

Running head: A ROADMAP INTO MENTAL SHAPE SPACE

**Subjectively interpreted shape dimensions as privileged and
orthogonal axes in mental shape space**

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Abstract

The shape of an object is fundamental in object recognition but it is still an open issue to what extent shape differences are perceived analytically (i.e., by the dimensional structure of the shapes) or holistically (i.e., by the overall similarity of the shapes).

The dimensional structure of a stimulus is available in a primary stage of processing for separable dimensions, although it can also be derived cognitively from a perceived stimulus consisting of integral dimensions. Contrary to most experimental paradigms, the present study asked participants explicitly to analyze shapes according to two dimensions. The dimensions of interest were aspect ratio and medial axis curvature, and a new procedure was used to measure the participants' interpretation of both dimensions (Part I, Experiment 1). The subjectively interpreted shape dimensions showed specific characteristics supporting the conclusion that they also constitute perceptual dimensions with objective behavioral characteristics (Part II): (i) the dimensions did not correlate in overall similarity measures (Experiment 2), (ii) they were more separable in a speeded categorization task (Experiment 3), and (iii) they were invariant across different complex 2-D shapes (Experiment 4). The implications of these findings for shape-based object processing are discussed.

Keywords:

There is a long-standing debate concerning the holistic or analytic nature of objects in the world and in the mind of humans, which was started by philosophers centuries ago (e.g., Democritus' atomism and Plato's ontology), and which continues today as an important theme in vision research (e.g., Garner, 1974; Gibson, 1979). The holistic-analytic debate in vision research has led to two mutually exclusive positions. In the analytic view, objects are believed to be perceived by first analyzing their properties, features and dimensions that constitute the object. Once these properties are known, the object can be derived. In contrast, according to the holistic view, objects are processed as one entity and their properties, features and dimensions are derived subsequent to this. . For this view, the features and dimensions of objects are the secondary, not primary properties on which visual object processing takes place. The distinction between holistic and analytic modes of processing has been proposed to reflect the underlying integral and separable dimensional stimulus structures, respectively (for seminal work, see Garner, 1974; Shepard, 1964). A prototypical example of a pair of integral dimensions in the literature is color brightness and saturation (Foard & Kemler Nelson, 1984; Lockhead, 1972; Melara, Marks, & Potts, 1993b), while shape and color have been considered separable dimensions (e.g., Garner, 1974; Handel & Imai, 1972).

However, the different characteristics of separable and integral dimensions are not defined in an all-or-none fashion. A number of sometimes contradicting results have led to the view that there is a continuum of integrality rather than a simple dichotomy (e.g., Foard & Kemler Nelson, 1984; Ward, Foley, & Cole, 1986). For instance, the existence of particular directions in a parametric shape space on which participants perform differently, also called privileged axes, has been considered as an operational definition for separable dimensions within speeded classification. Foard

and Kemler Nelson (1984) reported a failure to find privileged axes when unpracticed participants classified stimuli by saturation or brightness, while they did find privileged axes after practice. Their contradictory results clearly depend on particular experimental settings (see also Ward et al., 1986) and has led to the consideration of converging operations in order to accumulate evidence towards separable or integral interactions for particular dimensions and to take into account the specificity of the task context in which the evidence emerged. Converging operations (Garner, Hake, & Eriksen, 1956) stress the importance of investigating a psychological phenomenon by different operations to minimize the chance that an outcome depends on one particular operation. Among the converging operations for dimensional integrality, Garner (1974) found (i) interference in speeded classifications when participants had to attend selectively to a single dimension while the other was varying; (ii) a redundancy gain in classification speed when the classification criterion involved correlated dimensions; (iii) stimuli are grouped together according to overall similarity relations rather than shared dimensional components; and (iv) similarity ratings are better modeled by embedding them in an Euclidean space instead of a Minkowski space with privileged directions. Separable dimensions, in contrast, lead to neither interference nor redundancy gains, stimuli are rather classified according to dimensional resemblance and similarity ratings are better fit in a Minkowski space with privileged axes constituting the separable dimensions.

An important issue related to the integrality continuum mentioned earlier is whether separability and integrality are stimulus concepts or processing concepts. More concretely, are these concepts related to some existential structure in the nature of the stimulus that form part of the “real” world or are these structures some products emerging from the visual process itself? Naturally, any presumed existential structure

is a processed structure because we can only know the “real” world through observations, and observations necessarily comprise perceptual processing at some point. Therefore, it is important to be able to identify the dimensions that participants perceive as the ones that constitute the stimulus structure. A fruitful way to proceed is by using these dimensions in different kinds of tasks that would presumably call on different modes of processing. Instances of this are an unspeded and a speeded categorization task, or a task probing for overall similarity between stimuli like similarity ratings and one probing for a dimensional structure by asking participants explicitly to handle the object in terms of a particular dimension, like ordering the objects by their surface luminance. When the processed structures of the stimuli are consistent over a broad range of tasks calling on different processing mechanisms then we may infer that the dimensional structure has some existential relevance (stimulus related) or at least some psychological relevance (process related) that generalizes over different kinds of tasks, in line with the idea of converging operations.

The present study. In the present study, we will introduce a method that can reveal the participants’ preferred interpretations of two-dimensional (2-D) shape attributes. We will refer to these dimensions as the “participant-defined dimensions” and we will show that these dimensions constitute “privileged axes”, a concept introduced by Smith and Kemler Nelson (1978, see also Foard & Kemler Nelson, 1984). We adopted the method of adjustment under conditions that prompt analysis and force participants to form their own subjective criterion for adjusting the stimuli without time constraints. The introduced adjustment task actually provides a parametric measure of the participants’ interpretations of the two dimensions and

allows a comparison between the participant-defined and the experimenter-defined parameterizations.

We applied the method to a specific pair of shape dimensions, namely aspect ratio and medial axis curvature, which has been studied before and revealed a primarily separable dimensional structure (Arguin & Saumier, 2000; Op de Beeck, Wagemans, & Vogels, 2001, 2003). Additionally, these dimensions are key dimensions in Biederman's (1987) theory of "Recognition By Components" (RBC) for the identification of geon types. In RBC theory, the visual representation of the object consists of volumetric parts, called geons, which resemble the parts of the object under view. Because geons constitute a restricted set of volumetric shapes, only the detection of a few properties is sufficient to select one particular geon from the set. Aspect ratio and curvature are examples of the dimensions involved in the detection of a geon from a 2-D retinal image. Stankiewicz (2002) has studied these dimensions using a noise masking paradigm, which is a similar approach to Garner interference explained above. In both paradigms, one dimension is varied randomly while participants carry out a speeded categorization task on another dimension. When the random variation of the irrelevant dimension does not slow down or affect task performance, the dimensions are called separable. Stankiewicz found that aspect ratio and medial axis curvature of 3-D bar shapes were processed independently from each other. Also neuronal tuning for aspect ratio and curvature has been demonstrated (Kayaert, Biederman, Op de Beeck, & Vogels, 2005).

The present study consists of two parts. In Part I, we will explain the new task by which we can derive the participants' interpretations of the dimensions of aspect ratio and medial axis curvature (participant-defined dimensions). We will propose a transformation that can align the parametric stimulus space to an orthogonal basis,

which corresponds to making the participant-defined dimensions orthogonal, and we will test and validate the procedure (Experiment 1). Experimental tasks that depend on subjective interpretations are often suspected to provide biased measurements, and therefore, relatively little effort has been put forward to study the subjective interpretations of shape dimensions explicitly. Our study provides a critical advance in attempting to specify the subjective interpretation of shape. In Part II, we will use more traditional experimental paradigms and we will test whether the participant-defined dimensions also appear to be effective in these tasks. In the first task (Experiment 2), we probed for overall similarity by asking participants to adjust a stimulus to a reference stimulus without time pressure. In the second task (Experiment 3), we used a speeded categorization task to verify whether variation on an irrelevant dimension would interfere with the categorization speed on a relevant dimension. Finally, in the third task (Experiment 4), we varied the initial parametric implementations of aspect ratio and curvature and we applied them on three different contour shapes. We discuss the results in the light of the arguments concerning the holistic and analytic modes of processing.

PART I: A METHOD TO EXTRACT PARTICIPANTS' PREFERRED SHAPE DIMENSIONS AND TO MAKE THEM ORTHOGONAL

In Figure 1, we introduced the stimulus set for Experiment 1. Variations in shape were created along two dimensions, aspect ratio and curvature, represented as the horizontal and vertical dimensions in the shape space in Figure 1, respectively. Similar dimensions have been used before in behavioral studies dealing with 3-D

object representations and separable dimensions in 2-D shapes (e.g., Arguin & Saumier, 2000; Biederman, 1987; Op de Beeck et al., 2003; Stankiewicz, 2002; Stankiewicz & Hummel, 1996). For the exact mathematical description of the dimensions that we applied in Figure 1, we refer to the method section of Experiment 1.

[Insert Figure 1 about here]

We will refer to the representation of Figure 1 as consisting of the initial, parametric or experimenter-defined dimensions. The goal of the procedure is to measure the two orientations in this 2-D shape space with aspect ratio and curvature that comply with the participant's interpretations of these dimensions. These two new orientations can also serve as a basis for the creation of a new stimulus set. In the remainder of the study, we will refer to this new stimulus space as consisting of the participant-defined dimensions.

