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A Qualitative Calculus for Moving Point Objects Constrained by Networks

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"Notre nature est dans le mouvement…"

("Our nature consists in motion…")

Blaise Pascal

Een Kwalitative Calculus voor Bewegende Puntobjecten Beperkt door Netwerken

Samenvatting (Dutch Summary)

Continu bewegende objecten vormen een belangrijk studieobject in een groot aantal domeinen (Laube et al. 2005). Enkele voorbeelden zijn: een bioloog die het verplaatsinggedrag van een kudde dieren wil bestuderen, een verkeersplanner die de bewegingen van auto's wil volgen en een sportwetenschapper die de onderlinge interacties van voetballers tijdens een wedstrijd wil analyseren. Vanuit geometrisch standpunt, concentreren de meeste toepassingen zich op de positionele beweging van het voorwerp zelf waardoor bewegende objecten meestal tot punten worden vereenvoudigd. De recente evoluties in diverse plaatsbepalingstechnieken (GPS, GSM, ...) laten toe grote hoeveelheden dergelijke bewegende puntobjecten op te meten en op te slaan (Laube et al. 2005; Zeimpekis et al. 2003). Er is al heel wat onderzoek verricht in het genereren (Brinkhoff 2002; Pfoser and Theodoridis 2003), indexeren (Agarwal et al. 2003; Saltenis et al. 2000), en modelleren en bevragen (Erwig et al. 1999; Sistla et al. 1997) van bewegende objecten in tijdruimtelijke databanken. Redeneren over de relaties tussen bewegende puntobjecten echter vormt nog maar sinds kort het voorwerp van onderzoek, vooral het redeneren binnen een kwalitatief kader (Cohn and Renz 2007; Van de Weghe 2004). Nieuwe technieken binnen informatiesystemen, zoals Geografische Informatiesystemen (GIS), zouden echter veel meer kwalitatieve methodes moeten hanteren (Egenhofer and Mark 1995b). Aangezien mensen verkiezen te communiceren in kwalitatieve termen (Freksa 1992b), zouden dergelijke systemen dichter komen bij de manier waarop informatie wordt meegedeeld (Renz et al. 2000). Wat GIS betreft, passen deze ideeën volledig binnen het onderzoeksdomein van de Naïeve Geografie (Naive Geography) (Egenhofer and Mark 1995b). Door hun populariteit, wordt een GIS niet alleen meer door domeinspecialisten gebruikt (b.v. Google Earth, systemen voor autonavigatie). Het gebruik van kwalitatieve methodes binnen informatiesystemen zou de toegankelijkheid moeten verzekeren voor een brede waaier gebruikers (Egenhofer and Mark 1995b).

Aangezien redeneren over bewegingen een belangrijk onderdeel vormt van het alledaagse menselijke kennisvermogen (Forbus 1980), is er een duidelijke behoefte om een kwalitatieve 'bewegingscalculus' te ontwikkelen. In het domein van kwalitatief ruimtelijk redeneren is Mereotopologie het meest onderzochte studiegebied (Bennett 1997). Volgens het 9-Intersectie Model (*9-Intersection Model*) echter zijn er slechts twee triviale topologische relaties tussen twee puntobjecten: de objecten zijn ofwel co-incident ofwel disjunct (Egenhofer and Herring 1991). Aangezien bewegende objecten in de realiteit meestal niet samenvallen, en topologische modellen geen verder onderscheid kunnen maken tussen disjuncte objecten, zijn deze calculi in het geval van bewegende puntobjecten niet expressief genoeg. Een typisch voorbeeld is het geval waar twee vliegtuigen zich in een gescheiden relatie bevinden. Het is noodzakelijk om te weten of deze beide vliegtuigen in deze relatie kunnen blijven, zoniet kunnen de gevolgen catastrofaal zijn. De Kwalitatieve Traject Calculus (*Qualitative Trajectory Calculus*: QTC), geïntroduceerd door Van de Weghe (2004), is op dit vlak expressiever. QTC beschrijft en redeneert over kwalitatieve relaties tussen disjuncte continu bewegende puntobjecten. In Van de Weghe (2004), worden twee soorten QTC geïntroduceerd. De basiscalculus (*QTC-Basic*: QTC_B) beschrijft de onderlinge relaties tussen bewegende puntobjecten met behulp van afstandsvergelijkingen, terwijl QTC-Dubbel Kruis (*QTC-Double Cross*: QTC_C) de relaties beschrijft via een referentieframe bestaande uit drie referentielijnen in de vorm van een dubbel kruis.

Moreira et al. (1999) maken een onderscheid tussen twee soorten bewegende objecten: voorwerpen die in de vrije ruimte kunnen bewegen (b.v. een vogel die door de lucht vliegt) en voorwerpen die in hun bewegingsvrijheid beperkt worden (b.v. een trein kan enkel op het spoorwegnetwerk bewegen). Een groot aantal bewegingen worden duidelijk begrensd door een netwerk (binnenschepen kunnen enkel varen op kanalen en sommige rivieren, auto's rijden op straatnetwerken, enz.). Daarom is de hoofddoelstelling van dit proefschrift het uitbreiden van de QTC theorie naar objecten die enkel op netwerken kunnen bewegen. Met andere woorden, het doel is een kwalitatieve calculus op te stellen die het mogelijk maakt om relaties tussen bewegende puntobjecten die enkel op netwerken kunnen bewegen te beschrijven en te onderzoeken: De Kwalitative Traject

Caculus op Netwerken (QTC_N). Een tweede doelstelling bestaat erin om een eerste aanzet te geven tot de taalkundige en cognitieve bruikbaarheid en geschiktheid van QTC.

