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ORIGINAL ARTICLE

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The role of working memory in carrying and borrowing

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Abstract The present study analyzed the role of phonological and executive components of working memory in the borrow operation in complex subtractions (Experiments 1 and 2) and in the carry operation in complex multiplications (Experiments 3 and 4). The number of carry and borrow operations as well as the value of the carry were manipulated. Results indicated that both the number of carry/borrow operations and the value of the carry increased problem difficulty, resulting in higher reliance on phonological and executive working-memory components. Present results are compared with those obtained for the carry operation in complex addition and are further discussed in the broader framework of working-memory functions.

Introduction

Suppose you have to buy some presents for family and friends. If you buy two presents at €38 each and two presents at €29 each, you have to execute two multiplications (i.e., $2\times38=76$ and $2\times29=58$) and an addition (i.e., $76+58=134$) to know the total sum you have to pay. When you pay €140, the cashier will return €6 to you, after having executed the subtraction $140-134$. This example nicely illustrates the frequency with which we need to solve complex arithmetic tasks in daily life. While solutions to simple forms of mental arithmetic (e.g., $7+5$ or 4×8) often can be retrieved from long-term memory (e.g., Cooney, Swanson, & Ladd, 1988; Siegler, 1988), complex forms of mental arithmetic (e.g., $28+43$ or 12×17) require other processes. Two of these extra processes are carrying and borrowing. Returning to our example, carry operations were needed in both multiplications. Since 2×8 gives 16 and 2×9 gives 18, a 1 had to be carried from the units to the tens. As the sum of 6 and 8 is 14, the addition required a carry operation from the units to the tens as well. The borrow operation, on the other hand, can be seen as the 'reverse' of the carry operation. In borrowing, we do not have a surplus, but a shortfall. Indeed, when subtracting 134 from 140, a 1 has to be borrowed from the tens in order to subtract 4 from 0.

The frequent use of carrying and borrowing notwithstanding, little is known about the functional mechanisms that are at the heart of these cognitive processes. Furthermore, most studies concerning complex arithmetic were executed on the carry operation in complex addition. These studies showed that the presence of carry operations increases problem difficulty. That is to say, the time that is needed to mentally calculate the solution of complex arithmetic problems strongly correlates with the number of carry operations (e.g., Ashcraft & Faust, 1994; Ashcraft & Kirk, 2001; Ashcraft & Stazyk, 1981; Dansereau & Gregg, 1966; Faust, Ashcraft, & Fleck, 1996; Imbo, Vandierendonck, & De Rammelaere, in review; Widaman, Geary, Cormier, & Little, 1989). After a step in which a digit has to be carried, there has to be an extra step wherein this information is put into working memory (WM). In a later step, this information has to be retrieved from WM. When this information is lost, errors emerge. Inefficient carry procedures indeed have been shown to be one of the most frequent causes of errors in mental arithmetic (e.g., Fürst & Hitch, 2000; Hitch, 1978; Noël, Désert, Aubrun, & Seron, 2001). Though very interesting, these studies leave questions concerning less investigated operations unanswered. For this reason, the present study investigated the carry operation in complex multiplications and the borrow operation in complex subtractions. For example, one might question whether the number of carry/borrow operations will influence problem difficulty of multiplications and subtractions in the same way. Due to methodological

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difficulties and lower ecological validity, the present study does not consider complex division problems.

Although the *borrow operation in subtractions* can be seen as the inverse of the carry operation in additions, no study investigated similarities and differences across both operations. However, there is evidence that the borrow operation increases problem difficulty as well. In a developmental study executed by Brown and Burton (1978), errors in subtraction problems were frequently caused by bugs in the procedural knowledge of borrow rules. For example, some children answered 41 to the problem 42−3, reflecting a misunderstanding in the borrow rule "subtract the smaller from the larger value". An even more subtle bug is apparent in the next example: 801−158 = 553. Here, the bug avoids borrowing from zero, by taking both borrows from the leftmost column, the 8 in 801. Apparently, children's mistakes are related to the procedural aspects of calculating, such as borrow rules, rather than to the simple arithmetic facts. In a more recent study with a brain-damaged patient, Sandrini, Miozzo, Cotelli, and Cappa (2003) also demonstrated the importance of borrow procedures. These researchers investigated number processing of an aphasic patient with relatively preserved numerical abilities (i.e., the patient had a good numerical comprehension). The patient's performance on multi-digit problems, however, was characterized by "a selective impairment of the borrowing procedure" (pp. 85). More specifically, the type of errors she made was consistent across all complex subtractions and was called the "smaller-from-larger bug". This bug appeared in problems where the digit-to-subtract-from was smaller than the digit-to-be-subtracted (e.g., 2 and 8, respectively, in the problem 132−18). In such a case, the patient preferred to invert the operation and subtract the 2 from the 8, which resulted in the incorrect response 116. As noted above, this bug is also typically observed in children learning to calculate. This case study further provides evidence for the dissociation between conceptual knowledge and procedural knowledge, showing that either one can be impaired while the other is preserved. A final study that highlights the importance of the borrow operation is the one conducted by Geary, Frensch, and Wiley (1993). They investigated simple and complex subtraction performance in younger and older adults. Complex subtractions requiring a borrow operation were observed to be solved much slower than those not requiring a borrow operation. The presence of a borrow operation interacted with age, however. Younger and older adults did not differ for no-borrow latencies, but the older adults were faster at executing the borrow operation than the younger adults. Geary et al. (1993) explain this unexpected result by arguing that the older adults have practiced complex subtractions more than the younger ones.

Another operation resembling addition is multiplication, since multiplication can be seen as repeated addition (e.g., 4×8 = 8 + 8 + 8 + 8). Moreover, *carry operations in multiplications* resemble those in addition problems. Consider the example provided earlier in this paper (2×38). As it has been shown that both complex additions and complex multiplications are processed columnwise (Geary, Widaman, & Little, 1986), most people will first multiply 2×8 = 16. This yields a value of 6 for the units and a 1 which must be held in WM during the multiplication 2×3. Then, this remembered value has to be carried and added to the temporary product (6) to complete the problem. Even though it has been suggested that complex additions and complex multiplications include similar carrying processes (e.g., retaining intermediary results in WM), not much research has been carried out to investigate the carry operation in complex multiplication.

In order to better understand these arithmetic operations, the aim of the present study was to investigate carry operations (in multiplications) and borrow operations (in subtractions) more thoroughly. As both carry and borrow operations comprise processes that might rely on WM, we focused on the role of different WM components in solving complex subtractions and complex multiplications. Before presenting the study we conducted, a short overview of WM and its functions in mental arithmetic is provided.

The role of working memory

An efficient implementation of carry and borrow operations requests (among other things) the temporary storage of intermediary results, the use of problem-solving skills, and the use of rule-based procedures (e.g., Geary, 1994; Geary & Widaman, 1987; Hope & Sherill, 1987) which all rely on WM resources. WM is a capacity-limited system that is responsible for storing and processing information in a variety of cognitive tasks. Although there exist many WM models (see Miyake & Shah, 1999, for an overview), the present study uses the multi-componential WM model of Baddeley and Hitch (1974), Baddeley (1986, 1992), and Baddeley and Logie (1999) as conceptual framework, as this model is dominantly used in mental-arithmetic research (DeStefano & LeFevre, 2004). This WM model comprises three components: a central-executive and two subordinate slave systems. The executive WM component is responsible for the supervision and coordination of the two slave systems, namely the phonological loop and the visuo-spatial sketchpad (Baddeley, 1986; Gilhooly, Logie, Wetherick, & Wynn, 1993; Logie, 1993). The phonological loop is able to store and manipulate phonologically coded verbal information (Baddeley & Logie, 1992; Baddeley, Thomson, & Buchanan, 1975; Salamé & Baddeley, 1982), whereas the visuo-spatial sketchpad is able to store and manipulate information in visual and spatial codes (Baddeley & Lieberman, 1980; Farmer, Berman, & Fletcher, 1986; Logie, 1986, 1989, 1991). The executive WM component is also responsible for control and decision processes, reasoning and language comprehension and production, on-line cognitive processing (e.g., problem solving and calculating), and task switching.

