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WORKING PAPER

Pension reform in an OLG model with heterogeneous abilities

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First version : August 2012
This version : December 2014

2012/810

Published in
Journal of Pension Economics and Finance, 2017

D/2012/7012/43

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29 December 2014

Abstract

We study the effects of pension reform on hours worked, human capital, income and welfare in an open economy populated by four overlapping generations: three active generations (the young, the middle aged and the older) and one generation of retired. Within each generation we distinguish individuals with high, medium or low ability to build human capital. Our simulation results prefer an intelligent pay-as-you-go pension system above a fully-funded private system. This pay-as-you-go system conditions pension benefits on past individual labor income, with a high weight on labor income earned when older and a low weight on labor income earned when young. Uncorrected, however, such a system implies welfare losses for current low-ability generations and rising inequality. Complementing or replacing it by basic and/or minimum pension components is negative for aggregate employment and welfare. Better is to maintain the tight link between individual labor income and the pension also for low-ability individuals, but to strongly raise their replacement rate. An additional correction improving the welfare of low-ability individuals would be to maintain for these individuals equal weights on past labor income.

Keywords: employment by age; retirement; pension reform; heterogeneous abilities; overlapping generations

JEL Classification: E62; H55; J22; J24

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0. Introduction

Growing concern for the long-run financial viability of public pension systems has put pension reform high on the agenda of policy makers and researchers. Many countries legislated changes to their pension system during the last two decades. At the same time, the literature on pension economics expanded rapidly. To face the pension challenge, there now seems to be general agreement on the need for higher employment, especially among older individuals, and higher productivity and growth. Several institutional and policy variables may play a role here. The design of the pension system is certainly one of them. Another concern is to provide adequate retirement benefits for everyone, so as to avoid old-age poverty. Optimal pension design should therefore not only serve the objective of higher employment and productivity, but also safeguard the welfare of households with low earnings capacity.

A huge literature has studied the influence of the pension system on employment and/or growth (see e.g. Lindbeck and Persson, 2003, and the many papers that we refer to below). In Buyse *et al.* (2013) we took this literature as far as we could. We studied the effects of pension reform in an OLG model for an open economy where hours worked by young, middle aged and older individuals, education of the young, the retirement decision of older workers, and aggregate per capita growth, are all endogenous. The model also contains a rich fiscal block to assess the effects of pension reform on the public budget. Simulating the model, our results preferred an intelligent pay-as-you-go (PAYG) system above a fully-funded private system. We found positive effects on employment, growth and welfare to be the strongest in a PAYG system that conditions pension benefits on past individual labor income, with a high (low) weight on labor income earned when older (young) to compute the pension assessment base. Pension reform in this direction encourages young individuals to study and build human capital, which promotes productivity and per capita income. Furthermore, it encourages older workers to postpone retirement. Strengthening the link between one's future old-age pension, on the one hand, and one's human capital and labor supply when older, on the other, introduces strong financial incentives which may bring about important changes in behavior.

An important weakness of our model in Buyse *et al.* (2013), however, is that it assumes equal ability and capacity to learn for all people. Reality is different, however. Data reveal that in 2010 26% of the 25-64 year old population in the OECD had no upper secondary degree. About 44% had an upper secondary degree but no tertiary degree. The fraction of people with a tertiary degree was 30%. Among young cohorts (age 25 to 34), educational attainment is higher. Yet, the fraction that does not complete upper secondary education is still close to 20% on average (OECD, Education at a Glance, 2012, Tables A1). The simple fact that innate ability as for example reflected by IQ varies across people, implies that one can never expect everyone to succeed at the secondary, let alone the tertiary level. The challenge that emerges from these facts is clear. If an 'intelligent' pension reform requires a tighter link between one's future pension and one's individual human capital and labor income (especially labor income earned at older age), welfare losses for individuals with low innate ability may be unavoidable. The incidence of old-age poverty among these individuals can be expected to rise. At the same time, individuals with high innate ability will experience welfare gains.

One may thus expect a substantial increase in welfare inequality. Our main research questions in this paper arise from these expectations. What pension reform is optimal if the objective is not only to improve employment and aggregate productivity and efficiency, but also to avoid welfare losses for low-ability individuals and rising inequality in welfare? If some redistribution in the pension system is necessary, should this then be achieved by a minimum pension, a basic pension or by an earnings related pension paired with a higher replacement rate for the low educated?

To answer these questions we study optimal pension reform and the induced income and welfare distribution in a general equilibrium OLG model with endogenous employment and human capital that also incorporates heterogeneity in ability. More precisely, we extend our model in Buyse *et al.* (2013) by defining in each generation individuals that are born with high, medium or low innate ability. Individuals with higher ability enter the model with more human capital. They are also more productive in building additional human capital when they allocate time to (tertiary) education. Calibrating and simulating the model, our findings highlight the major importance of accounting for heterogeneous abilities and for the distributional consequences of pension reform. We confirm the aggregate efficiency of the 'intelligent' PAYG system advocated in Buyse *et al.* (2013) compared to a fully-funded private system, but also demonstrate the significant welfare losses that moving to this system would impose on current generations of low-ability individuals who cannot study and who earn low wages. Intragenerational welfare inequality would rise strongly. To avoid this, additional policy measures targeting low-ability individuals will be necessary. Investigating various alternatives, we learn the following. First, the introduction of a minimum pension does promote the welfare of the current and future low-ability generations. It is negative, however, for aggregate welfare, employment and per capita output. The main reason is that labor supply and employment among low-ability individuals would fall sharply. Eligibility to a pension above the level that these individuals can ever collect from their own labor kills an important incentive to work. Together with a rise in public pension expenditures, these negative employment effects undermine the public budget, and force the government to raise taxes. Second, the alternative of introducing a basic or flat pension for all citizens has even worse effects. Flat pensions imply a reduction in the return to working for all individuals and to education for individuals with higher and medium ability. Overall negative effects on employment, human capital and productivity would in the end make everyone worse off in absolute terms, including the individuals with low ability. The only positive effect may be that inequality declines. Third, a much more efficient response to the distributional challenge imposed by the PAYG system advocated in Buyse *et al.* (2013) is to maintain the tight link between individual labor income and the pension also for low-ability individuals, but to significantly raise their replacement rate. Moreover, since these individuals cannot study (at the tertiary level), it makes much less sense for them to reduce (raise) the weight attached to labor income earned as a young (older) worker to compute the pension assessment base.

Many studies have documented how the pension system may affect the incentives of individuals of different ages to work (e.g. Sheshinski, 1978; Auerbach *et al.*, 1989; Gruber and Wise, 2002; Sommacal, 2006; Cigno, 2008; Fisher and Keuschnigg, 2010; Jaag *et al.*, 2010; de la Croix *et al.*, 2013;

Fehr *et al.*, 2013). Others have investigated the relationship between the pension system and investment in human capital, as a major determinant of productivity and growth (e.g. Zhang, 1995; Kemnitz and Wigger, 2000; Docquier and Paddison, 2003; Zhang and Zhang, 2003; Le Garrec, 2012). Most recently, Ludwig *et al.* (2012), Buyse *et al.* (2013) and Kindermann (2015) made progress by studying pension reform in OLG models where both employment by age and human capital are endogenous. Although these three studies differ in the way they model growth (exogenous or endogenous), each of them demonstrates the importance of modelling the many mutual relationships between key variables. For example, if policy can make people postpone retirement and work longer, the return to investment in education will rise, and so may human capital and productivity. Conversely, policies that promote investment in human capital will also encourage people to work longer since they will then get a higher return from their investment. Also, if pension reform discourages employment of the young, it may still be efficient if this contributes to education. For a proper assessment of the effects of pension reform it is important to take such interactions into account. As we have mentioned above, we take the model developed in Buyse *et al.* (2013) as our starting point, but extend it by modeling individuals with heterogeneous abilities.

Many researchers have introduced heterogeneous abilities in OLG models before. Some have done this to study the effects of the pension system on inequality, as one of the dependent variables. The way in which heterogeneity is introduced differs, however. Some authors model individuals with different human capital (or skill) levels when they enter the model (e.g. Sommacal, 2006; Fehr *et al.*, 2013). Others introduce individuals with the same initial human capital, but different learning abilities (e.g. Docquier and Paddison, 2003; Kindermann, 2015). Another assumption to make is whether or not human capital and productivity are subject to idiosyncratic shocks during life, as for example in Fehr *et al.* (2013). In our model individuals with higher ability will have both higher initial human capital and be more productive in building additional human capital when they allocate time to (tertiary) education. Individuals with low ability will enter the model with low human capital and have zero productivity to study and build additional human capital. We abstain, however, from shocks to individual human capital and productivity during individuals' life. This set of assumptions may offer the best match to recent findings by Huggett *et al.* (2006, 2011) and Keane and Wolpin (2007) that heterogeneity in human capital endowment at young age and learning abilities, rather than shocks to human capital, account for most of the variation in lifetime utility. Our approach also matches findings that innate learning ability and human capital at the age of 23 are strongly positively correlated (Huggett *et al.*, 2011). A final important element is the relationship between the human capital of subsequent generations. In the main part of this paper, we follow Ludwig *et al.* (2012) and Kindermann (2015) among others, and assume that human capital is predetermined and generation-invariant. Growth will then be exogenous. In a short robustness section we will, however, assume that when people enter the model, they inherit a fraction of the human capital of the previous generation, as in Azariadis and Drazen (1990) and Buyse *et al.* (2013). Individuals with higher ability inherit a larger fraction. Different generations then start with different (ability-specific) human capital, and growth becomes endogenous.

The structure of this paper is as follows. In Section 1 we document differences in employment by age, education of the young, the effective retirement age of older workers, and per capita growth across 13 OECD countries before the financial crisis (1995-2007). Section 2 sets out our basic model with predetermined and generation-invariant human capital. Next to the pension system, we introduce a fiscal policy block. The government in the model sets tax rates on labor, capital and consumption. It spends its revenue on (non-productive) goods, 'non-employment' benefits (including early retirement benefits), old-age pensions, and interest payments on outstanding debt. In Section 3 we calibrate the model on actual data. Section 4 gives more insight into the reality behind the key pension policy parameters and the key fiscal policy parameters in our model. We report data for the same 13 OECD countries. In Section 5 we confront the model's predictions (using the country-specific policy parameters) with the facts described in Section 1. Section 6 includes the results of a range of model simulations. We investigate the steady state employment, education, output and welfare effects of various reforms of the pension system. We study effects per generation and per ability group. In Section 7 we investigate (and confirm) the robustness of our findings to allowing an intergenerational transfer of human capital and endogenous growth. Section 8 concludes the paper.

1. Cross-country differences in employment, tertiary education and per capita growth

Table 1 contains key data on employment, education and growth in 13 OECD countries in 1995-2007. One would like a reliable model to match the main cross-country differences reported here. The employment rate in hours (n) indicates the fraction of potential hours that are actually being worked by the average person in one of three age groups (20-34, 35-49, 50-64). Comparable data for hours worked by ability type (skill level) are not available. Potential hours are 2080 per person per year (52 weeks times 40 hours per week). The observed employment rate rises if more people in an age group have a job, and if the employed work more hours. The employment rate in the age group of 50 to 64 is also affected by the average age at which older workers withdraw from the labor force. We include the effective retirement age as the fourth data column in the Table. In most countries, this age is well below the official age to receive old-age pensions (65 in most countries, 60 in France and Italy). The education rate (e) is our proxy for the fraction of time spent studying by the average person of age 20-34. It has been calculated as the total number of students in full-time equivalents, divided by total population in this age group. Our data for (average annual) real per capita growth concern real potential GDP per person of working age. We refer to Appendix A for details on the calculation of our data, and on the assumptions that we have to make.

As is well known, middle aged individuals work most hours, followed by the young. The older generation works the lowest number of hours. Average employment rates across countries in these three age groups are 55.0%, 63.7% and 43.6% respectively. Furthermore, the data reveal strong cross-country differences. We observe the highest employment rates in each age group in the US. Employment rates are much lower in the core countries of the euro area. The Nordic countries take intermediate positions, although they are close to the core euro area for the younger generation. The latter, however, seems to be related to education. Young people's effective participation in

education is also by far the highest in the Nordic countries. These countries also show the highest potential per capita growth rates. On average, growth in the core euro area and the US was more than 0.5 percentage points lower in the period under consideration. The US and the other Anglo-Saxon countries tend to have the lowest participation in education among people of age 20 to 34. Finally, we note that the effective retirement age also varies across countries. The retirement age is quite low in Belgium (57.9) and France (58.8). By contrast, individuals in Nordic or Anglo-Saxon countries participate longer. Unsurprisingly, correlation between the effective retirement age and the employment rate among older workers (n_3) is very high (0.89).

Table 1

Employment rate in hours (n) by age, effective retirement age, education rate (e) and per capita growth in OECD countries (1995-2006/7)

	n_1 (20-34)	n_2 (35-49)	n_3 (50-64)	<i>Effective retirement age</i>	e	<i>Annual real per capita growth</i>
Austria	59.9	64.3	34.7	59.5	12.5	2.06
Belgium	51.1	56.8	29.3	57.9	14.1	1.77
France	48.7	60.3	38.0	58.8	14.9	1.54
Germany	49.7	55.2	34.9	61.1	17.2	1.56
Italy	50.1	61.9	33.8	60.1	12.6	1.30
Netherlands	50.8	54.6	34.2	60.0	14.7	2.20
Core euro area average	51.7	58.8	34.2	59.6	14.3	1.74
Denmark	56.2	66.7	49.6	62.2	21.7	1.81
Finland	55.6	69.0	47.3	60.2	23.1	2.72
Norway	51.9	60.9	50.6	63.1	18.1	2.29
Sweden	53.6	66.1	55.4	63.4	17.7	2.18
Nordic Average	54.3	65.6	50.7	62.2	20.2	2.25
US	65.6	74.2	59.6	64.2	12.8	1.54
UK	60.8	68.4	49.4	62.0	12.3	2.13
Canada	60.9	69.5	50.4	62.1	13.6	1.68
All country Average	55.0	63.7	43.6	61.1	15.8	1.91

Data sources: OECD (see Appendix A); data description: see main text and Appendix A. The data for employment and growth concern 1995-2007. The data for education and the effective retirement age are averages for 1995-2006. All data are in percent, except the retirement age.

2. The model with exogenous growth

Our analytical framework borrows heavily from Buyse *et al.* (2013). It consists of a computable four-period OLG-model for a small open economy with endogenous employment and human capital. New in this paper is that we realistically take into account differences in individuals' innate abilities.

2.1. Basic setup and demographics

We consider three active adult generations, the young, the middle aged and the older, and one generation of retired agents. Individuals enter the model at age 20. Each period of life is modeled to last 15 years. Within each generation we assume three types of individuals with different ability: a group H with high ability, a group M with medium ability and a group L with low ability. We normalize each ability group to 1, so that the size of a generation is 3, and total population is 12, and constant. Differences in ability are reflected both in the amount of human capital with which individuals enter the model and in their productivity of schooling (at the tertiary level) when young. Low ability individuals enter with the lowest human capital and will never go into tertiary education. They only work or have 'leisure' (including other non-market activities). High and medium ability young people enter the model with more human capital and will also invest a fraction of their time in tertiary education. Middle aged and older individuals do not study anymore. Whatever their innate ability, they only work or have 'leisure'. The statutory old-age retirement age in our model is 65. Individuals may however optimally choose to leave the labor force sooner in a regime of early retirement.

Output is produced by domestic firms acting on competitive markets. These firms employ physical capital together with existing technology and effective labor provided by the three active generations. In the spirit of Buiter and Kletzer (1993), physical capital is internationally mobile, whereas labor and human capital are immobile.

In what follows, we concentrate on the core elements of the model: the optimizing behavior of individuals, the formation of human capital, the behavior of domestic firms and the determination of aggregate output, capital and wages.

2.2. Individuals: preferences and time allocation

An individual with ability a ($a = H, M, L$) reaching age 20 in period t maximizes an intertemporal utility function of the form:

$$U_a^t = \sum_{j=1}^4 \beta^{j-1} \left(\ln c_{ja}^t + \frac{\gamma_j}{1-\theta} (\ell_{ja}^t)^{1-\theta} \right) \quad \forall a = H, M, L \quad (1)$$

with $0 < \beta < 1$, $\gamma_j > 0$, $\theta > 0$ ($\theta \neq 1$). Superscript t indicates the period of youth, when the individual comes into the model. Subscript j refers to the j th period of life and a refers to ability. Lifetime utility depends on consumption (c_{ja}^t) and enjoyed leisure (ℓ_{ja}^t) in each period of life. The parameters β , γ and θ define the discount factor, the relative value of leisure versus consumption,

and the inverse of the intertemporal elasticity to substitute leisure. These parameters are common across ability types. The preference parameter γ may, however, be different in each period of life. Except for the latter assumption, our specification of the instantaneous utility function is quite common in the macro literature (e.g. Rogerson, 2007; Erosa *et al.*, 2012).

