime cost (*OT*). Social factors are the percentage of non-violated physician constraints. For every metric the best approach is underlined.

Results show that the integrative approach is always better or equal than the iterative approach. Our proposed integrative model indicate a significant decrease in overtime work while processing more and rejecting less patients than the iterative approach. There is also less impact on the physician preferences when using the integrative approach. Only ctN and ctW yield similar levels of physician satisfaction.

4.2. Rejection policy

In the previous experiment the rejection policy stated that patients are rejected after 2 consecutive weeks not being scheduled. In the following experiment, we perform what-if analysis on the rejection policy. A limitation on the number of consecutive weeks (*Rej*) is the measurable parameter in the experiment. We repeat the DES analysis of the previous section for $Rej = 2, 4, 6$.

Table 3 displays the average economic and social metrics for a DES simulation of 52 weeks for the integrative approach. For $Rej = 2$, the results are the same as in Table 2. For $Rej = 4$, more patients get processed (*#P*) and consequentially less rejected (*#R*). The trade-off mechanism shows a slight increase in average overtime cost (*OT*) and social measures for the physician. If we set $Rej = 6$, the number of rejected patients disappear. Calculating the influence of the rejection policy is a decision support tool for the hospital manager.

5. Discussion

In this paper we propose a new management policy for OR planning with the use of DES. This new policy introduces the integration of the physician to patient assignment into the OR planning decision. We believe this integration is necessary in order to align two of the most important problems for hospital management, capacity planning and staff planning. In the simulation both economic performances as well as social performances are measured. The simulation of 52 weeks using the integrative scenario shows the positive impact on both throughput metrics and staff preferences. Therefore, we can conclude that the simulated performance of the OR department can be improved, when applying the integrated model.

Moreover, we simulate the proposed scenario having another rejection policy. We study the impact of prolonging the rejection date in order to decrease the number of cancellations, while only slightly forcing the waiting time to go higher. The integrated approach shows to be more resilient against these negative effects.

Our optimization model was able to solve a problem containing the assignment of 40 surgeries to 3 ORs by 7 physicians on a weekly basis. However, to achieve optimal results, the computation times can rise exponentially, when the problem instances size rises linearly. Consequently, we were not able to solve operating theatres of more than three rooms under similar circumstances in acceptable computation time (24 h). In practice, large hospitals consist of up to 20 ORs or more, serving hundreds of patients on a weekly basis. Our proposed integrated optimization model is not fit to generate the schedule under these requirements. However, if the procedure is replaced by a heuristic approach (Van Huele and Vanhoucke, 2014a), good feasible results could be attained in shorter computation time and the simulation procedure could be repeated.

The integration and alignment of different entities in hospitals are shown to have a positive effect on both productivity as well as staff-related social measures. In this work, the integration of the environmental aspect of the Triple Bottom Line of sustainability is a direct consequence of the economical aspect. Indeed, the optimization of the resources in a hospital has both economic and ecological impact. Hence no further attention is given to the environmental aspect in the manuscript.

Future research would encompass going further in this direction and integrating more aspects of the hospital resource modelling process. We suggest working towards a holistic approach in modelling the entire scheduling process in an operating theatre by integrating the bed assignment decision. In a second step, the entire surgical team (surgeons, nurses and anaesthesiologists) could be integrated to avoid suboptimal results and stimulate alignment.

References

Günal M and Pidd M (2010). Discrete event simulation for performance modelling in health care: A review of the literature. *Journal of Simulation* **4**(1): 42–51.

Harper P and Gamlin H (2003). Reduced outpatient waiting times with improved appointment scheduling: A simulation modelling approach. *OR Spectrum* **25**(2): 207–222.

Jun JB, Jacobson SH and Swisher JR (1999). Application of discrete-event simulation in health care clinics: A survey. *Journal of the Operational Research Society* **50**(2): 109–123.

Litvak E and Long MC (2000). Cost and quality under managed care: Irreconcilable differences. *American Journal of Managed Care* **6**(3): 305–312.

Macario A, Vitez TS, Dunn B and McDonald T (1995). Where are the costs in perioperative care?: Analysis of hospital costs and charges for inpatient surgical care. *Anesthesiology* **83**(6): 1138–1144.

Mallor F and Azcrate C (2011). Combining optimization with simulation to obtain credible models for intensive care units. *Annals of Operations Research* **221**(1): 255–271.

Persson M and Persson J (2010). Analysing management policies for operating room planning using simulation. *Health Care Management Science* **13**(2): 182–191.