Many empirical studies demonstrated the role of WM in mental arithmetic. The *executive* WM component has been shown to play an important role in simple additions and multiplications (Ashcraft, Donley, Halas, & Vakali, 1992; De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; De Rammelaere & Vandierendonck, 2001; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996). Logie, Gilhooly, and Wynn (1994) were the first to show that the executive WM component is also crucial to perform complex forms of mental arithmetic. More recently, the crucial role of this WM component in complex arithmetic has been confirmed, both for additions (Fürst & Hitch, 2000) and multiplications (Seitz & Schumann-Hengsteler, 2000, 2002). The *phonological* loop, in contrast, would only be indispensable in complex additions and multiplications (e.g., Fürst & Hitch, 2000; Noël et al., 2001; Seitz & Schumann-Hengsteler, 2000, 2002; Trbovich & LeFevre, 2003), but not in simple additions and multiplications (e.g., De Rammelaere et al., 1999, 2001; Seitz & Schumann-Hengsteler, 2000, 2002). Finally, the role of the *visuo-spatial* sketch pad in mental arithmetic remains unclear until now. In most studies, no evidence was found for a role of this WM component in mental arithmetic (e.g., Logie et al., 1994; Noël et al., 2001; Seitz & Schumann-Hengsteler, 2000; but see Lee & Kang, 2002, for an exception).

Despite the allegedly important role of WM in many processes required in carry and borrow operations, only a few studies explicitly investigated which WM components play a role in these operations. As noted before, the presence of carry operations decreased latency and accuracy performance in complex additions. Several studies demonstrated the important role of the executive WM component in executing the carry operation (but see Logie et al., 1994, for an exception). Fürst and Hitch (2000), for example, did not only observe that the number of carry operations increased problem difficulty, they also observed an interaction between the number of carry operations and executive WM load, indicating that executive processes contribute to carrying. Comparable observations (i.e., a main effect of number of carry operations and an interaction with executive WM load) were made by Ashcraft and Kirk (2001). Furthermore, Seitz and Schumann-Hengsteler (2002) observed that error rates of additions that involved carry operations increased under executive WM load. Finally, a more recent study confirmed the role of the executive WM component in carry operations using two distinct manipulations of problem difficulty (Imbo et al., in review). The first one was by increasing the number of carry operations, as previous research had done. The second one was by increasing the value to be carried, a variable that has never been manipulated in previous research. The value to be carried can be augmented by constructing additions where more than two numbers have to be added. In $175+261+182=618$, for example, a 2 has to be carried from the tens to the hundreds. Results showed that both number and value increased the difficulty of addition problems. Moreover, executive WM load was shown to disrupt the calculation performance strongly, and especially when more carry operations had to be executed or when the value of the carry was larger.

Overview of the present study

The aim of the present study was to collect evidence regarding the contribution of executive and phonological components of WM to the carry operation in complex multiplications and the borrow operation in complex subtractions. The contribution of the visuo-spatial sketch pad was not investigated though, since the evidence for a role of this WM component is very sparse, as outlined in the Introduction section. To pursue our line of research, we also tried to manipulate both the number of carry/borrow operations and the value of the carry/borrow. Pilot studies showed, however, that it is fairly difficult to manipulate the value of the borrow in subtraction problems. Therefore, we decided to manipulate only the number of borrow operations in the subtraction experiments (Experiments 1 and 2). Although the role of WM in complex subtractions has never been investigated before, we expected an important role of the executive WM component. This assumption can be extrapolated from the observation that the executive WM component is needed in both complex additions (as noted above) and simple subtractions (e.g., Seyler, Kirk, & Ashcraft, 2003). In analogy with the carry operation in complex additions, we further expected that the role of the executive WM component would grow larger as more borrow operations had to be executed. Since borrow operations also require temporary storage of information, the phonological loop was expected to play a role in complex subtractions as well.

In the second part of the study, the role of WM was investigated in two multiplication experiments. In the first one (Experiment 3), only the value of the carry was manipulated, and in the second one (Experiment 4) both the number of carry operations and the value of the carry were manipulated. Based on previous research (e.g., Seitz & Schumann-Hengsteler, 2000, 2002), we expected that executive WM load would influence multiplication performance negatively. Moreover, we hypothesized this influence to grow larger with the difficulty of the carry operations; the difficulty being determined by the value of the carry (Experiments 3 and 4) and the number of carry operations (Experiment 4). Whether phonological WM resources are needed in complex multiplication problems is difficult to predict: Seitz and Schumann-Hengsteler (2000) did observe a negative influence of phonological load on complex multiplications, whereas Seitz and Schumann-Hengsteler (2002) did not.

Experiment 1

Method

Participants

Twenty first-year psychology students—16 women and 4 men—with a mean age of 19.5 years participated in the present experiment for course requirements and credits.

Stimuli

All stimuli had the same format, and consisted of two 2-digit numbers. When the second number was subtracted from the first number, another 2-digit number was obtained. One hundred and sixty experimental and 12 practice subtractions were designed, which were divided into two types of stimuli: (a) no borrow operation, and (b) one borrow operation with value 1 (examples can be found in Table 1). For each type, correct answers were distributed evenly between 11 and 30, between 31 and 50, between 51 and 70 and between 71 and 89. This approach avoided the size of the correct answer being an interfering variable. *T* tests indeed confirmed that the size did not differ significantly across both types of stimuli.

Instruments and procedure

All participants were tested individually. Each problem was shown at the centre of a computer screen in columnwise Arabic notation. The problem remained visible until the participant responded. Participants were asked to type in the correct answer by first typing the units and then the tens. In this way, variability in strategy use was eliminated (e.g., Hitch, 1978). When participants typed in a number, they saw it appear on the screen. The measurement of response times (RTs) was accurate up to one millisecond and started as soon as the subtraction appeared on the screen and stopped when the participant typed in the last digit of his/her answer. Accuracy and speed were emphasized equally strongly, although no time limit was set, nor feedback provided. The intertrial interval was 1,000 ms.

All participants participated in three conditions of which the order was counterbalanced: (a) Control: participants had to solve the arithmetical problems without a secondary task. (b) Articulatory suppression: participants solved the arithmetical problems while saying ‘‘de’’ (‘‘the’’ in Dutch) continuously. This task was meant to load the phonological loop. (c) Random two-choice reaction time task (CRT-R task): a task interfering with executive functioning, without putting an important load on the subordinate systems (Szmalec, Vandierendonck, & Kemps, 2005). In this task, a series of low (262 Hz) and high (524 Hz) tones was presented at randomized intervals. The interval between two subsequent tones was either 900 or 1,500 ms. Participants had to say ‘‘hoog’’ (‘‘high’’ in Dutch) when they heard a high tone and ‘‘laag’’ (‘‘low’’ in Dutch) when a low tone was presented. The duration of each tone was 200 ms.

The experiment started with four practice problems, to get used to the apparatus and the procedure. The experiment further consisted of three blocks, one for each condition. These blocks each comprised an explanation of the secondary task, practicing the execution of the primary task in combination with the secondary task (two items), and solving 40 randomly presented experimental problems (consisting of 20 items of each problem type, with 5 of each size interval). Performance of the secondary tasks was measured as well. The spoken responses of the participants in the articulatory suppression condition were recorded and analyzed afterwards. For the CRT-R task, the experimenter checked online whether the responses of the participants were right or wrong. The participants also performed the secondary task alone for 2 min (‘‘single secondary task control condition’’). Performance in these conditions was also measured.

Table 1 Examples of stimuli used in Experiments 1 and 2 (subtraction)

Number of borrow operations			
Zero	One	Two	Three
Experiment 1 (Subtraction)			
68	64	(No stimuli)	(No stimuli)
25	18		
43	46		
Experiment 2 (Subtraction)			
8437	5856	6542	4123
2124	1638	3714	2745
6313	4218	2828	1378

Results and discussion

ANOVAs were performed to investigate the role of WM load and the number of borrow operations. Stepwise regression analyses were carried out to determine the best predictors of arithmetic performance. Finally, the secondary task performance was analyzed. In all results, unless otherwise stated, an α level of 0.05 was used. This holds for the subsequent experiments as well.