Figure 1 shows the individuals' time allocation over the life-cycle. Equations (2)-(5) describe how this is reflected in enjoyed leisure ℓ_{ja}^t . Time endowment in each period is normalized to 1.

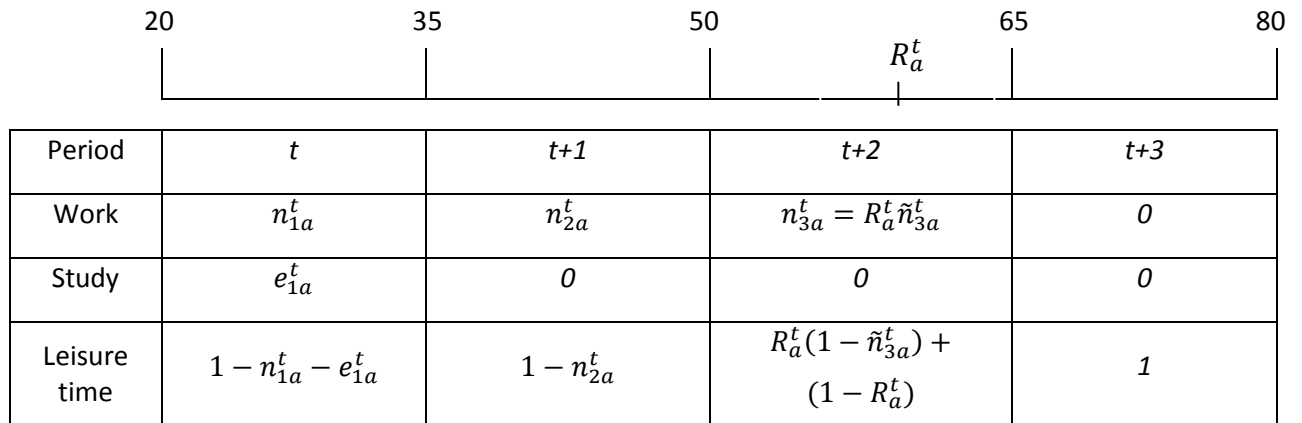
$$\ell_{1a}^t = 1 - n_{1a}^t - e_{1a}^t, \quad \text{with } e_{1L}^t = 0. \quad (2)$$

$$\ell_{2a}^t = 1 - n_{2a}^t \quad (3)$$

$$\ell_{3a}^t = \Gamma \left(\mu (R_a^t (1 - \tilde{n}_{3a}^t))^{1-\frac{1}{\zeta}} + (1 - \mu) (1 - R_a^t)^{1-\frac{1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \quad (4)$$

$$\ell_{4a}^t = 1 \quad (5)$$

Figure 1. Life-cycle of an individual of generation t and ability a



Note: $e_{1L}^t = 0$.

In the first period of active life (Equation 2), leisure falls in labor supply (n_{1a}^t) and in education time (e_{1a}^t). Only the low ability individuals do not study ($e_{1L}^t = 0$). In the second and third period, no one studies. Individuals only work or have leisure (Equations 3 and 4). Following the approach in Buysse *et al.* (2013), part of the individuals' optimal choice of leisure in the third period of their life concerns the determination of early retirement. Individuals choose R_a^t which relates to the optimal effective retirement age and which is defined as the fraction of time between age 50 and 65 that the individual participates in the labor market; $(1 - R_a^t)$ is the fraction of time in early retirement. Assuming that labor market exit is irreversible and post-retirement employment is not allowed, the relationship between the fraction of time devoted to work between 50 and 65 (n_{3a}^t) and the fraction of time devoted to work before early retirement but after 50 (\tilde{n}_{3a}^t), is as follows: $n_{3a}^t = R_a^t \cdot \tilde{n}_{3a}^t$. Leisure time in the third period therefore consists of two parts: non-employment time before the effective retirement age $R_a^t (1 - \tilde{n}_{3a}^t)$, and time in early retirement after it $(1 - R_a^t)$. Equation (4) then describes composite enjoyed leisure of an older worker as a CES-function of both parts. Like

Buyse *et al.* (2013), we assume imperfect substitutability between the two leisure types. The idea is that leisure time after and between periods of work is not the same as leisure time in periods when individuals are not economically active anymore.¹ Equation (4) expresses that individuals prefer to have a balanced combination of both rather than an extreme amount of one of them (and very little of the other). In this equation ζ is the constant elasticity of substitution, μ is a usual share parameter and Γ is added as a normalization constant such that the magnitude of ℓ_{3a}^t corresponds to the magnitude of total leisure time ($1 - n_{3a}^t$). The latter assumption allows us to interpret γ_3 as the relative value of leisure versus consumption in the third period, comparable to γ_1 and γ_2 . The main results in this paper are not in any way influenced by the magnitude of μ , Γ or ζ .

2.3. Individuals: budget constraints

Equations (6)-(10) describe the budget constraints that individuals are subject to. We briefly explain these constraints, paying particular attention to the determinants of the old-age pension benefit that individuals receive, and its relationship to employment and human capital in earlier periods.

$$(1 + \tau_c)c_{1a}^t + \Omega_{1a}^t = w_{a,t}h_{1a}^t n_{1a}^t (1 - \tau_w) + bw_{a,t}h_{1a}^t (1 - \tau_w)(1 - n_{1a}^t - e_{1a}^t) \quad (6)$$

$$(1 + \tau_c)c_{2a}^t + \Omega_{2a}^t = w_{a,t+1}h_{2a}^t n_{2a}^t (1 - \tau_w) + bw_{a,t+1}h_{2a}^t (1 - \tau_w)(1 - n_{2a}^t) + (1 + r_{t+1})\Omega_{1a}^t \quad (7)$$

$$(1 + \tau_c)c_{3a}^t + \Omega_{3a}^t = w_{a,t+2}h_{3a}^t R_a^t \tilde{n}_{3a}^t (1 - \tau_w) + bw_{a,t+2}h_{3a}^t (1 - \tau_w)R_a^t (1 - \tilde{n}_{3a}^t) + b_{er}w_{a,t+2}h_{3a}^t (1 - \tau_w)(1 - R_a^t) + (1 + r_{t+2})\Omega_{2a}^t \quad (8)$$

$$(1 + \tau_c)c_{4a}^t = (1 + r_{t+3})\Omega_{3a}^t + pp_a^t \quad (9)$$

$$pp_a^t = \rho_{wa} \sum_{j=1}^3 (p_j w_{a,t+j-1} h_{ja}^t n_{ja}^t (1 - \tau_w)) + \rho_{fa} \left(\frac{1}{9}\right) \sum_{j=1}^3 \sum_{a=H,M,L} (w_{a,t+3} h_{ja}^{t+4-j} n_{ja}^{t+4-j} (1 - \tau_w)) \quad (10)$$

with: $0 \leq p_j \leq 1$

$$\sum_{j=1}^3 p_j = 1$$

$$n_{3a}^t = R_a^t \tilde{n}_{3a}^t$$

The LHS of Equations (6)-(9) shows that individuals allocate their disposable income to consumption (including consumption taxes, τ_c) and to the accumulation of non-human wealth. We denote by Ω_{ja}^t the stock of wealth held by a type a individual of generation t at the end of the j th period of his life. Individuals start adult life with zero assets. As is clear from Equation (9), they also finish life with zero assets. During the three periods of active life, disposable income at the RHS includes after-tax labor

¹ The former may be particularly valuable from the perspective of relaxation and time to spend on personal activities of short duration. The latter may be valuable to enjoy activities that take more time and ask for longer term commitment (e.g. long journeys, non-market activity as a volunteer).

income and non-employment benefits. From the second to the fourth period, it may also include interest income. We denote by $w_{a,k}$ the real wage per unit of effective labor supplied at time k by an individual with ability a and by r_k the exogenous (world) real interest rate at time k .

Effective labor of an individual with ability a depends on hours worked (n_{ja}^t) and human capital (h_{ja}^t). Given the tax rate on labor income τ_w , young individuals earn an after-tax real wage equal to $w_{a,t} h_{1a}^t n_{1a}^t (1 - \tau_w)$. After-tax labor income of middle aged and older workers in Equations (7) and (8) is determined similarly. For the fraction of time that young, middle aged and older individuals are inactive, they receive a non-employment benefit from the government. Older individuals may be eligible to two kinds of benefits: standard non-employment benefits (analogous to what young and middle aged workers receive) as long as they are on the labor market, and early retirement benefits after having withdrawn from the labor market. All benefits are defined as a proportion of the after-tax wage of a full-time worker. The net replacement rate for standard non-employment benefits is b , for early retirement benefits it is b_{er}^2 .

After the statutory retirement age (65) individuals have no labor income and no non-employment benefits anymore. They earn interest income from accumulated non-human wealth, and they receive an old-age pension benefit (pp_a^t). We assume a public PAYG pension system in which pensions in period k are basically financed by contributions from the active generations in that period k (see below). As described by Equation (10), individual net pension benefits consist of two components. A first one is related to the individual's earlier net labor income. It is a fraction of his so-called pension base, i.e. a weighted average of net labor income in each of the three active periods of life. The net replacement rate is ρ_{wa} . The parameters p_1, p_2 and p_3 represent the weights attached to each period. This part of the pension rises in the individual's hours of work n_{ja}^t and his human capital h_{ja}^t . It will be lower when the individual retires early (lower R_a^t). The second component of the pension is a flat-rate or basic pension. Every retiree receives the same amount related to average net labor income in the economy at the time of retirement. Here, the net replacement rate is ρ_{fa} .

Note that we allow ability-specific pension replacement rates ρ_{wa} and ρ_{fa} . This specification is in line with the data in many countries. The importance of own-income related versus flat components may be very different depending on people's earned income, and therefore ability (see Section 4 and Table 5 below). For other policy variables like labor tax rates such differences are much smaller (Heylen and Van de Kerckhove, 2013). The introduction of ability-specific pension replacement rates also allows a richer policy analysis.

2.4. Individuals: human capital formation

Individuals enter our model at the age of 20 with a predetermined level of human capital. This level is generation-invariant, but it rises in innate ability. The latter reflects for example that higher innate

² As explained in greater detail by Buyse *et al.* (2013, footnote 5), the approach to model early retirement benefits as a function of a worker's last labor income, similar to standard non-employment benefits, reflects regulation and/or common practice in many countries.

ability makes it easier for individuals to learn and accumulate knowledge at primary and secondary school. In Equation (11) we normalize the human capital of a young individual with high ability to h_0 . A young individual with medium ability enters the model with only a fraction ε_M of this. A young worker with low ability enters with an even lower fraction ε_L . These fractions will be calibrated.

$$h_{1a}^t = \varepsilon_a h_0 \quad \forall a = H, M, L \quad (11)$$

with $0 < \varepsilon_L < \varepsilon_M < \varepsilon_H = 1$.

During youth, individuals with high and medium ability will invest a fraction of their time to expand their human capital, making them more productive in the second and third period. We adopt in Equation (12.a) a human capital production function similar to Lucas (1990), Glomm and Ravikumar (1998), Bouzahzah *et al.* (2002) and Docquier and Paddison (2003). The production of new human capital by these individuals rises in the amount of time they allocate to education (e_{1a}^t) and in their initial human capital (h_{1a}^t). We assume a common elasticity of time input (σ) and a common efficiency parameter (ϕ) for both ability types. Individuals with low innate ability do not study. In Equation (12.b) their human capital remains constant. Finally, we assume in Equation (13) that the human capital of all individuals remains unchanged between the second and the third period. We have in mind that learning by doing in work may counteract depreciation. The same assumption explains the lack of depreciation in Equation (12). In no way does this assumption affect our main results in this paper.

$$h_{2a}^t = h_{1a}^t (1 + \phi (e_{1a}^t)^\sigma) \quad \forall a = H, M \quad (12.a)$$

$$h_{2L}^t = h_{1L}^t \quad (12.b)$$

$$h_{3a}^t = h_{2a}^t, \quad \forall a = H, M, L \quad (13)$$

with $0 < \sigma \leq 1, \phi > 0$.

2.5. Individuals: optimization and the role of the pension system

Individuals will choose consumption, labor supply in each period of active life, education when young (for the medium and high ability individuals), and their effective retirement age to maximize Equation (1), subject to Equations (2)-(13). Substituting Equations (2)-(5) for ℓ_{ja}^t and (6)-(9) for c_{ja}^t into (1), and maximizing with respect to $\Omega_{1a}^t, \Omega_{2a}^t, \Omega_{3a}^t, n_{1a}^t, n_{2a}^t, \tilde{n}_{3a}^t, R_a^t$ and e_{1a}^t , yields eight first order conditions for the optimal behavior of an individual with ability a entering the model at time t . Equation (14) expresses the law of motion of optimal consumption over the lifetime. Equations (15.a), (15.b) and (15.c) describe the optimal labor-leisure choice in each period of active live. Individuals supply labor up to the point where the marginal utility of leisure equals the marginal utility gain from work. The latter consists of two parts. Working more hours in a particular period raises additional resources for consumption both in that period and when retired. The marginal utility gain from work rises when the marginal utility of consumption ($1/c_{ja}^t$) is higher, and when an extra hour of work yields more extra consumption. Higher human capital (and its underlying

determinants), lower taxes on labor, lower taxes on consumption and lower non-employment benefits contribute to the gain from work. Extra consumption during retirement rises in the own-income- related pension replacement rate (ρ_{wa}) and in the weight attached to the relevant period when computing the pension base (p_j). Equations (15.a)-(15.c) highlight positive substitution effects from the pension replacement rate ρ_{wa} . To the extent that higher replacement rates raise individuals' consumption possibilities (c_{ja}^t), they also cause adverse income effects on labor supply. Basic pensions (ρ_{fa}) do not directly occur in Equations (15), but they do affect employment via this income effect.

$$\frac{c_{j+1,a}^t}{c_{ja}^t} = \beta(1 + r_{t+j}), \quad \forall j = 1,2,3 \quad (14)$$

$$\frac{\gamma_1}{(\ell_{1a}^t)^\theta} \frac{-\partial \ell_{1a}^t}{\partial n_{1a}^t} = \frac{w_{a,t} h_{1a}^t (1-\tau_w)(1-b)}{c_{1a}^t (1+\tau_c)} + \beta^3 \frac{\rho_{wa} p_1 w_{a,t} h_{1a}^t (1-\tau_w)}{c_{4a}^t (1+\tau_c)} \quad (15.a)$$

$$\begin{aligned} \frac{\gamma_2}{(\ell_{2a}^t)^\theta} \frac{-\partial \ell_{2a}^t}{\partial n_{2a}^t} &= \frac{w_{a,t+1} (1+\phi(e_{1a}^t)^\sigma) h_{1a}^t (1-\tau_w)(1-b)}{c_{2a}^t (1+\tau_c)} \\ &+ \beta^2 \frac{\rho_{wa} p_2 w_{a,t+1} (1+\phi(e_{1a}^t)^\sigma) h_{1a}^t (1-\tau_w)}{c_{4a}^t (1+\tau_c)} \end{aligned} \quad (15.b)$$

$$\begin{aligned} \frac{\gamma_3}{(\ell_{3a}^t)^\theta} \frac{-\partial \ell_{3a}^t}{\partial \tilde{n}_{3a}^t} &= \frac{w_{a,t+2} (1+\phi(e_{1a}^t)^\sigma) h_{1a}^t R_a^t (1-\tau_w)(1-b)}{c_{3a}^t (1+\tau_c)} \\ &+ \beta \frac{\rho_{wa} p_3 w_{a,t+2} (1+\phi(e_{1a}^t)^\sigma) h_{1a}^t R_a^t (1-\tau_w)}{c_{4a}^t (1+\tau_c)} \end{aligned} \quad (15.c)$$

Equation (16) describes the first order condition for the optimal effective retirement age. The LHS represents the utility loss from postponing retirement. Later retirement reduces enjoyed leisure as early retiree, but raises enjoyed leisure in between periods of work for given work time \tilde{n}_{3a}^t . The RHS shows the marginal utility gain from postponing retirement. This marginal gain follows from consuming the extra labor income (vis-à-vis the early retirement benefit) in the third period, and the higher future old-age pension after 65. The latter effect rises in ρ_{wa} and p_3 .

