Memory properties in a Landau-Lifshitz hysteresis model for thin ferromagnetic sheets

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The paper deals with a two-dimensional numerical model for the evaluation of the electromagnetic hysteretic behavior of thin magnetic sheets when applying a unidirectional magnetic field. The time variation of the magnetization vector **m** in each space point obeys the Landau-Lifshitz equation. The effective field is the result of several contributions: the applied field, the magnetostatic field, the anisotropy field, and the exchange field. Microstructural features, such as grain size and crystallographic texture, are introduced in the micromagnetic model by dividing the geometry in subregions, each with its own magnetic preferable directions. In the article, numerical experiments are presented aiming at low-frequency applications. The presented micromagnetic model is used to study magnetic memory material properties. © *2006 American Institute of Physics*.
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I. INTRODUCTION

Ferromagnetic polycrystalline material can be magnetized by an externally imposed time varying magnetic field. However, between the magnetization of the material and the applied magnetic field, there exists no unique relation: no single-valued magnetization curve is described, but instead a magnetization loop or a hysteresis loop. The shape of these magnetization loops is determined mainly by the imposed magnetic field on the one hand and the composition and microstructural material properties on the other. In particular, these properties concern the density of dislocations, the size, and size partitioning of the present crystal grains, crystallographic texture, etc. Indeed, during the magnetization process, the mobility of the magnetic domain walls is affected by these microstructural quantities.

The presented two-dimensional micromagnetic model is based on the Landau-Lifshitz formalism,[1] which describes the electromagnetic action at the level of magnetic dipoles, resulting in the damped precession movement of the local magnetization vector with constant amplitude. We recall that, by means of the local effective field, the formalism allows us to account for both the magnetic short-distance effect (e.g., exchange field and anisotropy field) and the magnetic long-distance effects (magnetostatic field, imposed magnetic field), after defining the abovementioned microstructural properties in each point of the material.

Starting from a predefined microstructural state, the micromagnetic model must permit us to describe the macroscopic magnetic behavior in the form of magnetization loops, caused by a time varying magnetic field. The resulting macroscopic magnetic behavior as well as the magnetic dipole configuration at each time point is used to study magnetic memory material properties included in the micromagnetic model.

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II. MAGNETIC MEMORY BEHAVIOR

Hysteresis modeling handles the problem of how to construct (predict) the transition curves, which correspond to any changes of the magnetic field **H**. In spite of the variety of characteristics among different magnetic materials some general features are observed in their magnetization processes. These features have been described already in 1905,[2] and are known as Mandelung's rules. Considering the hysteresis curves in Fig. 1, these experimentally established rules can be stated as follows:

(1) The path of any transition (reversal) curve is uniquely determined by the coordinates of the reversal point, from which this curve emanates.
(2) If any point 4 of the curve 3-4-1 becomes a new reversal point, then the curve 4-5-3 originating at point 4 returns to the initial point 3 ("return-point memory").
(3) If the point 5 of the curve 4-5-3 becomes the newest reversal point and if the transition curve 5-4 extends beyond the point 4, it will pass along the part 4-1 of curve 3-4-1, as if the previous closed loop 4-5-4 did not exist at all ("wiping-out property").

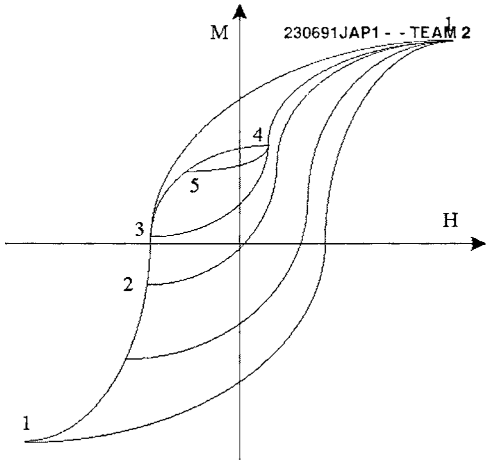

FIG. 1. Transition curves illustrating Mandelung's rules.