ANOVA on solution latency

A 3 (WM load: none, phonological, executive) $\times$ 2 (Borrow operations: zero or one) ANOVA on times of correct responses was used, with repeated measures on both factors (see Fig. 1). The main effect of WM load was significant, $F(2,18) = 14.1$. Planned comparisons showed that, under executive WM load, subtractions

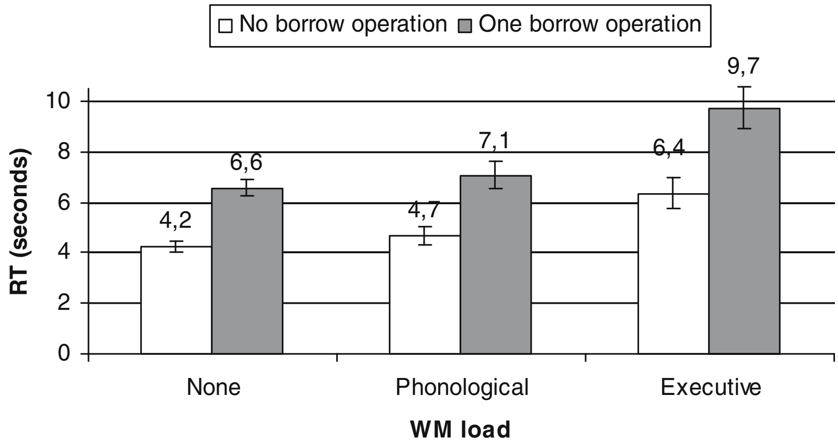

Fig. 1 Response times (seconds) in Experiment 1 (subtraction) as a function of working-memory load and number of borrow operations. Standard errors are denoted by *error bars*

were solved slower than in the control condition [$F(1,19) = 22.4$], and than under phonological load [$F(1,19) = 29.6$]. Subtractions were also solved slower under phonological load than in the control condition [$t(19) = 1.7$, one-tailed]. The main effect of Borrow operations [$F(1,19) = 92.1$] showed that RTs were significantly larger when a borrow operation had to be performed than when no such operation had to be performed. The interaction between WM load and Borrow operations also reached significance [$F(2,18) = 3.8$] and showed that the rise in RT between subtractions without and with borrow operations was especially larger when WM was under executive load than when it was not loaded [$F(1,19) = 7.2$] and than when it was under phonological load [$F(1,19) = 5.7$].

ANOVA on accuracy

The same 3×2 ANOVA design was applied to percentages of correctly solved subtractions (see Fig. 2). As with the RTs, the main effect of WM load was significant [$F(2,18) = 19.1$], with lower accuracy under executive load than in the control condition [$F(1,19) = 38.4$], and than under phonological load [$F(1,19) = 14.5$]. A further planned comparison also showed lower accuracy under phonological WM load than in the control condition [$t(19) = 2.0$; one-tailed]. The main effect of Borrow operations showed that accuracies decreased significantly when borrow operations had to be performed [$F(1,19) = 19.9$]. The interaction between WM load and Borrow operations just failed to reach significance [$F(2,18) = 3.2$, $p = 06$]. Although the effect of Borrow operations was significant in all conditions, it was significantly larger when WM was under executive load than when it was not loaded [$F(1,19) = 6.8$].

Regression analyses

In order to find the most meaningful predictors of subtraction performance, stepwise regression analyses were performed on the mean times of correct responses and the mean accuracy per item (in the control condition only; see Table 3). The predictors were: (1) the number of borrow operations, (2) the correct solution of the subtraction problem, (3) correct unit, and (4) correct ten. The number of borrow operations was the only significant predictor of both RT data ($R^2 = 0.61$) and accuracy data ($R^2 = 0.10$). RTs and error rates were higher when a

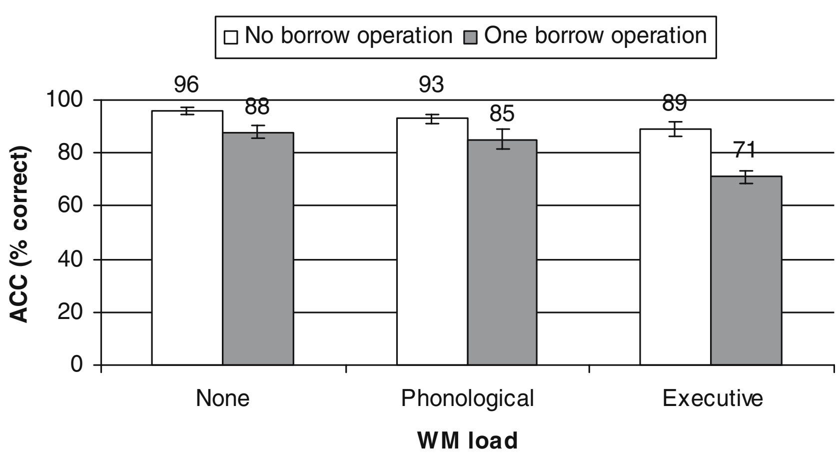

Fig. 2 Accuracies (% correct) in Experiment 1 (subtraction) as a function of working-memory load and number of borrow operations. Standard errors are denoted by *error bars*

borrow operation had to be performed than when no borrow operation had to be performed.

Analyses of secondary task performance

In the articulatory suppression condition, participants did significantly slow down their rate of saying "the" while calculating as compared to a single secondary task control condition (respectively, 87.9 and 94.0 words per minute, $t(19) = 2.6$). For the CRT-R task, one participant's data were lost due to a computer bug. The participants made more errors while calculating in comparison to CRT-R only [respectively, 42.9% correct responses vs. 67.8%, $t(18) = 6.7$]. These results show that when few WM resources were left, performance tended to be impaired not only on the primary task but also on the secondary task (see also Hegarty, Shah, & Miyake, 2000). This indicates that there was no trade-off between both tasks. More specifically, a bad performance on the primary task was not compensated by a better performance on the secondary task. Therefore, secondary-task effects could be taken for real.

Discussion

Results showed that solving complex subtractions relied heavily on executive WM resources, as predicted. Moreover, the phonological WM component was shown to play a role in solving complex subtractions as well: when this WM component was loaded, people calculated slower and less accurately. Furthermore, borrow operations were shown to increase problem difficulty: calculation was slower and less accurate when a borrow operation had to be performed than when no such operation had to be performed. The executive WM component was needed to perform these borrow operations fast and correctly, as observed in the significant executive load × borrow interactions. The importance of borrow operations was also confirmed in the regression analyses: the number of borrow operations was the only significant predictor of calculation performance. In the next experiment, the complex subtractions were made even more difficult (i.e., containing zero, one, two, or three borrow operations) in order to investigate the role of executive and phonological WM components more thoroughly.

Experiment 2

Method

Participants

Twenty volunteers—4 men and 16 women (with a mean age of 22.4 years)—participated in the present experiment. None of them had participated in Experiment 1.

Stimuli

All stimuli had the same format, and consisted of two 4-digit numbers. When the second number was subtracted from the first number, another 4-digit number was obtained. Seventy-two subtractions were designed, which were divided into four types of stimuli: (a) no borrow operation, (b) one borrow operation, (c) two borrow operations, and (d) three borrow operations (examples can be found in Table 1). The value that had to be borrowed was always 1. For each type, 18 stimuli were constructed. Within these 18 stimuli, three had a correct answer in the one-thousands, three in the two-thousands, three in the three-thousands, three in the four-thousands, three in the five-thousands, and three in the six-thousands. This approach avoided that the size of the correct answer would be an interfering variable. *T* tests indeed confirmed that the size of the correct answer did not differ significantly across the four types of stimuli. For the subtractions with one or two borrow operations, the place of the operation was controlled, with as many borrows from the tens, hundreds, and thousands.

Instruments and procedure

Instruments and procedure of the second experiment were almost identical to those in Experiment 1. However, since stimuli and responses consisted of 4-digit numbers, participants had to type in the correct answer by first typing the units, then the tens, then the hundreds, and finally the thousands (UTHT). There were three conditions of which the order was counterbalanced: (a) control, (b) articulatory suppression, and (c) CRT-R task. The experiment started with three practice problems, and three more practice problems were offered in each condition. There were three blocks, one for each condition. In each block, 20 items (5 of each problem type) were presented in randomized order.