$$\begin{aligned} \frac{\gamma_3}{(\ell_{3a}^t)^\theta} \frac{-\partial \ell_{3a}^t}{\partial R_a^t} &= \frac{w_{a,t+2} (1+\phi(e_{1a}^t)^\sigma) h_{1a}^t (1-\tau_w) (\tilde{n}_{3a}^t + b(1-\tilde{n}_{3a}^t) - b_{er})}{c_{3a}^t (1+\tau_c)} \\ &+ \beta \frac{\rho_{wa} p_3 w_{a,t+2} (1+\phi(e_{1a}^t)^\sigma) h_{1a}^t \tilde{n}_{3a}^t (1-\tau_w)}{c_{4a}^t (1+\tau_c)} \end{aligned} \quad (16)$$

Finally, Equation (17) imposes for high and medium ability individuals that the marginal utility loss from investing in human capital when young equals the total discounted marginal utility gain in later periods from having more human capital. Individuals will study more the higher future versus current after-tax real wages and the higher the marginal return of education ($\sigma \phi(e_{1a}^t)^{\sigma-1}$). Labor taxes

during youth therefore encourage individuals to study, whereas labor taxes in later periods of active life discourage them. Notice also that high benefit replacement rates in later periods, and a high income-related pension replacement rate (ρ_{wa}), combined with high weights p_2 and p_3 , will encourage young individuals to study. The reason is that any future benefits and the future pension rise in future labor income, and therefore human capital. A final interesting result is that young people study more – all other things equal – if they expect to work harder in later periods (n_{2a}^t , $n_{3a}^t = R_a^t \tilde{n}_{3a}^t$).

$$\frac{\gamma_1}{(\ell_{1a}^t)^\theta} \frac{-\partial \ell_{1a}^t}{\partial e_{1a}^t} - \frac{1}{c_{1a}^t} \frac{\partial c_{1a}^t}{\partial e_{1a}^t} = \beta \frac{1}{c_{2a}^t} \frac{\partial c_{2a}^t}{\partial e_{1a}^t} + \beta^2 \frac{1}{c_{3a}^t} \frac{\partial c_{3a}^t}{\partial e_{1a}^t} + \beta^3 \frac{1}{c_{4a}^t} \frac{\partial c_{4a}^t}{\partial e_{1a}^t} \quad \forall a = H, M \quad (17)$$

with:

$$\frac{\partial c_{1a}^t}{\partial e_{1a}^t} = - \frac{b w_{a,t} h_{1a}^t (1-\tau_w)}{1+\tau_c}$$

$$\frac{\partial c_{2a}^t}{\partial e_{1a}^t} = \sigma \phi(e_{1a}^t)^{\sigma-1} \frac{w_{a,t+1} h_{1a}^t (1-\tau_w) [n_{2a}^t + b(1-n_{2a}^t)]}{1+\tau_c}$$

$$\frac{\partial c_{3a}^t}{\partial e_{1a}^t} = \sigma \phi(e_{1a}^t)^{\sigma-1} \frac{w_{a,t+2} h_{1a}^t (1-\tau_w) [R_a^t (\tilde{n}_{3a}^t (1-b) + b - b_{er}) + b_{er}]}{1+\tau_c}$$

$$\frac{\partial c_{4a}^t}{\partial e_{1a}^t} = \rho_{wa} \sigma \phi(e_{1a}^t)^{\sigma-1} \frac{\sum_{j=2}^3 (p_j n_{ja}^t w_{a,t+j-1} h_{1a}^t (1-\tau_w))}{1+\tau_c}$$

The pension system in our model is of the PAYG type as we see it in most OECD countries. Expenditures are basically financed by contributions from workers (labor taxes). However, since we do not define a strictly separate budget for the pension system, the government may also support it using other resources from its general budget (see Section 2.7.). It will be obvious from our discussion of Equation (10) and the first order conditions in this section that *for a given way of financing* the specific organization of pension benefits may have strong effects on behavior in earlier periods of life. We summarize them here. Both income and substitution effects occur:

- A higher replacement rate ρ_{wa} raises the return to working (n , for all ability groups) and to building human capital (e, h , for high and medium-ability individuals) in earlier periods. It will encourage individuals to work and to invest in education.
- Changes in the particular weights of the periods that constitute the pension assessment base to which ρ_{wa} applies, may modify these incentive effects. The return to working in a particular period rises in the weight attached to that period. A shift in weight from p_1 to p_3 brings strong incentives to work less when young, and to work more and longer when old. This shift also includes a strong incentive to invest in human capital. The net return to education rises in p_2 and p_3 , but falls in p_1 .
- Pension systems that encourage individuals to work more when middle aged or older, also stimulate them to study when young (at least when they have medium or high innate ability). The reason is that an increase in n_2 or n_3 raises the return to education. Conversely, individuals who

invest more in human capital when young will also prefer to work more and longer at higher age. The reason here is that a higher level of human capital raises wages and the return to working.

- Higher replacement rates ρ_{wa} do not only bring about substitution effects, however. Raising individuals' consumption possibilities, they also cause adverse income effects on labor supply.
- The story is different when old-age benefits are of the basic pension type (ρ_{fa}). These cause no substitution effects, and thus no incentive effects to work or study. They only affect employment (negatively) via the income effect. Since lower employment in later periods affects the return to education, a basic pension system would also discourage investment in education. Shifting from an own-earnings related to a basic pension system is bad for efficiency.

Obviously, for a proper assessment of the effects of pension systems and reforms, one cannot disregard the issue of financing. In this respect, it has been shown in the literature that if an increase of the replacement rate ρ_{wa} and the future pension benefit is associated with an increase in the tax rate on labor, the positive effect on labor supply disappears. In most cases, i.e. when the present discounted value of benefits is lower than the value of the contributions, the effect may turn negative (see e.g. also Cigno, 2008; Fisher and Keusschnigg, 2010). The positive effect on education will not disappear, however. A pension system with earnings-related benefits will always encourage individuals to invest in education when young. The reason is that when the present value of future benefits is lower than the value of the contributions, an implicit tax structure results that has high tax rates on labor income in the first period of active life and lower tax rates towards the end. This subsidizes human capital formation (see also Kindermann, 2015). Raising individuals' future wages, a higher level of human capital will then recreate positive incentive effects for individuals to work when middle aged and older. All these interactions between endogenous labor and endogenous human capital, supplied by individuals of different generations and ability, clearly highlight the need for a larger scale numerical analysis of pension reform. We carry out this analysis in Section 6.

2.6. Domestic firms, output and factor prices

Firms act competitively on output and input markets and maximize profits. All firms are identical. Total domestic output (Y_t) is given by the production function (18). Production exhibits constant returns to scale in aggregate physical capital (K_t) and labor in efficiency units ($A_t H_t$), so that profits are zero in equilibrium. Technology A_t is growing at an exogenous and constant rate x : $A_{t+1} = A_t(1 + x)$. Equation (19) defines total effective labor as a CES aggregate of effective labor supplied by the three ability groups. In this equation s is the elasticity of substitution between the different ability types of labor and η_H, η_M and η_L are the input shares. We will impose that $\eta_H = 1 - \eta_M - \eta_L$.

$$Y_t = K_t^\alpha (A_t H_t)^{1-\alpha} \quad (18)$$

$$H_t = \left(\eta_H H_{H,t}^{1-\frac{1}{s}} + \eta_M H_{M,t}^{1-\frac{1}{s}} + \eta_L H_{L,t}^{1-\frac{1}{s}} \right)^{\frac{s}{s-1}} \quad (19)$$

Equation (20) specifies effective labor per ability group. Within each ability group we assume perfect substitutability of labor supplied by the different age groups.

$$\begin{aligned} H_{a,t} &= n_{1a}^t h_{1a}^t + n_{2a}^{t-1} h_{2a}^{t-1} + n_{3a}^{t-2} h_{3a}^{t-2} \\ &= (n_{1a}^t + n_{2a}^{t-1} \psi_a^{t-1} + n_{3a}^{t-2} \psi_a^{t-2}) \varepsilon_a h_0 \quad \forall a = H, M, L \end{aligned} \quad (20)$$

To derive Equation (20) we make use of Equations (12) and (13) where we define:

$$1 + \phi(e_{1a}^t)^\sigma \equiv \psi_a^t, \text{ where } \psi_L^t = 1 \quad (21)$$

It then follows that: $h_{3a}^{t-j} = h_{2a}^{t-j} = \psi_a^{t-j} h_{1a}^{t-j} \quad \forall a = H, M, L$.

Furthermore, we exploit the result that

$$h_{1a}^t = h_{1a}^{t-1} = h_{1a}^{t-2} = \varepsilon_a h_0 \quad (22)$$

Substituting Equation (20) for $a = H, M$ and L into (19), and recognizing differences in the capacity ε_a to inherit human capital as indicated by Equation (11), yields Equation (23).

$$H_t = \left[\sum_{a=H,M,L} \eta_a \varepsilon_a^{1-\frac{1}{s}} (n_{1a}^t + n_{2a}^{t-1} \psi_a^{t-1} + n_{3a}^{t-2} \psi_a^{t-2})^{1-\frac{1}{s}} \right]^{\frac{s}{s-1}} h_0 \quad (23)$$

Competitive behavior implies in Equation (24) that firms carry physical capital to the point where its after-tax marginal product net of depreciation equals the world real interest rate. Physical capital depreciates at rate δ_k . Capital taxes are source-based: the tax rate τ_k applies to the country in which the capital is used, regardless of who owns it. The (world) real interest rate being given, firms will install more capital when the amount of labor in efficiency units increases or the capital tax rate falls. In that case the net return to investment in the home country rises above the world interest rate, and capital flows in. Furthermore, perfect competition implies equality between the real wage and the marginal product of effective labor for each ability type (Equation 25). Workers of a particular ability type will earn a higher real wage when their supply is relatively scarce, when the level of technology is higher, and when physical capital per unit of aggregate effective labor is higher.

$$\left[\alpha \left(\frac{A_t H_t}{K_t} \right)^{1-\alpha} - \delta_k \right] (1 - \tau_k) = r_t \quad (24)$$

$$(1 - \alpha) A_t^{1-\alpha} \left(\frac{K_t}{H_t} \right)^\alpha \eta_a \left(\frac{H_t}{H_{a,t}} \right)^{\frac{1}{s}} = w_{a,t} \quad \forall a = H, M, L \quad (25)$$

Our assumptions of constant population and of individuals entering the model with a predetermined and generation-invariant level of human capital imply that in steady state effective labor will be constant. Physical capital, output and real wages by contrast will all grow at the exogenous technology growth rate x .

2.7. Government

Equation (26) describes the government's budget constraint. Demand for goods G_t , benefits related to non-employment B_t (including early retirement benefits), old-age pension benefits PP_t , and interest payments $r_t D_t$ are financed by taxes on labor T_{nt} , taxes on capital T_{kt} , and taxes on consumption T_{ct} and/or by new debt ΔD_{t+1} . We define D_t as outstanding public debt at the beginning of period t .

$$\Delta D_{t+1} = D_{t+1} - D_t = G_t + B_t + PP_t + r_t D_t - T_{nt} - T_{kt} - T_{ct} \quad (26)$$

with: $G_t = gY_t$

$$\begin{aligned} B_t &= \sum_{a=H,M,L} \left((1 - n_{1a}^t - e_{1a}^t) b w_{a,t} h_{1a}^t (1 - \tau_w) + (1 - n_{2a}^{t-1}) b w_{a,t} h_{2a}^{t-1} (1 - \tau_w) \right. \\ &\quad \left. + R_a^{t-2} (1 - \tilde{n}_{3a}^{t-2}) b w_{a,t} h_{3a}^{t-2} (1 - \tau_w) + (1 - R_a^{t-2}) b_{er} w_{a,t} h_{3a}^{t-2} (1 - \tau_w) \right) \\ PP_t &= \sum_{a=H,M,L} \left(\rho_{wa} \sum_{j=1}^3 \left(p_j w_{a,t+j-4} h_{ja}^{t-3} n_{ja}^{t-3} (1 - \tau_w) \right) \right. \\ &\quad \left. + \rho_{fa} \left(\frac{1}{9} \right) \sum_{j=1}^3 \sum_{a=H,M,L} \left(w_{a,t} h_{ja}^{t+1-j} n_{ja}^{t+1-j} (1 - \tau_w) \right) \right) \\ T_{n,t} &= \tau_w \sum_{a=H,M,L} \left(\sum_{j=1}^3 n_{ja}^{t+1-j} w_{a,t} h_{ja}^{t+1-j} \right) \\ T_{kt} &= \tau_k (\alpha Y_t - \delta_k K_t) \\ T_{ct} &= \tau_c \sum_{j=1}^4 \left(c_{jH}^{t+1-j} + c_{jM}^{t+1-j} + c_{jL}^{t+1-j} \right) \end{aligned}$$

Note our assumption that the government claims a given fraction g of output. Goods bought by the government have no effect on private sector productivity, nor do they directly affect individuals' utility. Non-employment benefits (B_t) are an unconditional source of income support related to inactivity (leisure) and non-market household activities as in Rogerson (2007) and Dhont and Heylen (2009). Although it may seem strange to have such transfers in a model without involuntary unemployment, there is clear practical relevance. Unconditional or quasi unconditional benefits to structurally non-employed people are a fact of life in many European countries. Note also our assumption that the pension system is fully integrated into government accounts. We do not impose a specific financing of the PAYG pension plan. The government can use resources from the general budget to finance pensions.

2.8. Aggregate equilibrium and the current account

Optimal behavior by firms and households and government spending underlie aggregate domestic demand for goods in the economy. Our assumption that the economy is open implies that aggregate domestic demand may differ from supply and income, which generates international capital flows and imbalance on the current account. Equation (27) describes aggregate equilibrium as it can be derived from the model's equations. The LHS of (27) represents national income. It is the sum of domestic output Y_t and net factor income from abroad $r_t F_t$, with F_t being net foreign assets at the beginning of t . The aggregate stock of wealth Z_t accumulates wealth held by individuals who entered the model in $t-1$, $t-2$ and $t-3$. At the RHS of (27) CA_t stands for the current account in period t .

$$Y_t + r_t F_t = C_t + I_t + G_t + CA_t \quad (27)$$

$$\begin{aligned}
\text{with: } F_t &= Z_t - K_t - D_t \\
CA_t &= F_{t+1} - F_t = \Delta Z_{t+1} - \Delta K_{t+1} - \Delta D_{t+1} \\
I_t &= \Delta K_{t+1} + \delta_k K_t
\end{aligned}$$

3. Parameterization

The economic environment described above allows us to simulate the effects on employment, education, output and welfare of various changes in the pension system. Our main contribution in this paper is that we model and assess differential effects for individuals with different ability. This simulation exercise requires us first to parameterize and solve the model. Table 2 contains an overview of all parameters. Many have been set in line with the existing literature. Others have been calibrated to match key data.

We set the rate of time preference at 1.5% per year, the (exogenous and constant) world real interest rate at 4.5% per year and the physical capital depreciation rate at 8% per year. Considering that periods in our model last 15 years, this choice implies a discount factor $\beta = 0.8$, an interest rate $r = 0.935$ and physical capital depreciation $\delta_k = 0.714$. In the production function for goods we assume a capital share coefficient α equal to 0.3. The elasticity of substitution s between the different ability types of effective labor is set equal to 1.5. Our values for the rate of time preference, the capital share and capital depreciation are well within the range of values imposed in the literature (e.g. Altig *et al.*, 2001; Heijdra and Romp, 2009; Ludwig *et al.*, 2012). So is the value for s . The empirical labor literature consistently documents values between 1 and 2 (see Caselli and Coleman, 2006). For the value of the intertemporal elasticity of substitution in leisure ($1/\theta$) we follow Rogerson (2007, p. 12). He puts forward a reasonable range for θ from 1 to 3. In line with this, we impose θ to be equal to 2. This choice implies an elasticity of labor supply which is much higher than the very low elasticities typically found in micro studies. Given our macro focus, however, these micro studies may not be the most relevant ones (see Rogerson and Wallenius, 2009; Fiorito and Zanella, 2012).

