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The Bohm sheath criterion in strongly coupled complex plasmas

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Abstract. A modification of the classical Bohm sheath criterion is investigated in complex plasmas containing Boltzmann electrons, cold fluid ions and strongly coupled microparticles. Equilibrium is provided by an effective 'temperature' associated with electrostatic interactions between charged grains. Using the small-potential expansion approach of the Sagdeev potential, a significant reduction of the ion Bohm velocity is obtained for complex plasma parameters relevant for experiments. The result is of consequence for all problems involving ion drag on microparticles, including parametric instability, structure formation, wave propagation, etc.

Contents

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1. Introduction

Progress in experimental studies of complex plasmas has rekindled interest in some basic aspects of plasma descriptions. One of these is the proper description of the Bohm sheath criterion for complex plasmas (containing besides electrons and ions also highly charged microparticles). This problem becomes especially important because some experiments have revealed a significant effect of the microparticles on the sheath structure [1, 2]. Interest in the ion Bohm velocity in complex plasmas is also dictated by its importance for the formation of the sheath structure as a whole, and for estimates of the ion fluxes to the microparticles and to the electrode/wall surfaces, in particular.

The first paper to discuss the influence of heavy massive and charged dust grains on the Bohm sheath criterion was by Hellberg *et al* [3]. Later the Bohm sheath criterion has been generalized to multispecies plasmas, by assuming that the heavier plasma components are described by cold fluid theory [4]. Recently, attention has also turned to the influence of charge variations and the existence of electronegative ions (usually important in processing plasmas) on the ion Bohm velocity [5]. It is important to note that all studies of the critical Bohm velocity considered the microparticles either as immobile or governed by cold fluid theory.

In many experiments involving complex plasmas (under microgravity as well as in laboratory conditions), the particles are strongly coupled and remain in an equilibrium state, which results from a self-consistent distribution of all plasma parameters within the discharge. If the discussion is limited to the cases when the gravitational force is negligibly small (e.g. in microgravity experiments or when dealing with tiny, submicron/nanometer particles), we show that the strongly coupled structures, common to many experiments, allow us to introduce an effective ‘temperature’ associated with the electrostatic interactions between similarly charged microparticles. Introducing such a ‘temperature’, which is usually a few orders of magnitude higher than the real kinetic dust temperature, permits us to reduce the problem to the standard scheme of a Sagdeev potential. Using the small-potential expansion approach of Bohm, we obtain results that can significantly modify the classical requirement that the ion velocity exceeds the ion-acoustic speed at the sheath edge [6].

2. Analytical model

To study the influence of the charged microparticles on the sheath structure, we consider a simplified model of the one-dimensional collisionless sheath. The sheath edge at $x = 0$ separates the quasineutral plasma ($x < 0$) and the sheath region ($x > 0$). At the sheath edge, the electric potential is taken to be zero, $\varphi = 0$, and the charge neutrality condition requires

$$n_{e0} + Z_d n_{d0} - n_{i0} = 0, \tag{1}$$

with n_{e0}, n_{i0} and n_{d0} being the electron, ion and dust equilibrium number density, respectively, and Z_d denoting the particle charge number in the quasineutral plasma, for $x \leqslant 0$. In the sheath, the electrons are assumed to be Boltzmann distributed,

$$n_e = n_{e0} \exp(e\varphi/T_e), \tag{2}$$

where T_e and e refer to the electron temperature and charge, respectively.

Contrary to the electrons, the positive ions are accelerated by the sheath electric field to velocities higher than the ion thermal velocity, and hence the cold fluid model is an appropriate approximation. Assuming moreover that the discharge pressure is low enough so that ion–neutral and ion–dust collisions can be neglected, the ion velocity is then determined from the steady-state momentum and continuity equations

$$m_i V_i \frac{\mathrm{d}V_i}{\mathrm{d}x} = -e\,\frac{\mathrm{d}\varphi}{\mathrm{d}x}, \tag{3}$$

$$n_{i0} V_{i0} = n_i V_i. \tag{4}$$

Here n_i and V_i are the ion density and fluid velocity within the sheath, respectively, while V_{i0} is the ion velocity at the edge of the sheath, viz at $x = 0$, and m_i refers to the ion mass. Note that the continuity equation (4) assumes the ion losses to dust and ion sources to be negligible.