III. MICROMAGNETIC MODEL

The conventional theory of ferromagnetic materials is based on the assumption, following Landau and Lifshitz,[1] that the magnetization of magnetic dipoles $\mathbf{m}$ varies with the position, but that it has a fixed temperature-dependent magnitude $|\mathbf{m}|=M_s$ (below Curie temperature). The evolution of $\mathbf{m}$ is governed by the Landau-Lifshitz (LL) equation

$$\frac{\partial \mathbf{m}}{\partial t} = \frac{\gamma_G}{1+\alpha^2}\mathbf{m}\times\mathbf{H}_{\text{eff}} - \frac{\alpha\gamma_G}{1+\alpha^2}\frac{\mathbf{m}}{M_s}\times(\mathbf{m}\times\mathbf{H}_{\text{eff}}) \tag{1}$$

with α and γ_G the damping constant and the gyromagnetic constant, respectively. The static micromagnetic equilibrium condition is usually formulated as $\mathbf{m}\times\mathbf{H}_{\text{eff}}=\mathbf{0}$.

For the present study we considered a *two-dimensional* model of a thin ferromagnetic sheet with thickness d. Here, we assume all quantities being invariant in the z direction, while keeping the three-dimensional character of the LL equation (1). The x axis is chosen orthogonal to the sheet. Consequently, for a *static micromagnetic equilibrium*, the z component of $\mathbf{m}$ and $\mathbf{H}_{\text{eff}}$ must be zero in order to avoid magnetic charges at infinity on the z direction. In equilibrium, the walls appearing in the domain structure will be of Néel type, resulting in large magnetic charges in the walls and high corresponding magnetostatic fields. To overcome these high magnetostatic fields, large external magnetic fields must be applied in order to run through hysteresis loops.

The effective field $\mathbf{H}_{\text{eff}}$ is given by

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_{\text{exch}} + \mathbf{H}_{\text{ani}} + \mathbf{H}_{\text{ms}} + \mathbf{H}_a \tag{2}$$

with the exchange and anisotropy fields

$$\mathbf{H}_{\text{exch}} = \frac{2A}{\mu_0 M_s}\nabla^2(\gamma_x\mathbf{e}_x + \gamma_y\mathbf{e}_y + \gamma_z\mathbf{e}_z), \tag{3}$$

$$\mathbf{H}_{\text{ani}} = -\frac{1}{\mu_0 M_s}\left(\frac{\partial\phi_{\text{ani}}}{\partial\gamma_x}\mathbf{e}_x + \frac{\partial\phi_{\text{ani}}}{\partial\gamma_y}\mathbf{e}_y + \frac{\partial\phi_{\text{ani}}}{\partial\gamma_z}\mathbf{e}_z\right) \tag{4}$$

with A the exchange stiffness and the anisotropy energy $\phi_{\text{ani}}=K_1(\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2+\alpha_3^2\alpha_1^2)+K_2(\alpha_1^2\alpha_2^2\alpha_3^2)$.

Here, α_i is the direction cosine of the $\mathbf{m}$ vector with respect to the ith crystallographic preferable direction (cubic structure), while γ_j ($j=x$, y, or z) refer to the direction cosine of $\mathbf{m}$ with respect to the (x,y,z)-coordinate system. K_1 and K_2 are anisotropy constants.

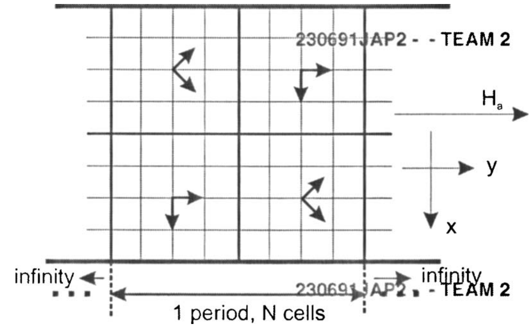

FIG. 2. Scheme of the micromagnetic model of an thin ferromagnetic sheet. Periodical structure in the y direction is shown.

The magnetostatic field follows from $\nabla\cdot\mathbf{H}_{\text{ms}}=-\nabla\cdot\mathbf{m}$ and $\nabla\times\mathbf{H}_{\text{ms}}=0$ using two-dimensional Green's functions:

$$\mathbf{H}_{\text{ms}} = -\frac{1}{2\pi}\int_S\left(\frac{[m_x,m_y,0]}{|\boldsymbol{\rho}-\boldsymbol{\rho}'|^2} - 2\frac{(\mathbf{m}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}'))(\boldsymbol{\rho}-\boldsymbol{\rho}')}{|\boldsymbol{\rho}-\boldsymbol{\rho}'|^4}\right)d\rho'. \tag{5}$$

Here, $\boldsymbol{\rho}=x\mathbf{e}_x+y\mathbf{e}_y$ and $\boldsymbol{\rho}'=x'\mathbf{e}_x+y'\mathbf{e}_y$. S is the area for which $0<x<d$, $-\infty<y<+\infty$. The applied field $\mathbf{H_a}$ is the quasistatic unidirectional field, enforced along the y direction.