Dit proefschrift is onderverdeeld in negen hoofdstukken, na de inleiding in Hoofdstuk 1, wordt in Hoofdstuk 2 een algemeen overzicht gegeven van gerelateerd werk binnen het domein van kwalitatief redeneren. Aangezien een beweging zowel een ruimtelijke als een tijdsdimensie bevat, worden de verschillende benaderingen voor het voorstellen van en redeneren over kwalitatieve relaties gegeven met betrekking tot tijd, ruimtelijke en tijdruimtelijke informatie. Omdat QTC_B de basis vormt voor QTC_N, is er een significant deel aan gewijd. Een relatie tussen twee objecten in QTC_B wordt hoofdzakelijk beschreven op basis van afstandsveranderingen. Belangrijk hierbij is dat QTC_B de beweging van beide objecten ten opzichte van elkaar weergeeft. Om de relatie tussen twee objecten *k* en *l* op een tijdstip *t* te beschrijven wordt telkens één object in de tijd gefixeerd (vb. object *l*). De beweging van object *k* ten opzichte van object *l* wordt bepaald door de afstand tussen *k* en *l* op tijdstip *t* te vergelijken met de afstand tussen *k* op t^- (net voor *t*) en *l* op *t* en de afstand tussen *k* op t^+ (net na *t*) en *l* op *t*. Deze twee vergelijkingen bepalen of *k* beweegt '*naar*' (afstand wordt kleiner), *'weg van'* (afstand wordt groter), of stabiel blijft (afstand verandert niet) ten opzichte van *l*. Analoog kan de beweging van object *l* ten opzichte van object *k* bepaald worden. De beweging van beide objecten ten opzichte van elkaar kan uitgedrukt worden door de verzameling kwalitatieve waarden {− ('*naar*'), + (*'weg van'*), 0 ('*stabiel*')}. Een relatie tussen twee objecten kan bijgevolg voorgesteld worden aan de hand van een tekenreeks bestaande uit twee karakters. Dit leidt tot een verzameling van 9 (3^2) verschillende bewegingsmogelijkheden. Dit zijn de zogenaamde QTC niveau 1 bewegingen. Het toevoegen van een derde karakter dat de relatieve snelheid voorstelt tussen beide objecten aan het label breidt het aantal mogelijkheden uit tot 27 (3^3), aangezien de snelheid van *k* ofwel lager, ofwel gelijk, ofwel groter is dan de snelheid van *l*. Deze verschillende bewegingen behoren tot de QTC niveau 2 relaties.

Hoofdstuk 3 bestaat uit twee delen. Een eerste deel definieert de bewegende objecten en het netwerk waarop ze bewegen. In het tweede deel worden de QTC_N relaties formeel gedefinieerd. De relaties tussen twee objecten die op een netwerk bewegen worden gedefinieerd op basis van het kortste pad tussen deze twee objecten. Een belangrijk voordeel van een dergelijke definitie is dat de afstandgebaseerde definitie kan vervangen worden door een eenvoudigere definitie die de relaties bepaalt aan de hand van de al dan niet veranderende topologische relatie tussen het bewegende object en het kortste pad tussen beide objecten, aangezien een object enkel naar een ander object kan bewegen als en slechts als het langs het kortste pad beweegt. Alle 27 theoretisch mogelijke QTC_N relaties bestaan in de realiteit, in tegenstelling tot objecten die enkel in één dimensie kunnen bewegen (QTC_{B1}) waar slecht 17 relaties mogelijk zijn. Niet alle relaties kunnen echter over een interval aanhouden, in tegenstelling tot objecten die vrij in een twee dimensionale ruimte kunnen bewegen (QTC_{B2}).

De Hoofdstukken 4 tot en met 7 richten zich op de uitwerking van gekende technieken binnen het kwalitatief redeneren op QTC_N relaties.

Hoofdstuk 4 gaat na hoe er extra kennis kan afgeleid worden uit de compositie van twee QTC_N relaties. Anders geformuleerd: gesteld dat je de relatie tussen de objecten *k* en *l* kent en tevens de relatie tussen *l* en *m*, wat weet je dan over de relatie tussen *k* en *m*? Een compositietabel (*Composition Table*: CT) geeft een overzicht van alle mogelijke composities tussen twee relaties. Een compositietabel voor de 27 QTC_N relaties blijkt echter onbruikbaar, aangezien deze geen nieuwe kennis oplevert. Indien men extra kennis heeft over de relatie tussen de twee objecten (vb. een object bevindt zich op het kortste pad tussen de twee andere objecten) worden de compositietabellen bruikbaarder.

In Hoofdstuk 5 wordt onderzocht hoe de QTC_N relaties in de tijd kunnen veranderen, indien de objecten continu bewegen en veranderen van beweging. Er zijn drie gebeurtenissen die de QTC_N relatie tussen twee objecten kunnen veranderen: een snelheidsverandering, een knooppasage, en een verandering van het kortste pad tussen de twee objecten in de relatie. Een Conceptueel Burendiagram (*Conceptual Neighbourhood Diagram*: CND) dat de mogelijke veranderingen (door deze drie gebeurtenissen) in de tijd weergeeft werd opgesteld.