Rohleder T, Bischak D and Baskin L (2007). Modeling patient service centers with simulation and system dynamics. *Health Care Management Science* **10**(1): 1–12.

Roland B and Riane F (2011). Integrating surgeons' preferences in the operating theatre planning. *European Journal of Industrial Engineering* **5**(2): 232–250.

Van Huele C and Vanhoucke M (2014a). Decomposition based heuristics for the integrated surgery and physician scheduling problem. *Health Systems*, advance online publication 5 December, doi:10.1057/hs.2014.27.

Van Huele C and Vanhoucke M (2014b). Analysis of the integration of the physician rostering problem and the surgery scheduling problem. *Journal of Medical Systems* **38**(6): 1–16.

VanBerkel P and Blake J (2007). A comprehensive simulation for wait time reduction and capacity planning applied in general surgery. *Health Care Management Science* **10**(4): 373–385.

Virtue A, Kelly J and Chaussalet T (2011). Using simplified discrete-event simulation models for health care applications. In: *Proceedings of the Winter Simulation Conference*, WSC '11, pp 1154–1165.

Appendix

Mathematical model

Sets:

S	Surgeries	$s = 1, \ldots, \lvert S \rvert$
S'	Surgeries of previous week	$s = 1, \ldots, \lvert S' \rvert$
T	Time instances	$t = 1, \ldots, \lvert T \rvert$
D	Days	$d = 1, \ldots, \lvert D \rvert$
R	Operating rooms	$r = 1, \ldots, \lvert R \rvert$
P	Physicians	$p = 1, \ldots, \lvert P \rvert$

Cost parameters:

c_p^{cons} consecutive working days violation cost for physician p
c_p^{week} weekly work violation cost for physician p
c_p^{day} daily work violation cost for physician p
c_{prs}^{skills} skill assignment violation cost for physician p
c_{pdt}^{free} unavailability violation cost for physician p

Parameters:

$$A_{sr} = \begin{cases} 1 & \text{if surgery } s \text{ can take place in OR } r \\ 0 & \text{otherwise} \end{cases}$$

$$O_{pdt} = \begin{cases} 1 & \text{if physician } p \text{ is unavailable on time instance } t \text{ and on day } d \\ 0 & \text{otherwise} \end{cases}$$

B_d available beds in the ward on day d
b_s LOS belonging to surgery s
d_s duration of surgery s
$N_p^{\min}$ minimum amount of hours that physician p has to work
$N_p^{\max}$ maximum amount of hours that physician p has to work
$W_p^{\min}$ minimum consecutive days that physician p has to work
$W_p^{\max}$ maximum consecutive days that physician p has to work
t_{rd} maximum overtime at operating room r and on day d
Q_p maximum surgery time per day for physician p
ES_s earliest starting day of surgery s
LS_s latest starting day of surgery s
f_{st} throughput parameter for surgery s and time instant t

Goal variables:

o_p^{1+} penalty of positive daily physician work content constraint
o_p^{1-} penalty of negative daily physician work content constraint

o_p^{2a+} penalty of positive maximum weekly physician work content constraint
o_p^{2a-} penalty of negative maximum weekly physician work content constraint
o_p^{2b+} penalty of positive minimum weekly physician work content constraint
o_p^{2b-} penalty of negative minimum weekly physician work content constraint
o_{pdt}^3 penalty of physician unavailabilities constraint
o_{rs}^4 penalty of OR type constraint
o_p^{5a+} penalty of positive maximum consecutive working days
o_p^{5a-} penalty of negative maximum consecutive working days
o_p^{5b+} penalty of positive minimum consecutive working days
o_p^{5b-} penalty of negative minimum consecutive working days

Decision variables:

$$x_{stdrp} = \begin{cases} 1 & \text{if surgery } s \text{ starts at day } d \text{ and time instant } t \text{ in OR } r \text{ by physician } p. \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{x}_{dp} = \begin{cases} 1 & \text{if the physician } p \text{ works on day } d. \\ 0 & \text{otherwise} \end{cases}$$