We applied the method of adjustment to measure the participant-defined dimensions after providing them with only an ambiguous description of the shape dimensions of interest. For example, we could define the dimension of aspect ratio as the ratio between height and width and curvature as the global curvature of the stimulus. These descriptions are ambiguous because a whole range of orientations in the parametric space in Figure 1 are compatible with the descriptions. It is easy to verify in Figure 1 that the description of aspect ratio fits with each vertical and tilted orientation going from left to right. Assuming that participants would be able to extract the specific directions in the parametric space that would maximize the changes with respect to the specified dimensions, the definitions are still vague from a geometric point of view, because the contours of the silhouettes in Figure 1 have different height profiles going from left to right and different width profiles going

from top to bottom (hampering a judgment of height and width, resp.). Extracting the axis of global curvature from such stimuli is also not trivial.

Once the dimensional concepts were provided to the participant, two stimuli were selected from the stimulus space that then served as reference stimuli in the adjustment task. Additionally, one probe stimulus was randomly selected from the stimulus space and was freely adjustable in the domain of the introduced stimulus space. When the stimulus set is ordered in correspondence to the introduced dimensions of aspect ratio and curvature, as seen in Figure 1, the participant was asked to adjust the probe stimulus to a fixed reference stimulus (labeled with number 1 in Figure 1) until it appears perceptually equal with respect to the curvature dimension. Additionally, the participant was asked to adjust the same probe stimulus to a second fixed reference stimulus (labeled with number 3 in Figure 1) with respect to the aspect ratio dimension. The participant could freely switch between the two adjustment criteria and terminate the trial when satisfied with the settings on both criteria.

[Insert Figure 2 about here]

A possible outcome of this procedure is illustrated in Figure 2. Note that the adjustment responses are mapped in a 2-D Cartesian coordinate system where the squared grid corresponds to the stimulus samples in Figure 1. While the probe stimulus was adjusted to F_3 with respect to aspect ratio (horizontal dimension), it was adjusted to F_1 with respect to curvature (vertical dimension). The dashed lines connect one reference stimulus and one adjustment response and can be denoted as the interpolated path that the participant characterized as a constant magnitude for aspect

ratio (slightly tilted away from vertical) or curvature (slightly tilted away from horizontal).

In a 2-D Cartesian coordinate system, a standard basis is formed by two orthogonal unit vectors composing orthogonal axes. Such a mathematical description makes only sense when it relates to orthogonal or independent dimensions. Therefore, one could argue that the thick tilted lines, the average of the dashed lines, are more suitable orientations to be drawn orthogonally (as axes in the Cartesian coordinate system) than any arbitrary set of experimenter-defined parametric dimensions. In Appendix 1, an algorithm is proposed that calculates the average angles, α and β , between the orientations of the experimenter-defined parametric dimensions and the orientations of the participant-defined dimensions. Additionally, a transformation \mathbf{T} is provided that will be used in all experiments of the present study to transform the experimenter-defined dimensions into the participant-defined dimensions. It should be noted that the transformation we used is limited to a linear recombination of the initial introduced dimensions. The introduced dimensions are probably curvilinear paths in the parametric space but it would ask for a different and more exhaustive approach to measure all necessary parameters of the curvilinear paths. Our approach will lead to a linear approximation that is sufficient for the goals of this study. By repeating the approach at different locations and by using a locally weighted smoothing procedure a curvilinear approximation could be achieved.

Experiment 1A

In Experiment 1A, we used the task to derive the participant-defined dimensions as explained by the procedure in the previous section and we used the transformation as explained in Appendix 1. We applied the procedure two times: in a

first session to determine the transformation and to redefine the shape dimensions, and in a second session to evaluate the orthogonality of the new idiosyncratic derived dimensions of the first session. If the method is able to capture the dimensions that participants attend intentionally and use to express the perceived similarities between shapes, then applying the same procedure once again on a stimulus space composed of participant-defined dimensions should result in orientations for the new participant-defined dimensions close to horizontal and vertical (i.e., α and β denoted in Appendix 1 should approach 0).

Methods

Participants. Two naive volunteers from the psychology graduate program at the University of Leuven (S2 and S3) participated in addition to author BO (S1). The indices for the participants are arbitrary but we used the same indices for participants that participated in multiple experiments.

Stimuli. Stimuli were presented as light grey silhouettes (37 cd/m²) on a dark grey background (24 cd/m²). The variable stimuli in the adjustment task could be adjusted by the arrow keys along both dimensions in small incremental steps, barely visible to the human eye.

Aspect ratio and curvature are defined by the image deformation:

$$I: (x, y) \rightarrow f_1(x, y) = \left(cx + \left[1 - \cos\left(\frac{y}{h}r\right) \right] \frac{h}{2c \sin\left(\frac{r}{2}\right)}, \frac{h \sin\left(\frac{y}{h}r\right)}{2c \sin\left(\frac{r}{2}\right)} \right)$$

with parameter c (column in Figure 1) and r (row in Figure 1) expressing the amount of aspect ratio and curvature, respectively, and the constant h expressing the height of the original image on which the deformations are applied (see Figure 1 and Figure 6A).

In order to obtain equally distinctive perceptual steps along the continuum, both parameters c and r were scaled in a pilot study ($N = 5$) and two new scales c' and r' were introduced: $c = 1.0015^{103c'}$ and $r = 3.2561(e^{0.2026r' + 0.3377} - 1)$. In this constellation, the rows and columns in Figure 1 have c' - and r' -values of -1.5, -1, -0.5, 0, 0.5, 1, 1.5. The encircled stimuli have coordinates $c', r' = 1, 1$, $1, -1$, $-1, -1$, $-1, 1$. It is important to mention that the formulas and the chosen scale are only working instruments and have no specific psychological meaning or consequences for the aim and conclusions of the study. Note also that the grids in Figure 6 serve only an illustrative purpose in this paper but were not presented to the participants in any experiment.

Apparatus. The stimuli (maximum size, $5^\circ \times 4^\circ$) were presented on a 17" Dell monitor with a resolution of 1600 by 1200 pixels, a color depth of 16 bits per gun and an average refresh rate of 75 Hz for these settings. The PC was a Dell Optiflex Gx620, Pentium IV, 2.8 GHz. The distance between the participants and the screen was about 80 cm. Participants had to indicate their responses by means of an AZERTY keyboard on which the four arrow keys allowed them to adjust the stimulus and the numerical "0" key allowed them to terminate the trial when they were pleased with their adjustments.

Procedure. The experiment involved two sessions: one before the transformation T (see Appendix 1) and one after the transformation. In the first session, the same dimensions were used for each participant, while in the second session, the transformation for each participant led to slightly different dimensions depending on the results in the first session.

Although we explained the procedure in the introduction by means of one pair of reference stimuli, we have chosen two different pairs of reference stimuli in the actual experiments. In the previously introduced task description, the probe stimulus had to be adjusted to each fixed stimulus with respect to one specific dimension. In the actual experiments, we also switched the dimension according to which the probe stimulus had to be matched. Combining two ways of comparing with two pairs of fixed stimuli led to four different kinds of adjustment trials. The two pairs of fixed stimuli were positioned in the corners of a square-shaped configuration in the stimulus space. In Figure 1, the encircled stimuli were used as fixed reference stimuli. In a P_1 trial, the curvature is adjusted to the fixed stimulus indicated by 4 in Figure 1 and the aspect ratio is adjusted to the fixed stimulus numbered by 2. Adjusting the curvature to the stimulus numbered by 3 and the aspect ratio to the stimulus numbered by 1 produced a P_2 trial. Adjusting the curvature to the stimulus numbered by 2 and the aspect ratio to the stimulus numbered by 4 produced a P_3 trial. Finally, adjusting the curvature to the stimulus numbered by 1 and the aspect ratio to the stimulus numbered by 3 produced a P_4 trial. To make sure that the participants did not adjust the variable stimuli to the wrong dimension (e.g., adjusting a P_1 trial as if it was a P_3 trial), an error message appeared when the participant tried to terminate the trial too far outside the correct quadrant. The initial position of the variable stimulus was chosen randomly in the correct quadrant. Given our particular quadrilateral configuration, we would expect the variable stimulus in a P_1 , P_2 , P_3 and a P_4 trial to be located in the neighborhood of the stimulus numbered with 1, 2, 3, and 4, respectively. In total, we collected 16 trials for each kind of adjustment trial. Each session thus consisted of 64 trials and lasted approximately 30 minutes. The first and second session were separated by at least one day. In the second session, we used the same positions in the

stimulus space for the fixed stimuli. However, the two pairs of fixed stimuli were slightly different in appearance than the two pairs in the first session because of the transformed dimensions that constituted a new square of positions in the parametric space for reference stimuli in the adjustment task.