Four parameters relate to human capital production. For the elasticity with respect to education time (σ) we choose a conservative value of 0.3. This value is within the range considered by Bouzahzah *et al.* (2002) and Docquier and Paddison (2003), but much lower than the elasticity of 0.80 that we see in Lucas (1990) or Glomm and Ravikumar (1998). The choice of a conservative value for σ excludes that our main findings in the next sections might be due to an overestimation of the returns to education³. The literature provides much less guidance for the calibration of the relative initial human capital of medium and low ability individuals (relative to the initial human capital of high ability individuals, ε_M and ε_L). To determine these parameters we rely on PISA science scores. These scores leave no doubt. In about all OECD countries the science test score of students at the 17th percentile varies between 65% and 69% of the test score of students at the 83th percentile, while the science test score of students at the 50th percentile varies between 82.5% and 85.5% of the test

³ Imposing higher values for σ would only reinforce our main conclusions in this paper.

score of students at the 83th percentile⁴. The differences across countries in these relative scores are extremely small. We can take them as objective indicators of the relative cognitive capacity of low and medium ability individuals, and will correspondingly set ε_L equal to 0.67 and ε_M equal to 0.84. Last but not least, the efficiency parameter ϕ in the human capital production function has been determined by a calibration procedure that we discuss now.

We determined eight parameters by calibration. Next to the efficiency parameter in human capital production (ϕ), these are the exogenous technology growth rate (x), two share parameters in aggregate effective labor (η_M and η_L , where η_H follows as $1 - \eta_L - \eta_M$), three taste for leisure parameters ($\gamma_1, \gamma_2, \gamma_3$) and the elasticity of substitution (ζ) in the composite leisure function in Equation (4). The calibration target values are reported at the bottom of Table 2. Six of them concern Belgium: three employment rates, the effective retirement age, aggregate participation in tertiary education, and growth. We choose Belgium since it is a small open economy (and therefore matches key assumptions of our model) and since in Belgium public pension benefits are calculated exactly as we model them⁵. The other two target values are the relative wages of young workers with below upper secondary education or with upper secondary education in the US compared to workers with tertiary education. Although in practice a whole system of simultaneous equations is solved in which each target value is important for each parameter to be calibrated, it may be useful for our exposition here to bring some more structure. Certain parameters are clearly more than others linked to certain target values. The calibrated growth rate of technology (x) reflects total per capita output growth over a period of 15 years, annual growth in Belgium being 1.77%. The leisure parameters, including the elasticity of substitution in the composite leisure function (4), are basically determined so that with observed levels of the policy variables (tax rates, non-employment benefit replacement rates, pension replacement rates, etc.) in Belgium, the model correctly predicts Belgium's employment rates by age (n_1, n_2, n_3) and effective early retirement age (R). By the same approach the efficiency parameter in human capital production (ϕ) is mainly determined to correctly predict participation in education (e). We find that the taste for leisure rises with age ($\gamma_1 = 0.074, \gamma_2 = 0.147, \gamma_3 = 0.258$) and observe a stronger degree of substitutability than in the Cobb-Douglas case between the two types of leisure for older workers ($\zeta = 1.54$). The efficiency parameter ϕ turns out to be 1.21. Finally, calibration of the share parameters η_M and η_L is mainly driven by the values for relative wages of young workers in the US. They are determined so that with observed levels of the policy variables in the US, and given the whole set of other parameters, the model correctly predicts these relative wages. As shown by Equation (25), the share parameters are important determinants of the relative productivity of labor. Actual wages are informative if a close

⁴ The data that we report are averages of the PISA results for the years 2000, 2003 and 2006. Ideally, for our parameterization, we dispose of PISA test scores for students aged 19. The available data concern students aged 15.

⁵ Public pensions are proportional to average annual labor income earned over a period of 45 years, with equal weights to all years. In our model this comes down to $\rho_{wa} > 0$, with $p_1 = p_2 = p_3 = 1/3$. Only individuals with labor income below about 75% of the mean receive an additional social assistance benefit, which in our model can be expressed as a 'basic pension' for the low ability individuals. So, $\rho_{fL} > 0$, while $\rho_{fM} = \rho_{fH} = 0$. We provide more details in the next section.

link can be assumed between wages and productivity. This condition is much more likely fulfilled in the US than in Europe, which explains the introduction here of US relative wages rather than Belgian ones. We provide more detail on our calibration procedure to obtain η_L and η_M in Appendix B. The results imply $\eta_L = 0.19$, $\eta_M = 0.33$ and $\eta_H = 0.48$.

Finally, we had no ex ante indication on the remaining parameters in the composite leisure function in Equation (4). We impose equal weight for both leisure types ($\mu=0.5$). The normalization parameter Γ equals 2. The size of this parameter has no impact at all on our results.

Table 2. Parameterization and benchmark equilibrium

Technology and preference parameters					
Goods production (output)	$\alpha = 0.30, s = 1.5, \eta_H = 0.48, \eta_M = 0.33, \eta_L = 0.19$				
Exogenous technology growth	$x = 0.301$				
Human capital	$\phi = 1.21, \sigma = 0.3$				
Initial human capital	$\varepsilon_M = 0.84, \varepsilon_L = 0.67$				
Preference parameters	$\beta = 0.80, \theta = 2, \gamma_1 = 0.074, \gamma_2 = 0.147, \gamma_3 = 0.258$ $\mu = 0.5, \zeta = 1.54, \Gamma = 2$				
World real interest rate	$r = 0.935$				
Capital depreciation rate	$\delta_k = 0.714$				
Fiscal policy and pensions policy parameters ^(a)					
$\tau_w = 67.2\%, \tau_c = 13.4\%, \tau_k = 27.1\%, b = 59.6\%, b_{er} = 79.0\%,$ $\rho_{wL} = 55.4\%, \rho_{wM} = 63.1\%, \rho_{wH} = 42.7\%, \rho_{fL} = 17.2\%, \rho_{fM} = \rho_{fH} = 0\%$					
Target values for calibration					
Employment, education and growth ^(b)					
n_1	n_2	n_3	R	e	<i>Annual per capita growth</i>
51.1%	56.8%	29.3%	57.9	14.1%	1.77%
Relative wages of young workers, US ^(c)					
$w_L h_{1L} / w_H h_{1H}$		$w_M h_{1M} / w_H h_{1H}$			
0.43		0.63			

Notes: (a) Values for Belgium. For a detailed description of these policy parameters, see Section 4 in this paper;

(b) Values for Belgium, see Table 1 and Appendix A.

(c) As a proxy for the relative wage of low-ability (medium-ability) young workers, we use available data on earnings of workers of age 25-34 with below upper secondary education (with secondary education) in the US relative to earnings of workers with a tertiary degree. The data concern 2007. Data source: OECD Education at a Glance, 2009, Table A7.1a.

4. Fiscal policy and pension policy in 13 OECD countries

Tables 3 and 4 describe key characteristics of fiscal policy in 1995-2001/2004. Our proxy for the tax rate on labor income concerns the total tax wedge, for which we report the marginal rate in %. The data cover personal income taxes, employee and employer social security contributions payable on

wage earnings and payroll taxes. The OECD publishes these marginal tax data for eight family and income situations. Our data for τ_w in Table 3 are the average of all these situations. Belgium, Germany, Italy, Sweden and Finland have marginal labor tax rates above 55% or even 60%. The US have marginal labor tax rates below 40%. Capital tax rates are effective marginal corporate tax rates reported by the Institute for Fiscal Studies (their EMTR, base case). Germany and Belgium have the highest rates. In contrast to labor (and consumption), capital is taxed relatively little in the Nordic countries. As to consumption taxes, we follow Dhont and Heylen (2009) in computing them as the ratio of government indirect tax receipts (net of subsidies paid) to total domestic demand net of indirect taxes and subsidies. Our simplifying assumption is that consumption tax rates correspond to aggregate indirect tax rates. The Nordic countries stand out with the highest consumption tax rates, the US with the lowest. The utter right column in Table 3 shows the average ratio of gross government debt to GDP in the period that we study. The data range from less than 50% in Norway and the UK to more than 100% in Belgium and Italy.

Table 3 Fiscal policy: Tax rates and government debt

	tax rate on labor income (in %)	consumption tax rate (%)	tax rate on capital income (%)	Public debt (% of GDP)
Proxy for :	τ_w	τ_c	τ_k	D/Y
Austria	54.9	13.2	17.3	69.6
Belgium	67.2	13.4	27.1	111.7
France	52.9	17.1	21.7	68.9
Germany	60.4	11.1	34.4	63.1
Italy	55.2	14.7	14.9	122.1
Netherlands	52.0	12.2	24.3	68.2
Denmark	48.6	18.9	22.5	60.3
Finland	56.2	15.2	17.2	54.1
Norway	50.8	16.4	22.1	40.4
Sweden	56.0	17.9	16.1	67.2
UK	44.9	14.5	21.2	46.6
US	37.4	7.2	23.6	61.9
Canada	46.4	14.5	24.8	83.8
Overall average	52.5	14.3	22.1	70.6

Notes: Labor tax rates are data for the total tax wedge, marginal rate (OECD, Taxing Wages). Data are for 2000-2004. Earlier data are not available. For details, see Appendix A. Capital tax rates are effective marginal corporate tax rates (Institute for Fiscal Studies, their EMTR, base case; data are for 1995-2001, see also Devereux *et al.*, 2002). Consumption tax rates are from Dhont and Heylen (2009). Data are for 1995-2001.

Table 4 summarizes our data for the expenditure side of fiscal policy. A first variable is our proxy for the net non-employment benefit replacement rate b . Since in our model non-employment is a structural or equilibrium phenomenon, the data that we use concern net transfers received by structurally or long-term unemployed people. They include social assistance, family benefits and

housing benefits in the 60th month of benefit receipt. They also include unemployment insurance or unemployment assistance benefits if these benefits are still paid, i.e. if workers can be structurally unemployed for more than five years without losing benefit eligibility⁶. The data are expressed in percent of after-tax wages. In line with our approach to determine labor tax rates, we again compute the average of data reported by the OECD for a wide range of family and income cases to determine b (see Appendix A). Overall, the euro area countries and the Nordic countries pay the highest net benefits on average. Transfers to structurally non-employed people are by far the lowest in the US. A related variable is our proxy for the net early retirement benefit replacement rate b_{er} . The data are again expressed in percent of after-tax final wages. To assess the generosity of early retirement we integrate the information available via b and data for the implicit tax rate on continued work in the early retirement route as provided by Duval (2003) and Brandt *et al.* (2005). For details, see Appendix A. We observe a very generous early retirement regime in Belgium and Finland, whereas net early retirement benefits in Anglo-Saxon countries are much lower.

Table 4 Fiscal policy: net benefit replacement rates

	Non-employment benefit (net replacement rate, %)	Early retirement benefits (net replacement rate, %)
Proxy for :	b	b_{er}
Austria	56.3	71.6
Belgium	59.6	79.0
France	46.0	63.8
Germany	64.7	70.8
Italy	17.0	55.7
Netherlands	55.0	68.1
Denmark	61.9	43.2
Finland	61.3	73.8
Norway	56.9	39.9
Sweden	55.4	39.0
UK	51.1	39.4
US	30.5	18.3
Canada	44.4	27.0
Overall average	50.8	53.0

Notes: A description of all variables is given in the main text. For more details, see Appendix A. The data for net benefit replacement rates are an average for 2001-2004 (earlier data are not available).

Government spending on goods is the residual category in Equation (26). Its value is endogenously determined to satisfy the government budget constraint. The values that we obtain are in general very close to actual values of the sum of government consumption and productive government

⁶ In the period that we study, this is the case in Austria, Belgium, France, Germany, Finland, Ireland, and the UK. Workers cannot be structurally non-employed and still receive unemployment benefits in the Netherlands, Italy, Denmark, Norway, Sweden, Spain, Portugal, Switzerland and the US (OECD, 2004, www.oecd.org/els/social/workincentives, Benefits and Wages, country specific files).

spending for 2001-04 as reported in Heylen and Van de Kerckhove (2013, their Table 4). For example, for the US we obtain an endogenous G equal to 19.1% of GDP, while Heylen and Van de Kerckhove report 19.5%. For the core euro area average the numbers are 26.8% and 24.9% respectively.

Table 5 contains our data for the pension policy parameters ρ_{wa} and ρ_{fa} , and shows how they may be different for people with low, medium and high earnings potential. In this way we give more insight into the reality behind the key pension policy parameters in our model. The data have been taken or computed from OECD (2005). They include only (quasi-)mandatory pension programs⁷. In line with our specification in Equation (10), ρ_{wa} is expressed as a percentage of an individual's average lifetime net labor income, while ρ_{fa} is expressed as a percentage of average economy-wide net labor income at the time of retirement. We consider individuals at 50 percent of mean earnings as representative for the low ability group, individuals with mean earnings as representative for the medium ability group, and individuals at twice the mean earnings as representative for the high ability group. In the majority of countries individuals with mean or higher earnings only receive earnings-related pensions ($\rho_{wa} > 0, \rho_{fa} = 0$ for $a = M, H$). Among these countries, Austria and Italy pay the highest net replacement rates ($\rho_{wM} > 85\%$), Belgium and the US the lowest ($\rho_{wM} < 65\%$)⁸. Five countries also pay basic pensions to individuals with mean or higher earnings: the Netherlands, Denmark, Norway, the UK and Canada. For individuals with low earnings, the situation is somewhat the opposite. Their pension includes a significant basic (or similar) component in most countries. Unsurprisingly, the Netherlands, Denmark and the UK pay the highest 'basic' amounts⁹.

As a final remark, we bring to the attention that the straightforward way in which the OECD computes the pension replacement rates, in percent of an individual's average lifetime labor income, comes down to assuming in our model that the weights p_1, p_2 and p_3 are all equal to 1/3. For reasons of consistency we will therefore make this assumption for all individual countries when we derive our model's predictions. We are aware, however, that equal weights do not fully match practice in all countries. Some deviate from this prototype, to varying degrees¹⁰. When we compare

⁷ In most countries mandatory programs are public. For Denmark, the Netherlands and Sweden the data also include benefits from mandatory private systems. These benefits are earnings-related and included as part of ρ_{wa} . Voluntary, occupational pensions are not included in our data.

⁸ Next to the pension level, differences exist also in the precise organization of the earnings-related system. Some countries have pure defined-benefit systems (e.g. Belgium, Finland, US), others have so-called point systems (Germany) or notional-account systems (Italy, Sweden). Although these three systems can appear very different, OECD (2005) shows that they are all similar variants of earnings-related pension schemes.

⁹ As we explain in detail in Buyse *et al.* (2014, Appendix A), it should be mentioned that our proxy for ρ_{fa} also includes targeted and minimum pensions. Basic pensions pay the same amount to every retiree. Targeted plans pay a higher benefit to poorer pensioners and reduced benefits to better-off ones. Minimum pensions are similar to targeted plans. Their main aim is to prevent pensions from falling below a certain level (OECD, 2005, p. 22-23). Our main motivation to merge these three categories in our proxy for ρ_{fa} is that they are not (or even inversely) linked to earnings.

¹⁰ In Austria, Norway and France earnings-related pensions are not calculated from average lifetime income but from average income during the final working years or a number of years with the highest earnings. Ideally, one would impose different weights p_1, p_2 and p_3 . However, the pension replacement rate reported by the OECD would then no longer be reliable since it is based on the assumption of equal weights.

our model's predictions for these countries to the facts in the next section, we should take this into account. Assuming equal weights may slightly bias our predictions.

Table 5. Net pension replacement rates

Proxy for:	Net earnings-related pension replacement rate (% of average earned net labor income)			Net basic pension replacement rate (% of economy-wide average net labor income)		
	Low	Medium	High	Low	Medium	High
	ρ_{wL}	ρ_{wM}	ρ_{wH}	ρ_{fL}	ρ_{fM}	ρ_{fH}
Austria	88.7	88.9	75.9	0.0	0.0	0.0
Belgium	55.4	63.1	42.7	17.2	0.0	0.0
France	62.9	68.8	59.2	23.2	0.0	0.0
Germany	60.4	71.8	67.0	0.8	0.0	0.0
Italy	89.3	88.8	89.1	0.0	0.0	0.0
Netherlands	0.0	42.1	62.9	46.4	42.1	36.2
Denmark	15.3	11.0	10.0	43.6	43.1	42.2
Finland	82.3	78.8	78.3	4.9	0.0	0.0
Norway	36.4	43.0	38.4	26.4	22.1	20.3
Sweden	64.6	65.9	74.3	13.6	2.3	0.0
UK	0.0	5.0	8.0	43.6	42.6	41.2
US	61.4	51.0	39.0	0.0	0.0	0.0
Canada	31.6	33.9	18.1	31.5	23.2	23.3
Overall average	49.9	54.8	51.0	19.3	13.0	12.6

Notes: Pension replacement rates have been taken or computed from OECD (2005, p. 52 and part II).