Aangezien niet alleen de objecten door hun beweging van plaats kunnen veranderen, maar de netwerken zelf ook kunnen veranderen, wordt de invloed van deze veranderingen op QTC_N relaties bestudeerd in Hoofdstuk 6. Indien de netwerken continu veranderen, blijkt dat de relaties tussen twee objecten kunnen veranderen alsof ze vrij in twee dimensies bewegen. Indien ook discontinu netwerkveranderingen toegelaten worden, blijkt dat nog altijd niet alle transities van een relatie naar elke andere bestaan. De hoofdreden hiervoor is dat over de ganse QTC theorie objecten verondersteld worden continu te bewegen.

In Hoofdstuk 7 wordt aangetoond dat een QTC_N relatie slecht kan omgevormd worden naar een zogenaamde RTC_N relatie. Een RTC_N relatie bepaalt of de afstand tussen twee objecten gelijk blijft of kleiner of groter wordt. Deze eigenschap van QTC_N relaties is enerzijds belangrijk omdat ze toelaat extra kennis af te leiden uit een QTC_N relatie en anderzijds omdat een dergelijke unieke transformatie niet geldt voor objecten die vrij in de ruimte kunnen bewegen (QTC_{B2}).

Indien men een kwalitatieve calculus wil gebruiken in termen van Naive Geografie is het noodzakelijk om na te gaan hoe geschikt deze calculus is om bepaalde ruimtelijke taalkundige uitdrukkingen weer te geven, door middel van empirische testen. In Hoofdstuk 8 wordt een eerste aanzet gegeven om de taalkundige en cognitieve bruikbaarheid en geschiktheid van QTC na te gaan. Voorlopig wordt enkel QTC_B behandeld omdat deze calculus intuïtief bewegingen '*naar*' en '*weg van*' een object beschrijft. De testen beperken zich ook tot objecten die in één dimensie kunnen bewegen (i.e. QTC_{B12} relaties). Het belangrijkste besluit uit Hoofdstuk 8 is dat objecten enkel '*naar*' ('*towards*') of '*weg van*' ('*away from*') een ander object kunnen bewegen als en slechts als de afstand tussen beide objecten respectievelijk verkleint of vergroot. Met andere woorden, objecten die in de richting van een ander object bewegen maar door een tragere snelheid terrein verliezen op dat object kunnen niet als een beweging '*naar*' dat object gecommuniceerd worden of omgekeerd een object dat in de andere richting van het referentie object beweegt maar dreigt ingehaald te worden kan niet gecommuniceerd worden als een beweging '*weg van*' dat object. Daarenboven hoeven bewegingen '*naar*' een object niet noodzakelijk te eindigen op dezelfde plaats als het referentie object en objecten die '*weg van*' een object bewegen kunnen starten op dezelfde plaats als het referentie object.

Tot slot worden de algemene conclusies besproken in Hoofdstuk 9, aangevuld met mogelijkheden voor verder onderzoek.

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Chapter 3: Section 3.3; Chapter 5: Section 5.3

Bogaert, P., Van de Weghe, N., and De Maeyer, P. (2004). "Description, Definition and Proof of a Qualitative State Change of Moving Objects along a Road Network." M. Raubal, A. Sliwinski, and W. Kuhn, eds., *Geoinformation and Mobility - Geoinformation and Mobility (GI-days 2004)*, Münster, Germany, 239-248.

Chapter 3: Section 3.3; Chapter 5: Section 5.3

Bogaert, P., Van de Weghe, N., Cohn, A. G., Witlox, F., and De Maeyer, P. (2006). "Reasoning about Moving Point Objects on Networks." M. Raubal, H. J. Miller, A. U. Frank, and F. Goodchild, eds., *4th International Conference on Geographic Information Science (GIScience 2006)*, Münster, Germany, 29-32.

Chapter 3: Sections 3.2, 3.3

Bogaert, P., Van de Weghe, N., Cohn, A. G., Witlox, F., and De Maeyer, P. (2007). "The Qualitative Trajectory Calculus on Networks." *Lecture Notes in Artificial Intelligence*, 4387, 20-38.

Chapter 3: Section 3.2, 3.3; Chapter 5: Section 5.2, 5.3

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Chapter 6: Sections 6.3

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Furthermore, parts of this work are under review or are planned to be submitted:

Bogaert, P., Cohn, A. G., Van de Weghe, N., Delafontaine, M., Witlox, F., and De Maeyer, P. "QTC_N: A Calculus for querying moving points on a network." *International Journal of Geographical Information Science* (under review).

Chapter 3: Sections 3.2, 3.3; Chapter 7: Sections 7.1, 7.2

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Chapter 8: Sections 8.2, 8.3, 8.4, 8.4

Special thanks go out to the co-authors for their contribution.