Each trial consisted of one variable stimulus positioned in the middle of the screen and two fixed stimuli positioned in the upper left and upper right corner of the screen. The position of each fixed stimulus determined the specific comparison: The variable probe stimulus had to be adjusted towards the fixed reference stimulus in the upper left corner with the horizontal arrows and to the fixed stimulus in the upper right corner with the vertical arrows. To remind the participant of this arrangement, the labels “horizontal” and “vertical” were written above the corresponding fixed stimuli. In half of the trials of each session (chosen randomly), the horizontal arrow keys adapted the contour towards more (left arrow) or less (right arrow) elongation, while the vertical keys adapted the shape towards more (up arrow) or less curvature (down arrow). In the other half of the trials, the horizontal keys were linked to more (right arrow) or less (left arrow) curvature and the vertical keys to more (down arrow) or less (up arrow) elongation. The link between experimenter-defined dimensions and key assignments switched between trials. Switching prevented that key directions could bias the interpretation of a dimension towards the initial experimenter-defined dimensions. Although we do not believe that such a link would bias the adjustments substantially, we wanted to prevent it anyway.

Instructions. The procedure was explained to each participant at the beginning of the experiment. The dimension of aspect ratio was described as the height of the stimulus relative to the width. Curvature was described as the bending of the whole figure. For complex stimuli like the ones we used in this experiment, these

definitions are inadequate (underdetermined). Therefore, we explained to the participants that the dimensions of the stimulus could not be explained more exactly without becoming very technical and that the dimensions were implemented in the arrow keys, so they could rely on the arrow keys to develop an understanding of each shape dimension. The participants were also told to focus on the global shape instead of just one local feature like a corner or a small part of the stimulus. The participants first completed five practice trials to become familiar with the procedure.

Results

The main dependent variables of interest were the two angles α and β , derived from the Equations (3) and (4) in Appendix 1. The main independent variable of interest is the coordinate system, consisting of the initial implemented dimensions (session 1) versus the transformed dimensions corresponding to the participant's interpretation of equal aspect ratio and equal curvature between the adjusted stimulus and the reference stimuli (session 2).

In Figure 3, the adjustments are localized in the stimulus space for each session (*session 1* and *session 2*) and for each subject (S_1 , S_2 , and S_3) separately, and labeled with the trial type P_1 , P_2 , P_3 and P_4 . The upper row of three panels shows the settings for the first session and the bottom row of three panels represents the second session. In each panel, the settings for the P_1 , P_2 , P_3 and P_4 trials are indicated by diamonds, plusses, squares and crosses, respectively. The locations of the average adjustments are indicated by the unfilled circles and the hypothetical perfect adjustment positions that would result from the aimed orthogonal horizontally and vertically aligned dimensions are indicated by the filled circles. Clearly, the transformation in-between session 1 and 2 helped considerably to approach the

hypothetical perfect case. The unfilled circles are not always visible in the bottom row because they are plotted behind the filled circles.

The straight grids in the panels of the first session (upper row) are also plotted in the graphical representation of the second session (bottom row) and show how the affine transformation \mathbf{T} led to new coordinates c' and r' . It is interesting to observe how similarly sheared the transformed grids were between participants, indicating how consistent the subjective dimensions are among human observers. This is not obvious, in light of the large degree of freedom given to the participants in determining what to use in their adjustments.

[Insert Figure 3 about here]

From the locations of the mean adjustment for each type of trial, the angles for the transformation were calculated. The transformation parameters are depicted in Figure 4 (left panel for α and right panel for β). The black bars indicate the value of the rotations for the settings in the first session (expressed in radians). Filling in those angles in Equations (3) and (4) led to two new dimensions applied in the second session. The grey bars specify the directions of the attended dimensions in the second session. As can be seen, all dimensions in the second session had angles closer to zero. The error bars indicate the 95% confidence interval obtained by bootstrapping. In the bootstrap procedure, we sampled the data set with replacement to create 100,000 artificial datasets and we calculated the angles. The 2.5 and the 97.5 percentile identify the lower and upper limit of the 95% confidence interval. There was no overlap between the confidence intervals of the angles in the first compared to

the second session. Therefore, all angles shifted significantly closer to zero and were in several cases even not significantly different from zero in the second session.

[Insert Figure 4 about here]

Although the transformation from the first to the second session showed the same trend for all the participants (as noted in Figure 3), the orientations (α and β) in the first session were significantly different between participants. The confidence intervals of the black bars in Figure 4 do not overlap. However, it is useful to point out that the differences were numerically rather small (a few degrees). Moreover, after applying the transformation, the differences between observers disappeared (the confidence intervals between grey bars overlap).

Discussion

The results displayed in Figures 3 and 4 show that the procedure succeeded in uncovering the dimensions that participants interpreted and used when judging dimensional shape similarities. The dimensions derived from their adjustments differ systematically from the dimensions manipulated by the arrow keys in the first session. (compare the bottom row to the top row in Figure 3). When we transformed the parametric shape space separately for each observer, in effect, creating their own individual shape space with the dimensions that they used spontaneously, the dimensions of aspect ratio and curvature became aligned vertically and horizontally in the second session. The evidence is not necessarily strong because the three participants started from the same initial parametric space and by administering a similar task, similar results are rather likely, which might have resulted coincidentally in more orthogonality. To add more evidence and to validate the procedure more

profoundly, we conducted a similar experiment (1B), in which we started each session with different initial parametric dimensions for each participant.

Experiment 1B

In two separate sessions the parametric space was composed of an arbitrarily chosen linear combination of the dimensions of aspect ratio and curvature. If the orientations derived by the method provide an indication of the dimensions spontaneously interpreted by the participants, then the derived dimensions should differ in both sessions from the introduced dimensions and, moreover, after applying the corresponding transformation in each session, the same resulting pair of dimensions should be brought forward by the procedure for the two different sessions. In other words, if we plot the participants' dimensions from both sessions in the same coordinate system, they should all fall on top of each other. Experiment 1B can, therefore, provide evidence that the experimenter-defined dimensions applied in the key dynamics and presented in the instructions as examples of curvature and aspect ratio modifications did not bias the adjustments or the subjective criteria of both dimensions in the mind of the participants.

Methods

Participants. Three naive volunteers from the psychology graduate program at the University of Leuven participated in this study. Two of them did not participate in the Experiment 1A (S_4 and S_5).

Stimuli. The same stimulus space from the previous experiment was used but the parameter coordinates (c' , r') in session 1 and session 2 were arbitrarily

transformed by the affine transformations $\begin{bmatrix} 0.9900 & 0.1659 \\ -0.0595 & 0.9900 \end{bmatrix}$ and $\begin{bmatrix} 1.0021 & 0.0261 \\ 0.0803 & 1.0021 \end{bmatrix}$

for the first participant (S_2), $\begin{bmatrix} 1.0021 & 0.0261 \\ 0.0803 & 1.0021 \end{bmatrix}$ and $\begin{bmatrix} 1.0389 & 0.1741 \\ 0.2323 & 1.0389 \end{bmatrix}$ for the second

participant (S_4), $\begin{bmatrix} 0.9984 & 0.0260 \\ -0.0600 & 0.9984 \end{bmatrix}$ and $\begin{bmatrix} 1.0389 & 0.1741 \\ 0.2323 & 1.0389 \end{bmatrix}$ for the third participant

(S_5). The introduced dimensions differed by 0.14 radians to 0.28 radians from the initial dimensions in the previous experiment. Both S_2 and S_5 had one transformation in common with S_4 and one transformation not shared by S_4 .

Apparatus. The stimuli were presented on the same monitor and the same computer as in Experiment 1A. Stimuli were presented as light grey silhouettes (37 cd/m²) on a dark grey background (24 cd/m²) at a distance of 80 cm (max. size of 5° x 4°).

Procedure. The procedure and verbal instructions were identical to Experiment 1A.

Results

In Figure 5, the dimensions that participants used for their interpretations of equal aspect ratio and curvature are plotted as black dashed (session 1) and solid (session 2) lines. The introduced dimensions in the key dynamics are plotted as grey lines. For all three subjects, the attended dimensions of the first session are corresponding very closely to the attended dimensions in the second session. From bootstrapping the data in the same way as in Experiment 1A, the attended dimensions in both sessions did not differ significantly in four cases ($p > 0.05$) and in two cases there was a significant difference of only a few degrees. The two vertical preferred axes for the first participant (S_2) and the two horizontal preferred axes for the third participant (S_5) were significantly different. It is obvious that the attended directions

in the mind of the participant did not correspond to the introduced dimensions in the key dynamics (black axes are different from gray axes). This finding rules out the possibility that participants simply adopted the experimenter-defined dimensions in the key dynamics. Moreover, the participant-defined dimensions seemed to correspond largely in both sessions.

[Insert Figure 5 about here]

Discussion

Although different dimensions were assigned to the key dynamics in session 1 and 2, the method seemed to be consistent in pointing out the same parametric interpretations in the stimulus space. In other words, participants were very consistent concerning their own subjective criteria for the dimensions of aspect ratio and curvature. There was no significant difference between the participant-defined dimensions in the first and the second session for the majority of cases and when there was a difference, it was quite small.

The pattern of results presented in Experiment 1A and 1B provides substantial evidence that the procedure is useful to derive the participant-defined dimensions. A natural way to proceed from here is to verify whether these subjective dimensions would relate to any objective measures of psychological relevance. In the following section we will test independence and separability for the participant-defined dimensions, respecting the idea of converging operations by calling on different tasks.