The data concern 2002. For more details, see Appendix A.

5. Our model's predictions and the facts in 13 OECD countries

Can our model match the facts that we have reported in Table 1? In this section we confront our model's predictions with the true data for 1995-2007. Clearly, one should be aware of the serious limitations of such an exercise. First of all, our model is highly stylized and may (obviously) miss potential determinants of employment or education. Second, even if we compute the true data in Table 1 as averages over a longer period, these averages need not be equal to the steady state. Countries may still be moving towards their steady state. Third, this exercise only concerns the last 10 to 15 years before the financial crisis. Due to lack of data – especially with respect to marginal labor tax rates and non-employment benefits before the mid 1990s – it is impossible for us to relate changes in performance to changes in policy within countries over longer time periods. In spite of all this, if one considers the extreme variation in the predictions of existing calibrated models investigating for example the effects of fiscal policy in the literature (see Stokey and Rebelo, 1995), even a minimal test of the 'goodness of fit' of our model is informative. This information is important to assess the value of the simulations that we present in the next section, and their reliability for policy analysis. In most papers in the literature a test of the external validity of the model is missing.

Our calibration implies that our model's prediction matches the employment rates by age, the effective retirement age of older workers and participation in education in Belgium. The test of the model's validity is whether it can also match the data for the other countries, and cross-country differences. Before one uses a model for policy analysis, one would like to see for example that the model does not overestimate, nor underestimate the performance differences related to observed cross-country policy differences. Our test is tough since we impose the same preference and technology parameters, reported in the upper part of Table 2, on all countries. Only fiscal policy variables and the pension replacement rates differ. Moreover, assuming perfect competition, we disregard differences in labor and product market institutions, which some authors consider of crucial importance (e.g. Nickell *et al.*, 2005). Still, we find that the model matches the facts remarkably well for a large majority of countries, especially the facts related to employment and labor market participation among older workers. Basically, we here confirm earlier findings by e.g. Ohanian *et al.* (2008) and Dhont and Heylen (2008) that once one controls for fiscal policy differences, variation in taste for leisure or different market rigidities are not critical to explain cross-country variation in labor market performance.

Underlying our model's predictions for each country, is the assumption of a constant debt to GDP ratio at the level reported for that country in Table 3. Government spending G adjusts endogenously in Equation (26) to obtain this equilibrium debt to GDP ratio.

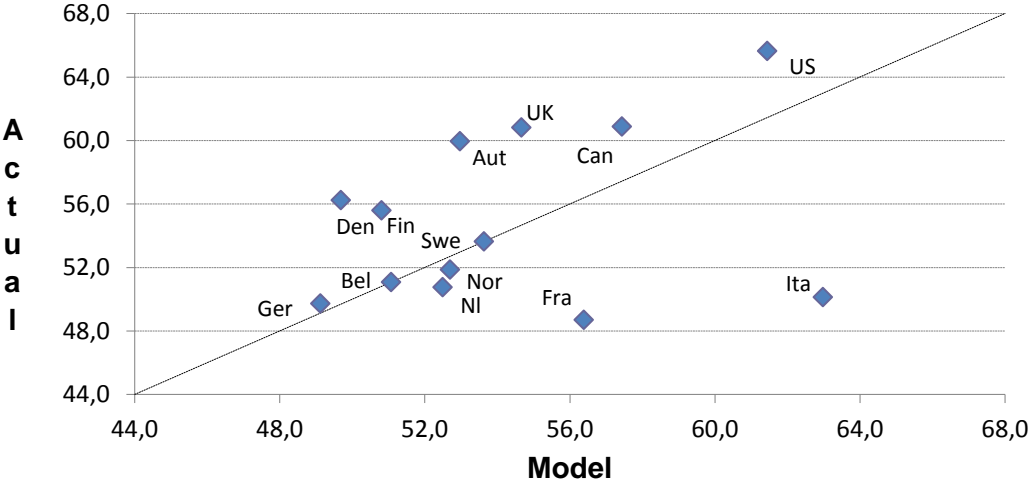
Figures 2 to 4 relate our model's predictions to actual observations for three employment rates by age (aggregated over the three ability groups). Figure 5 compares predictions and facts for the effective retirement age. We add the 45°-line to assess the absolute differences between predictions and facts, as well as the coefficient of correlation between predictions and facts. Our model performs quite well. In each age group, it correctly predicts relatively high employment rates in the US and Canada and relatively low employment in Germany and the Netherlands. For young workers it also correctly predicts relatively low employment in the Nordic countries. For older workers it has relatively high employment right in Sweden, Norway and the UK. Overall correlation between the model's predictions and the actual data varies between 0.29 in Figure 2, 0.61 in Figure 4 and even 0.92 in Figure 5. We call these results quite good, all the more so since there are good reasons for the main deviations between predictions and facts. One of these deviations concerns Italy for employment. A major element behind the deviation for this country seems to be underestimation of the fallback income position for structurally non-employed workers, especially young workers. OECD data show very low replacement rates in Italy. However, as shown by Reyneri (1994), the gap between Italy and other European countries is much smaller than it seems when family support as an alternative to unemployment benefits is taken into account.¹¹ A second large deviation can be observed for Denmark and Finland in Figure 3 and – to a lesser extent – Figure 4. The main explanation here is related to our model's substantial underestimation of participation in tertiary

¹¹ Fernández Cordon (2001) shows that in Italy young people live much longer with their parents than in other countries.

education in these two countries. As we explain when we discuss Figure 6, again there are good reasons. The point here is that underestimating education among young people, the model will also underestimate the return to work when individuals are at middle age and older. It then comes as no surprise to see low predictions for n_2 and n_3 in these two countries¹².

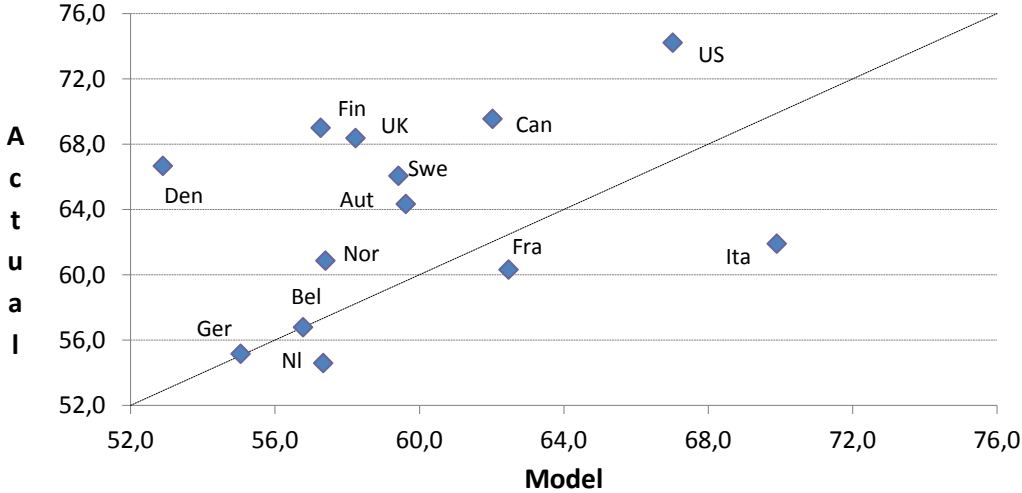
In Figure 6 we relate our model’s predictions to the facts for education. While the model predicts the facts quite well in the core euro area and the Anglo-Saxon countries, it has major difficulty explaining high participation in tertiary education in the Nordic countries, especially Finland and Denmark. A comparison with results reported by Heylen and Van de Kerckhove (2013) reveals the main reason for this: our specification of the human capital production in this paper. In addition

Figure 2. Employment rate in hours of young individuals in 13 countries, in %, 1995-2007



Note: The dotted line is the 45°-line. Correlation between actual data and the model’s predictions is 0.29. Excluding Italy correlation rises to 0.60.

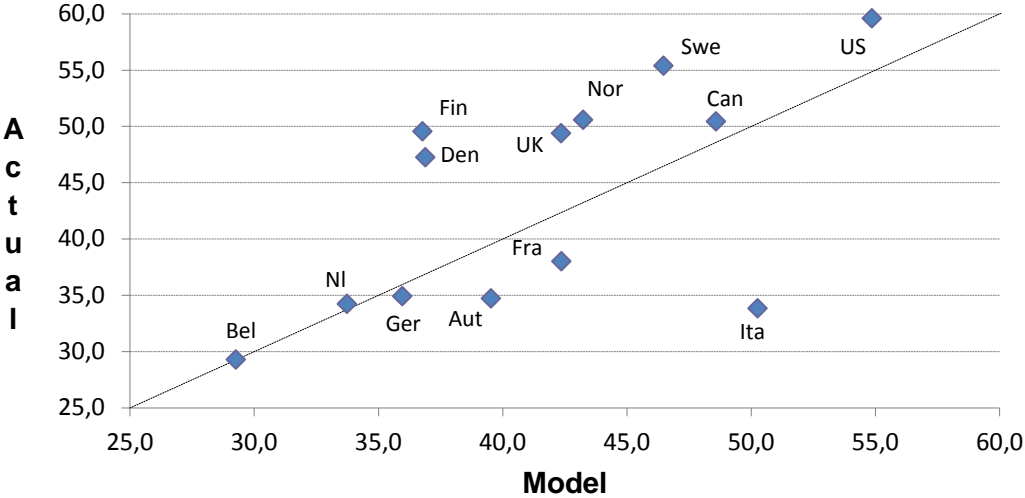
Figure 3. Employment rate in hours of middle aged individuals in 13 countries, in %, 1995-2007



Note: Correlation between actual data and the model’s predictions is 0.32. Excluding Italy correlation is 0.51.

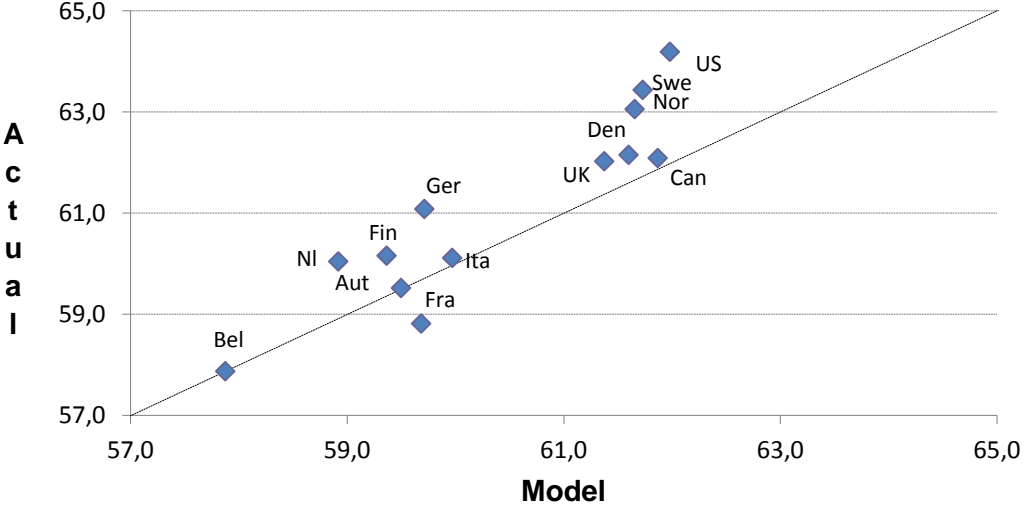
¹² Over all 13 countries the correlation between our model’s prediction errors for e and n_2 (n_3) is 0.54 (0.68).

Figure 4. Employment rate in hours of older individuals in 13 countries, in %, 1995-2007



Note: Correlation between actual data and the model’s predictions is 0.61. Excluding Italy correlation is 0.81.

Figure 5. Effective retirement age in 13 countries, in %, 1995-2006

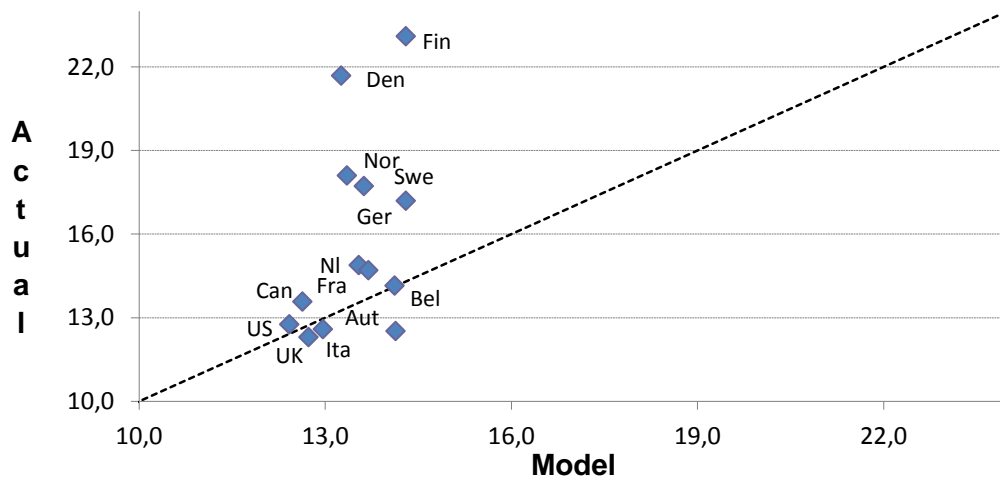


Note: The dotted line is the 45°-line. Correlation between actual data and the model’s predictions is 0.92.

to individual time in education, Heylen and Van de Kerckhove also take into account government education spending (a fixed fraction of output) and the quality of education (proxied by country-specific PISA scores) as factors in the human capital production function. Moreover, they specify a more flexible CES function for human capital production. Their model predicts much higher participation rates of around 20% on average in the Nordic countries (see their Figure 6). We have deliberately chosen a simple human capital function. Moreover, we have adopted a conservative value for the elasticity of human capital with respect to education time (σ). The reason for these choices was to exclude that our main findings in the next sections might be due to an overestimation

of the returns to education. Given the lack of hard empirical evidence and no consensus about the determinants of human capital, nor about the underlying functional form and parameter values, caution was appropriate. Narrow cross-country variation in our model's predictions for e is therefore an expected result.

Figure 6. Participation rate in tertiary education in 13 countries, in %, 1995-2006



Note: The dotted line is the 45°-line. Correlation between actual data and the model's predictions is 0.42.

6. Public pension reform

In this section we study the effects of various reforms of the pension system on employment, education (human capital), income and welfare. We report steady state aggregate effects and effects per generation and per ability group. To solve our model and to perform our policy simulations, we choose an algorithm that preserves the non-linear nature of the model. We follow the methodology basically proposed by Boucekine (1995) and implemented by Juillard (1996) in the program Dynare. We use Dynare 4.4. Throughout all our policy simulations we assume that the government maintains a constant debt to GDP ratio in each period. To reach this goal, it adjusts the consumption tax rate. For a proper understanding of timing, it will be our assumption that the economy is in steady state at time $t=-1$. Reform is announced at time $t=0$ and implemented with a delay of 1 period, i.e. at time $t=1$. Hence, reforms apply to everyone except the generation of retirees at $t=0$, since they are no longer able to adapt their behavior¹³.

Tables 6 and 7 and Figure 7 report the results. We focus on seven (permanent) reforms in key features of the pension system. Table 6 shows the steady state effects on employment rates by age ($j=1,2,3$) and by ability ($\alpha=H,M,L$), aggregate employment, the effective retirement age of older

¹³ Current retirees will therefore not experience a change in their pension replacement rate(s), nor in the rules behind the computation of their pension assessment base. Their disposable income can change, however, when the government adjusts consumption taxes to keep the ratio of public debt to GDP constant, or when the aggregate average net wage (to which the basic pension replacement rate ρ_{fa} applies) changes.

workers, average participation in education, and per capita output. Following Buyse *et al.* (2013), the benchmark from which we start, and against which all policy shocks are evaluated, is the average of six core euro area countries. The parameters describing the benchmark pension system are indicated in the upper left corner of the table and in a first note below the table. Individual earnings-related replacement rates vary in the benchmark between 59% (ρ_{wL}) and 71% (ρ_{wM}). They are applied to a pension base where each active period has equal weight ($p_{ja}=1/3$). Basic pensions take values between 6% (ρ_{fH}) and 15% (ρ_{fL}) of aggregate average net labor income. There is no particular minimum pension (MP=0). The percentage point change in the consumption tax rate to maintain a constant debt to GDP ratio is indicated at the bottom of the table.

