Preconditioner for a Multilevel Fast Multipole Method (MLFMM) accelerated intrusive Stochastic Galerkin Method (SGM) based scattering solver

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***Abstract*— We present a preconditioner for an intrusive Stochastic Galerkin Method (SGM) based scattering solver that also leverages the Multilevel Fast Multipole Method (MLFMM). The proposed preconditioner is essential in developing a general and intrusive SGM method. The simulation results were obtained for a canonical scattering structure with perfect electrically conducting (PEC) strips with statistically varying geometry. Results are reported for the number of iterations, with and without using a preconditioner, and for the time required to setup the preconditioner.**

I. Introduction

Stochastic modeling of electromagnetic structures that exhibit inherent variability has been studied in recent years. Methods based on polynomial chaos expansion (PCE) have better accuracy and efficiency over traditional Monte Carlo (MC) analysis [1]. These methods can be divided into two classes: non-intrusive ones, which rely on reusing a traditional deterministic solver, and intrusive solvers, which require modification of the computational algorithm. Both types were combined with the Method of Moments (MoM) for solving Boundary Integral Equation (BIE) scattering problems [2]. The basic idea of all PCE based methods is to describe the random variations by a linear combination of polynomials. The number of polynomials K grows rapidly with the number of stochastic parameters and the polynomial order. The intrusive SGM approach results in a large linear system of equations that needs to be solved. To decrease the computational time needed to solve such a large system, the Multilevel Fast Multipole Method (MLFMM) is invoked [3]. It was shown that the calculation of matrix element interactions through a plane wave decomposition of the Green's function remains applicable in the stochastic case if the variations of sources residing in sufficiently separated boxes are mutually independent. The total complexity of one matrix-vector product is shown to be equal to the deterministic MLFMM complexity scaled by a factor that depends on K. To further decrease the solution time, the number of iterations in the iterative solver should be reduced.

We consider the same structure as in [3], which is a standard structure for analyzing novel methods. PEC strips are organized in a periodic two-dimensional, but finite array, as in Fig. 1. For each strip, its width w is considered as a random variable, as well as its y-coordinate of the position of its center, which is described by its relative offset h w.r.t. the nominal value. The nominal positions of the centers are equally spaced with a constant spacing T. The variability is described with vectors of widths w and heights h, which are chosen to be independent uniformly distributed random variables. The widths vary between $0.5T - \lambda/20$ and $0.5T + \lambda/20$, while heights vary between $-\lambda/20$ and $\lambda/20$, where $T = 0.5\lambda$, and λ is the wavelength.

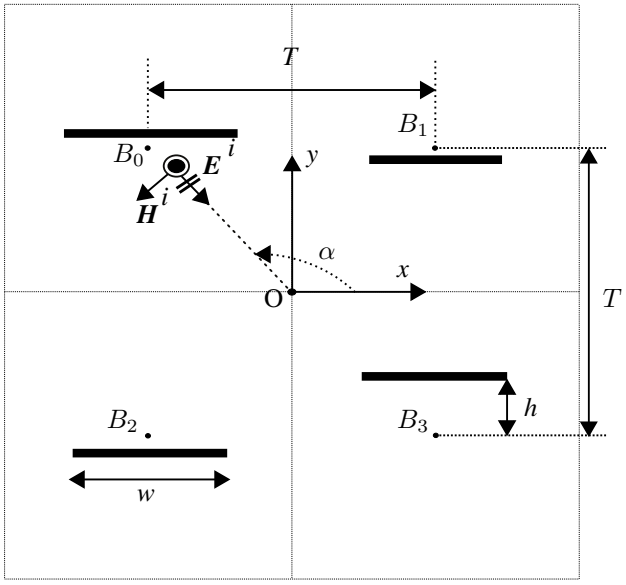

Fig. 1. Periodic 2D array of PEC strips. Widths w and heights h are random. The figure is taken from [3]

In order to show the benefits of the SGM-MLFMM, in [3], the 2×2 structure in Fig. 1 was taken as a starting point and expanded into a 4×4 and 8×8 structure, keeping the same rectangular organization of the strips. It was shown that the crossover point, when a SGM-based MoM and MLFMM are compared, is the same as in the deterministic case.