PART II: ORTHOGONAL SHAPE DIMENSIONS ARE PERCEIVED AS INDEPENDENT AND SEPARABLE

Experiment 2

In Experiment 2, participants were asked to adjust a variable stimulus in all its aspects to one fixed stimulus. Such a task prompts a comparison in overall similarity between the variable and the fixed stimulus. The fixed stimulus was the most central stimulus in the shape space of Figure 1. Each adjustment trial will provide a location in the shape space that the participant judges to consist of the same shape as the reference stimulus. From all positions, we can estimate the probability mass function for each position in the shape space to become identified as the reference stimulus.

This procedure has been introduced by Alfonso-Reese (2001) to estimate perceptual noise in the context of General Recognition Theory (Ashby & Perrin, 1988; Ashby & Townsend, 1986). In GRT, each presentation of a stimulus elicits one point in the perceptual space of the stimulus set. Unfortunately, any percept is affected by noise, and therefore, the position of the elicited point is variable. In the common version of GRT, the noise probability distribution has a Gaussian profile. In the adjustment task introduced by Alfonso-Reese, the covariance estimation of the noise is carried out by the covariance estimation of all end locations of the variable stimuli in the adjustment task.

In Experiment 2, perceptual independence of the introduced and the derived dimensions were verified. More concretely, we examined whether both dimensional components correlated in the adjustments on overall similarity and we compared the

correlation for the initial introduced dimensions (see Figure 1) and the participant-defined dimensions.

Methods

Participants. S_1 , S_4 , and two naïve volunteers, S_8 , S_9 , participated in this study. The transformation T was already derived for the first two participants. For the last two participants (S_8 , S_9), the transformation was derived by the same procedure as in Experiment 1A.

Stimuli. In the first condition, the originally imposed dimensions were used in the key assignment during the adjustments (see Experiment 1A). The adjustments led to a gradual manipulation of the contour of the variable stimulus, a change that was perceived in a smooth and continuous way by the participants. In the second condition, the dimensions, extracted from the participant's mental shape space using the method of Experiment 1, were applied in the horizontal and vertical key dynamics.

Apparatus. The stimuli were presented on a 17" Dell monitor with a resolution of 1600 by 1200 pixels and a color depth of 16 bit (60 Hz for these settings). The PC was a Dell computer Gx270, Pentium IV, 2.8 GHz. Stimuli were presented as light grey silhouettes (37 cd/m^2) on a dark grey background (24 cd/m^2) at a constant distance of 68 cm with the aid of a chin rest. The stimuli were maximally 5° high and 2° wide.

Procedure. There were 64 adjustment trials for each condition and each condition had a different key assignment during the adjustments: in 64 trials, the keys manipulated the initial dimensions and in another 64 trials, the keys manipulated the dimensions chosen by the participant. The 128 trials from both conditions were randomly intermingled in one session. The naïve participants were not aware that the dimensions in the key assignment could differ from trial to trial.

In each trial, the fixed stimulus was displayed in the middle of the upper half of the screen and the variable stimulus was displayed in the middle of the lower half of the screen with an independent random offset of ± 120 pixels, horizontally and vertically.

Results

The difference in key dynamics was not visible to the human eye and in Experiment 1B it was actually shown that the adjustments were not affected by which dimensions were implemented in the arrow keys. Therefore, all 128 trials were grouped together to increase power and the correlations were computed based on all trials, either represented in the coordinate system of the initial dimensions (r_E) or represented in the coordinate system of the participant's chosen dimensions (r_P).

In Table 1, the Pearson correlations are given for each of the four participants (rows) in each of the two coordinate systems: the first column presents the one based on the Experimenter dimensions and the second column presents the one based on the Participant dimensions. In the first column, all correlations were significantly different from zero, while in the second column, all correlations were close to zero and none differed significantly from zero. The third column presents the statistical significance for the difference between the two correlations per participant.

[Insert Table 1 about here]

Discussion

In the initial coordinate system, all correlations were significantly different from zero, indicating that these dimensions are not perceptually independent (as

defined in the GRT framework). Of course, the reason why the initial dimensions did correlate is because the initial dimensions were not complying with the dimensional structure in the stimuli that participants extracted. In the GRT framework, the initial dimensions of the stimulus space would not have constituted perceptual dimensions. However, in the participant's coordinate system, all correlations were close to zero indicating that the dimensions in the mind of the participant were perceptually independent. Although the dimensions in the participants' mind thus appeared to be independent, the differences between the correlations were only significant for the first participant and marginally significant for two other participants. However, because the arbitrary initial dimensions are geometrically rather similar to the participant-defined dimension, the small difference in correlation is rather normal. It is obvious that the improvement could be much larger if we started from initial dimensions that deviated much more from the participant-defined dimensions, although it is impossible to know a priori how much the initial and the derived dimensions will deviate from each other.

From the results in Experiment 2, we can conclude that the dimensions that the participants have chosen to use in expressing the perceived shape similarities were independent in the neighborhood of the centre of the stimulus space.

Experiment 3

In Experiment 3, we will address the more specific question whether the internal dimensions of curvature and aspect ratio in the mind of the participants are perceived more separably than the somewhat arbitrarily defined dimensions introduced by the experimenter in Experiment 1.

From the previous experiments, we know that the difference between both dimensional structures is rather modest and thus a sensitive method is needed. A sensitive method is the filter and redundancy task in constrained speeded classification (Ashby & Maddox, 1994; Garner & Felfoldy, 1970). By using the filter task, we will evaluate whether speeded categorization responses for stimuli differing on one dimension are affected by variations on an irrelevant dimension, a phenomenon also called Garner interference. We will evaluate Garner interference for speeded classifications on the initial dimensions and on the participant-defined dimensions.

In previous speeded classification tasks, participants were asked to classify the stimuli as fast as possible according to one relevant dimension. Depending on the subset of stimuli that needed to be classified, there were usually three conditions. First, in the one-dimensional condition, a subset of two stimuli was used that varied only on one dimension which was relevant for classification (e.g., for classifications on curvature, possible subsets in Figure 1 were stimulus 1 and 2 in one block of trials and stimulus 3 and 4 in another block). Second, in the correlated condition, a subset of two stimuli differing on both dimensions had to be classified. Although the stimuli differed on both dimensions, only one dimension was instructed to be relevant for the classification task while the second dimension was redundant (e.g., stimuli 1 and 3 in one block or 2 and 4 in another block). Third, in the orthogonal condition, all four stimuli within the same block were involved in the classification task. However, only one dimension was relevant for the classification task, so two stimuli belonged to one category and the other two stimuli belonged to the second category. When stimuli involved a separable dimensional structure, it was assumed that in the orthogonal

condition, the variation on the irrelevant dimension did not affect the categorization speed compared to the categorization speed in the one-dimensional condition.

In the version of the method that we employed in Experiment 3, the orthogonal condition was compared to the one-dimensional condition between two configurations of stimuli. The two configurations consisted of identical positions in the stimulus space but in reference to slightly different coordinate systems (the one introduced by the experimenter in the method section of Experiment 1 and the one derived from the participant's settings). The dependent variable of interest was the difference in reaction times (RTs) between the one-dimensional and the orthogonal conditions for the original introduced dimensions ("Experimenter") compared to the participant-defined dimensions ("Participant"). If the dimensions derived from the participant were more separable, then the RTs between the one-dimensional and the orthogonal condition should converge more in the conditions using those perceptually defined dimensions than with the original defined dimensions.

Methods

Participants. S_1 , S_3 , S_4 and S_8 , participated in this study. The transformation T was already derived for all these participants in former experiments.

Stimuli. The original imposed dimensions are similar to Experiment 1 and 2. Six stimuli were selected in an orthogonal configuration as displayed in Figure 6. In the one-dimensional condition, there are three subsets each consisting of one pair of stimuli (framed by a dashed rectangle in Figure 6). Each pair differed only on the relevant difference for classification as it was instructed to the participant. Each pair occurred only in one block of trials. In the orthogonal condition, however, all six stimuli were mixed randomly within a block but the classification rule remained the same. As can be seen in Figure 6, the variation along the irrelevant axis of curvature

was three times larger than the variation along the relevant axis of aspect ratio. By inducing more Garner interference, we hoped to obtain more powerful measurements. The same configuration rotated by an angle of 90° (not displayed) was also used for measuring classification performance on the dimension of curvature. Both configurations and conditions were repeated in each dimensional structure (the one preferred by the participant and the one introduced by the experimenter).

[Insert Figure 6 about here]

Apparatus. The stimuli were presented on a 17" Dell monitor with a resolution of 1024 by 768 pixels and a color depth of 16 bit (75 Hz for these settings). The PC was a Dell computer Gx270, Pentium IV, 2.8 GHz. Stimuli were presented as light grey silhouettes (37 cd/m^2) on a dark grey background (24 cd/m^2) at a constant distance of 68 cm with the aid of a chin rest. The stimuli were maximally 7° high and 3° wide. Responses were collected on a response box 200A with a baud rate of 19.2 Kbs. The most left and right keys on the response box were used in the categorization task.