The fluid equations (3) and (4) can be integrated exactly, with the boundary conditions φ, $\mathrm{d}\varphi/\mathrm{d}x \to 0$ and $V_i \to V_{i0}$, as $x \to 0$. After elimination of the velocity one finds for the ion density

$$n_i = \frac{n_{i0}}{\sqrt{1 - 2e\varphi/\left(m_i V_{i0}^2\right)}}. \tag{5}$$

The equilibrium state of the heavy plasma component—the charged particles—results from the balance of all the forces acting on the microparticles, including the electrostatic, plasma drag, gravitational, pressure gradient forces, etc. An important issue is also that a complex plasma is a thermodynamically open system, and hence the microparticles generally affect all plasma parameters, including the electric potential profile. As a result, the determination of a self-consistent steady state particle distribution becomes, in general, a very complicated problem and all theoretical and numerical attempts to reconstruct the dust distribution within the discharge plasma require some simplifying assumptions (e.g. [7, 14, 19]).

In this paper, however, we address the situation when the gravitational force acting on the particles is negligible, the particles are strongly coupled and the dust cloud extends over the discharge plasma occupying the volume up to a boundary presheath/sheath. Such cases are relevant for studies of complex plasmas under microgravity conditions (see e.g. [8]–[11]), for thermophoretically levitated systems [12], for experiments on nanoparticle coagulation [13] and for processing plasmas dealing also with very small (submicron/nanometer sized) particles [13]–[15].

On the periphery of the discharge, the microparticles usually do not show a directed motion, but vibrate near their equilibria. For grains carrying a charge $Q_d = Z_d e$, two forces are most important for the particle equilibria: the electrostatic force, $F = eZ_d \mathrm{d}\varphi/\mathrm{d}x$, and the force due to the gradient of the internal 'electrostatic pressure', P_d, originating from the repulsion of similarly charged microparticles. For Yukawa-type interacting grains, in the nearest-neighbor approximation, P_d can be approximated by [7]

$$P_d \simeq \frac{N_{nn}}{3} \Gamma T_d n_d (1+\kappa) \mathrm{e}^{-\kappa}, \tag{6}$$

where n_d and T_d are the particle number density and kinetic temperature, respectively, N_{nn} is determined by the dust structure and corresponds to the number of nearest neighbors (e.g. in the crystalline state, $N_{nn} = 12$ for the fcc and hcp lattices and $N_{nn} = 8$ for the bcc lattice). Furthermore, Γ is the coupling parameter defined by

$$\Gamma = \frac{Z_d^2 e^2}{T_d \Delta}.$$

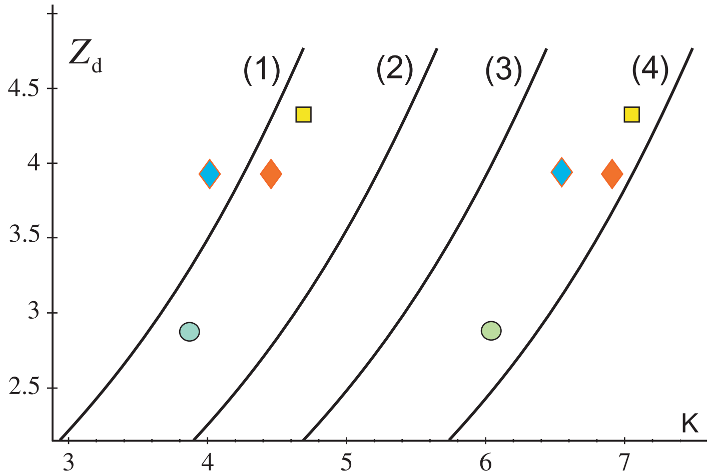

Figure 1. Curves determining the upper limit of the particle separation κ_{cr} versus Z_d according to (7), for different values of the screening length: $\lambda_D = 400\,\mu$m (curve 1); $\lambda_D = 150\,\mu$m (curve 2); $\lambda_D = 80\,\mu$m (curve 3) and $\lambda_D = 30\,\mu$m (curve 4). Symbols correspond to κ_{cr} computed in two limits $\lambda_D = \lambda_{De}$ and $\lambda_D = \lambda_{Di}$ for particles with radius $a = 3.4\,\mu$m, used in specific microgravity experiments of [10, 11] (diamonds), of [17] (squares) and of [18] (circles).