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List of Abbreviations

ATIS	Advanced Traveler Information Systems
CND	Conceptual Neighbourhood Diagram
CT	Composition Table
DC	is disconnected from
EC	is externally connected to
EQ	is identical with
GIS	Geographical Information Systems
GIS-T	Geographical Information Systems for Transportation
HCI	Human Computer Interaction
JEPD	jointly exhaustive and pairwise disjoint
NTPP	is a nontangential proper part of
NTPPI	is the inverse of a nontangential proper part of
$OPRA_m$	Oriented Point Relation Algebra
PO	partially overlaps
QR	Qualitative Reasoning
QSR	Qualitative Spatial Reasoning
QSTR	Qualitative Spatiotemporal Reasoning
QTC	the Qualitative Trajectory Calculus
QTC_C	the Qualitative Trajectory Calculus - Double Cross
QTC_{C2}	the Qualitative Trajectory Calculus-Double Cross in two dimensions
QTC_{C21}	the Qualitative Trajectory Calculus-Double Cross in two dimensions at level 1
QTC_{C22}	the Qualitative Trajectory Calculus-Double Cross in two dimensions at level 2
QTC_B	the Qualitative Trajectory Calculus-Basic
QTC_{B1}	the Qualitative Trajectory Calculus-Basic in one dimension
QTC_{B11}	the Qualitative Trajectory Calculus-Basic in one dimension at level 1
QTC_{B12}	the Qualitative Trajectory Calculus-Basic in one dimension at level 2
QTC_{B2}	the Qualitative Trajectory Calculus-Basic in two dimension
QTC_{B21}	the Qualitative Trajectory Calculus-Basic in two dimension at level 1
QTC_{B22}	the Qualitative Trajectory Calculus-Basic in two dimension at level 2
QTC_N	the Qualitative Trajectory Calculus on Networks
$QTC_{N'}$	the Qualitative Trajectory Calculus on Changing Networks

$QTC_{CN'}$	the Qualitative Trajectory Calculus on Changing Networks affected by Continuous Changes
$QTC_{DN'}$	the Qualitative Trajectory Calculus on Changing Networks affected by Discontinuous Changes
RTC	the Relative Trajectory Calculus
RTC_N	the Relative Trajectory Calculus on Networks
TMS	Traffic Management Systems
TPP	is a tangential proper part of
TPPI	is the inverse of a tangential proper part

List of Symbols

$=$	is equal to
$\neq$	is not equal to
$<$	is smaller than
$>$	is larger than
$\leq$	is smaller than or equal to
$\geq$	is larger than or equal to
$\sum$	summation
$\exists k$	there exists a k
$\forall k$	for all k
$\neg k$	not k
$k \wedge l$	k and l
$k \vee l$	k or l
$k \rightarrow l$	k implies l
$k \leftrightarrow l$	k is equivalent to l
$\{k,l\}$	a set whose elements are k and l
$\{x \mid p(x)\}$	The set of all x such that $p(x)$ is true
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^2$	$\mathbb{R}^2$ x $\mathbb{R}^2$
$\in$	is an element of
$\notin$	is not an element of
$\|A\|$	The number of elements in A
$\emptyset$	the empty set
$A \subset B$	A is a proper set of B
$A \subseteq B$	A is a subset of B
$A \cap B$	the intersection of A and B
$A \cup B$	the union of A and B
t	the time point t
t^-	the time point temporally before t
t^+	the time point temporally after t

$i(t^{-}\ t^{+})$	an interval beginning at t^{-} and ending at t^{+}
$\asymp$	the temporal connection
$t_1 \prec t_2$	t_1 is temporally before t_2
$k\vert t$	the position of an object k at time t
v_k	the speed of k
$v_k\vert t$	the speed of x at time t
$\mathrm{d}(k,l)$	the distance between two positions k and l
$\angle(k,l)$	the angle between k and l
$\mapsto v_k$	the velocity vector of object k
δA	the boundary of A
A°	the interior of A
A^{-}	The exterior of A
RL	the reference line through k and l
$RL\perp 1$	the first perpendicular reference line
$RL\perp 2$	the second perpendicular reference line
S	a spatial framework
curves(S)	the set of simple non closed curves in S
c	a simple non closed curves in S
len(c)	the length of c
end(c,x)	is true if x is an end point of a curve c
subcurve(c,x,y)	the subcurve of c, between (and including) x and y
W	a network
nodes(W)	the set of nodes in W
edges(W)	the set of edges in W
loc(n)	the spatial location of a node n in S
deg(n)	the degree of a node n
loc(e)	the spatial location of an edge e in S
seg(e,x,y)	a segment of *edge segment* between (and including) x and y
p	a path in W
$\vert p\vert$	the length of a path p
SP^{t}_{Wkl}	a shortest path in a network W between two different objects k and l at time t

$R(k,l)$	a relation between k and l
$R_1 \otimes R_2$	the weak composition of R_1 and R_2
$R_1 \circ R_2$	the strong composition of R_1 and R_2

Chapter 1
Introduction

1.1 Background and Problem Statement

As Hazarika (2005, p.1) stated: *"Moving around the environment is one of the primary tasks which human beings and animals accomplish equally well"*. Therefore, it is not surprising that continuously moving objects are prevalent in many domains such as human movement analysis (traffic planning or sports scene analysis) and animal behaviour science (Laube et al. 2005). From the geometrical point of view, most applications focus on the positional movement of the object itself, therefore these objects are commonly simplified into points. Recent advances in various positioning technologies (GPS, wireless communication, …) allow capturing and storing large amounts of such moving point objects (Laube et al. 2005; Zeimpekis et al. 2003). Research has been done in generating (Brinkhoff 2002; Pfoser and Theodoridis 2003), indexing (Agarwal et al. 2003; Saltenis et al. 2000), modelling and querying (Erwig et al. 1999; Sistla et al. 1997) moving objects in spatiotemporal databases. However, only recently work has been conducted in reasoning about the relations between moving point objects, especially in a qualitative framework (Cohn and Renz 2007; Van de Weghe 2004). Yet, it is argued that new techniques within information systems, such as Geographical Information Systems (GIS), should increasingly employ qualitative methods (Egenhofer and Mark 1995b), in order to come closer to the way information is communicated (Renz et al. 2000), since humans prefer to communicate in a qualitative way (Freksa 1992b). In terms of GIS, these ideas completely fit within the scope of Naive Geography (Egenhofer and Mark 1995b). Due to the popularity of these systems, they are not only used by domain specialists (e.g. Google Earth, car navigation systems). Using qualitative methods should ensure information systems to be easily accessible to a large range of users (Egenhofer and Mark 1995b).