To reduce the number of iterations, we introduce a preconditioner that is based on a block-Jacobi preconditioner. This

type of a preconditioner was introduced in [4]. The SGM matrix exhibits a block structure where the diagonal blocks are, in general, equal to the average (mean) matrix $\overline{\boldsymbol{Z}}_0$ of the structure. The random variations are to be found in the other blocks. This means that the resulting matrix is diagonally dominant, and for small relative variations, this block diagonal matrix with $\overline{\boldsymbol{Z}}_0$ on the diagonal is a good representation of the whole matrix and thus it can be used as a preconditioner. However, when using the MLFMM, the entire $\overline{\boldsymbol{Z}}_0$ with size $N \times N$ is never stored. Therefore, the preconditioner is based on the diagonal blocks within the $\overline{\boldsymbol{Z}}_0$ matrix that correspond to the near interactions, in our case, near interactions between points in one box on the lowest MLFMM level that contain one PEC strip. The organization of this matrix is presented schematically in Fig. 2. The black squares within the diagonal block correspond to the near interactions within one box and in the MLFMM approach these blocks are actually stored. The size of the blocks could be increased at the cost of a higher setup time, since the number of sources and random variables inside the block are increased. Moreover, more near interactions in the MLFMM tree would be involved which affects the solution time.

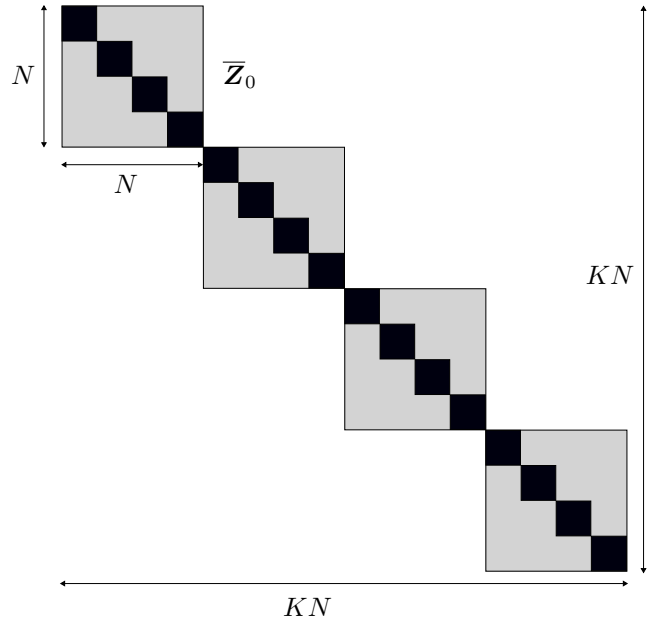

Fig. 2. Block diagonal organization of the preconditioner matrix. Due to the lack of the space, the simple situation for the 2×2 array with $K = 4$ is presented.

II. Simulation results

Here we provide results for two types of the structures, one involving 4×4 and another involving 8×8 strips. The first structure is represented with 32 random variables and discretized with $N = 320$ unknowns. The second one is described with 128 random variables and discretized with 1280 unknowns. Simulations are obtained with total polynomial orders 1 and 2, as presented in Table I. The iterative precision is set to 10^{-8} and a stabilized biconjugate gradient iterative solver is used [5]. To compare results, we will focus on the total setup time t_{se}, solution time t_{so} and the number of iterations N_{iter} needed to obtain the predefined accuracy. The last column in Table I indicates whether the preconditioner was used or not. The total number of unknowns is $N_{stoc} = KN$.

TABLE I
Simulation results

N_{stoc}	K	t_{se} [s]	t_{so}[s]	N_{iter}	Preconditioner
10 560	33	16.4	3	49	yes
10 560	33	15.7	12.9	225	no
179 520	561	76	100	56	yes
179 520	561	74.4	450	258	no
165 120	129	90	249	190	yes
165 120	129	88.6	987	778	no
10 732 800	8 385	423	31 114	212	yes
10 732 800	8 385	422	116 816	818	no

It is clear from the table that the number of iterations when using a preconditioner is smaller than without preconditioner. This significantly reduces the solution time, even though the time for a single iteration is increased due to the application of the preconditioner. We can see small differences in setup time, which now involves additional calculation of the inverse of the block matrix. However, this difference is negligible compared to the total time. It is clear that this type of the preconditioner, although simple, remains effective for the SGM-MLFMM solver. Even for small electromagnetic structures, this preconditioner is needed, since complexity grows fast with the number of polynomials K.

III. Conclusions

Developing an efficient preconditioner is essential in the construction of general and intrusive MLFMM-based stochastic methods. We have shown that a simple block-Jacobi preconditioner can serve for this purpose. The choice of the preconditioner is based on the particular properties of the SGM matrix and the geometry. One should be careful when determining the size of the blocks on which preconditioning will be applied, especially so for large structures. Further research is needed to combine this preconditioner with large electromagnetic structures and to study their effectiveness.

References

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