Model:

$$\max \quad \sum_{s\in S}\sum_{p\in P}\sum_{t\in T}\sum_{r\in R}\sum_{d\in D} f_{st}x_{stdrp} \tag{A.1}$$

$$-\sum_{p\in P} c_p^{day} o_p^{1+} - \sum_{p\in P} c_p^{week}\left(o_p^{2a+} + o_p^{2b-}\right) - \sum_{p\in P}\sum_{d\in D}\sum_{t\in T} c_{pdt}^{free} o_{pdt}^3 \tag{A.2}$$

$$-\sum_{p\in P}\sum_{r\in R}\sum_{s\in S} c_{prs}^{skills} o_{prs}^4 - \sum_{p\in P} c_p^{cons}\left(o_p^{5a+} + o_p^{5b-}\right) \tag{A.3}$$

$$\sum_{p\in P}\sum_{s\in S}\sum_{\tau=t-d_s+1}^{t} x_{s\tau drp} \leqslant 1 \qquad \forall r\in R,\ \forall d\in D,\ \forall t\in T \tag{A.4}$$

$$\sum_{r\in R}\sum_{s\in S}\sum_{\tau=t-d_s+1}^{t} x_{s\tau drp} \leqslant 1 \qquad \forall t\in T,\ \forall d\in D,\ \forall p\in P \tag{A.5}$$

$$x_{stdrp} = 0 \qquad \forall s\in S,\ \forall r\in R,\ \forall p\in P,\ \forall d\in D,\ \forall t\in T \mid (t+d_s) > \mid T \mid \tag{A.6}$$

$$\sum_{d\in D}\sum_{t\in T} x_{stdrp} \leqslant A_{sp} \qquad \forall s\in S,\ \forall r\in R,\ \forall p\in P \tag{A.7}$$

$$x_{stdrp} = 0 \qquad \forall s\in S,\ \forall r\in R,\ \forall p\in P,\ \forall t\in T, \forall d\notin [ES_s, LS_s] \tag{A.8}$$

$$\sum_{d'=d}^{d+b_s} z_{sd'} \geqslant b_s x_{stdrp} \qquad \forall p\in P,\ \forall d\in D,\ \forall s\in S,\ \forall r\in R,\ \forall t\in T \tag{A.9}$$

$$\sum_{s\in S} z_{sd} \leqslant B_d \qquad \forall d\in D \tag{A.10}$$

$$\sum_{s\in S}\sum_{r\in R}\sum_{t\in T} d_s x_{stdrp} = Q_p + o_p^{1+} - o_p^{1-} \quad \forall p\in P,\ \forall d\in D \tag{A.11}$$

$$\sum_{t\in T}\sum_{d\in D}\sum_{s\in S}\sum_{r\in R} d_s x_{stdrp} = N_p^{\max} + o_p^{2a+} - o_p^{2a-} \quad \forall p\in P \tag{A.12}$$

$$\sum_{t\in T}\sum_{d\in D}\sum_{s\in S}\sum_{r\in R} d_s x_{stdrp} = N_p^{\min} + o_p^{2b+} - o_p^{2b-} \quad \forall p\in P \tag{A.13}$$

$$\sum_{r\in R}\sum_{s\in S} O_{pdt}\left[\sum_{\tau=t-d_s+1}^{t} x_{s\tau drp}\right] = o_{pdt}^3 \qquad \forall p\in P, \forall d\in D, \forall t\in T \tag{A.14}$$

$$\sum_{d\in D}\sum_{t\in T}\sum_{p\in P} x_{stdrp} = A_{sr} - o_{rs}^4 \qquad \forall r\in R,\ \forall s\in S \tag{A.15}$$

$$\sum_{n=d}^{W_p^{\min}+d+1} \bar{x}_{np} \geq W_p^{\min}\left(\bar{x}_{dp} - \bar{x}_{(d-1)p}\right) + o_p^{5b+} - o_p^{5b-} \quad \forall t\in T,\ \forall d\in D\setminus\{1\},\ \forall p\in P,\ \forall s\in S,\ \forall r\in R \tag{A.16}$$

$$\sum_{n=d}^{W_p^{\max}+d+1} \bar{x}_{np} \leqslant W_p^{\max} + o_p^{5a+} - o_p^{5a-} \qquad \forall t\in T, \forall d\in D,\ \forall p\in P,\ \forall s\in S,\ \forall r\in R \tag{A.17}$$

$$\bar{x}_{dp} \geqslant x_{stdrp} \qquad \forall d\in D,\ \forall p\in P,\ \forall s\in S,\ \forall r\in R \tag{A.18}$$

$$x_{stdrp} \in \{0,\ 1\} \quad \forall p\in P,\ \forall r\in R,\ \forall d\in D,\ \forall t\in T,\ \forall s\in S \tag{A.19}$$