For the *space discretization*, we considered a polycrystal structure of the thin ferromagnetic sheet consisting of a large number of interacting basis cells and periodic in the y direction. Each cell contains one single magnetization vector $\mathbf{m}$ and has predefined easy axes for the magnetization, as, e.g., shown in Fig. 2. Notice that two preferable directions are always lying in the xy plane, while the third is along the z direction. This is favorable as the z component of $\mathbf{m}$ must be zero for a *static micromagnetic equilibrium* in order to avoid magnetic charges at infinity in the z direction. The space derivatives appearing in Eqs. (3) and (4) are approximated by a classical finite difference method. Using the periodicity in the y direction, the magnetostatic field [Eq. (5)] is calculated by fast Fourier transforms in order to save computational time.[3]

For the *time discretization* the quasistatic applied field $\mathbf{H}_a$ is approximated with a piecewise constant time function. It is assumed that at the moment the applied field $\mathbf{H}_a$ jumps from a constant value to the next one, the material is in static micromagnetic equilibrium. Using the LL equation (1), the magnetization dynamics in each basis cell is computed through time stepping until a new static micromagnetic equilibrium is obtained corresponding with the new constant value for the applied field. The magnetization dynamics is evaluated analytically at $[t_i,t_{i+1}]$ by introducing, *in each basis cell*, a local (u,v,w) system, with an axis along $\mathbf{H}_{\text{eff}}(t_i)$. In the local (u,v,w) system at time step t_i, one has $\mathbf{H}_{\text{eff}}(t_i)=a\mathbf{e}_u$ and $\mathbf{m}(t_i)=u_i\mathbf{e}_u+v_i\mathbf{e}_v+w_i\mathbf{e}_w$. At the next time step $t_{i+1}=t_i+\Delta t$, we obtain $\mathbf{m}(t_{i+1})=u_{i+1}\mathbf{e}_u+v_{i+1}\mathbf{e}_v+w_{i+1}\mathbf{e}_w$ from Eq. (1) with

$$u_{i+1} = M_s\frac{e^{acM_s\Delta t}(M_s+u_i) - e^{-acM_s\Delta t}(M_s-u_i)}{e^{acM_s\Delta t}(M_s+u_i) + e^{-acM_s\Delta t}(M_s-u_i)}, \tag{6}$$

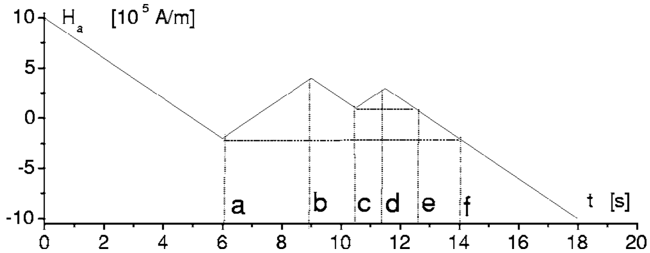

FIG. 3. The applied *quasistatic* magnetic field H_a along the y direction.

$$v_{i+1} = M_s \frac{2[v_i \cos(a\Delta t) - w_i \sin(a\Delta t)]}{e^{acM_s\Delta t}(M_s + u_i) + e^{-acM_s\Delta t}(M_s - u_i)}, \tag{7}$$

$$w_{i+1} = M_s \frac{2[v_i \sin(a\Delta t) + w_i \cos(a\Delta t)]}{e^{acM_s\Delta t}(M_s + u_i) + e^{-acM_s\Delta t}(M_s - u_i)}, \tag{8}$$

where $\mathbf{H}_{\text{eff}}(t) = \mathbf{H}_{\text{eff}}(t_i)$ for $t_i < t < t_{i+1}$, $a = \gamma_G H_{\text{eff}}(t_i)/(1+\alpha^2)$, and $c = \alpha/M_s$. In order to improve the rate of convergence, α is chosen to be 1.