Here Δ denotes the mean interparticle distance, and κ refers to its normalized value through $\kappa = \Delta/\lambda_D$, with λ_D being the screening length of the complex plasma.

Dealing with the nearest-neighbor approximation in (6) implies a limitation on the particle separation, with at least $\kappa > 1$. On the other hand, if we focus on the crystalline structures, then $\Gamma > \Gamma_{cr}$, where the curve $\Gamma_{cr}(\kappa)$ separates the crystal and liquid states in the phase diagram. This crystal–liquid curve can reasonably be approximated as $\Gamma_{cr} \simeq 106\exp(\kappa)/(1+\kappa+\kappa^2/2)$ [16]. For given values of λ_D and Z_d, the condition $\Gamma > \Gamma_{cr}$ restricts κ from above, requiring $\kappa < \kappa_{cr}$, where κ_{cr} obeys

$$\frac{Z_d^2 \times 10^{-3}}{2\lambda_D[\mu\text{m}]} \simeq \frac{\kappa_{cr}\exp(\kappa_{cr})}{1+\kappa_{cr}+\kappa_{cr}^2/2}. \tag{7}$$

Here we have put the particle temperature equal to the neutral gas (room) temperature, viz $T_d \simeq 0.03$ eV.

For realistic experimental conditions, the κ_{cr} following from (7) typically lie between 4 and 7 (see figure 1). As a result, the considerations that follow relate to the range $1 < \kappa < 7$, outside which our assumptions lose their validity.

We now introduce the effective dust 'temperature', arising from the electrostatic interactions between the strongly coupled particles, as

$$T_d^{(\text{eff})} = \frac{N_{nn}}{3}\,\Gamma T_d (1+\kappa)e^{-\kappa}. \tag{8}$$

In microgravity experiments, Γ is typically $\Gamma \gtrsim 10^3$, while $\kappa \simeq 2$–4, leading to an effective 'temperature' of 1–10^2 eV. For processing plasmas, when dealing with smaller, e.g. 100 nanometer-sized particles [14, 15], $\Gamma \gtrsim 10^2$ and $\kappa \simeq 1$–3, thus leading to $T_d^{(\text{eff})} \simeq 1$–10 eV. Therefore, the effective dust temperature for strongly coupled plasmas, equation (8), is always a few orders of magnitude higher than the kinetic temperature of the particles, T_d.

The quantity $T_{\rm d}^{\rm (eff)}$ is, in general, a weak function of the local plasma parameters and the electric potential (the latter follows from the dependence $T_{\rm d}^{\rm (eff)}$ on values of the screening length and the interparticle distance). The screening length is determined by a combination of the ion and electron Debye lengths, and both of those imply a weak dependence on the plasma densities, of the kind $\lambda_{\rm Di} \propto 1/\sqrt{n_{\rm i}(\varphi)}$ and $\lambda_{\rm De} \propto 1/\sqrt{n_{\rm e}(\varphi)}$. The interparticle separation is given by the local dust density as $\Delta \simeq n_{\rm d}^{-1/3}(\varphi)$. The derivations we address here relate to significantly small perturbations of the plasma characteristics (in particular, the potential is assumed to be $\varphi \ll T_{\rm e}/e$), which make the discrepancies between the local Δ and $\lambda_{\rm D}$ and those corresponding to the sheath edge at $x \simeq 0$ really small (a second-order effect) and therefore can be neglected to simplify the treatment. The definition of $T_{\rm d}^{\rm (eff)}$ (8) thus involves fixed $\Delta \simeq n_{\rm d0}^{-1/3}$, and the screening length, $\lambda_{\rm D}$, corresponding to the sheath edge, $x = 0$.