Results and discussion

ANOVA on solution latency

A 3 (WM load: none, phonological, executive) × 4 (Borrow operations: 0, 1, 2, 3) ANOVA on times of correct responses was used, with repeated measures on both factors (see Fig. 3). The main effect of WM load was significant [$F(2,18) = 19.2$]. Further planned comparisons showed that, under executive WM load, subtractions were solved slower than in the control condition [$F(1,19) = 33.6$] and than under phonological load [$F(1,19) = 39.3$]. There was no significant difference in RTs between the control condition and the condition with phonological load [$F(1,19) < 1$]. The main effect of borrow operation shows that RTs increased with the number of borrow operations [$F(3,17) = 109.5$]. However, both

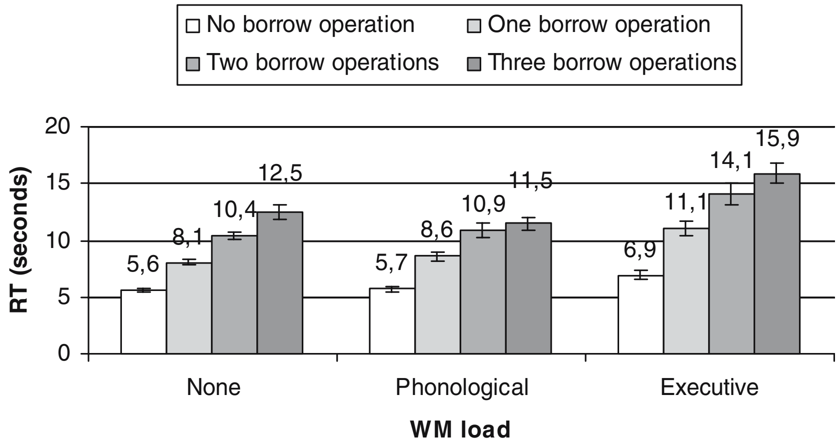

Fig. 3 Response times in Experiment 2 (subtraction) as a function of working-memory load and number of borrow operations. Standard errors are denoted by *error bars*

the linear component [F(1,19) = 321.4] and the quadratic component [F(1,19) = 13.0] appeared to be significant. The linear component clearly shows the rising trend: higher RTs for more borrow operations, whereas the quadratic component shows that this rise became less steep at the end. Finally, the interaction was significant as well [F(6,14) = 5.2]. Although the rise in RTs between subtractions without and with borrow operations was significant in all conditions, this effect was significantly larger when WM was under executive load than when it was not loaded [F(1,19) = 20.2].

ANOVA on accuracy

The same 3×4 ANOVA design was applied to percentages of correctly solved subtractions (see Fig. 4). As with the RTs, the main effect of WM load was significant [F(2,18) = 17.1]. Planned comparisons showed lower accuracies under executive load than in the control condition [F(1,19) = 35.9] and than under phonological load [F(1,19) = 16.0]. In the latter condition, accuracies were lower than in the control condition [F(1,19) = 6.7]. There was also a main effect of borrow operation [F(3,17) = 13.3]. This effect corresponded to a linear effect [F(1,19) = 43.7], showing that accuracy decreased with the number of borrow operations. The interaction between WM load and Borrow operations was also significant [F(6,14) = 4.3], showing that the decrease in accuracy from subtractions without borrow operations to subtractions with borrow operations was higher in the conditions with a phonological or executive load than in the control condition [F(1,19) = 6.3 and F(1,19) = 13.2, respectively]. The linear decrease in accuracy with the number of borrow operations was also steeper when WM was under phonological or executive load than when it was not loaded [t(19) = 1.8 and t(19) = 4.4, respectively].

Regression analyses

In order to find the most important predictors of the subtraction performance, stepwise regression analyses were performed on the mean times of correct responses and the mean accuracy per item (in control condition

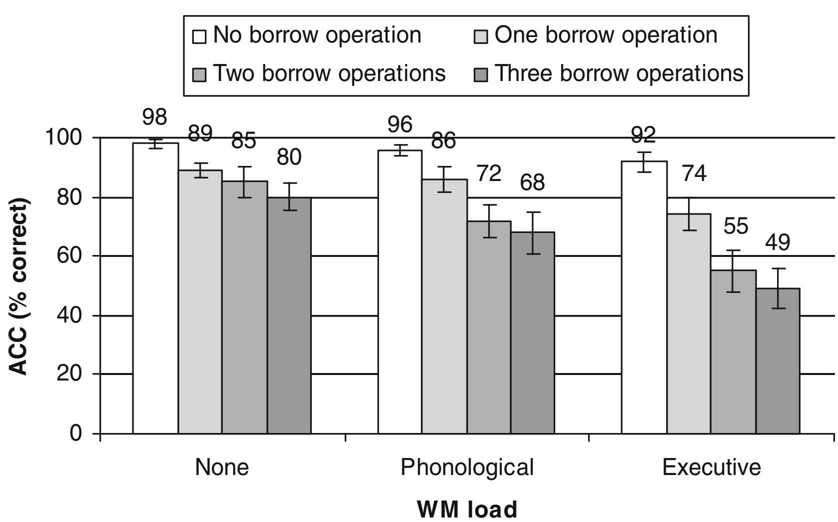

Fig. 4 Accuracies (% correct) in Experiment 2 (subtraction) as a function of working-memory load and number of borrow operations. Standard errors are denoted by *error bars*

only; see Table 3). The predictors were: (1) the number of borrow operations, (2) the correct solution of the subtraction problem, (3) correct unit, (4) correct ten, (5) correct hundred, and (6) correct thousand. The number of borrow operations and the Correct unit were the most important predictors of RT ($R^2 = 0.74$). The number of borrow operations predicted 69%, and the correct unit added 5%. For the accuracy data, the number of borrow operations was the only significant predictor ($R^2 = 0.14$).

Error analyses

Since the number of borrow operations was more extensively manipulated than in Experiment 1, the present data allowed us to investigate the relationship between the committed errors, the WM load, and the borrow procedure. We first checked whether the errors were evenly distributed over the units, tens, hundreds and thousands. For the units, there was an error in 2.9% of the cases, for the tens in 7.3% of the cases, for the hundreds in 9.1% of the cases, and for the thousands in 8.7% of the cases. Since participants were instructed to calculate from right to left (UTHT), WM load was the lowest for the units and grew as the calculation continued, which may explain the low error percentage for the units. We also tested the numerical distance between the erroneous and the expected digits (e.g., in 4,561−1,218 = 3,353 instead of 3,343, there is a distance of 1 in the tens). This distance was 1.9 for the units, 1.3 for the tens and the hundreds, and 1.2 for the thousands. The decrease in the numerical distance as we move toward the leftmost position can be explained by global estimation strategies: a distance error is less detrimental if it appears in the units, rather than in the tens, hundreds, or thousands (Noël et al., 2001). Finally, errors were observed especially when borrow operations had to be performed. Of all errors on the units, tens, hundreds, and thousands, 94.3, 82.8, 70.2, and 82.7%, respectively, was made when borrowing was required from or to that specific unit, ten, hundred, or thousand.

Analyses of secondary task performance

In the articulatory suppression condition, participants did not slow down their rate of saying "the" while calculating as compared to a single secondary task control condition (respectively, 75.8 and 76.9 words per minute, $t(19) < 1$). These data (i.e., a bad primary-task performance under phonological WM load and an equal articulatory-suppression performance with and without primary task) indicate that there was no trade-off between primary and secondary task. For the CRT-R task, participants made more errors while calculating in comparison to CRT-R-only [respectively, 90.0% correct responses vs. 99.1%, $t(19) = 3.9$], and were thus impaired on both the primary and secondary task.

Discussion

The present results confirm (a) that complex subtractions need executive WM resources to be solved fast and accurately and (b) that phonological WM resources are indispensable to solve complex subtractions accurately. As in the previous experiment, calculation performance was affected by the number of borrow operations, as shown in the ANOVAs, the regression analyses, and the error analyses. The executive WM component was further shown to guarantee correct and fast performance of the borrow operation: The disturbance caused by loading this WM component grew larger as the number of borrow operations increased. Moreover, the phonological loop did interact with the number of borrow operations as well, although only for accuracy data.

Thus far, results showed that the executive WM component is indispensable to solve carry operations in complex additions (e.g., Imbo et al., in review) and borrow operations in complex subtractions (Experiments 1 and 2 of the present paper) fast and accurately. The phonological WM component appeared to be important in carry and borrow operations as well, although its contribution seemed to be more associated to accuracy. So far, the role of these WM components in the carry operation in complex multiplications has not been tested. The next two experiments aimed to investigate this unexplored domain. In Experiment 3, we only manipulated the value of the carry, whereas in Experiment 4, both the value of the carry and the number of carry operations was manipulated.

Experiment 3

Method

Participants

Twenty-four volunteers—9 men and 15 women—with a mean age of 23.5 years participated in the present experiment. None of them had participated in Experiments 1 or 2.