As reasoning about motion is an important part of common sense knowledge (Forbus 1980), there is a need to develop qualitative motion calculi. Mereotopology is the most developed area of qualitative spatial reasoning (Bennett 1997). However, according to the 9-intersection model there are only two trivial topological relations between two point objects: equal and disjoint (Egenhofer and Franzosa 1991). Since in the real world most moving objects, have disjoint relations, and topological models cannot further differentiate between disjoint objects nor indeed can any purely topological representation, these calculi are not expressive enough. An obvious example is the case of two airplanes, in which it is imperative to know whether both airplanes are likely to stay in a disjoint relation; if not, the consequences are catastrophic. A more expressive calculus, able to describe and reason about continuously moving objects is the Qualitative Trajectory Calculus (QTC) introduced by Van de Weghe (2004). This calculus deals with qualitative relations between two disjoint, moving point objects. In Van de Weghe (2004), two types of QTC are defined. The Qualitative Trajectory Calculus – Double Cross (QTC_C) examines relations between moving point objects based on three reference lines forming a so called double cross (Van de Weghe et al. 2005a; Van de Weghe et al. 2005b). The Qualitative Trajectory Calculus – Basic (QTC_B) copes with these relations by comparing differences in distance (Van de Weghe et al. 2006; Van de Weghe and De Maeyer 2005).

Moreira et al. (1999) differentiate between two kinds of moving objects: objects that have a completely free trajectory, only constrained by the dynamics of the object itself (e.g. a bird flying through the sky) and objects that have a constrained trajectory (e.g. a train on a railway track). Clearly, a large number of human movements are tied to a network. For that reason, in this thesis the focus is on extending QTC for moving point objects constrained by networks, i.e. the Qualitative Trajectory Calculus on networks (QTC_N): a calculus for representing and reasoning about qualitative relations between two disjoint objects moving along a network.

1.2 Thesis Structure

This thesis is divided into nine chapters. After this introduction (Chapter 1), a general overview of related work in the field of Qualitative Reasoning is given in Chapter 2. As a movement has a spatial as well as a temporal dimension, the different approaches to temporal, spatial and spatiotemporal qualitative representation and reasoning are mentioned. Special attention is paid to the Qualitative Trajectory Calculus, as it is the more general theory in which the Qualitative Trajectory Calculus on Networks (QTC_N) fits.

In Chapter 3, QTC_N is introduced. There are two main sections in this chapter. First of all, a definition concerning the network and the objects moving on it, is stated. Afterwards, the focus in on the formal definition of a relation in QTC_N and the different canonical cases are presented.

The next four chapters focus on well known reasoning techniques within Qualitative Reasoning. Chapter 4 addresses the composition of QTC_N relations and different Composition Tables are built. Chapter 5 examines how QTC_N relations change over time, assuming continuous motion. The different events causing a QTC_N relation to change are given and the Conceptual Neighbourhood Diagram is constructed. Since not only the moving objects, but the network itself on which the objects move, can be subject to change, the effect of changes to the network on QTC_N relations are studied in Chapter 6. The different transitions in QTC_N relations caused by continuous and discontinuous changes to the network and the combination of both are examined and different Conceptual Neighbourhood Diagrams are constructed. Chapter 7 focuses on the transformation of QTC_N into a purely relative distance calculus, the Relative Trajectory Calculus on Networks (RTC_N). In contrast to QTC, which computes distances between objects at different times (e.g. computing the distance between object k at time point t_1 and object l at time point t_2), the Relative Trajectory Calculus (RTC) defines relations based on the relative motion of an object k against an object l at the same moment in time. Just as the composition of QTC_N relations, this unique transformation allows inferring additional knowledge from QTC_N relations.

If qualitative calculi are to be used in terms of Naive Geography (e.g. as a means to overcome information overload or in the domain of Human Computer Interaction), empirical evidence is mandatory in order to express usefulness or strength of a qualitative calculus in these domains. Therefore, in Chapter 8, the first steps to reveal the cognitive and linguistic semantics of QTC_B are set. The focus is on QTC_B, since this calculus is (intuitively) assumed to describe the prepositions *'towards'* and *'away from'*. At this initial stage, the empirical tests are limited movements defined in QTC_{B12}, thus, objects which have a constrained linear trajectory.

Finally, conclusions and directions for future research are given in Chapter 9.

Chapter 2
Qualitative Representation and Reasoning

2.1 Introduction

Reasoning is the act of using reason to derive a conclusion from certain premises. It can be performed on qualitative as well as on quantitative information. Frequently, the view on time and space is quantitative (Freksa and Berendt 1995): "The train to the airport leaves at *5 P.M.*", "The distance between Brussels and Ghent is *55 km*", "The car drives at *30 km/h*". Quantitative information is 'measured by quantity' (Galton 2000). Typically, a predefined unit of a quantity is used (Goyal 2000). Qualitative information, on the other hand, is concerned with information which "depends on a quality" (Galton 2000): "Breakfast is *before* lunch", "The meadow is *next to* the stable", "The car is *fast*".