The microparticle steady state requires

$$n_{\rm d} e Z_{\rm d} \frac{{\rm d}\varphi}{{\rm d}x} = \frac{{\rm d}P_{\rm d}}{{\rm d}x}, \tag{9}$$

and therefore

$$n_{\rm d} = n_{\rm d0} \exp\left(eZ_{\rm d}\varphi/T_{\rm d}^{\rm (eff)}\right). \tag{10}$$

The set of equations (2), (5) and (10) is closed by Poisson's equation for the plasma potential in the region $x > 0$,

$$\frac{{\rm d}^2\varphi}{{\rm d}x^2} = -4\pi e n_{\rm i0}\left(\frac{V_{\rm i0}}{\sqrt{V_{\rm i0}^2 - 2e\varphi/m_{\rm i}}} - \frac{n_{\rm e0}}{n_{\rm i0}}\exp\left(e\varphi/T_{\rm e}\right) - \frac{Z_{\rm d} n_{\rm d0}}{n_{\rm i0}}\exp\left(eZ_{\rm d}\varphi/T_{\rm d}^{\rm (eff)}\right)\right). \tag{11}$$

This equation can be integrated in the traditional way to yield

$$\frac{1}{2}\left(\frac{{\rm d}\phi}{{\rm d}X}\right)^2 + V(\phi, M) = 0, \tag{12}$$

with the Sagdeev potential $V(\phi, M)$ given by

$$V(\phi, M) = M^2\frac{n_{\rm i0}}{n_{\rm e0}}\left(1 - \sqrt{\frac{2\phi}{M^2} + 1}\right) + \left(1 - {\rm e}^{-\phi}\right) + \frac{n_{\rm d0}}{n_{\rm e0}}\gamma\left(1 - \exp\left(-Z_{\rm d}\phi/\gamma\right)\right). \tag{13}$$

In expressions (12) and (13), we have used dimensionless variables for the potential $\phi = -e\varphi/T_{\rm e}$, the coordinate $X = x/\lambda_{\rm De} = x/\sqrt{T_{\rm e}/\left(4\pi e^2 n_{\rm e0}\right)}$, $\gamma = T_{\rm d}^{\rm (eff)}/T_{\rm e}$ and the Mach number $M = V_{\rm i0}/\sqrt{T_{\rm e}/m_{\rm i}} = V_{\rm i0}/V_{\rm s}$ characteristic for pristine electron–ion plasmas.

3. Bohm sheath criterion

Equation (12) represents the energy integral for a classical particle of unit mass moving with velocity ${\rm d}\phi/{\rm d}X$ in a potential well $V(\phi, M)$ (where M appears as a parameter). Expanding the Sagdeev potential to second order in ϕ gives

$$V(\phi, M) \simeq \frac{1}{2}\left[\frac{1}{M^2(1-p)} - \frac{p}{1-p}\frac{Z_{\rm d}}{\gamma} - 1\right]\phi^2 = A\phi^2, \tag{14}$$

where $p = Z_d n_{d0}/n_{i0}$ denotes the Havnes parameter. For exponential decay of the sheath one needs A to be negative, which leads to

$$M^2 > \frac{\gamma}{\gamma(1-p) + pZ_d}. \tag{15}$$

The classical Bohm condition of supersonic ions ($M > 1$), valid for usual electron–ion discharges ($p = 0$), is now modified by the presence of microparticles. In a formal way, the structure of the sheath criterion for strongly coupled complex plasmas (15) corroborates the Bohm condition for electronegative plasmas [21] if we treat the charged microparticles as negative ions with $Z_d = 1$ and put $\gamma = T_i/T_e$.

Inserting the explicit expression for the effective temperature of the dust species (8) in γ, we rewrite the Bohm criterion for strongly coupled complex plasmas (15) as

$$M > M_{\min} = \sqrt{\frac{\kappa^2(1+\kappa)}{(1-p)[\kappa^2(1+\kappa) + s^2\exp(\kappa)]}}, \tag{16}$$

where the quantity s is mainly specified by a relation between the electron Debye length, λ_{De}, and the screening length near the sheath edge, λ_D, through $s^2 = 12\pi\lambda_{De}^2/(N_{nn}\lambda_D^2)$. Note that expression in the form of (16) assumes $\gamma \neq 0$ and is not appropriate for the limiting case of usual electron–ion plasmas.