Stimuli

All stimuli had the same format, and comprised a 2-digit number that had to be multiplied with a 1-digit number. The correct product of both numbers always was a 3-digit number. There always was one digit that had to be carried from the units to the tens, and the value of that digit was 1, 2, 3 or 4 (examples can be found in Table 2). Strong restrictions were imposed on the construction of the multiplications so as to avoid resolving them with a short-cut rule (e.g., "everything multiplied with a zero is zero"). The 1-digit number never was 0, 1, 2, 5 or 9, and the 2-digit number never ended in 0, 1, 5 or 9. All possible stimuli that met these restrictions were

Table 2 Examples of stimuli used in Experiments 3 and 4 (multiplication)

Number of carry operations							
One				Two			
Value of the carry				Value of the carry			
1	2	3	4	1	2	3	4
Experiment 3 (Multiplication)							
32	38	46	17	(No stimuli)			
8	3	6	6				
256	114	276	102				
Experiment 4 (Multiplication)							
314	114	117	(No stimuli)	132	144	188	(No stimuli)
3	6	5		6	6	4	
942	684	585		792	864	752	

used in the present experiment—a total of 73. This problem pool was divided into four types: (a) value 1, (b) value 2, (c) value 3, and (d) value 4, with, respectively, 20, 23, 10 and 20 multiplications each. *T* tests confirmed that the size did not differ significantly across the four types of stimuli.

Instruments and procedure

As in the previous experiments, all participants were required to use the same procedure. They first had to calculate the product of the units of the 2-digit number with the 1-digit number. Next, they had to calculate the product of the tens of the 2-digit number with the 1-digit number. There were three conditions of which the order was counterbalanced: (a) control, (b) articulatory suppression, and (c) CRT-R task. The experiment started with one practice problem, to get used to the apparatus and the procedure. After the explanation of the secondary task, the execution of the primary task in combination with the secondary task was practiced too (two problems per secondary task). After these practice problems, the three blocks (with 22 multiplication problems each) were presented. Each participant thus solved a total of 73 problems, 7 practice trials and 66 experimental trials. Instruments and procedure were equal to those used in the previous experiment.

Table 3 Summary of the regression analyses for Experiments 1, 2, 3, and 4: The successive significant predictors, the corresponding R^2 values, and the standardized *Beta* values (for the additional predictor only)

		R^2	*Beta*
Response time			
Experiment 1	Number of borrow operations	0.612	0.782
Experiment 2	Number of borrow operations	0.693	0.671
	Correct unit	0.735	0.262
Experiment 3	Value of the carry	0.292	0.479
	Problem size	0.411	0.351
Experiment 4	Number of carry operations	0.150	0.387
Accuracy			
Experiment 1	Number of borrow operations	0.097	−0.312
Experiment 2	Number of borrow operations	0.138	−0.372
Experiment 3	Value of the carry	0.125	−0.398
	Correct unit	0.275	−0.391
Experiment 4	Number×value	0.061	−0.246

Results and discussion

ANOVA on solution latency

A 3 (WM load: none, phonological, executive) × 4 (value of the carry: 1, 2, 3, 4) ANOVA on times of correct responses was used, with repeated measures on both factors (see Fig. 5). The main effect of WM load was significant [$F(2,22) = 17.4$]. Further planned comparisons showed that calculation was slower under executive WM load than under phonological WM load [$F(1,23) = 31.9$] or than in the control condition [$F(1,23) = 35.6$]. There was no significant difference between RTs in the control condition and the condition with phonological WM load [$F(1,23) < 1$]. The main effect of value showed that RTs increased linearly with the value of the carry [$F(1,23) = 14.4$]. The interaction between both factors was not significant [$F(6,18) = 1.4$].

ANOVA on accuracy

The same 3×4 ANOVA was run on percentages of correctly solved multiplications (see Fig. 6). The main effect of WM load was significant [$F(2,22) = 3.3$]. Further planned comparisons showed that more errors were made under executive WM load than under phonological load [$F(1,23) = 3.2$, with $p = 0.09$)] and than in the control condition [$F(1,23) = 6.7$]. Accuracy did not differ between the control condition and the condition with phonological WM load. The main effect of value was significant as well [$F(3,21) = 15.5$], and was composed of a significant linear component [$F(1,23) = 22.1$] and a significant quadratic component [$F(1,23) = 7.7$]. Accuracy decreased as the value of the carry grew larger, but this decreasing pattern became less steep at the end. The interaction between WM load and value was not significant [$F(6,18) < 1$].

Regression analyses

Stepwise regression analyses were performed on the mean time of correct response and the mean accuracy per item (in the control condition only) in order to find the most important predictors of the multiplication performance (see Table 3). The predictors used were: (1) the value of the carry, (2) the correct solution of the multiplication problem, (3) correct unit, (4) correct ten, and (5) correct hundred. In the RT data, value and correct solution turned out to be the most important predictors ($R^2 = 0.41$). Value predicted 29%, and correct

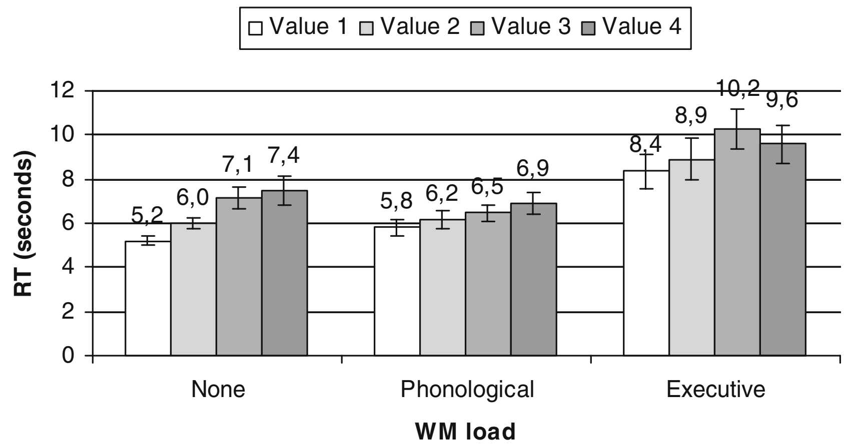

Fig. 5 Response times (seconds) in Experiment 3 (multiplication) as a function of working-memory load and value to be carried. Standard errors are denoted by *error bars*

solution added 12%. Value and correct unit were the most important predictors for accuracy ($R^2 = 0.28$). Value predicted 13%, and correct unit added 15%.

Error analyses

Most errors were made on the tens: they were incorrect in 17.7% of the cases. For the units, there was an error in 5.7% of the cases, and for the hundreds in 8.5% of the cases. The low error percentages in the units can be explained by the lower WM load (due to the UTH order of calculation), whereas the low error percentage in the hundreds can be explained by the small variation in possible answers (the correct product always was a one-, two-, or three-hundred number). Furthermore, the carry operation always occurred from the units to the tens, which explains the high error percentage for the tens. The numerical distance between the erroneous and expected digits decreased from the left to the right: it was largest for the units (3.5), smaller for the tens (2.2), and the smallest for the hundreds (1.7). According to Noël et al. (2001), this pattern can be explained by global estimation strategies: errors are worse on hundreds than they are on units.

Analyses of the secondary task performance

In the articulatory suppression condition, participants did not slow down their rate of saying ‘‘the’’ while calculating as compared to a single secondary task control condition (respectively, 76.2 and 72.3 words per minute, $t(23) = 1.4$), indicating no trade-off between primary and secondary task. For the CRT-R task however, participants made more errors while calculating in comparison to CRT-R-only [respectively, 76.4% correct responses vs. 98.6%, $t(23) = 6.8$], indicating impairment on both the primary and secondary task.

Discussion

The results of the present experiment show the important role of the executive WM component in complex multiplication problems (see also Seitz & Schumann-Hengsteler, 2000, 2002). Multiplications were solved more slowly and less accurately when less executive WM resources were available. Phonological WM load, however, did not affect multiplication performance. This result contradicts the results of Seitz and Schumann-Hengsteler (2000), where these researchers did observe

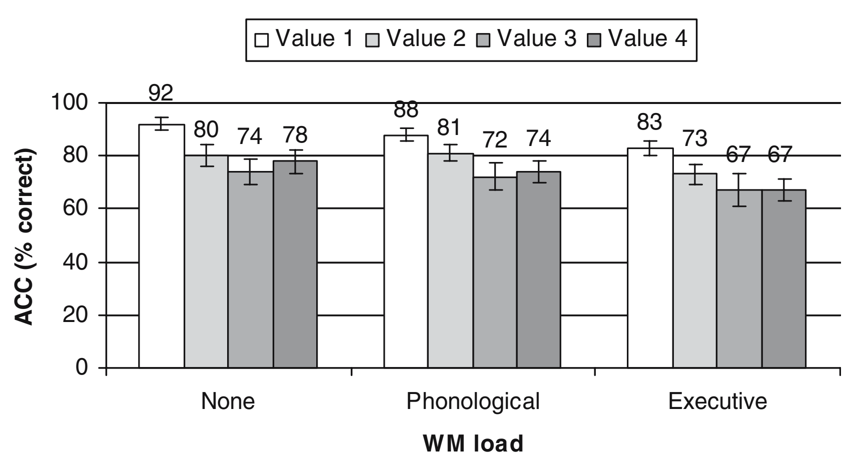

Fig. 6 Accuracies (% correct) in Experiment 3 (multiplication) as a function of working-memory load and value to be carried. Standard errors are denoted by *error bars*

slower performance on complex multiplications under phonological WM load; but agrees with their results of 2002, where they did not observe slower performance on complex multiplications under phonological WM load (although the error rates were slightly higher under phonological WM load). In the General discussion, we elaborate on the null result of present experiment, and review some explanations.