Scientists have become aware that the human way of thinking is qualitative in nature (Freksa 1992b), especially when we do not use any measuring tool (Escrig and Toledo 2002). Qualitative information can, therefore, be more efficient and more meaningful than quantitative information. This can be illustrated by a quote from Clementini et al. (1997, p.318): "*Saying that Alaska is 1 518 800 km² is sufficiently exact quantitative information about size and distances in Alaska but very likely it is not meaningful in relation to the spatial knowledge of the average listener. On the other hand saying that Alaska alone is bigger than all the states of the East coast from Maine to Florida is cognitively more immediate*".

Of particular interest in describing qualitative information, are representations that form a finite set of jointly exhaustive and pairwise disjoint (JEPD) relations. In a set of JEPD relations, any two entities are related by exactly one of these relations, they can be used to represent definite or complete knowledge with respect to the given level of granularity.

Incomplete or partial knowledge can be specified by unions of possible JEPD relations (Renz and Nebel 2007).

A key topic, concerning qualitative information, is to find ways to represent continuous aspects of the world (space, time, quantity, etc.) by a small set of symbols (Cohn and Hazarika 2001; Forbus 1997). In the qualitative approach, continuous information is being quantised or qualitatively discretised by landmarks separating neighbouring open intervals, resulting in discrete quantity spaces (Weld and de Kleer 1990). A distinction is only introduced if it is relevant to the current research context (Clementini et al. 1997; Cohn and Hazarika 2001). For example, if one does not know the exact location of a bucket and a water source, but one knows that the bucket is further away from the observer than the water source, one can label this relation with the qualitative value '+'. One could also say that the bucket is closer to the observer than the water source, by representing the relation with the qualitative value '−'. Finally, the bucket and the water source can be equally far from the observer, resulting in a qualitative value '0'. Only the essence of information is studied, represented by a small set of symbols such as the quantity space {–, 0, +} consisting of the landmark value '0' and its neighbouring open intervals '–' and '+'.

Since a quantity space usually has a natural ordering associated with it, arithmetic algebras are regularly devised (Clementini et al. 1997; Iwasaki 1997). Worth mentioning is that qualitative arithmetic operators do not always lead to a unique solution. For example, if one knows that Andrew is older Max and Andrew is also older than Tony, it is impossible to determine if Max is older, younger or equally old as Tony.

An example given by Freksa shows that although reasoning with qualitative information (i.e. qualitative reasoning (QR)) can sometimes lead to partial answers, this answer is sometimes better than having no answer at all: *"... we know that X was born before Y's death and that X died after Y. We do not know who was born first. From this information we can conclude that Y lived during X' lifetime or he started X' lifetime or his life overlapped with X' life. Although we can not infer who was the older artist or which was the period when they both lived, at least we know that there was a common period".* (Freksa 1992a, p.206)

Reasoning with qualitative information is often easier than its quantitative counterpart, since it is less informative, in a certain sense (Freksa 1992b). An example illustrating this statement is given by Goyal (2000, p.2): "*If a faucet is discharging water in a bathtub and the rate of water entering the tub is more than the water leaving the tub, the tub will eventually overflow. To arrive at this conclusion, no elaborate equations were used*". In line with this statement qualitative information can often provide, at an early stage of research, an ideal way to deliver insights in order to identify quickly potential problems that warrant more detailed quantitative analyses (Iwasaki 1997). Since the goal of a reasoning process usually is a qualitative rather than a quantitative result, i.e. a decision (Freksa 1992b) and information systems such as Geographical Information Systems (GIS) are built to aid people in making decisions concerning (spatial) problems, the success of such a system therefore partially depends on its ability to answer qualitative questions, without making people learn about the internal data representation (Goyal 2000).

Another important aspect of QR is that change in qualitative values is assumed continuous (Cohn and Hazarika 2001). Using the example of the bucket and the water source, it is clear that when the bucket is moved, it can not change its relation, with respect to the observer, from being further away directly into being closer from the water source without being equally far first. To put it in Forbus' (1988, p.268) words: "*Continuity is a formal way of enforcing the intuition that things change smoothly. A simple consequence of continuity, respected by all systems of qualitative physics, is that, in changing, a quantity must pass through all intermediate values. That is, if $A < B$ at time t_1, then it cannot be the case that at some later time t_2 $A > B$ holds, unless there was some time t_3 between t_1 and t_2 such that $A = B$*".

Since space and time are two important aspects of geographical information, these aspects in relation with qualitative reasoning are presented in more detail below.

2.2 Qualitative Temporal Representation and Reasoning

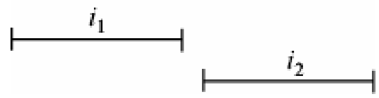
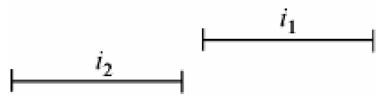
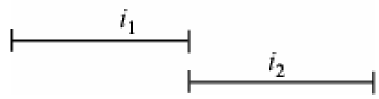
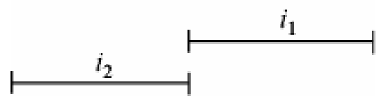
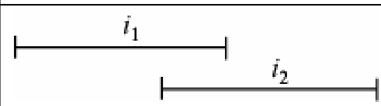
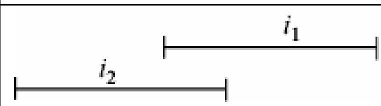
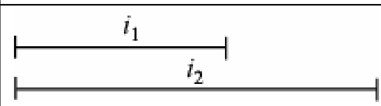
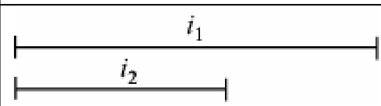
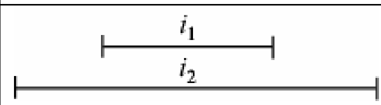
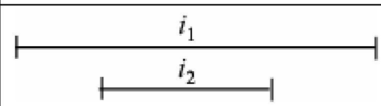
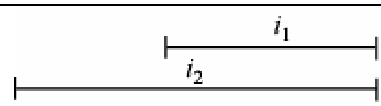
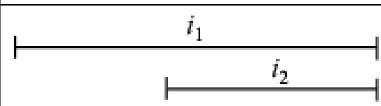
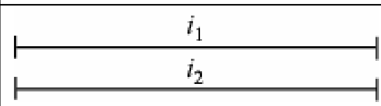
Qualitative temporal reasoning (QTR) is the subfield of QR which deals with representing and reasoning about temporal information (Pani and Bhattacharjee 2001; Vila 1994). Since time is a scalar entity, it is very well suited for a qualitative approach.