For plasma discharges, one of the main difficulties lies in the proper estimation of the plasma screening length at the sheath boundary and within the sheath. The standard approach implies that in the bulk plasma the screening is mainly due to the thermal ions, i.e. $\lambda_D \to \lambda_{Di}$. In the sheath, the shielding becomes more electron-like, although the value of λ_D could be also affected by non-thermal ions [20]. It is then reasonable to suggest that the real λ_D near the sheath edge lies somewhere between the two limiting values λ_{Di} and λ_{De}.

To have a manageable discussion we take s as a variable parameter and plot $M_{\min}$ as a function of κ for different s, covering a total range from $s_{\min} = \sqrt{\pi}$ in the limit $\lambda_D \to \lambda_{De}$, to $s_{\max} = \sqrt{100\pi}$ when $\lambda_D \to \lambda_{Di}$ (the latter is taken for typical temperature ratios $T_e/T_i \simeq 100$ and $N_{nn} = 12$). The resulting dependencies $M_{\min}(\kappa)$ are shown in figure 2 for two values of the Havnes parameter p. As an example, we have addressed observations of the dust density perturbations excited near the sheath edge at low gas pressures [10, 11]. The dust particles have radius $a = 3.4\,\mu$m and carry a charge $Z_d = 4000$. The dust number density, $n_d = 5\times10^4\,\text{cm}^{-3}$, and temperature, $T_d = 0.025\,\text{eV}$, give $\Gamma \simeq 3.4\times10^3 > \Gamma_{cr}$. The ion densities used are $n_i = 3\times10^8\,\text{cm}^{-3}$ [10] and $n_i = 10^9\,\text{cm}^{-3}$ [11], thus leading, respectively, to $p = 0.65$ and 0.2, as indicated in figure 2.

Generally, we find smooth functions $M_{\min}(\kappa)$ with a reduction in the Mach number and hence a lower ion Bohm velocity due to the presence of highly charged dust grains (compared to the case of a pristine discharge sheath where $M_{\min} = 1$). Two factors determine the ion Bohm velocity—the Havnes parameter p and the quantity s. But $s \propto \lambda_{De}/\lambda_D$ is the most crucial parameter responsible for the ion velocity reduction. The larger s is, the more $M_{\min}(\kappa)$ is reduced. In the extreme case $s = s_{\max}$ (the screening length at the sheath edge is close to the ion Debye length in the bulk plasma), the ion Bohm velocity becomes comparable or can be even slightly less than the ion thermal velocity, thus violating our initial assumptions about cold ions. In real discharges, however, the plasma screening length at the sheath edge can be expected to be larger than the ion Debye length in the bulk plasma, and the ion Bohm velocity most probably corresponds to one of the curves (2)–(4) shown in figure 2. All these plots demonstrate the