The results also showed the importance of value of the carry: as this value grew larger, calculation was slower and less accurate. Regression analyses confirmed the importance of this variable, since value always was one of the most important predictors of multiplication latencies and accuracies. Surprisingly, there was no interaction between WM load and value of the carry, indicating that WM did not play a specific role in the carry operation. As we assumed that the multiplication problems used in the present experiment might have been too simple, an additional experiment was run. In this fourth and final experiment of the present study, the role of WM in complex multiplications was studied by extending the scope of number of carry operations. Problem difficulty was thus determined by the number of carry operations *and* the value of the carry.

Experiment 4

Method

Participants

Twenty-three volunteers—9 men and 14 women—with an average age of 23.9 years participated in the present experiment. None of them had participated in Experiments 1, 2, or 3.

Stimuli

All stimuli had the same format, and consisted of one 3-digit number that had to be multiplied by a 1-digit number. The number of carry operations was one or two, and the value that had to be carried was 1, 2, or 3. This resulted in six problem types: (a) one carry with value 1, (b) one carry with value 2, (c) one carry with value 3, (d) two carries with value 1, (e) two carries with value 2, and (f) two carries with value 3 (examples can be found in Table 2). The 1-digit number never was a 0, 1, or 9, so as to avoid the use of short-cut rules. For each problem type, 13 multiplications were designed, except for types (e) and (f), for which (due to the constraints), respectively, 12 and 10 multiplications were designed. This resulted in a total of 74 problems. In the multiplications with one carry operation, the place of this operation was controlled for: carries were equally frequent from the units to the tens and from the tens to the hundreds. *T* tests confirmed that the size did not differ significantly across the six types of stimuli.

Instruments and procedure

Instruments and procedure were equal to those used in the previous experiment. There were three conditions of which the order was counterbalanced: (a) control, (b) articulatory suppression, and (c) CRT-R task. The present experiment started with two practice problems, to get used to the apparatus and the procedure. After the explanation of the secondary task, the execution of the primary task in combination with the secondary task was practiced too (one problem per secondary task). After these practice problems, the three blocks (with 23 multiplication problems each) were presented. Each participant thus solved a total of 74 problems: 5 practice trials and 69 experimental trials.

Results and discussion

ANOVA on solution latency

A 3 (WM load: none, phonological, executive) × 2 (number of carry operations: one or two) × 3 (value to be carried: 1, 2, 3) ANOVA was run on RTs of correctly solved multiplications, with repeated measures on all factors (see Fig. 7). The main effect of WM load was significant [$F(2,21) = 9.4$]. Further planned comparisons showed that multiplications under an executive WM load were solved more slowly than those solved in the control condition [$F(1,22) = 18.6$] and than those solved under phonological load [$F(1,22) = 19.1$]. Performance did not differ between the control condition and the condition with phonological WM load [$F(1,22) < 1$]. The significant main effect of number of carry operations [$F(1,22) = 57.7$] shows that multiplications with one carry operation were solved faster than multiplications with two carry operations. Finally, the main effect of value was significant [$F(2,21) = 4.5$] and comprised both a linear component [$F(1,22) = 4.2$] and a quadratic component [$F(1,22) = 6.7$], showing that RTs rose when the value to be carried was larger, but that this rising trend became less steep. No significant interaction effects were observed.

ANOVA on accuracy

The same 3×2×3 ANOVA was run on percentages of correctly solved multiplications (see Fig. 8). The main effect of WM load was significant [$F(2,21) = 11.4$] and showed significantly lower accuracies under executive load than in the control condition [$F(1,22) = 16.5$] and than under phonological load [$F(1,22) = 18.8$]. Accuracies did not differ between the control condition and the condition with phonological WM load [$F(1,22) < 1$]. The main effect of number of carry operations [$F(1,22) = 5.2$] showed that multiplications with two carry operations were solved less accurately than multiplications with only one carry operation. Finally, the main effect of value was linearly significant [$F(1,22) = 6.0$], with lower accuracies as the value of the carry grew larger. The interaction between WM load and number did not reach significance, although one planned comparison showed that the effect of number of carry operations tended to be worse under executive load than in the control condition [$t(22) = 1.3$, $p = 0.10$; one-tailed]. The interaction between WM load and value, however, was significant [$F(4,19) = 2.6$] and showed that under executive WM load, accuracies decreased especially when a 3 had to be carried, in comparison with the carrying of a 2 [$F(1,22) = 4.6$] or a 1 [$F(1,22) = 8.6$].

Regression analyses

As in previous experiments, stepwise regression analyses were performed on the mean time of correct response and the mean accuracy per item (in the control condition only), in order to find the most important predictors of the subtraction performance (see Table 3). The predictors were: (1) the number of carry operations, (2) the value of the carry, (3) number×value: the product of the number of carry operations with the value to be carried (for example, when two carry operations of value 3 had to be performed, this predictor had value 6), (4) the correct solution of the multiplication problem, (5) correct unit, (6) correct ten, and (7) correct hundred. In the RT data, the number of carry operations turned out to be the most important predictor ($R^2 = 0.15$), whereas number×value was the only significant predictor for accuracy ($R^2 = 0.06$).

Error analyses

Percentages of errors varied across the position, with 2.6% of the units being wrong, 7.5% of the tens, and 7.7% of the hundreds. The low error percentage in the units can again be explained from the lower WM load in this processing stage, since the order in which participants had to calculate was UTH. The numerical distance between the erroneous and expected digits decreased from left to right: it was the largest for the units (3.2), smaller for the tens (2.2), and the smallest for the hundreds (1.9). Global estimation strategies may be responsible for this pattern (Noël et al., 2001). Next, we analyzed whether the errors were due to a malfunctioning carry procedure. The difficulties inherent to the carry operation were indeed expressed in the committed errors. The units, tens, and hundreds were mainly wrong when a carry operation had to be performed from or to

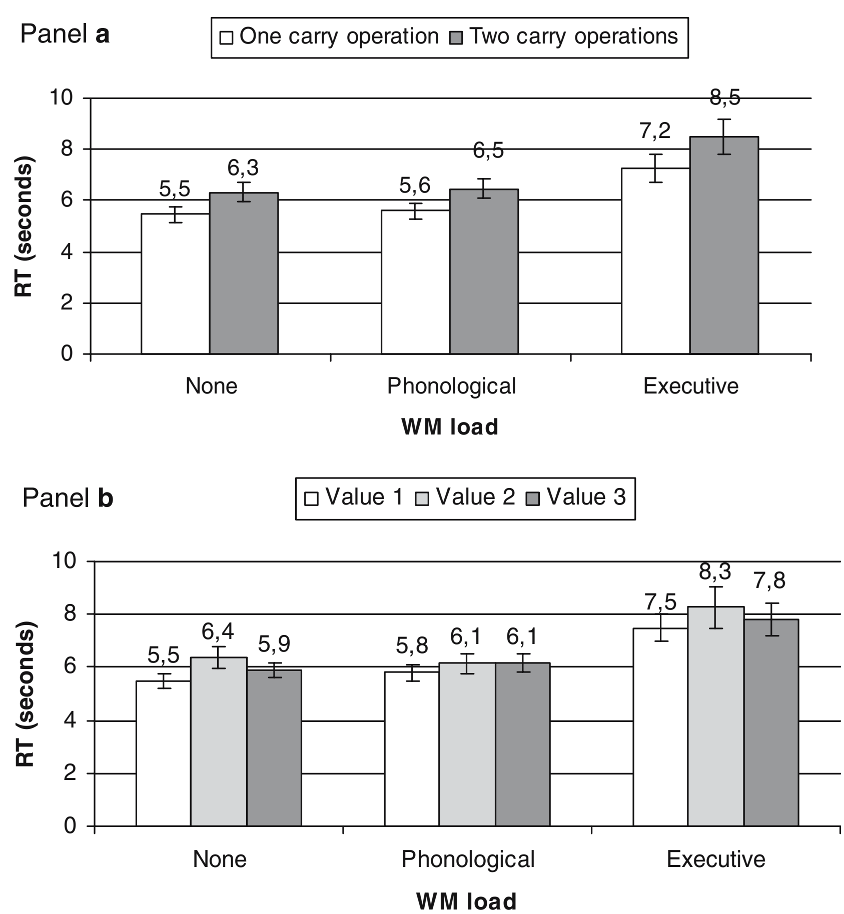

Fig. 7 Response times (seconds) in Experiment 4 (multiplication) as a function of working-memory load and number of carry operations (**a**) and as a function of working-memory load and value to be carried (**b**). Standard errors are denoted by *error bars*