Procedure. For two participants, there were six sessions in the following order: first, a practice session on the original dimensions, then a practice session on the derived dimensions, followed by a session on the original dimensions and a session on the derived dimension, all in one day. The next day the order was switched by first applying a session on the derived dimensions and then a session on the original dimensions. For the other half of the participants, the session order between original and derived dimensions was switched.

Each session consisted of twelve blocks of 54 trials each: six blocks for curvature and six blocks for aspect ratio. Three blocks involved the one-dimensional

condition (successive blocks for each separate stimulus pair) and three blocks involved the orthogonal condition (all three stimulus pairs intermixed). In each block, the stimuli were displayed in random order. To prevent a possible learning effect on our results as much as possible, we introduced some practice sessions before the start of the experiment, we used the same order of blocks within each session and counterbalanced the order of sessions between participants. The practice sessions were identical to the experimental sessions but each block contained only 6 trials instead of 54. Over two days, we obtained 324 trials for the orthogonal condition and 324 trials for one-dimensional condition per dimension in each axes configuration.

In each trial, a small black cross was presented in the middle of the screen for 250 ms followed by a blank interval of 300 ms. Stimuli appeared in the centre of the screen with a random horizontal and vertical offset ranging between -70 to 70 pixels. To avoid that participants would respond in a fixed cadence (i.e., with more or less the same speed on each trial, thereby shifting the potential effects from RTs to accuracy), the anticipation of the onset of a trial was disrupted by randomly changing the interval between the end of one trial and the beginning of the next trial within a range of 1050 to 1500 ms.

Results

In Table 2, the results for the four participants are summarized. The table contains the median of the RTs (in ms) for each participant (row) and each condition (column). Because we did not want to rely on the normality assumption concerning the distribution of the RTs, we used a bootstrap procedure in which we resampled the data and created 10,000 artificial datasets with the same number of experimental trials. For each dataset (the one obtained with the Experimenter-defined dimensions and the one with the Participant-defined dimensions), the median per condition was

calculated and the difference was computed between the orthogonal and the one-dimensional condition. These differences, d_E and d_P , respectively, provide a measure for the separability of the applied dimensions. The lower d_E and d_P , the more separable the dimensions are. Subsequently, the difference between d_E and d_P , and the confidence interval around it, were calculated by bootstrapping. Hence, $d_E - d_P$ designates the difference in separability between the initial dimensions and participant-defined dimensions (derived individually for each participant from adjustment settings, as explained in Experiment 1). Finally, P shows the relative frequency in the bootstrap procedure that the participant-defined dimensions have a higher measure for separability (less difference between the orthogonal and the one-dimensional condition) than the dimensions introduced by the experimenter. The relative strength of the improvement by using the participant's dimensions is expressed by $d_E - d_P$. In nearly all cases, the effect is positive (except for α in S_3) and we even obtained a significant improvement within multiple participants (β in S_1, S_4, S_8 , and marginal significant in S_3 and α in S_1).

[Insert Table 2 about here]

Discussion

Except for the curvature dimensions of participant S_3 , the results showed a clear tendency towards greater separability for the participant-defined dimension compared to the dimensions initially introduced by the experimenter. The improvements for the direction of constant curvature were rather modest, except for the first participant. A modest improvement could have been expected because the angle α was rather small. The angle β , in contrast, was larger and therefore more

improvement in separability could be expected for the dimensional direction of constant aspect ratio. In other words, by rotating the introduced dimensions, aspect ratio was perceived more constant and Garner interference on curvature classifications was reduced to a greater extent. Participants seemed to rely more on the dimensional structure of the stimuli when the irrelevant variation was induced by the participant-defined dimensions.

Experiment 4

In the last quest for independence, we used the same adjustment task as the one introduced in Part I, but in the light of converging operations (Garner et al., 1956), we investigated different operational definitions of the introduced dimensions and applied them on different basic contour shapes. As we argued in the introduction, aspect ratio and curvature are rather abstract dimensions in the context of rather complex stimuli with variable features and contour profiles. The different stimulus sets in Experiment 4 were created by a factorial combination of three different complex shapes (shown in the legend of Figure 8) with six slightly different geometric definitions of aspect ratio and curvature (see $f_1(x,y)$ to $f_6(x,y)$ in Figure 7).

The goal of Experiment 4 was to test the stability of the preferred participant's definitions of aspect ratio and curvature for different shapes and different parametric definitions of aspect ratio and curvature. If a preferred basis to judge similarity among shapes exists and if our method is indeed capable of extracting the way in which participants attend to variations in aspect ratio and curvature among complex shapes, then the present experiment should yield the same pair of dimensions, regardless of the different contour shapes on which the initial dimensions of aspect ratio and

curvature were applied. Moreover, this consistency should be observed with the different geometric parameterizations presented to the participants.

Methods

Participants. Two naive volunteers from the psychology and neuroscience graduate programs at the University of Leuven (S_6 and S_7) participated in addition to author BO (S_1) in this experiment. The two naïve subjects had not participated in the previous experiments.

Stimuli. There were 18 stimulus sets in this experiment, created by applying six slightly different definitions of the dimensions of aspect ratio and curvature to each of three basic shapes. Each different definition led to slightly different image deformations. In Figure 7, the image deformations are shown for one of the three shapes. The first panel adopts the image deformation that was also used in Experiment 1A; in the following panels (from 2 to 6) the image deformations can be expressed by:

$$I: (x, y) \rightarrow f_2(x, y) = \left(c^2x + [1 - \cos(\frac{y}{h}r)] \frac{h}{2c \sin(\frac{r}{2})}, \frac{h \sin(\frac{y}{h}r)}{2c \sin(\frac{r}{2})} \right),$$

$$I: (x, y) \rightarrow f_3(x, y) = \left(cx + [1 - \cos(\frac{y}{h}r)] \frac{h}{cr}, \frac{h \sin(\frac{y}{h}r)}{cr} \right),$$

$$I: (x, y) \rightarrow f_4(x, y) = \left(cx + [1 - \cos(\frac{y}{h}r)] \frac{ch}{r}, \frac{h \sin(\frac{y}{h}r)}{cr} \right),$$

$$I: (x, y) \rightarrow f_5(x, y) = \left(cx + [1 - \cos(\frac{y}{h}r)] \frac{ch}{2 \sin(\frac{r}{2})}, \frac{h \sin(\frac{y}{h}r)}{2c \sin(\frac{r}{2})} \right), \text{ and}$$

$$I: (x, y) \rightarrow f_6(x, y) = \left(x + [1 - \cos(\frac{y}{h}r)] \frac{h}{2 \sin(\frac{r}{2})}, \frac{h \sin(\frac{y}{h}r)}{2c^2 \sin(\frac{r}{2})} \right), \text{ respectively.}$$

These formulas have no specific psychological value and should be considered purely as tools to investigate the consistency of the derived dimensions for different image deformations applied to different shapes in the present study. (Other studies

have been devoted specifically to investigating the psychophysical dimension of curvature in simplified stimuli; see Foster, Simmons, & Cook, 1993; Foster & Wagemans, 1993). Note, however, that Panels A and B, as well as Panels E and F, have identically shaped stimuli, except for a difference in size change when going from left to right in each shape space. It is easy to verify in the equations above that the coordinates differ only in scale by the parameter c . Such a correspondence helps to detect if participants are looking at absolute metric measures or rather at relative, scale-independent measures such as the ratio between height and width, for instance.

Apparatus. The same apparatus was used as in Experiment 1. Stimuli also had the same size and luminance as in Experiment 1.

Procedure. Six different combinations of the dimensions of aspect ratio and curvature, applied to three basic stimuli, resulted in 18 different stimulus spaces. A square configuration of reference stimuli was chosen in each of these shape spaces. Eight trials per variable stimulus per participant (instead of 16 as in the previous experiments) were collected, leading to 576 trials per participant, distributed over eight sessions, each lasting approximately one hour. The sequence was completely randomized and all different shapes and image deformation schemes were mixed. The task and the instructions were exactly the same as in Experiment 1.

[Insert Figure 7 about here]

Results

The dependent variables are the orientations derived from the adjustments. They are displayed in Figure 8 for each type of image deformation (6 rows), each

participant (3 columns), and each basic shape (3 different colors in the plots), which were the three independent variables in the analyses. An indication of the consistency of the dimensional orientations is given by the effect size for these independent variables. A measure of the effect size is the contribution of these variables to the explained variance in the full ANOVA model. We treated the variable “participant” as a factor just like the other two variables because we are dealing with individualized (possibly idiosyncratic) mental shape spaces, derived from adjustments by individual participants.

For the dependent variable α (i.e., the orientation in which curvature is perceived as constant), the variable “image deformation” determines 84% of the explained variance, while the variables “participant” and “shape” contributed only 2.8% and 3.2% of the explained variance, respectively. Moreover, the interactions between “shape” and “participant”, and between “shape” and “image deformation”, explained only 1.8% and 4.4% of the variance, respectively.