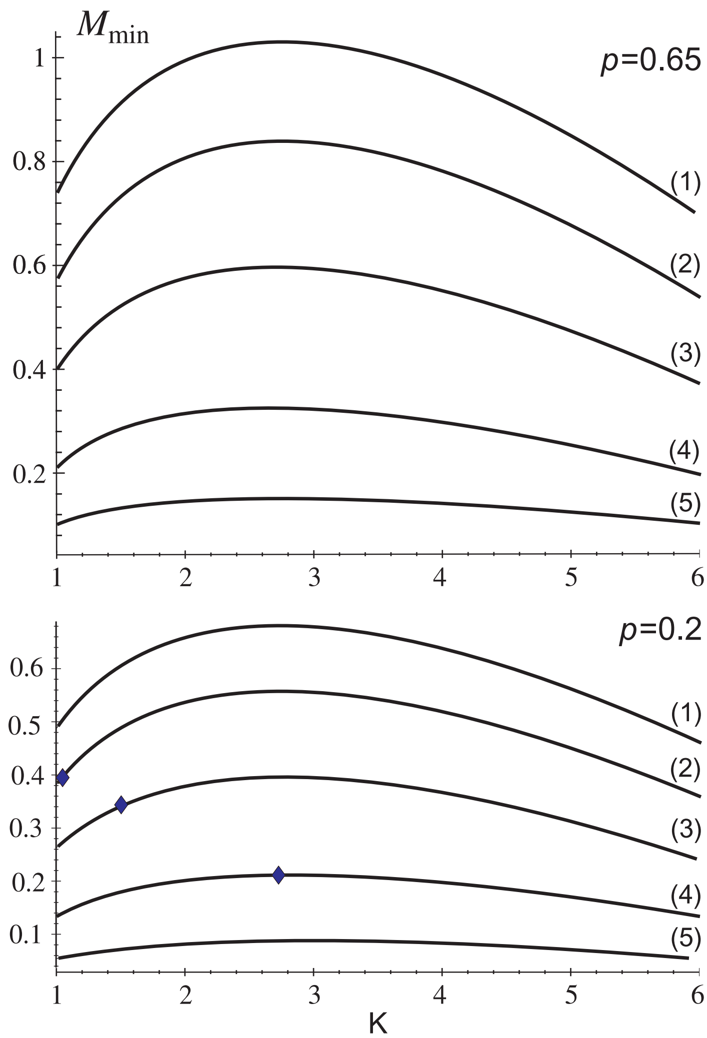

Figure 2. The minimal Mach number M_{min} as a function of the normalized interparticle distance κ, for various s and for two values of the Havnes parameter, $p = 0.65$ and 0.2. Curve 1 is for $s = \sqrt{\pi}$ ($\lambda_D = \lambda_{De}$); curves 2–4 correspond to $s = 4\sqrt{\pi}/3$ ($\lambda_D = 3/4\lambda_{De}$); $s = 2\sqrt{\pi}$ ($\lambda_D = 1/2\lambda_{De}$); $s = 4\sqrt{\pi}$ ($\lambda_D = 1/4\lambda_{De}$), respectively. Curve 5 is for the limiting case $s = \sqrt{100\pi}$ ($\lambda_D = \lambda_{Di}$). Symbols in the case $p = 0.2$ indicate M_{min} calculated for the plasma parameters of [11].

significant reduction of the Bohm velocity, by factors of 2–5. Physically, such an effect can be caused by a decrease of the spatial gradient of the electric potential inside the plasma due to the presence of a dense dust structure. Note that a similar tendency in the behavior of the plasma potential follows from figure 5 of [19] presenting the calculations of the plasma potential profile in a weakly collisional discharge for two different values $p = 0.5$ and 0.85.

4. Conclusions

In this paper, we have derived a new Bohm sheath criterion characteristic for complex plasmas containing highly negatively charged microparticles, besides the electrons and ions.

Assuming a simplified model for the equilibrium state for the plasma species, we have shown that the strongly coupled structures introduce an effective 'temperature' associated with the electrostatic interactions between the particles having likely charges. Such a 'temperature' is a few orders of magnitude higher than the real kinetic dust temperature. This leads to specific modifications of the standard Sagdeev potential. Using the small-potential expansion approach of Bohm, we have obtained a criterion that modifies the classic requirement that the ion velocity exceeds the ion-acoustic speed at the sheath edge. Our theory predicts a significant reduction of the ion Bohm velocity by factors of 2–5. The findings are discussed in the light of realistic plasma parameters relevant for complex plasma experiments performed under microgravity.

Note that the results thus obtained could be of importance, not only for understanding some basic processes in complex plasmas involving ion drag on microparticles, but also for processing plasmas, where the numerous small contaminations by submicron- and nanometer-sized particles can lead to enormous densities (up to $5 \times 10^7\,\mathrm{cm}^{-3}$) that carry high electric charges (e.g. for nanometer grains Z_d up to 400) [14, 15]. One can expect therefore a significant change in the sheath structure of such dust-loaded plasmas and a possible reduction in the ion Bohm velocity, which ultimately influences the deposition rate, the quality of thin films and related effects.

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