Figure 7 shows the welfare effects of these policy changes for high and for low-ability individuals of current and future generations. The results for medium-ability individuals are in general close to those for the high-ability group. We report on the vertical axis the welfare effect on individuals of the generation born k periods after the announcement of the policy reform, where k is indicated on the horizontal axis. So, the data at $k=0$ for example concern the young in the period of the policy announcement. The data at $k=-3$ concern the retirees in that period¹⁴. Our welfare measure is the (constant) percentage change in benchmark consumption in each period of remaining life that individuals should get to attain the same lifetime utility as after the policy shock (see also King and Rebelo, 1990). To compute this percentage change we keep employment rates at the benchmark. For example, policy 1 implies a welfare gain for the current high-ability young ($k=0$) equal to 2.4% of benchmark consumption. It implies a welfare loss for the current older low-ability individuals ($k=-2$) equal to 3% of their benchmark consumption. In Table 7 we integrate the welfare effects induced by each policy reform into a single aggregate summary measure. For each individual we first compute the present discounted value of the total consumption change over life that is required in the benchmark to make him equally well off as under the policy reform. The basis of our computation is the data that we report in Figure 7. But now we also take into account differences in the length of remaining life. For young individuals the data in Figure 7 apply to four periods, whereas for retired individuals they only apply to one remaining period. Next, we impose that all those who lose under the new policy are compensated by the winners. Our summary measure is the present discounted value of the net aggregate consumption gain of all winners after having compensated the losers, in percent of initial GDP. The first row in Table 7 includes all current and four future generations of all three ability types into the computation. The second row includes only those generations that live at the moment the reform is announced.

The starting point of our discussion is policy 1, which introduces for all individuals an increase in p_3 , and a fall in p_1 , along the lines preferred by Buyse *et al.* (2013). To compute the pension base, the weight of labor income earned as an older worker rises to 2/3, the weight of labor income earned when young falls to 0. Our results confirm the important positive effects of such a reform for aggregate hours worked, for hours worked by older workers, for human capital formation by the

¹⁴ Consistent with footnote 13, these retirees are only indirectly affected by the policy change.

Table 6. Steady state effects of pension reform – Effects for a benchmark of 6 core euro area countries (Austria, Belgium, France, Germany, Italy and the Netherlands).

Initial values:	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5	Policy 6	Policy 7
$p_{1a}=1/3$ $p_{2a}=1/3$ $p_{3a}=1/3$ $MP=0$	$p_{1a}=0$ $p_{2a}=1/3$ $p_{3a}=2/3$	$MP=60\%$	$\rho_{wa}=0$ $\rho_{fa}=75\%$	$p_{1a}=0$ $p_{2a}=1/3$ $p_{3a}=2/3$ $MP=60\%$	$p_{1a}=0$ $p_{2a}=1/3$ $p_{3a}=2/3$ $\rho_{wL}=85\%$	$p_{1MH}=0$ $p_{2MH}=1/3$ $p_{3MH}=2/3$ $\rho_{wL}=85\%$	Fully Funded
Effect ^(a) :							
Δn_1	-4.73	-0.43	-0.86	-4.55	-4.87	-4.11	3.82
Δn_2	0.06	-1.03	-3.32	-0.90	0.31	0.27	2.17
Δn_3	7.30	-3.73	-11.31	1.15	8.66	5.62	2.38
ΔR ^(c)	0.89	-0.51	-1.52	0.08	1.05	0.70	0.40
Δe	2.53	0.00	-0.78	2.53	2.53	2.53	-1.19
Δn ^(a, b)	0.53	-1.61	-4.80	-1.55	0.97	0.33	2.79
$\Delta\%$ total hours ^(d)	1.00	-3.03	-9.06	-2.92	1.83	0.63	5.27
Δn_H	-0.04	0.00	-3.91	-0.04	-0.04	-0.04	2.74
Δn_M	-0.12	0.00	-4.48	-0.12	-0.12	-0.12	2.64
Δn_L	1.75	-4.82	-6.01	-4.48	3.07	1.16	3.00
$\Delta\%$ per capita output ^(d)	4.41	-1.53	-10.7	2.34	4.83	4.17	3.95
$\Delta\tau_c$ ^(e)	-2.71	1.35	14.8	-1.25	-2.25	-1.31	-5.75

Notes: Initial policy values: $\rho_{wL}=59.4\%$, $\rho_{wM}=70.6\%$, $\rho_{wH}=66.1\%$, $\rho_{fL}=14.6\%$, $\rho_{fM}=7.0\%$, $\rho_{fH}=6.0\%$. Initial steady state (benchmark): $n_1 = 55.1\%$, $n_2 = 61.3\%$, $n_3 = 39.9\%$, $R = 59.4$, $e = 13.7\%$, $n = 53.0\%$, $n_H = 52.1\%$, $n_M = 52.2\%$, $n_L = 54.7\%$, $\tau_c = 13.6\%$.

(a) difference in percentage points between the new steady state and the benchmark, except for total hours worked, per capita output and R .

(b) change in (weighted) aggregate employment rate in hours, change in percentage points.

(c) change in optimal effective retirement age, in years.

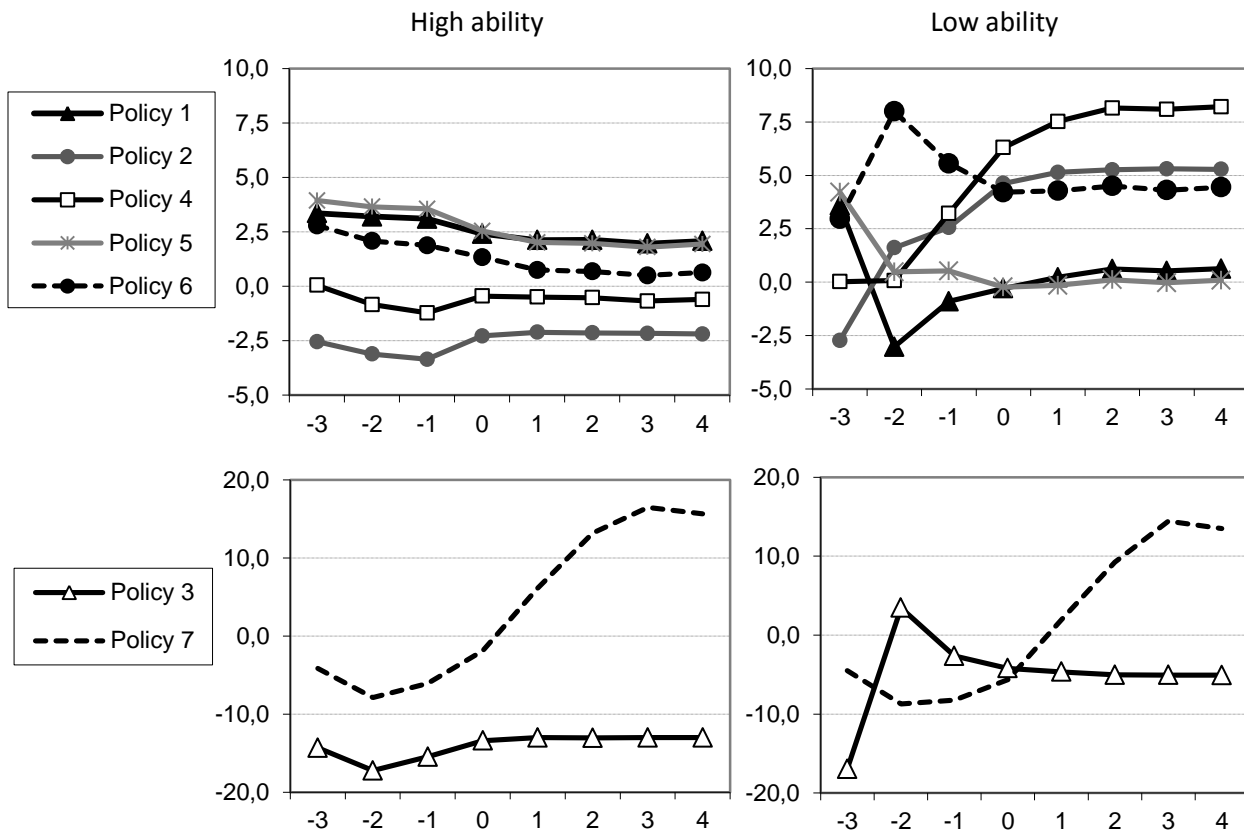
(d) difference in percent between new steady state and the benchmark.

(e) change in consumption tax rate in percentage points to keep the ratio of debt to GDP constant.

young, and for per capita output. The higher (lower) marginal utility from work when older (young) makes it interesting to shift work from the first period of active life to the third, and to postpone effective retirement (n_3 and R rise, n_1 falls). The positive effect that we observe on R and n_3 is fully in line with earlier arguments by Sheshinski (1978) and Gruber and Wise (2002), among others. Jaag *et al.* (2010) also predict a shift from n_1 to n_3 when p_1 falls and p_3 rises. Unlike in Jaag *et al.*, however, the role of endogenous education in our model strongly qualifies the fall in young workers' labor supply. As is clear from Table 6, participation in tertiary education (e) increases. Young individuals – at least those of high and medium ability – are encouraged to study because the lifetime rate of return to building human capital rises. This follows first from the reduction of the opportunity cost of studying when young, second from the perspective of working longer, and third from the greater importance of effective human capital when old in the calculation of the pension. Extra schooling reinforces incentives to work at older age. Individuals of low innate ability do not have the option to study and to enjoy higher human capital. These individuals can only respond to the new policy by working more and longer ($\Delta n_L=1.75$). In the end they are the only ones to work more over their lifetime. The individuals with medium or high ability do not ($\Delta n_H, \Delta n_M \approx 0$). As a final positive effect of policy 1 we observe a significant improvement in the overall government budget.

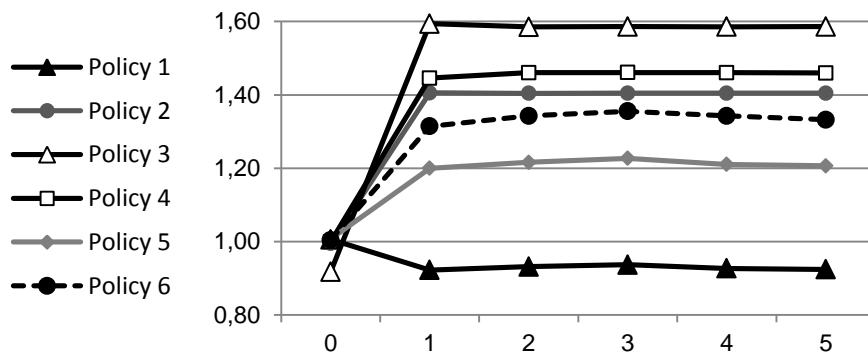
The bottom row of Table 6 reveals that the government will be able to maintain a constant public debt to GDP ratio with a reduced consumption tax rate (-2.71%-points).

Figure 7. Welfare effects for individuals belonging to current and future generations after pension reform



Note: The vertical axis indicates the welfare effect for individuals belonging to the generation born k periods after the announcement of permanent pension reform. The horizontal axis indicates k . Negative numbers for k point at generations born before the (announcement of the) reform.

Figure 8. Pension level (relative to the benchmark) of low-ability retirees at time t (where $t=0$ is when the policy reform is announced, and $t=1$ when it is implemented)



Note: Policy 7 is not included. This policy implies a gradual reduction of public pensions to zero.

Table 7. Net welfare effect after compensating welfare transfers (expressed as % of initial GDP)

Included generations	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5	Policy 6	Policy 7
All current + 4 future	2.06	-1.53	-10.9	0.23	2.35	2.00	-0.33
All current	1.51	-1.27	-7.70	-0.04	1.87	1.60	-3.47

Note: for a description of the computation of these data, see main text.

A quick comparison with the other policies in Table 6, to be discussed immediately, reveals that most of these policies are less effective than policy 1 when it comes to promoting employment of older workers, investment in human capital and per capita output. A major disadvantage of policy 1, however, is the welfare loss that it imposes on all current generations of low-ability individuals (Figure 7, upper panel, RHS). These individuals work more, but can hardly consume more. Even if policy 1 may be part of the solution to the overall challenge of employment and productivity in today's economies, and in that sense contribute to safeguard the welfare state in the future, it may also worsen conditions for many lower-ability individuals. Moreover, it may offer no solution to the problem of old-age poverty faced by many. Figure 8 shows an important fall of about 7% relative to the benchmark in the pension level of all generations of low-ability individuals to come. These observations make it also politically difficult to impose such a policy.

Policies 2 and 3 tackle the problem of low pensions and welfare for low-ability individuals. Policy 2 maintains all benchmark replacement rates, but also introduces a minimum pension. Individuals are sure of a pension equal to at least 60% of the average net labor income per worker in the economy. In practice the latter implies a strong increase in the pension level for the low-ability group (see also Figure 8), but no ex-ante change for the other two groups. Their optimal behavior, given their human capital endowment and all policy variables, implies a pension that is above 60% of the average net wage from the beginning. We remind that none of the policy reforms that we discuss apply to the retired at the moment of the announcement of the reform, so they are not eligible to the minimum pension. As shown by Figure 7, all low-ability individuals experience welfare increases up to about 5% under policy 2. For the welfare of all other individuals, however, these policies have negative effects. A key element is the drastic drop in the employment rate among low-ability individuals. The perspective of a minimum pension weakens the incentives for them to work. In Table 6 we observe a drop in n_L of about 4.8%-points. The implied fall in aggregate employment and its negative effects on the government's budget, force the latter to raise consumption tax rates for all. Furthermore, medium and higher ability individuals can also expect a fall in their wage per unit of effective labor due to the reduction of low-ability labor supply¹⁵.

Policy 3 imposes a shift from own-earnings related pensions to 'basic' pensions on all individuals. Every retiree gets a basic pension equal to 75% of average net labor income per worker in the economy. In our model ρ_w goes to zero for all ability groups, ρ_f becomes 0.75. This policy

¹⁵ As a narrow alternative to policy 2, we also investigated the introduction of a minimum pension combined with an abolishment of all basic pensions. All effects were very similar. Only the required increase in the consumption tax rate was smaller, since the government could save money from ρ_{fa} going to 0.

basically goes one step further than policy 2. It breaks the relationship between the pension and an individual's human capital and labor supply also for the high and medium-ability groups. The fall in the return to studying and to working also for these groups is at the basis of an overall and strong fall in employment, education time and per capita output (see also Sommacal, 2006). Figure 7 reveals negative welfare effects almost across the board, especially for higher ability individuals and all future generations. Only current older low-ability individuals gain. They benefit most from higher pensions. Due to falling per capita output, this gain will not persist for the future low-ability generations, however. As a result, policy 3 shows among the worst net aggregate welfare effects in Table 7.

Policies 4, 5 and 6 are alternative attempts to combine the efficiency of policy 1 with the objective to raise everyone's welfare and to reduce the risk of old-age poverty for low-ability individuals. Policy 4 extends policy 1 with a minimum pension equal to 60% of the average net wage, like in policy 2. This policy is most beneficial for the welfare of all current young and future low-ability individuals (Figure 7). They enjoy both an immediate increase in their pension, for which they have to work less, and the benefits from increased human capital formation by the high and medium-ability groups. The latter immediately contributes to higher wages per person, also for the lower ability individuals. Like policy 2, however, policy 4 also imposes welfare losses on the current generations of high (and medium-ability) individuals, which reduces its chances politically. Net aggregate effects in Table 7 are still slightly negative for those generations alive when the policy change is announced.

Policy 5 tackles the problem of welfare losses and low income at old-age for the low-ability group by significantly raising their individual earnings-related pension replacement rate to 85% ($\Delta\rho_{wL}=25.6\%$ -points). This policy combines the efficiency gains from policy 1 with strong incentives for the low-ability group to work more and longer. In contrast to the disincentives induced by basic or minimum pensions, policy 5 raises the return to work since it yields more future pension. Among all the reforms that we discuss in Table 6 and that maintain the PAYG system, not one has more favorable effects on the aggregate employment rate ($\Delta n=0.97$), on the employment rate of low-ability individuals ($\Delta n_L=3.07$) and on the employment rate of older workers ($\Delta n_3=8.66$) than policy 5. Higher pensions can as a result be paid without the need for the government to raise consumption taxes. Given the strong rise in output and employment, τ_c can even be reduced by 2.25%-points. Compared to policy 1, welfare effects are better for all low-ability generations alive at the time of announcement of the policy reform, without hurting the medium and high-ability groups. Policy 5 induces the best net aggregate welfare effects in Table 7.