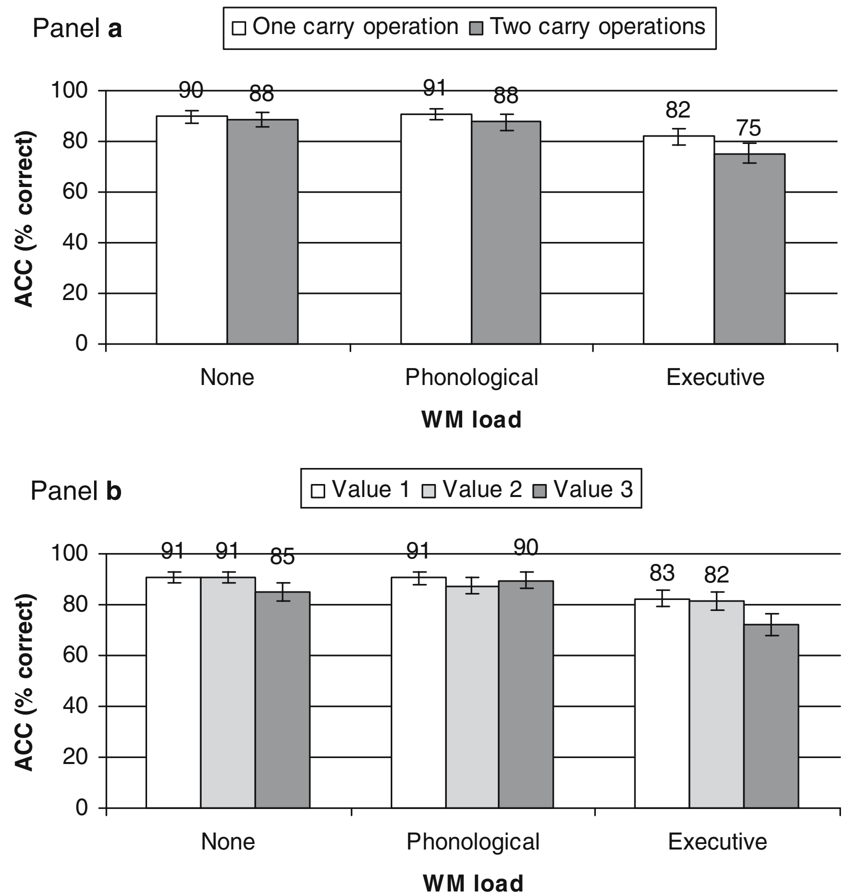

Fig. 8 Accuracies (% correct) in Experiment 4 (multiplication) as a function of working-memory load and number of carry operations (**a**) and as a function of working-memory load and value to be carried (**b**). Standard errors are denoted by *error bars*

the units, tens, and hundreds, with 90.2, 80.7, and 83.6%, respectively. Moreover, the value that had to be carried was reflected in the committed errors. The −1-errors (i.e., when the produced unit, ten, or hundred was 1 beneath the correct unit, ten, or hundred, e.g., when 236 was produced when the correct answer had to be 246) occurred equally frequently when a 1, 2, or 3 had to be carried. The −2-errors and −3-errors however, mirrored the value that had to be carried: the majority of all −2-errors (66.4%) occurred when a 2 had to be carried, and the majority of all −3-errors (85.7%) occurred when a 3 had to be carried; −2-errors occurred less frequently when a 1 or a 3 had to be carried, whereas −3-errors were very rare when a 1 or a 2 had to be carried.

Analyses of the secondary task performance

In the articulatory suppression condition, participants did not slow down their rate of saying "the" while calculating as compared to a single secondary task control condition (respectively, 81.1 and 78.8 words per minute, $t(22) < 1$), indicating no trade-off between primary and secondary task. For the CRT-R task, however, participants made more errors while calculating in comparison to CRT-R-only [respectively, 74.1% correct responses vs. 97.5%, $t(19) = 5.8$], indicating impairment on both the primary and secondary task.

Discussion

As in Experiment 3, the present results confirmed the important role of the executive WM component in complex multiplication. Moreover, there was some evidence that the executive WM component was especially important when more carry operations or carry operations with higher values had to be executed. No influence of phonological WM load was observed, an issue that is further elaborated in the General discussion. Both the number of carry operations and the value of the carry determined the difficulty of the calculation process. The regression analyses showed that a combination of both variables (i.e., the predictor 'Number×value') could explain most variance of the accuracy performance. Furthermore, error analyses showed that many errors were due to malfunctioning carry procedures. Indeed, most errors were committed when carry operations were needed. Forgetting to perform a carry operation was a very frequent error, which was reflected in the high percentages of −1, −2, and −3 errors where respectively, a 1, 2, or 3 had to be carried. Other frequently committed errors were the carrying of a wrong digit, which was reflected in the percentage of -1 errors where a 2 or a 3 had to be carried.

General discussion

Results of the present study showed that executive WM resources are needed to perform carry and borrow operations fast and correctly. Phonological WM resources, however, were only needed in borrow operations but not in carry operations. These results and additional considerations are further discussed below.

The executive WM component

The results confirmed the important role of the executive WM component in complex arithmetic. It is true that many of the functions ensured by the executive WM component are necessary in complex arithmetic, such as estimation processes (Logie et al., 1994), the sequencing of calculation steps (Fürst & Hitch, 2000), counting-based procedures (Hecht, 2002), maintaining order information and keeping track in multistep problems (Ashcraft & Kirk, 2001), and arithmetic strategy selection and strategy execution (Imbo, Duverne, & Lemaire, submitted). More importantly, however, is that the present research corroborated the significant role of the executive WM component in carry and borrow operations. In Experiments 1 and 2, a larger negative influence of executive WM load was observed as the number of borrow operations grew larger. In Experiment 4, executive WM resources were especially needed as the value to be carried grew larger, although there was also some preliminary evidence that the role of the executive WM component grew larger as more carry operations were needed. These results provide new insights into the role of the executive WM component, since its role in such operations was—until now—only shown in the carry operation in complex additions (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Imbo et al., in review).

One may first question the role of the executive WM component in executing more carry/borrow operations (as observed in Experiments 1, 2, and 4). As previously suggested (Fürst & Hitch, 2000; Imbo et al., in review), executive control might be needed in carry and borrow operations to inhibit the 'normal' order of operations during calculating. As we are more used to calculate *without* carry operations, the 'no-carry task set' will get automatically activated. When a carry operation is needed, however, the strongly activated 'no-carry task set' has to be suppressed, and the 'carry task set' must be activated, which takes much effort. As both task sets are competing with each other, this conflict has to be resolved under control of executive WM resources. When more carry operations have to be executed, conflicts between the 'no-carry task set' and the 'carry task set' occur even more often, and the resolution of these conflicts requires extra executive control. Obviously, this line of reasoning can also explain why so much executive control is needed to perform subtractions with more borrow operations fast and accurately. As noted above, the executive WM component also guarantees that succeeding steps (e.g., calculation procedures whether or not including retrieval) run in an ordered way. Since more carry/borrow operations imply more calculation steps, more executive WM resources will be needed to sequence these steps. A final explanation goes back to the very 'basic' conception of WM as a system devoted to the coordination of processing and storage (Baddeley & Hitch, 1974; Barrouillet, Bernardin, & Camos, 2004). As executive WM resources have limited capacity, trade-offs between processing and storage may occur. Carry/borrow operations probably increase both the storage load (i.e., the number of units of information that have to be retained in WM) and the processing load (i.e., the extra addition/subtraction operations that have to be executed). Consequently, resources that are devoted to storage are no longer available for processing (and the other way round), resulting in poorer performance.