For the angle β (i.e., the orientation in which aspect ratio is perceived constant), a different pattern emerged: the variable “image deformation” explained 64.9% of the total variance, while “participant” introduced 23.8% of the variance and “shape” only 1.4%. The interactions between “shape” and “participant”, and between “shape” and “image deformation” explained only 1.1% and 5.1% of the variance, respectively. However, the larger proportion explained variance by the variable “participant” to the values of β could be attributed completely to the difference between the third participant (S_7) and the other two participants. One should note that this deviation was fairly consistent in all six panels (compare the third column with the first two columns in Figure 8): The vertical dimension in the last column is always shifted with a constant angle (of about 0.15 radians in the counterclockwise direction

on average) compared to the other two participants. Running the same analysis without the third participant resulted in only 0.6% explained variance by “participant”, 90% by “image deformation” and 1.2% by “shape”.

[Insert Figure 8 about here]

Discussion

The angles α and β were fairly consistent for the independent variable “shape” and also to a smaller extent for the variable “participant”. For the orientation of the most horizontal dimension (α), all three participants were consistent; for the orientation of the most vertical dimension (β), especially the first two participants, the author BO (S_1) and a naïve participant (S_6), were consistent. The consistency in each participant for the different shapes appeared in each of the six parameterization contexts. The consistency found in Experiment 1 among participants is therefore not just a result of the specific parameterization or the task that we applied there. It generalizes to different shapes and parameterizations.

The stimuli from Figure 6A and 6B, and also from Figure 6E and 6F, have identical shape, differing only in size. As seen in Figure 8, the orientations of the axes in the mental shape spaces of our participants are very similar for rows A and B, as well as for rows E and F. This is an indication that the extracted dimensions are scale-invariant (as they should for true shape dimensions). It is also worth pointing out that the specific geometric definitions used in Panel B yielded the smallest angles, indicating that the specific physical dimensional structure imposed in those shape spaces best approached the shape dimensions attended to and used by the participants. Hence, the perceived dimensions of aspect ratio and curvature corresponded rather

closely to these physical dimensions. It is because of this property that we have also used this particular stimulus space in another study with single-cell recordings in macaque monkeys (De Baene, Ons, Wagemans, & Vogels, 2008).

The consistency of the variable “shape” can be seen in each panel of Figure 8: the axes in the mental shape spaces of each participant are more or less on top of each other for each different shape. A consistent pattern of axes between different shapes can only emerge when two conditions are fulfilled. First, participants need to be consistent in their criteria for deciding when the variable stimulus is equal to the fixed stimulus with respect to the shape dimensions of aspect ratio and curvature. Secondly, the variations in aspect ratio and curvature need to be perceived in a consistent way between different shapes. In other words, the preferred interpretation of aspect ratio and curvature are independent of the local contour shape differences. The consistency of the preferred dimensions in the shape spaces reveals that the dimensions have psychological significance.

GENERAL DISCUSSION

Main findings of the present study

Whether the processing of multidimensional stimuli is analytic or holistic is not an all-or-none issue but may depend also on stimuli, task, available processing time, and attentional factors (e.g., Ward et al., 1986). It was this consideration that led to Garner’s (1974) notion of the need for converging operations to evaluate the separable and integral nature of dimensions.

Building on the notion of converging operations, our study tried to shed light on the processing of a pair of shape dimensions: aspect ratio and curvature.

Stankiewicz (2002) showed that our visual system represents the dimensions defining a 3-D bar shapes (e.g., its aspect ratio and the main axis curvature) independently of the angle from which it is viewed (see also Demeyer, Zaenen, & Wagemans, 2007; Foster & Gilson, 2002). We applied similar dimensions to 2-D stimuli and we introduced a method to derive participants' intuitions of these dimensions parametrically through a linear combination of the introduced dimensions. Therefore, we were able to implement the participants' intuitions of these dimensions as new parametrical dimensions of interest in various tasks. We tested these dimensions for different kinds of independence and we found that the participant-defined dimensions possess psychological relevance in the sense that they did not correlate in the adjustments on overall similarity (see Experiment 2), because they had a higher degree of separability than the experimenter-defined dimensions (see Experiment 3) and because they were chosen consistently for different kinds of shapes (see Experiment 4).

Different views on the integrality continuum

Based on the differences between primary and secondary perceived stimulus structures (Garner, 1974; Kemler Nelson, 1993; Lockhead, 1972; Shepard, 1987; Shepp, 1989), we can distinguish different views on the integrality continuum.

According to the first view, the primary processing of integral dimensions is holistic and the primary processing of separable dimensions is analytic. Perceived integral dimensions like color brightness and saturation are secondary in the sense that they are derived from perceived stimuli when particular task demands foster a dimensional analysis. For instance, in a generalized identification task (Foard & Kemler Nelson, 1984), participants learned labels for three stimuli and were instructed

to generalize in line with the implemented axis in the stimulus space composed of the dimensions brightness and saturation. Performance was superior when the axes complied with the dimensions indicating that the dimensions can entail a privileged status when the task fosters dimensional analysis. Kemler Nelson (1993) indicated that privileged axes can be obtained for integral dimensions, although they are less apparent than for separable dimensions. Although the primary perceived stimulus structure for integral dimensions is based on overall similarity, the dimensional structure can be derived as well and becomes available when the task encourage analysis like, for instance, the task that we introduced in the first part of the present study. It is not clear whether the dimensional structure is primary or secondary in the present study because the task can probably also be used to infer participant-defined dimensions for integral dimensions as long as participants can form an understanding of the dimensions of interest. Although the participant-defined dimensions of aspect ratio and curvature seemed to provide more separable dimensions in the speeded categorization task in Experiment 3, a perfectly separable structure was not found as Garner interference was still obtained to some extent for the participant-defined dimensions. However, it should be noted that the variation on the irrelevant dimensions was two times larger than the variation on the relevant dimensions, and this may have strengthened an interference effect compared to the traditionally employed filter task where both variations were set equal (e.g., Garner & Felfoldy, 1970). In any event, these findings support the original idea (Garner, 1974; Lockhead, 1972; Shepard, 1964) that separability and integrality form a continuum instead of two separate classes.

The second view assumes a holistic primary organization for all stimuli (e.g., the “blobs” of Lockhead, 1972; see also Kemler Nelson’s view on integrality, 1989,

1993). The dimensional features become available only when the stimulus is further analyzed and analysis serves a different function than object recognition. The present study might provide an indirect clue to the need for analytic processing in a later processing stage. For instance, the thickness and degree of curvature of a stick or a tree branch do not matter for identification (at the basic category level) but they will affect strongly what one can do with it (e.g., turning it into a walking stick, spear, bow or arrow). In the holistic view, object identification is a holistic process and secondary analytic processing might serve this evolutionary gift of humans to manipulate, create and make objects (see also Gibson, 1979; Norman, 1988). In the present study, the task that probed for dimensions forced participants to manipulate the shape of the stimulus. The evidence in Experiment 4 indeed provides an indication that aspect ratio and curvature can be analyzed separately in a similar way for three differently shaped stimuli, allowing for a similar action-related handling of different objects.

In the third view, the claim is made that all objects are perceived through dimensions and analytic processes (e.g., Melara, Marks, & Potts, 1993a, 1993b). However, the dimensions that are processed analytically by a participant in an experiment might differ from the dimensions that the experimenter has put forward as the dimensions of interest. For example, manipulating rectangles by their height and their width might result in data that suggest an integral dimensional structure. However, in the same parametric space, two new dimensions emerge when height and width are combined in a specific, correlated way, namely, aspect ratio and size. Contrary to width and height, aspect ratio and size operate as separable dimensions (for emergent properties, see Cheng & Pachella, 1984; Pomerantz & Pristach, 1989; Pomerantz, Sager, & Stoeber, 1977). Integrality is then considered as an artifact of the

wrong choice of dimensions by the experimenter. Separability is then attributed to the privileged or preferred basis in the parametric stimulus space.

Deriving basic building blocks for shape perception

In the logic of the overall analytic view of processing, separable dimensions are closely related to separate units of processing that correspond to the coding units of the visual process. When the dimensions are processed by separate units, they can be attended selectively and, therefore, tasks investigating selective attention, like the filter task, are helpful to find the dimensions of the visual system. This logic has been put forward by, for instance, Pomerantz (1989) and has motivated multiple researchers to disentangle the “separable” dimensions of the visual system (Duncan, 1984; Kahneman & Henik, 1981; Op de Beeck et al., 2001; Pomerantz, 1981, 2003). The task introduced in Experiments 1 provided two dimensions that participants attended selectively as they adjusted only one dimension at a time and ignored the second one. The attended dimensions indeed led to more separability in the more classical paradigm of Experiments 3 and, therefore, the relation between selective attention and separability seems to hold. However, it is hard to prove that these dimensions are the basic building blocks that the visual system binds together in object perception.

Nevertheless, being able to reveal the basic building blocks of shape perception seems an important prerequisite for making progress in understanding shape perception, similarly to the way in which vision science has made progress in understanding low-level visual processing. One of the basic building blocks in the early stages of the visual system seems to correspond with Gabor filters (e.g., DeValois & DeValois, 1980; Marcelja, 1980), which respond maximally to a Gabor patch with optimal corresponding properties. (A Gabor patch is the product of a

sinusoidal luminance profile characterized by a particular orientation and a particular spatial frequency and a Gaussian envelope characterized by a particular size and a particular aspect ratio.) Progress in vision science has depended strongly on the use of such well-controlled stimuli and the manipulation of these characteristics or dimensions (e.g., Heeger, 1994; but for more recent discussions, see Carandini et al., 2005; Rust & Movshon, 2005).