Policy 6 reconsiders the basic choice made in policy 1 to raise the weight of labor income earned as an older worker in the computation of the pension assessment base, and to reduce the weight of labor income earned when young. One of the main advantages of this choice is that it promotes education and human capital formation. Given that low-ability individuals will never continue education at the tertiary level, however, one may question this change in weights for them. Policy 6 therefore maintains the much higher individual earnings-related replacement rate for the low-ability group ($\rho_{wL}=85\%$), but combines this with equal weights $p_j=1/3$ for this group. The shift to

$p_1=0$, $p_2=1/3$ and $p_3=2/3$ only applies to medium and high-ability individuals. The employment and output effects of policy 6 are a little less good than those of policy 1. So are the welfare effects for the individuals with high and medium ability. However, for the low-ability individuals, who work the highest fraction of their time while they are young, maintaining p_1 at $1/3$ in policy 6 implies a further increase in their pension benefit and in their welfare compared to policy 5. All in all, the welfare effects from policy 6 are among the best for the low-ability individuals, with only small cost imposed on the others. In Table 7 net aggregate welfare effects from policy 6 are comparable to those from policy 1 and only a little lower than those from policy 5. Policy makers with no aversion to inequality may therefore prefer policy 5. As soon as one attaches greater weight to the evolution of the welfare of low-ability (low income) groups, however, policy 6 may come out as preferable.

Policy 7 is a gradual shift from the PAYG system in the benchmark to a system with full private capital funding. This policy completely abolishes old-age pension benefits (ρ_{wa} , ρ_{fa}). For the government it implies a drastic cut in pension expenditures. We assume that this drop in expenditures feeds through into lower social security contributions for all workers such that, ex ante, the decline in total labor tax receipts in % of GDP is exactly the same as the drop in pension expenditures.¹⁶ We observe in Table 6 that this transition to a fully-funded private pension scheme is most beneficial for employment. The new steady state shows higher hours worked among all ability groups and all age groups. The aggregate employment rate n rises by about 2.8%-points. The rise in employment is the strongest among older workers and among individuals with lower innate ability. Aggregate per capita output also rises strongly (+ almost 4%) and the overall government balance improves. To maintain a constant debt to GDP ratio, the government can reduce the consumption tax rate by 5.75%-points. Considering existing literature (e.g. Ludwig *et al.*, 2012; Fisher and Keuschnigg, 2010), these positive effects come as no surprise. The same holds, however, for a number of negative effects from moving to a fully-funded private system. First of all, the steady state time allocated to education falls, confirming the theoretical expectations of Kemnitz and Wigger (2000), Buyse *et al.* (2013) and Kindermann (2015), among others. Next, Figure 7 reveals a strong intertemporal trade-off in the welfare effects from moving to a fully-funded system. Future generations gain, but current, transitional generations experience large welfare losses¹⁷. This result is also well known in the

¹⁶ In particular, the gradual decline in ρ_{wa} and ρ_{fa} is announced at time $t=0$ and implemented as follows. Pension benefits are not reduced for retirees at the moment of policy announcement ($t=0$), since retirees are not able to react to a pension reduction. In $t=1$ and $t=2$ the replacement rates are respectively reduced to $2/3$ and $1/3$ of their initial rates. From $t=3$ onwards, ρ_{wa} and ρ_{fa} are zero. At each moment, overall labor tax rates are reduced to ex ante compensate for the decline in pension expenditures. Like Buyse *et al.* (2013, policy 6b in their Table 5) we assume that net non-employment benefits remain unchanged when labor taxes are reduced.

¹⁷ The explanation for the welfare loss of current generations in our model is as follows. The announcement of the transition to a fully-funded system, and the perspective of a gradual fall in labor taxes during periods 1, 2 and 3, as described in footnote 16, makes individuals shift hours worked to the future. During transition the young will study more, but total effective labor falls. Since this reduces the marginal productivity of physical capital, it will also discourage investment. Capital flows out. The economy experiences a strong drop in aggregate output (and tax revenue), which will force the government to raise consumption taxes. In later periods the economy enjoys the benefits from higher employment, physical capital inflow and lower taxes.

literature. It applies to all three ability groups. Individuals with low ability in the transitional generations are hit the hardest though. The very substantial welfare gains for future generations that movement to a fully-funded system may bring about, also when compared to the future gains from e.g. policies 1 or 6, cannot wipe out these welfare losses in the shorter run. In Table 7, when we take into account welfare effects on all current and four future generations, a slight negative net result remains.

7. The model with endogenous growth

In this section, we extend the model presented above with an endogenous growth mechanism driven by education. Empirical research has shown that education is indeed one of the most important determinants of economic growth in the long run (see e.g. Hanushek and Woessmann, 2012). More specifically, we introduce the assumption that education generates a positive externality in the sense of Azariadis and Drazen (1990). Each young generation inherits a fraction of the average level of human capital of the middle aged generation. The higher an individual's ability, the larger the fraction he inherits. Equation (28) reflects this assumption. It replaces Equation (11). As a complement, technology is from now on assumed constant, i.e. A does not grow anymore.

$$h_{1a}^t = \varepsilon_a \pi (h_{2H}^{t-1} + h_{2M}^{t-1} + h_{2L}^{t-1})/3 \quad \forall a = H, M, L \quad (28)$$

$$0 < \pi, 0 < \varepsilon_L < \varepsilon_M < \varepsilon_H = 1 \quad (29)$$

The value of π is to be calibrated. In line with the procedure described in Section 3, we also calibrate it to Belgium. The key target value in the calibration process is average per capita economic growth. We obtain an inheritance parameter π equal to 0.87. Individuals with medium and lower ability inherit less ($\varepsilon_L < \varepsilon_M < 1$). The values for ε_L and ε_M are unchanged. So are the elasticity of human capital with respect to education time (σ), which we keep at 0.30, and most of the other calibrated parameters¹⁸. The model generates endogenous growth, with the growth rate rising in the fraction of time that young people (of high and medium ability) allocate to education (e), the human capital inheritance parameter (π) and the fractions ε_L and ε_M . More exactly, we derive in Appendix D that:

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) = \ln\left(\pi \frac{(1+\phi(e_{1H}^{t-1})^\sigma)^{\varepsilon_M} (1+\phi(e_{1M}^{t-1})^\sigma)^{\varepsilon_L}}{3}\right) \quad (30)$$

Does allowing for an endogenous growth mechanism affect the main conclusions that we have drawn in the previous section? Tables 8 and 9 and Figure 9 bring the answer. All in all, it will be clear that our main results are robust to the assumptions made about human capital formation and growth. Pension reform according to policy 1 again brings positive effects on aggregate employment, human capital, output (growth) and aggregate welfare. For the welfare of current generations of

¹⁸ Appendix C reports all calibrated parameters.

Table 8. Steady state effects of pension reform when growth is endogenous – Effects for a benchmark of 6 core euro area countries.

Initial values:	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5	Policy 6	Policy 7
$p_{1a}=1/3$	$p_{1a}=0$	$MP=60\%$	$\rho_{wa}=0$	$p_{1a}=0$	$p_{1a}=0$	$p_{1MH}=0$	Fully
$p_{2a}=1/3$	$p_{2a}=1/3$		$\rho_{fa}=75\%$	$p_{2a}=1/3$	$p_{2a}=1/3$	$p_{2MH}=1/3$	Funded
$p_{3a}=1/3$	$p_{3a}=2/3$			$p_{3a}=2/3$	$p_{3a}=2/3$	$p_{3MH}=2/3$	
$MP=0$				$MP=60\%$	$\rho_{wL}=85\%$	$\rho_{wL}=85\%$	
Effect ^(a) :							
Δn_1	-4.56	-0.59	-0.90	-4.70	-4.67	-3.97	3.72
Δn_2	0.36	-1.35	-3.40	-1.17	0.68	0.51	2.09
Δn_3	7.78	-4.34	-11.43	0.46	9.24	5.89	2.26
ΔR ^(c)	0.94	-0.58	-1.54	0.00	1.12	0.73	0.39
Δe	2.48	0.00	-0.76	2.48	2.48	2.48	-1.14
Δn ^(a, b)	0.84	-1.96	-4.88	-1.90	1.34	0.55	2.70
$\Delta\%$ total hours ^(d)	1.58	-3.70	-9.22	-3.58	2.53	1.03	5.09
Δn_H ^(a)	0.21	0.00	-3.87	0.21	0.21	0.21	2.61
Δn_M	0.13	0.00	-4.43	0.13	0.13	0.13	2.50
Δn_L	2.17	-5.88	-6.34	-6.03	3.69	1.30	2.98
Δ annual per capita growth rate ^(a)	0.15	0.00	-0.05	0.15	0.15	0.15	-0.08
$\Delta\tau_c$ ^(e)	-2.18	2.64	11.72	1.55	-1.84	-0.58	-5.82

Notes: Initial policy values: $\rho_{wL}=59.4\%$, $\rho_{wM}=70.6\%$, $\rho_{wH}=66.1\%$, $\rho_{fL}=14.6\%$, $\rho_{fM}=7.0\%$, $\rho_{fH}=6.0\%$. Initial steady state (benchmark): $n_1 = 55.1\%$, $n_2 = 61.2\%$, $n_3 = 39.9\%$, $R = 59.3$, $e = 13.8\%$, $n = 52.9\%$, $n_H = 52.7\%$, $n_M = 52.8\%$, $n_L = 53.3\%$, $\tau_c = 13.6\%$.

(a) difference in percentage points between the new steady state and the benchmark, except for total hours worked and R .

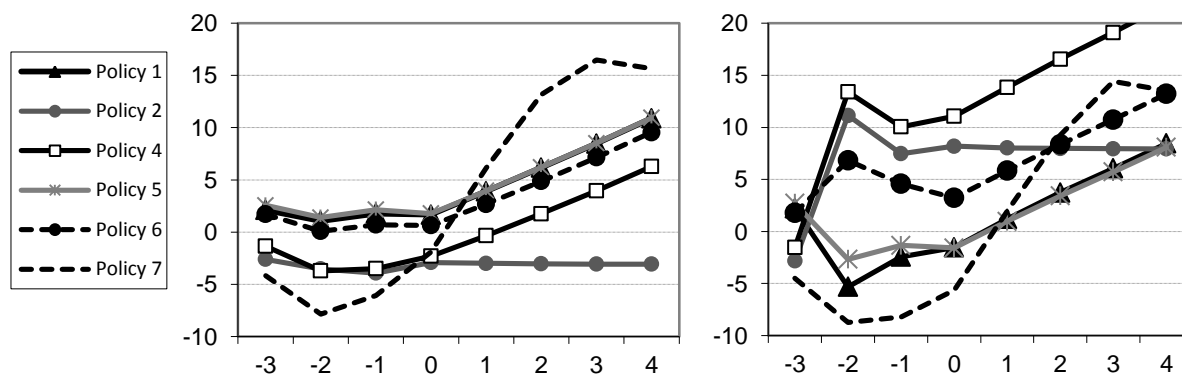
(b) change in (weighted) aggregate employment rate in hours, change in percentage points.

(c) change in optimal effective retirement age, in years.

(d) difference in percent between new steady state and the benchmark.

(e) change in consumption tax rate in percentage points to keep the ratio of debt to GDP constant.

Figure 9. Welfare effects for individuals belonging to current and future generations after pension reform when growth is endogenous.



Note: The vertical axis indicates the welfare effect for individuals belonging to the generation born k periods after the announcement of permanent pension reform. The horizontal axis indicates k . Negative numbers for k point at generations born before the (announcement of the) reform.

Table 9. Net welfare effect after compensating welfare transfers (expressed as % of initial GDP) when growth is endogenous

Included generations	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5	Policy 6	Policy 7
All current + 4 future	2.40	-1.38	-9.72	0.78	2.67	2.51	-0.34
All current	0.65	-1.03	-6.57	-0.52	0.92	0.84	-3.49

Note: for a description of the computation of these data, see main text.

low-ability individuals, however, this policy is again bad. Considering net aggregate welfare effects in Table 9, the most efficient way to tackle this disadvantage would be policy 5. If one assigns a higher weight to the welfare of these low-ability individuals, however, policy 6 may become preferable. As we can see in Figure 9, policy 6 brings much better welfare effects for these individuals than policy 5. Finally, moving to a fully-funded system again offers the best perspectives if the objective is to raise employment. When growth is endogenous and driven by human capital accumulation, however, a fully-funded private system no longer offers the best perspectives for the welfare of future generations. In the more distant future ($k > 4$) the aggregate welfare bonus of an intelligent PAYG system relative to a fully-funded system only increases. A key element is that a FF system lacks the incentives to promote human capital formation and growth inherent in an earnings related PAYG system, and even more so in our policies 1, 5 and 6. Due to the intergenerational transfer of human capital, investment in education by today's generations of individuals of high and medium ability is also beneficial for future low-ability individuals.

8. Conclusion

Growing concern for the long-run financial viability of public pension systems has put pension reform high on the agenda of policy makers and researchers. To face the challenge, there now seems to be general agreement on the need for higher employment, especially among older individuals, and higher productivity and growth. Another concern is to provide adequate retirement benefits for everyone, so as to avoid old-age poverty.

In this paper we study the effects of pension reform in a four-period OLG model for an open economy where hours worked by young, middle aged and older individuals, education and human capital, the retirement decision of older workers, per capita output, and welfare are all endogenous. As our main contribution we distinguish within each generation individuals with high, medium or low ability. Differences in ability show up in both a different initial level of human capital and a different learning ability. The extension allows us to investigate also the effects of pension reform on the income and welfare levels of different ability groups. Our specification of pension benefits includes both own-earnings related and flat-rate or basic components. The weight of each component may differ for individuals with different abilities. Next to the pension system, we introduce a rich fiscal policy block to assess the effects of pension reform on the public budget. The government sets tax rates on labor, capital and consumption. It spends its revenue on (non-productive) goods, 'non-

employment' benefits (including early retirement benefits), old-age pensions, and interest payments on outstanding debt.

We check the validity of our model and our calibration by simulating the model for 13 OECD countries and comparing its results with the true data. Imposing common technology and preference parameters but country-specific policy parameters, we find a convincing match between the predictions of our model and the main facts.

Our simulation results prefer an 'intelligent' PAYG system above a fully-funded private system. This PAYG system conditions pension benefits on past individual labor income, with a high weight on labor income earned when older and a low weight on labor income earned when young. This system generates the best effects on human capital, productivity and the overall welfare of current and future generations. It also has positive effects on the government budget. Recognizing realistic differences in initial human capital and learning ability across people, however, we find that uncorrected this PAYG system also implies significant welfare losses for current low-ability generations and rising inequality. Low-ability individuals cannot accumulate more human capital when young. Moreover, if the weight in the pension assessment base of earned labor income when young falls, these individuals will see their future pension fall. The incentives for them are then to work more and longer (at low wages).

Analyzing alternative responses to tackle the problem of rising inequality and welfare losses for low-ability individuals, while maintaining the aggregate efficiency gains of an intelligent PAYG system, we can conclude as follows. Complementing or replacing the PAYG system by basic and/or minimum pension components would reduce inequality, but it would also be negative for aggregate employment and aggregate welfare. Strong and direct negative effects on labor supply of low-ability individuals and higher pension expenditures would induce the government to raise taxes. Much better is to maintain the tight link between individual labor income and the pension also for low-ability individuals, but to strongly raise their replacement rate. An additional correction improving the welfare of low-ability individuals would be to maintain for these individuals equal weights on past labor income in the pension assessment base.

Acknowledgements

We thank David de la Croix, André Decoster, Alexander Ludwig, Dirk Van de gaer, Glenn Rayp, Grégory Ponthière and two anonymous referees for most valuable suggestions and comments. We also benefited from discussions during seminars at the Université catholique de Louvain (IRES, March 2011) and the University of Cologne (May 2012), and from comments at the Netspar International Pension Workshop (Paris, June 2012), the 11th Journées Louis-André Gérard-Varet - Conference in Public Economics (Marseille, June 2012), and the 2012 Annual Conference of the European Society for Population Economics (Bern, June 2012). We acknowledge support from the Flemish government (Steunpunt Fiscaliteit en Begroting - Vlaanderen). Tim Buyse acknowledges financial support from the Research Foundation – Flanders (FWO), Brussels, Belgium. Any remaining errors are ours.