Hence, qualitative temporal reasoning has emerged as a lively subfield of qualitative reasoning and generated a lot of research effort and important results (Augusto 2001; Renz and Nebel 2007). For an overview of the different possibilities within GIS we refer to Langran (1992) and Egenhofer and Golledge (1998). Two temporal calculi, which have influenced many researchers in temporal, spatial and spatiotemporal reasoning, are presented below. Of particular interest are two well known reasoning techniques introduced by these calculi: Composition Tables (see 4.1) and Conceptual Neighbours (see 5.1).

2.2.1 The Interval Calculus

Time can be represented in many different ways. An outline of these different representations is given by Frank (1998). A single linear continuous time line is the most popular way to conceive time (Frank 1994). This time line can be represented by the set of real numbers and it has a total order associated with it. Time points and intervals can be defined on this one-dimensional line. A time point t refers to a specific moment in time and has no duration. An interval i, on the other hand, has a certain duration and is bounded by time points. So called one piece intervals are bounded by exactly two time points and can be represented as a pair (t^-,t^+) in which t^- and t^+ are part of the set of real numbers and $t^-<t^+$, meaning that t^-, the starting point of the interval, is temporally before t^+, the end-point of the interval. The Interval Calculus, introduced by Allen (1983), examines relations between two one piece time intervals, $i_1(t_1^-,t_1^+)$ and $i_2(t_2^-,t_2^+)$. Due to the total order of the starting and end-points of these two intervals, they can be qualitatively compared by three qualitative relations: greater than (>), equal to (=) and smaller than (<). In theory this would lead to 3^4=81 possible relations between two intervals. But due to the fact that the starting point of an interval takes place before the end-point and that the relations <, = and > are transitive, a set of thirteen possible interval relations remain. These thirteen so called Allen relations are JEPD and are presented in Table 2.1. In order to conduct further reasoning, Allen provides operations which can deal with the composition, the intersection and the inverse of a set of base relations (Allen 1983).

Table 2.1 The thirteen Allen relations
(based on Allen (1983 p.834-835))

Relation	Example	Condition	Symbol
i_1 is before i_2		$t_1^+ < t_2^-$	<
i_1 is after i_2		$t_1^- > t_2^+$	>
i_1 meets i_2		$t_1^+ = t_2^-$	m
i_1 is met by i_2		$t_1^- = t_2^+$	mi
i_1 overlaps i_2		$t_1^- < t_2^- \wedge t_1^+ > t_2^- \wedge t_1^+ < t_2^+$	o
i_1 is overlapped by i_2		$t_1^- > t_2^- \wedge t_1^- < t_2^+ \wedge t_1^+ > t_2^+$	oi
i_1 starts i_2		$t_1^- = t_2^- \wedge t_1^+ < t_2^+$	s
i_1 is started by i_2		$t_1^- = t_2^- \wedge t_1^+ > t_2^+$	si
i_1 is during i_2		$t_1^- > t_2^- \wedge t_1^+ < t_2^+$	d
i_1 contains i_2		$t_1^- < t_2^- \wedge t_1^+ > t_2^+$	di
i_1 finishes i_2		$t_1^- > t_2^- \wedge t_1^+ = t_2^+$	f
i_1 is finished by i_2		$t_1^- < t_2^- \wedge t_1^+ = t_2^+$	fi
i_1 equals i_2		$t_1^- = t_2^- \wedge t_1^+ = t_2^+$	=

2.2.2 The Semi-Interval Calculus

The Semi-Interval Calculus, introduced by Freksa (1992a), is a generalisation of Allen's Interval Calculus. Allen uses one piece intervals as the basic units of knowledge. The starting and end-points of these intervals are known. In many temporal reasoning situations it is not always necessary to know both points bounding an interval. As Freksa (1992a, p.206) states: *"in order to determine that Newton lived before Einstein it is sufficient to know that Newton's death took place before Einstein's birth; it does not help if in addition we know when Newton was born or when Einstein died"*. Freksa introduces the concept of semi-intervals which captures the information about either the beginning or the ending of an interval, but not both. His approach leads to an additional set of eighteen coarse relations, which are shown in Table 2.2. Each coarse relation is a conjunction of two or more Allen relations. A relation is defined as coarse relation if the corresponding disjunction forms a conceptual neighbourhood (see 5.1) of at least two relations of a JEPD set (Freksa 1992a).