It is relatively simple and straightforward to manipulate and control individual physical dimensions of visual stimulation like luminance, contrast, orientation and spatial frequency, in order to characterize the detectors or filters used by the early visual system because all these dimensions are perceived as qualitatively different and they can be attended independently from each other. If we adopt the same principles for more complex stimuli, then finding dimensions that can be manipulated and attended independently could help to characterize the tuning properties of the neurons in the higher cortical regions. The method applied in this study could help to fine-tune shape dimensions towards more separability. In former behavioral methods, separability was tested against integrality only on dimensions explicitly introduced in the stimulus set and researchers therefore made a choice regarding what they assumed to be relevant shape dimensions. In contrast to former behavioral methods, this new approach provides the means to find privileged directions in the stimulus set and might help to develop better-controlled stimuli to reveal the basic properties of more complex stimuli and high-level cortical cells (e.g., De Baene et al., 2008; Kayaert et al., 2005).

Early stages in the visual system are mainly involved in spatial filtering operations at various frequencies and orientations (DeValois & DeValois, 1980; Marcelja, 1980). The higher stages of the visual system might try to detect and tap

patterns of diagnostic information from the stimulus descriptions at the lower stages and might group this information into high-level dimensions that are useful in the context of a particular task. Such organization can explain the independence between the global shape dimensions of aspect ratio and curvature and the more local shape features expressing the shape differences between the three basic stimuli. In Figure 9, the three stimuli from Experiment 2 were filtered with a low-pass spatial filter in the upper panels where mainly global shape information is presented and a high-pass spatial filter in the lower panels showing also local shape information. The link between various Gestalt phenomena and lower spatial frequencies has been established before (Ginsberg, 1986). The stimuli in Figure 9 have the same aspect ratio and curvature and they seem to be more similar in the upper panels where only low spatial frequencies are presented. However, it is still possible to provide a rudimentary estimate of the curvature and the aspect ratio of these low-passed filtered stimuli. Alternatively, the shape differences appear more salient in the lower panels in which the higher spatial frequencies are enhanced. Here, the points of maximal curvature are more illuminated and we know that these points are more salient (e.g., De Winter & Wagemans, 2008) and are useful in shape segmentation (e.g., De Winter & Wagemans, 2006).

Therefore, the idea that global shape dimensions emerge from particular properties in the spatial frequency domain at a lower bandwidth while local shape information emerge from properties at higher bandwidths could explain the independence between the global shape dimensions and the more detailed shape differences between the three basic stimuli.

[Insert Figure 9 about here]

Conclusion

It is still an open issue whether the processing of multidimensional stimuli is analytic or holistic. However, we do believe that the approach introduced and tested here has the potential to extract privileged axes from participants mental shape space. In doing so, we used a method that taps rather directly in their intentional and conscious way of expressing shape dimensions. Moreover, we showed that these dimensions constitute perceptual dimensions because they were perceived as fairly separable and as independent, using methods that were developed before to access the more automatic and unconscious way of processing shape. This convergence of results offers promise to study higher-order perception of complex shape and to link it to lower-order visual properties.

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Appendix 1

A stimulus set can be constructed based on a whole variety of physical attributes. When only two attributes are manipulated, the two relevant attributes could serve as a basis for ordering the stimulus set. If these attributes are quantifiable by a parametrical value within a real-valued open interval, the stimulus set can be represented by an open connected region in a plane and each stimulus can be related to one vector in this region, composed of a 2-tuple of coordinates, one coordinate function for each attribute. Such stimulus sets have been used frequently in visual perception as well as in categorization research where object representations have frequently been modeled by vectors (e.g., Ashby & Perrin, 1988; Edelman, 1999; Nosofsky, 1984; Shepard, 1987).

When the stimulus modulation during the adjustments cannot be programmed in real time, then the stimuli can be made beforehand for each position in a rectangular grid covering the stimulus space. The gaps between successive grid points should be sufficiently small to ensure that no discontinuities are perceived during the adjustments. During the adjustments, the stimulus moves from one grid point to a neighboring one in the stimulus space by pressing the arrow keys. Adjustments are carried out based on local similarity along the dimensions that participants picture in their mind. Extracting these dimensions from the participants' mental shape space is the goal of the procedure.

In Figure A1, there are three panels. Panel A represents the Cartesian coordinate system in line with the design of the stimulus set. For example, Dim1 might be the direction in which the introduced dimension of curvature is kept

constant. Inversely, Dim2 might represent the direction in which the parameter of aspect ratio is kept constant. Panel B represents a hypothetical example of a coordinate system more suitable for the cognitive dimensions in the mind of the participant. The horizontal thick line is the direction in which curvature is perceived as constant and the vertical thick line is the direction in which aspect ratio is perceived as constant. A subject will adjust the stimulus parallel to these directions as seen in panel B. In Panel C, the coordinate system is transformed inversely to the initially introduced coordinate system and the cognitive dimensions in the mind of the participant are plotted as slanted lines. These slants are the orientations of interest.

Two arrangements are chosen freely: First, an angle is positive in the counterclockwise orientation and, second, the origin is chosen in the middle of the two fixed stimuli in Figure A1. In Panel B of Figure A1, it is evident that Dim1 can be expressed as an oriented line with coordinates $1, \tan \alpha$ x and x is a perimeter running along the line and α is the angle between Dim1 and the horizontal axis. Likewise, Dim2 can be expressed by the coordinates $-\tan \beta, 1$ y and y is a perimeter running along Dim2 and β is the angle between Dim2 and the vertical axis. Note that $-\tan \beta$ is a positive number because β is negative. From the expressions of both implemented dimensions, any position (x, y) in the initially imposed space can be expressed by the sum $1, \tan \alpha$ $x + -\tan \beta, 1$ y , or by the tuple $(x - \tan \beta y, \tan \alpha x + y)$ in the space of the participant's mind. If the coordinates in the participant's mind are denoted by (x', y') , then the following equation transforms the locations of the initial space towards the participant's space:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}, \quad (1)$$

with

$$\mathbf{T} = \begin{bmatrix} 1 & -\tan\beta \\ \tan\alpha & 1 \end{bmatrix} \quad (2)$$

It is relative easy to express the desired parameters α and β in terms of the x', y' coordinates. However, only the initial coordinates x, y corresponding to the initial dimensional structure are known by the experimenter. Therefore, the experimenter would like to express the desired parameters α and β in terms of these initial coordinates. Suppose that the coordinates for P, F₁ and F₂ are denoted by

x_p, y_p , x_{F_1}, y_{F_1} and x_{F_2}, y_{F_2} , then the coordinates x'_p, y'_p , x'_{F_1}, y'_{F_1} and x'_{F_2}, y'_{F_2} in the mind of the participant can be found through (1).

A participant minimizes the horizontal distance between P' and F'₂ and once she shifts her attention to the other dimension she minimizes the vertical distance between P' and F'₁. The two distances that she minimizes (in absolute terms) are denoted by, $x'_p - x'_{F_2}$ and $y'_p - y'_{F_1}$, or in terms of the experimenter's coordinate system, the distances become: $(x_p - x_{F_2}) - \tan\beta (y_p - y_{F_2})$ and

$\tan\alpha (x_p - x_{F_1}) + (y_p - y_{F_1})$, respectively. In Figure A1, Panel B, both distances are

zero (the coordinates coincide on the X-axis and the Y-axis, respectively). However, a real participant will perform less ideally and these distances might differ slightly from zero. If a participant performs multiple trials, a good robust approximation for the distances is obtained by the average location of the adjusted probe stimuli for each

pair of fixed stimuli. The distances become: $\bar{x}_p - x_{F_2} - \tan\beta (\bar{y}_p - y_{F_2})$ and

$\tan\alpha (\bar{x}_p - x_{F_1}) + (\bar{y}_p - y_{F_1})$, respectively, for all trials related to F₁ and F₂. When

there are more average variable stimuli obtained from different pairs of fixed stimuli ($i = 1, 2, \dots, k$), then there are more similar distances that should be minimized, which we will do by minimizing the sum of the squared distances. The sums for the vertical distances and horizontal distances can be expressed by:

$$f(\tan \alpha) = \sum_{i=1}^k [\tan \alpha (\bar{x}_{pi} - x_{F1i}) + (\bar{y}_{pi} - y_{F1i})]^2 \text{ and}$$

$$f(\tan \beta) = \sum_{i=1}^k [(\bar{x}_{pi} - x_{F2i}) - \tan \beta (\bar{y}_{pi} - y_{F2i})]^2, \text{ respectively.}$$

[Insert Figure A1 about here]

We find the minima for $\tan \alpha$ and $\tan \beta$ by setting $\frac{d(f(\tan \alpha))}{d(\tan \alpha)} = 0$ and