$$\bar{x}_{dp} \in \{0,\ 1\} \qquad \forall p\in P,\ \forall d\in D \tag{A.20}$$

$$o_p^{1+}, o_p^{1-}, o_p^{2a+}, o_p^{2a-}, o_p^{2b+}, o_p^{2b-} \in \{0, 1, 2 \ldots\}$$
$$\forall p \in P, \forall r \in R, \forall d \in D, \forall t \in T, \forall s \in S \quad \text{(A.21)}$$

$$o_{pdt}^{3}, o_{prs}^{4}, o_p^{5a+}, o_p^{5a-}, o_p^{5b+}, o_p^{5b-} \in \{0, 1, 2 \ldots\}$$
$$\forall p \in P, \forall r \in R, \forall d \in D, \forall t \in T, \forall s \in S \quad \text{(A.22)}$$

Objective (A.1) is to maximize the throughput of the surgeries by scheduling as many surgeries as possible on a weekly basis in order to optimize operating time. Because of the structure of the cost coefficient c_{st}, the overtime is minimized as well. Objectives ((A.2) and (A.3)) maximize the physician goals. Constraint (A.4) ensures that no surgery starts before the end of the previous surgery in that OR. Constraint (A.5) is quite similar by forcing that no surgery starts before the end of the previous surgery performed by the same physician. No surgery can start before the end of the day minus the duration of that surgery, therefore all x variables for those time instances are set to 0 in Constraint (A.6). Constraint (A.7) is only present in model ITER and mimics the pre-defined assignment of physicians to patients (surgeries).

Every surgery s has a time window $[ES_s, LS_s]$ that determines their allocation on the right date (Equation (A.8)). After the surgery, patients are allowed to rest for b_s days (LOS) in one of the available beds B_d, represented in Equations (A.9) and (A.10).

The constraints concerned with physician specific restrictions follow next: Equation (A.11) defines the maximum allowed number of surgeries Q_p per physician per day (ctQ). Physicians are preferred to work between $N_p^{\min}$ and $N_p^{\max}$ time instances per week (ctN). The excess of the maximum and the deficit of the minimum number of hours is being minimized. These constraints are reflected in the Equations (A.12) and (A.13). Physicians can also choose their preferred free or unavailable moments in the parameter O_{pdt} (Equation (A.14)). If $O_{pdt} = 1$, then physician p is unavailable at (d,t). Consequently, a surgery with physician p cannot start up to $d_s - 1$ time blocks before (d,t). Otherwise the surgery could end at a time instant where the physician ought to be unavailable. Some operations can only be done in specific ORs that are equipped for those kind of surgeries (Equation (A.15). If $A_{sr} = 0$ which prohibits an assignment from surgery s to OR r, and an actual assignment is made, the goal variable o_{rs}^4 is therefore set to 1, indicating a penalty for this constraint. Let us define these four constraint types (daily work constraint, skill constraint, weekly work constraint and unavailability constraint) as the set of physicians' constraints (ctA, ctQ, ctW, ctO, ctN).

Finally, the x_{stdrp} variable determines the starting time of a surgery s at day d and time instant t in OR r by physician p and must comply to the integrality Constraint (A.19). The binary variable $\bar{x}_{dp}$ determines if physician p is working on day d and its integrality constraints are defined in Equation (A.20). The auxillary physician-related goal variables $o_p^{1+}, o_p^{1-} o_p^{2a+}, o_p^{2a-}, o_p^{2b+}, o_p^{2b-}, o_{pdt}^3, o_{prs}^4, o_p^{5a+}, o_p^{5a-}, o_p^{5b+}, o_p^{5b-}$, are defined in Equations (A.21) and (A.22).

Parameter settings

The cost parameters (fixed parameter values) are set to:

Cost	*Symbol*	*Value*
Consecutive days goal cost	c_p^{cons}	$1/P$
Working hours goal cost	c_p^{work}	$1/P$
Day work goal cost	c_p^{day}	$1/P$
Skills goal cost	c_{prs}^{skills}	$1/(PRS)$
Unavailable goal cost	c_{pdt}^{free}	$1/(PDT)$

The parameters in the physician goal constraints (ctA, ctO, ctN, ctW, ctQ) are set to a tightness of 25% according to the tightness metrics proposed in Van Huele and Vanhoucke (2014b). The same holds for the surgery-related Constraints ((A.8)–(A.10)). The throughput favouring constant K is set equal to the maximum of the physician constraint costs in order to have an equilibrium between the social and economical metrics.

Received 6 September 2013;
accepted 23 October 2014 after three revisions