IV. EVALUATION OF MEMORY BEHAVIOR FOR THE MICROMAGNETIC MODEL

For the evaluation of the magnetic memory behavior during the magnetization processes in the thin ferromagnetic sheet, several numerical experiments were performed. Consider the data $\mu_0 M_s = 2.16$ T, $A = 1.5 \times 10^{-11}$ J m^{-3}, $K_1 = 0.48 \times 10^5$ J m^{-3}, $K_2 = -0.50 \times 10^5$ J m^{-3}, $\gamma_G = -2.21 \times 10^5$ mA^{-1} s^{-1} [pure iron (see Ref. 3, p 518)]. We divide one period of the polycrystal structure into $N = 64$ basis cells—squares of 10 nm—and choose the microstructural features as shown in Fig. 2. Numerical data are given below for the case when the magnetic field of Fig. 3 is applied. The applied magnetic field is approximated by a piecewise constant function, considering 2800 equidistant time intervals. In each time interval, the applied field takes the constant value $H_{a,k}$. At the beginning of each time interval k, where the applied magnetic field jumps from $H_{a,k}$ to $H_{a,k+1}$, micromagnetic dynamics are calculated by time stepping equation (1), using $\Delta t = 10^{-12}$ s. After 45 steps (in an average way), the next micromagnetic equilibrium is reached. Figure 4 shows the state of the magnetic dipoles in the sheet at the time points of local extrema or at the time points when minor loops are closed. Notice the recovery of the microstructural state of time point c when reaching time point e and the state of time point a when reaching time point f. This recovery guarantees the same value for the macroscopic magnetization—the average of the dipole magnetization vectors in one period of the crystallographic structure—and consequently the closing of the minor loops.

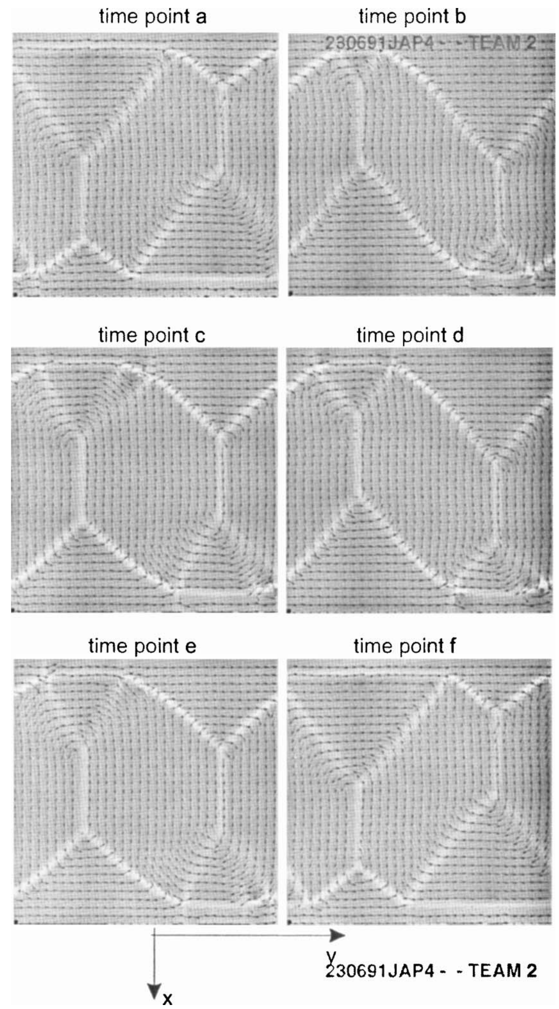

FIG. 4. The micromagnetic equilibria in the time points a, b, c, d, e, and f of Fig. 3. The arrows show the orientation of the local magnetic dipoles **m** while the gray scale gives the energy density (high density: white, low density: black).

V. CONCLUSIONS

For the presented micromagnetic model, we observe that when a time-varying magnetic field with local minima and maxima is applied, numerical experiments reveal that (1) minor hysteretic loops in the resulting macroscopic magnetization are indeed closed (return-point memory), (2) the closed minor loops are completely erased from the memory (wiping-out property), and (3) the magnetic domain structure at the starting point of a minor loop is recovered exactly when this loop is closed.

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