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Appendix A: Construction of data and data sources

In this appendix we provide more detail on the construction of some of our performance variables and policy variables.

Employment rate in hours (in one of three age groups, 1995-2007)

Definition: total actual hours worked by individuals in the age group / potential hours worked.

Actual hours worked = total employment in persons x average hours worked per week x average number of weeks worked per year

Potential hours = total population in the age group x 2080 (where 2080 = 52 weeks per year x 40 hours per week)

Data sources:

* Total employment and total population by age group: OECD Stat, Labour Force Statistics by Sex and Age. Data are available for many age groups, among which 20-24, 25-34, 35-44, 45-49, 50-54, 55-64. We constructed the data for our three age groups as weighted averages.

* Average hours worked per week: OECD Stat, Labour Force Statistics, Average usual weekly hours worked on the main job. These data are available only for age groups 15-24, 25-54, 55-64. We use the OECD data for the age group 15-24 as a proxy for our age subgroup 20-24, the OECD data for the age group 25-54 as a proxy for our age (sub)groups 25-34, 35-49 and 50-54.

* Average number of weeks worked per year: Due to lack of further detail, we use the same data for each age group. The average number of weeks worked per year has been approximated by dividing average annual hours actually worked per worker (total employment) by average usual weekly hours worked on the main job by all workers (total employment). Data source: OECD Stat, Labour Force Statistics, Hours worked.

Education rate of the young (age group 20-34, 1995-2006)

Definition: total hours studied by individuals of age 20-34 / potential hours studied

As a proxy we have computed the ratio: $(fts_{20-34} + 0.5 pts_{20-24} + 0.25 pts_{25-34}) / pop_{20-34}$

with: *fts* the number of full-time students in the age group 20-34

pts the number of part-time students in the age groups 20-24 and 25-34.

pop total population of age 20-34

Full-time students are assumed to spend all their time studying. For part-time students of age 20-24 we make the assumption (for all countries) that they spend 50% of their time studying, part-time students of age 25-34 are assumed to spend 25% of their time studying. Due to the limited number of part-time students, these specific weights matter very little.

Data sources:

* Full-time students in age groups 20-24, 25-29, 30-34: OECD Stat, Education and Training, Students enrolled by age (all levels of education, all educational programmes, full-time)

* Part-time students in age groups 20-24, 25-29, 30-34: OECD Stat, Education and Training, Students enrolled by age (all levels of education, all educational programmes). We subtracted the data for full-time students from those for 'full-time and part-time students'.

For those countries where data for specific years are missing, we computed period averages on the basis of all available annual data.

Average effective retirement age (1995-2006)

Definition: Average age of all persons (being 40 or older) withdrawing from the labor force in a given period.

Data source: OECD, Ageing and Employment Policies – Statistics on effective age of retirement.

Annual real potential per capita GDP growth rate (aggregate, 1995-2007)

Definition: Average annual growth rate of real potential GDP per person of working age

Data sources:

* real potential GDP: OECD Statistical Compendium, Economic Outlook, supply block, series GDPVTR.

* population at working age: OECD Statistical Compendium, Economic Outlook, labour markets, series POPT.

Tax rate on labor income (τ_w)

Definition: Total tax wedge, marginal tax rate in % of gross wage earnings. The data cover personal income taxes and social security contributions paid by employees on their wage earnings as well as social security contributions and payroll taxes paid by employers.

Data source: OECD, Statistical Compendium, Financial and Fiscal Affairs, Taxing Wages, Comparative tax rates and benefits (new definition).

The OECD publishes marginal labor tax rates for several family and income situations: single persons at 67%, 100% and 167% of average earnings (no children), single persons at 67% of average earnings (two children), one-earner married couples at 100% of average earnings (two children), two-earner married couples, one at 100% of average earnings and the other at 33 % (no children, 2 children), two-earner married couples, one at 100% of average earnings and the other at 67 % (2 children). Our data in Table 3 are the averages of these eight cases. Data for 2000-04.

Government debt (D_t)

Definition: General government gross financial liabilities.

Data source: OECD Statistical Compendium, Economic Outlook, N° 89, Government Accounts.

Net benefit replacement rate when young, middle aged and older before early retirement (b)

Definition: The data concern net transfers received by long-term unemployed people and include social assistance, family benefits and housing benefits in the 60th month of benefit receipt. They also include unemployment insurance or unemployment assistance benefits if these benefits are still paid, i.e. if workers can be structurally unemployed for more than five years without losing benefit eligibility. The data are expressed in % of after-tax wages. The OECD provides net replacement rates for six family situations and three earnings levels. Our data in Table 4 are the averages of these 18 cases. Data for 2001-2004.

Data source: OECD, Tax-Benefit Models, www.oecd.org/els/social/workincentives

Data adjustment: Original OECD data for Norway include the so-called “waiting benefit” (ventestønad), which a person could get after running out of unemployment benefits. Given the conditional nature of these “waiting benefits”, they do not match our definition of benefits paid to structurally non-employed individuals. We have therefore deducted them from the OECD data, which led to a reduction of net replacement rates by about 19 percentage points. For example, recipients should demonstrate high regional mobility and willingness to take a job anywhere in Norway. The “waiting benefit” was terminated in 2008. We thank Tatiana Gordine at the OECD for clarifying this issue with us.

Net early retirement replacement rates (b_{er})

To calculate our proxy for b_{er} we have focused on the possibility for older workers in some countries to leave the labor market along fairly generous early retirement routes. Duval (2003) and Brandt *et al.* (2005) provide data for the so-called implicit tax rate on continued work for five more years in the early retirement route at age 55 and age 60. The idea is as follows. If an individual stops working (instead of continuing for five more years), he receives a benefit (early retirement, disability...) and no longer pays contributions for his future pension. A potential disadvantage is that he may receive a lower pension later, since he contributed less during active life. Duval (2003) calculated the difference between the present value of the gains and the costs of early retirement, in percent of gross earnings before retirement. We use his data as a proxy for the gross benefit replacement rate for older workers in the early retirement route. To compute the net benefit replacement rate, we assume the same tax rate on early retirement benefits as on unemployment benefits. We call this net benefit replacement rate r_{er} . However, these implicit tax rates are only very rough estimates of the real incentive to retire embedded in early retirement schemes and are subject to important caveats (Duval, 2003, p. 15). The available implicit tax rates take into account neither the strictness of eligibility criteria nor the presence of alternative social transfer programs that may de facto be used as early retirement devices. Our assumption will be that a realistic replacement rate for the early retirement route (b_{er}) will be a weighted average of r_{er} and b , where we take the latter as a proxy for the replacement rate in alternative social transfer programs. If $r_{er} > b$, older workers will aim for the official early retirement route, but they may not all meet eligibility criteria and have to fall back on alternative programs. If $r_{er} < b$, workers will aim for the alternative, but again they may not be eligible. We propose that $b_{er} = \xi b + (1-\xi)r_{er}$. Underlying the data in Table 4 is the assumption that $\xi=0.5$. Correlation between b_{er} and r_{er} lies around 0.92. Cross-country differences roughly remain intact. Our results in the main text do not depend in any serious way on this assumption for ξ .

Data Source: OECD, Tax-Benefit Models, www.oecd.org/els/social/workincentives, Duval (2003), Brandt et al. (2005).

Net pension replacement rates (ρ_{wa} and ρ_{fa} for $a=L,M,H$)

OECD (2005, p. 52) presents net pension replacement rates for individuals at various multiples of average individual earnings in the economy. We consider the data for individuals at 50% of average earnings as representative for the low ability group, individuals with average earnings as

representative for the medium ability group, and individuals with twice average earnings as representative for the high ability group. Country studies in OECD (2005, part II) show the composition (sources) of this net replacement rate. This composition may be different for individuals with different income levels. Our proxy for ρ_{wa} includes all earnings-related pensions and mandatory occupational pensions when they depend on wages or hours worked. Our proxy for ρ_{fa} includes basic pensions, minimum pensions, targeted pensions, and old-age social assistance benefits, i.e. all categories that are not (or even inversely) related to individual earnings.

Since in our model ρ_{fa} is a percentage of the average net wage in the economy (Equation 10), whereas the above described OECD data are in percent of an individual's net wage, we multiply the OECD data with the ratio of the replacement in percent of average earnings to the replacement rate in percent of individual earnings to obtain our ρ_{fa} . This ratio can be derived from the 'pension modelling' tables in the individual country studies, at various multiples of average earnings.

Appendix B: Details on the calibration procedure to determine η_a (with $a = L, M, H$)

Given the data for US relative wages in Table 2, we have for the low-ability group that:

$$\frac{w_{L,t}h_{1L}^t}{w_{H,t}h_{1H}^t} = \frac{w_{L,t}\varepsilon_L h_{1H}^t}{w_{H,t}h_{1H}^t} = \frac{w_{L,t}}{w_{H,t}} \varepsilon_L = 0.43.$$

We also know from Equation (25) that $\frac{w_{L,t}}{w_{H,t}} = \frac{\eta_L}{\eta_H} \left(\frac{H_{H,t}}{H_{L,t}} \right)^{\frac{1}{S}}$, which implies for the US:

$$\frac{\eta_L}{\eta_H} \left(\frac{H_{H,t}}{H_{L,t}} \right)^{\frac{1}{S}} = \frac{0.43}{\varepsilon_L} = \frac{0.43}{0.67} = 0.64.$$

Similarly, it is easy to obtain for the medium ability group: $\frac{\eta_M}{\eta_H} \left(\frac{H_{H,t}}{H_{M,t}} \right)^{\frac{1}{S}} = \frac{0.63}{\varepsilon_M} = \frac{0.63}{0.84} = 0.75$.

If we finally take into account that $\eta_H = 1 - \eta_M - \eta_L$, and we introduce values for $H_{H,t}/H_{M,t}$ and $H_{H,t}/H_{L,t}$ which we simultaneously obtain elsewhere in the calibration (as functions of the employment rates, education rates, σ and ϕ), it is easy to see that we have three remaining equations in three unknowns (η_H, η_M, η_L) that can be solved.

Appendix C: Model with endogenous growth: parameterization.

Technology and preference parameters

Goods production (output)	$\alpha = 0.30, s = 1.5, \eta_H = 0.49, \eta_M = 0.33, \eta_L = 0.18$
Human capital	$\phi = 1.72, \sigma = 0.3$
Initial human capital	$\pi = 0.87, \varepsilon_M = 0.84, \varepsilon_L = 0.67$
Preference parameters	$\beta = 0.80, \theta = 2, \gamma_1 = 0.085, \gamma_2 = 0.145, \gamma_3 = 0.196$ $\mu = 0.5, \zeta = 1.54, \Gamma = 2$
World real interest rate	$r = 0.935$
Capital depreciation rate	$\delta_k = 0.714$

Fiscal policy and pensions policy parameters ^(a)

$\tau_w = 67.2\%, \tau_c = 13.4\%, \tau_k = 27.1\%, b = 59.6\%, b_{er} = 79.0\%,$ $\rho_{wL} = 55.4\%, \rho_{wM} = 63.1\%, \rho_{wH} = 42.7\%, \rho_{fL} = 17.2\%, \rho_{fM} = \rho_{fH} = 0\%$
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Target values for calibration

Employment, education and growth ^(b)

n_1	n_2	n_3	R	e	Annual per capita growth
51.1%	56.8%	29.3%	57.9	14.1%	1.77%

Relative wages of young workers, US ^(c)

$w_L h_{1L} / w_H h_{1H}$	$w_M h_{1M} / w_H h_{1H}$
0.43	0.63

Notes: See Table 2.

Appendix D: Derivation of Equation (30)

Starting point to derive our equation for the endogenous economic growth rate are Equations (18) and (19), where we now assume A to be constant.

$$Y_t = K_t^\alpha (AH_t)^{1-\alpha} \quad (18)$$

$$H_t = \left(\eta_H H_{H,t}^{1-\frac{1}{s}} + \eta_M H_{M,t}^{1-\frac{1}{s}} + \eta_L H_{L,t}^{1-\frac{1}{s}} \right)^{\frac{s}{s-1}} \quad (19)$$

Effective labor per ability group now becomes:

$$\begin{aligned} H_{a,t} &= n_{1a}^t h_{1a}^t + n_{2a}^{t-1} h_{2a}^{t-1} + n_{3a}^{t-2} h_{3a}^{t-2} \\ &= \left(n_{1a}^t + n_{2a}^{t-1} \frac{\psi_a^{t-1}}{x_{t-1}} + n_{3a}^{t-2} \frac{\psi_a^{t-2}}{x_{t-1} x_{t-2}} \right) h_{1a}^t \quad \forall a = H, M, L \end{aligned} \quad (31)$$

To obtain (31) we again define:

$$1 + \phi(e_{1a}^t)^\sigma \equiv \psi_a^t, \text{ with } \psi_L^t = 1 \quad (21)$$

so that $h_{3a}^{t-j} = h_{2a}^{t-j} = \psi_a^{t-j} h_{1a}^{t-j} \quad \forall a = H, M, L$.

Furthermore, assuming an intergenerational transfer of human capital according to Equation (28), we no longer use (22), but¹⁹ :

$$h_{1a}^t = x_{t-1} h_{1a}^{t-1} = x_{t-1} x_{t-2} h_{1a}^{t-2}, \quad (32)$$

where by definition: $x_t \equiv \pi \left(\frac{\psi_H^t + \varepsilon_M \psi_M^t + \varepsilon_L}{3} \right)$.

Substituting Equation (31) for $a = H, M$ and L into (19), and recognizing differences in the capacity ε_a to inherit human capital as indicated by Equation (29), yields Equation (33).

$$H_t = \left[\sum_{a=H,M,L} \eta_a \varepsilon_a^{1-\frac{1}{s}} \left(n_{1a}^t + n_{2a}^{t-1} \frac{\psi_a^{t-1}}{x_{t-1}} + n_{3a}^{t-2} \frac{\psi_a^{t-2}}{x_{t-1} x_{t-2}} \right)^{1-\frac{1}{s}} \right]^{\frac{s}{s-1}} h_{1H}^t \quad (33)$$

Substituting (33) for H_t and (24) for K_t/AH_t , we can rewrite (18) as

$$\begin{aligned} Y_t &= \left(\frac{K_t}{AH_t} \right)^\alpha AH_t \\ &= A \left[\frac{\alpha(1-\tau_k)}{r_t + \delta_k(1-\tau_k)} \right]^{\frac{\alpha}{1-\alpha}} \left[\sum_{a=H,M,L} \eta_a \varepsilon_a^{1-\frac{1}{s}} \left(n_{1a}^t + n_{2a}^{t-1} \frac{\psi_a^{t-1}}{x_{t-1}} + n_{3a}^{t-2} \frac{\psi_a^{t-2}}{x_{t-1} x_{t-2}} \right)^{1-\frac{1}{s}} \right]^{\frac{s}{s-1}} h_{1H}^t \end{aligned}$$

If we finally recognize that in steady state $r, \tau_k, \psi_a, e_{1a}$ and n_{ja} are constant, we obtain the long-run (per capita) growth rate of the economy as

$$\begin{aligned} \ln \left(\frac{Y_t}{Y_{t-1}} \right) &= \ln \left(\frac{h_{1H}^t}{h_{1H}^{t-1}} \right) = \ln(x_{t-1}) \\ &= \ln \left(\pi \frac{(1+\phi(e_{1H}^{t-1})^\sigma) + \varepsilon_M (1+\phi(e_{1M}^{t-1})^\sigma) + \varepsilon_L}{3} \right) \end{aligned} \quad (30)$$

¹⁹ Starting from Equation (28), and using (21) and (29), it is easy to see that:

$$h_{1H}^t = \pi \frac{h_{2H}^{t-1} + h_{2M}^{t-1} + h_{2L}^{t-1}}{3} = \pi \frac{\psi_H^{t-1} h_{1H}^{t-1} + \psi_M^{t-1} h_{1M}^{t-1} + \psi_L^{t-1} h_{1L}^{t-1}}{3} = \pi \frac{(\psi_H^{t-1} + \varepsilon_M \psi_M^{t-1} + \varepsilon_L)}{3} h_{1H}^{t-1} = x_{t-1} h_{1H}^{t-1}.$$

Human capital of the lower ability individuals ($a = M, L$) will grow at the same rate $\frac{h_{1a}^t}{h_{1a}^{t-1}} = \frac{\varepsilon_a h_{1H}^t}{\varepsilon_a h_{1H}^{t-1}} = \frac{h_{1H}^t}{h_{1H}^{t-1}}$ which explains the first part of Equation (32). Lagging this result by one period generates the second part.