A second question is why more executive WM resources were needed to carry *higher values*, as observed in Experiment 4. The task-set explanation described above may account for this observation as well: Since we are more used to carry small values, these task sets will get more readily activated compared to task sets for carry operations with higher values. A second explanation is based on interference effects. As the value of the carry differed across trials, it was possible that in the previous trial a 2 had to be carried while in the current trial a 3 has to be carried. Consequently, participants might suffer from interfering effects caused by the carry operation of the previous trial when executing the carry operation in the current trial. Executive control would be needed to restrain such interference effects. Thirdly, problem-size effects may also explain why executive WM resources are needed to carry higher values. Since mental arithmetic gets harder as the numbers get larger (e.g., Ashcraft, 1992, 1995; Ashcraft & Battaglia, 1978; Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Geary, 1996), one may assume that carrying high values requires more executive WM resources than carrying small values. However, these and alternative explanations about the role of the executive WM component in carrying are not mutually exclusive and should be put to further investigation.

The phonological WM component

A phonological WM load caused slower (Experiment 1) and less accurate (Experiments 1 and 2) performance on complex *subtractions*. Since the phonological WM component assures the temporary storage of intermediary results, it may guarantee accuracy during calculation processes (e.g., Fürst & Hitch, 2000; Hitch, 1978; Logie et al., 1994; Logie & Baddeley, 1987; Seitz & Schumann-Hengsteler, 2000, 2002). The effect of phonological load was indeed observed more clearly in accuracy analyses than in latency analyses (see also Hecht, 2002). In both *multiplication* experiments, however, no effect of phonological load was observed. As noted before, these results are in contradiction with those of Seitz and Schumann-Hengsteler (2000), who did observe a negative influence of phonological load on complex multiplication performance. Two points concerning their methodology should be mentioned, however. First, the correct product had to be produced orally, while the phonological loop was loaded by an articulatory suppression task. As participants had to switch between the suppression task and pronouncing their solution to the multiplication problem, the phonological load condition was not purely phonological. Second, since participants had to produce their solution orally at once, they were free to choose a strategy. They could have used short-cut rules and algorithms with additions or subtractions as sub-operations. For example, 9×28 can be solved doing $(9\times30)-(9\times2) = 270-18 = 252$, which not only includes multiplications, but also a complex subtraction. Therefore, it is impossible to conclude whether the phonological loop was needed in this subtraction and/or in the multiplications. Seitz and Schumann-Hengsteler (2000) admit that it is not clear whether their suppression task disrupted the multiplication process or a sub-process. In a follow-up study, however, Seitz and Schumann-Hengsteler (2002) did not observe any influence of phonological load on multiplication latencies (although error rates were slightly higher under phonological load). This is in agreement with our results, since we did not find an effect of phonological load on multiplication performance either. Moreover, in all experiments of the present study, participants were required to use the same procedure, which decreased the use of other strategies and thus excluded the use of complex additions and/or subtractions in the multiplication process.

But why was solving complex multiplications not affected by a phonological WM load? First, it is possible that the multiplication tasks used in the present study were not hard enough to require phonological WM resources. A 2- or 3-digit number had to be multiplied with a 1-digit number; a task that can be decomposed in easier ones. For example, the multiplication 32×8 can be broken up into 8×2 and 8×3. Since the phonological loop is not used in simple multiplications (De Rammelaere et al., 2001; Seitz & Schumann-Hengsteler, 2000, 2002), the problems used in the present multiplication experiments may have been rather simple than complex. Second, the experimental methodology might also have reduced the phonological WM load. As participants were asked to type in their calculations in the UTH order, they had to maintain only one digit at time. Future experiments in which participants have to produce the product once their calculation is completely finished, would probably observe effects of phonological WM load[1]. Finally, since multiplication processes are strongly trained skills in West Europe (Seitz & Schumann-Hengsteler, 2000, 2002), people use retrieval far more often in multiplication problems than in addition, subtraction or division problems (Campbell & Xue, 2001), which might also have decreased the need to rely on WM resources. It is, of course, still possible that no influence of phonological load was observed because the phonological loop is simply not used in complex multiplication processes; although future research is needed to confirm this null effect.

Finally, one of the goals of the present research was to further investigate the role of the phonological loop in *carrying and borrowing*. Evidence for a role of this WM component in the carry operation in additions was sparse (but see Imbo et al., in review; Fürst & Hitch, 2000; Noël et al., 2001); and its role in the carry operation in multiplications or in the borrow operation in subtractions was never studied yet. The results of Experiment 2 showed that a phonological load reduced accuracy of the borrow operation in subtraction problems. Thus, the phonological WM component became more important as the number of borrow operations grew. This can easily be explained as follows: as more borrow operations have to be performed, more results have to be kept temporary in WM, which is a role of the phonological loop. In Experiment 1, no interaction between phonological WM load and the number of borrow operations was observed, which was probably due to the less extended scope in Experiment 1 (zero or one borrow operations) as compared to the wide range in Experiment 2 (zero, one, two or three borrow operations).

The number of carry/borrow operations and the value to be carried

In the experiments where the *number* of carry/borrow operations was manipulated (Experiments 1, 2 and 4), a main effect of this variable was observed: calculation was slower and less accurate as more such operations had to be performed. Several studies showed the importance of this variable in additions (e.g., Ashcraft & Kirk, 2001; Faust et al., 1996; Fürst & Hitch, 2000; Logie et al., 1994; Noël et al., 2001) and in multiplications (e.g., Seitz & Schumann-Hengsteler, 2000), whereas the present study extended the importance of this variable to subtractions. The large influence of the

[1]Recently, a first attempt in this direction was made in an unpublished study in our lab. Complex multiplications (e.g., 16×8) were presented visually on which participants had to provide an oral response as soon as they had calculated the product. Phonological WM was loaded by presenting a five-letter string which participants had to repeat subvocally while calculating. An effect of phonological WM load was observed on accuracies but not on latencies. More specifically, accuracies tended to be lower under phonological WM load than in the control condition [$t(19) = 1.56$; $p = 0.07$; one-tailed]. Future research may elaborate on this issue.

number of carry/borrow operations on calculation performance was not only shown by ANOVAs. In the regression analyses, this variable always turned out to be one of the best predictors of both latency and accuracy data; and error analyses showed that most errors were conducted when a carry or borrow operation had to be performed.

The careful manipulation in Experiment 2 with zero, one, two or three borrow operations permitted us to investigate whether problem difficulty increased linearly with the number of borrow operations. Results showed that this was not the case: problem difficulty did not increase linearly as it increased more steeply between one and two borrow operations than between two and three borrow operations. This non-linear increase in problem difficulty can be explained by task set activation (see also Imbo et al., in review). As we are more used to calculate without carry operations, the 'no-carry task set' will get automatically activated. When the first carry operation occurs, the strongly activated 'no-carry task set' appears to be inappropriate, and the 'carry task set' has to be activated, which takes much effort. When there is another carry operation within the same problem, the 'carry task set' has the advantage of some rest activation and is more readily accessible. When successive carry operations within one problem are encountered, the rest activation stays reasonably high, which enhances the accessibility of the task set and reduces the effort to execute the carry operations. This augmenting rest activation explains the non-linear rise of problem difficulty with the number of carries: it becomes easier to access the 'carry task set'.

In both multiplication experiments, the *value* to be carried was manipulated; and a main effect of this variable was observed. Calculation performance was slower and more erroneous when larger digits had to be carried. Moreover, regression analyses showed that calculation performance was significantly predicted by the value of the carry; and error analyses confirmed that forgetting of the correct value to carry reduced accuracy. The extensive manipulation of the value to be carried (1, 2, 3, or 4 in Experiment 3, and 1, 2, or 3 in Experiment 4) allowed to investigate linearity effects, and showed that especially values larger than 1 caused the greatest difficulties. An explanation for this observation can be inferred from Hitch (1978), who states that a binary marker with value 0 (no carry operation) or 1 (carry operation) is stored in WM. The binary nature of the marker precludes any extra information about the carry (e.g., its value). It can be supposed that the default value is 1, which explains why carrying values higher than 1 is so difficult: Suppressing the default value of 1 takes more effort than connecting another value (2, 3, or 4) to the marker. Problem-size effects, however, could have played a role as well. Given that mental arithmetic gets harder as numbers get larger (Ashcraft, 1992, 1995; Ashcraft & Battaglia, 1978; Butterworth et al., 2001; Geary, 1996), one may assume that executing carry operations with high values is harder than executing carry operations with small values. Further research, however, will have to elaborate on these topics, so as to refine the preliminary conclusions provided in this paper.

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