$\frac{d(f(\tan \beta))}{d(\tan \beta)} = 0$. Filling in the single solutions in the second derivative leads to positive

numbers and therefore the solutions are minima. If we introduce the following vectors

$$\mathbf{X}_\alpha = [\bar{x}_{p1} - \bar{x}_{F11} \quad \bar{x}_{p2} - \bar{x}_{F12} \quad \dots \quad \bar{x}_{pk} - \bar{x}_{F1k}] \text{ and}$$

$$\mathbf{Y}_\alpha = [\bar{y}_{p1} - \bar{y}_{F11} \quad \bar{y}_{p2} - \bar{y}_{F12} \quad \dots \quad \bar{y}_{pk} - \bar{y}_{F1k}], \text{ then } \alpha \text{ can be obtained through:}$$

$$\tan \alpha = -\frac{\mathbf{X}_\alpha \cdot \mathbf{Y}_\alpha}{\mathbf{X}_\alpha \cdot \mathbf{X}_\alpha} \quad (3)$$

Similarity, introducing the vectors $\mathbf{X}_\beta = [\bar{x}_{p1} - \bar{x}_{F21} \quad \bar{x}_{p2} - \bar{x}_{F22} \quad \dots \quad \bar{x}_{pk} - \bar{x}_{F2k}]$

and $\mathbf{Y}_\beta = [\bar{y}_{p1} - \bar{y}_{F21} \quad \bar{y}_{p2} - \bar{y}_{F22} \quad \dots \quad \bar{y}_{pk} - \bar{y}_{F2k}]$, β can be obtained through:

$$\tan \beta = \frac{\mathbf{X}_\beta \cdot \mathbf{Y}_\beta}{\mathbf{Y}_\beta \cdot \mathbf{Y}_\beta} \quad (4)$$

where the operator “.” denotes the inner product of the two vectors. In (3) and (4), the solutions for the orientations are expressed in terms of the initially introduced coordinates.

From the solution, the transformation \mathbf{T} can be derived. The transformation \mathbf{T} allows us to go from the representation seen in Panel A to the representation in Panel B. The inverse transformation provides a means to find the directions attended by the participant in the initial space or coordinate system (see Panel C). These graphical representations and transformations are frequently used in the experiments reported in the body of the paper.

To infer confidence intervals for the angles α and β , a preferred procedure is to use simulations after bootstrapping and to determine the limits of the confidence interval that capture 95% of these simulations. Simulations are frequently used in the experiments reported in the body of the paper. A second way to proceed is by implementing a parametric statistical test based on some assumptions (i) that the errors are independently and normally distributed (ii) with constant variance in the parametric space. According to the introduced model, a participant minimizes the vertical distance between a variable stimulus P' and the corresponding fixed stimulus F'_1 . The sum of squared distances in the participants mind can be expressed by:

$\sum_i (y'_{Pi} - y'_{F1i})^2 = \sum_i [\tan \alpha (x_{Pi} - x_{F1i}) + (y_{Pi} - y_{F1i})]^2$ and in the absence of a transformation ($\alpha=0$), the sum of squared distances is: $\sum_i (y_{Pi} - y_{F1i})^2$.

The F-test to validate the improvement for angle α is:

$$F = \frac{(n-1) [\sum_i (y_{Pi} - y_{F1i})^2 - \sum_i [\tan \alpha (x_{Pi} - x_{F1i}) + (y_{Pi} - y_{F1i})]^2]}{\sum_i [\tan \alpha (x_{Pi} - x_{F1i}) + (y_{Pi} - y_{F1i})]^2}, \quad (5)$$

Analogously, the F-test to test the relative fit of the angle β is:

$$F = \frac{(n-1) \left[\sum_i (x_{pi} - x_{F2i})^2 - \frac{[\sum_i (x_{pi} - x_{F2i})(y_{pi} - y_{F2i})]^2}{\sum_i (y_{pi} - y_{F2i})^2} \right]}{\sum_i (x_{pi} - x_{F2i})^2 - \frac{[\sum_i (x_{pi} - x_{F2i})(y_{pi} - y_{F2i})]^2}{\sum_i (y_{pi} - y_{F2i})^2}} \quad (6)$$

The degrees of freedom for the restricted model are equal to the number of trials (n) and the degrees of freedom for the model using a transformation are equal to the number of trials minus the number of free parameters, namely, the estimated angle ($n-1$).

Tables

	(x,y) Experimenter, $H_0:r_E=0$				(x,y) Participant $H_0:r_P=0$				Difference, $H_0:r_E-r_P=0$		
	r_E	n	t	p	r_P	n	t	p	r_E-r_P	z	p
S_1^*	0.33	128	3.95	<0.01	0.05	128	0.59	0.56	0.28	2.31	<0.05
S_4^*	0.19	128	2.2	<0.05	-0.02	128	-0.23	0.82	0.21	1.7	<0.1
S_8^*	0.28	128	3.28	<0.01	0.07	128	0.84	0.43	0.21	1.69	<0.1
S_9^*	0.27	128	3.09	<0.01	0.14	128	1.56	0.12	0.13	1.06	0.29

* For S_1 , S_4 , S_8 and S_9 , α is -0.084 ($F_{(1,63)}=71$, $p<0.001$), -0.146 ($F_{(1,63)}=33$, $p<0.001$), -0.068 ($F_{(1,63)}=8$, $p<0.01$), and -0.0119 ($F_{(1,63)}=0.6$, $p>0.05$), respectively, and β is 0.233 ($F_{(1,63)}=188$, $p<0.001$), 0.324 ($F_{(1,63)}=343$, $p<0.001$), 0.164 ($F_{(1,63)}=28$, $p<0.001$) and 0.1621 ($F_{(1,63)}=19.6$, $p<0.001$), respectively (see Appendix 1 for deriving parametric values and F-tests)

Table 1: The correlations for the initial introduced dimensions (r_E), the derived dimensions for each individual participant (r_P), and the difference between both kinds of coordinate systems.

	β (direction of constant aspect ratio)								
	(x,y) Experimenter			(x,y) Participant				Difference	
	orth	One	d_E	orth	one	d_P	d_E-d_P	95%CI	$P(d_E>d_P)$
S_1	483	414	69	438	437	1	68	[40 107]	1
S_3^*	798	678	120	811	760	51	69	[-25 153]	0.917
S_4	570	495	75	518	502	16	59	[21 94]	0.999
S_8	876	578	298	714	520	194	104	[28 207]	0.997

	α (direction of constant curvature)								
	(x,y) Experimenter			(x,y) Participant				Difference	
	orth	One	d_E	orth	one	d_P	d_E-d_P	95%CI	$P(d_E>d_P)$
S_1	425	385	40	414	412	2	38	[12 65]	0.999
S_3^*	590	510	80	586	492	94	-14	[-54 41]	0.350
S_4	513	446	67	499	438	61	6	[-22 34]	0.626
S_8	655	552	103	620	531	89	14	[-24 54]	0.749

* For S_3 , the transformation and the relative improvement for each dimension are $\alpha=-0.121$, ($F_{(1,63)}=53$, $p<0.001$) and $\beta=0.104$ ($F_{(1,63)}=13$, $p<0.001$) (see Appendix 1). The relative improvements for S_1 , S_4 and S_8 are denoted in Table 1.

Table 2: The median RTs in ms. d_E and d_P are the difference between the orthogonal condition and the one-dimensional condition for the same configuration of stimuli

relative to the experimenter's initial dimensions and the derived dimensions for each participant, respectively.

Figure Legend

Figure 1: A two-dimensional ordered stimulus set with stimuli on equidistant positions in the stimulus space based on the dimensions of aspect ratio (varying along the horizontal dimension) and curvature (varying along the vertical dimension).

Figure 2: The representation of a hypothetical outcome of four adjustment trials in the stimulus space.

Figure 3: The locations of the adjustments from session 1 in the initially introduced stimulus spaces (top panels). The adjustments of session 2 in the mental shape space (perceived shape dimensions in the participant's mind), using the calculated transformation derived from their original adjustments (bottom panels). CU stands for curvature, AR for aspect ratio.

Figure 4: The transformation angles α and β (left and right panel, resp.) with 95% confidence intervals.

Figure 5: A comparison between the initial and participant-defined axes of session 1 and 2.

Figure 6: Three subsets of two stimuli at the same height in the stimulus space, serving as the subsets for the one-dimensional condition for classifications with respect to aspect ratio. A version of this space, rotated by 90° , was also used for classifications with respect to curvature.

Figure 7: Six different stimulus spaces, all based on slightly different definitions of curvature (CU) and aspect ratio (AR).

Figure 8: Orientations of the preferred axes in the mind of the participants plotted in the coordinate system of the initially introduced stimulus spaces. The initially introduced spaces by the experimenter are shown in Figure 6. The circles at one end of the colored axes are denoting the cases by which the procedure can improve the orthogonality of the dimensions significantly ($p < 0.05$). To determine this statistical significance, we used the F-test explained in Appendix 12.

Figure 9: The three different stimuli used in Experiment 2 from left to right, filtered by a low-pass spatial filter (upper panels) and a high-pass spatial filter (lower panels).

Figure A1: The representation of a hypothetical outcome of a distal adjustment trial in (A) the stimulus space represented by the experimenter, (B) the same solution in the representational space of the participant, and (C) the perceptual dimensions by the subject plotted in the initial space (see Appendix 1 for a description).