Table 2.2 Coarse relations based on Semi-Intervals (based on Freksa (1992a, p.220))

Relation	Conjunction of	Condition	Symbol
i_1 is younger than i_2	d f oi mi >	$t_1^- > t_2^-$	yo
i_1 is head to head with i_2	si = s	$t_1^- = t_2^-$	hh
i_1 is older than i_2	< m o fi di	$t_1^- < t_2^-$	ol
i_1 survives i_2	di si oi mi >	$t_1^+ > t_2^+$	sv
i_1 is tail to tail with i_2	fi = f	$t_1^+ = t_2^+$	tt
i_1 is survived by i_2	< m o s d	$t_1^+ < t_2^+$	sb
i_1 precedes i_2	< m	$t_1^+ \leq t_2^-$	pr
i_1 succeeds by i_2	mi >	$t_1^- \geq t_2^+$	sd
i_1 is contemporary of i_2	o fi di si = s d f oi	$t_1^- > t_2^+ \wedge t_1^+ < t_2^-$	ct
i_1 is born before the death of i_2	< m o fi di si = s d f oi	$t_1^- < t_2^+$	bd
i_1 died after the birth of i_2	o fi di si = s d f oi mi >	$t_1^+ > t_2^-$	db
i_1 is younger than and contemporary with i_2	d f oi	$t_1^- > t_2^- \wedge t_1^- < t_2^+$	yc
I_1 is older than and contemporary with i_2	o fi di	$t_1^- < t_2^- \wedge t_1^+ > t_2^-$	oc

i_1 survives and contemporary with i_2	di si oi	$t_1^+ > t_2^+ \wedge t_1^- < t_2^+$	sc
i_1 is survived by and contemporary with i_2	o s d	$t_1^+ < t_2^+ \wedge t_1^+ > t_2^-$	bc
i_1 is older and survived by i_2	< m o	$t_1^- < t_2^- \wedge t_1^+ < t_2^+$	sc
i_1 is younger and survives i_2	oi mi >	$t_1^- > t_2^- \wedge t_1^+ > t_2^+$	yc
Unknown	< m o fi di si = s d f oi mi >	none	?

As can be deducted from the definition of a coarse relation and Table 2.2, the eighteen semi-interval relations do not form a JEPD set. Still, the semi-interval relations enlarge the set of problems that can be represented and introduce a way to deal with uncertainty in temporal reasoning (Augusto 2001).

2.3 Qualitative Spatial Representation and Reasoning

Most spatial expressions in natural language are purely qualitative (Renz and Nebel 2007). From the domain of linguistics and cognitive linguistics, literature dealing with the link between language and space are manifold (Landau and Jackendoff 1993; Levinson 2003; Talmy 2000; Tversky and Lee 1998). Spatial expressions are used for describing direction (left, north, above …), distance (far, near …), size (large, extended …), shape (round, square …), etc. Our every day interaction with the physical world is mostly driven by qualitative abstractions of the (too precise) quantitative space (Cohn and Hazarika 2001). Not only humans' interaction with real-life, but also humans' interaction with information systems benefits from qualitative approaches (Clementini et al. 1997). In spite of these strong arguments, qualitative spatial reasoning (QSR), which is the subfield of QR dealing with representing and reasoning about spatial information, has only recently developed as an active research area. This is mainly due to the fact that space is multidimensional. Reasoning in more than one dimension leads to a higher degree of freedom and increases the possibility of describing entities and relations between entities (Renz and Nebel 2007). In their poverty conjecture, Forbus et al. (1987, p.431) have doubts about the fact that space can be dealt with using only qualitative methods: *"We suspect the space of representations in higher dimensions is sparse; that for spatial reasoning almost nothing weaker than numbers will do"* . The main reason leading to this statement is that there is no total order in higher dimensions : *"Quantity spaces don't work in more than one dimension, leaving little hope of concluding much by combining weak information about spatial properties"*. Over the last couple of years an increasing amount of research in QSR tends to counter the poverty conjecture (Cohn and Hazarika 2001). Because of the richness of space and its multiple aspects, most work in QSR focuses on single aspects of space (Renz and Nebel 2007). The most important aspects of space are topology, direction, and distance. This is the order in which humans acquire spatial notions (Piaget and Inhelder 1948).

2.3.1 Topology

Topology is the study of topological transformations (or homeomorphisms) and the geometrical properties that are left invariant by them (Worboys 1995). Consequently, topological properties and topological relations are properties and relations which are preserved by homeomorphisms. Transformations such as scaling, rotation, and translation are homeomorphisms. Contrary, tearing, puncturing, joining or inducing self intersection, are not (Stahl 2005). An intuitive notion to understanding topological properties or relations is often given by an example of drawing objects on a rubber sheet. The rubber sheet can be twisted, bent, stretched but can withstand it without being ripped or torn. *"If a polygon were drawn upon the sheet and a point was drawn inside the polygon, then after any amount of stretching the point would still be inside the polygon; on the other hand, the area of the polygon may well have changed. We say that the property of 'insideness' is a topological property (because it is invariant under rubber sheet transformation) while 'area' is not a topological property"* (Worboys 1995, p.111). As a consequence, topology earned the nickname 'rubber sheet geometry' (Bennett 1997; Henle 1979; Johnson and Glenn 1960).

Topology is very well suited for QSR, because topological distinctions are inherently qualitative (Cohn and Hazarika 2001; Renz and Nebel 2007). In mathematics, there is a substantial amount of literature concerning topology. However, most of the works are not very well suited for QSR, since they are far too abstract to be relevant for 'every day' qualitative spatial descriptions (Gotts et al. 1996). Nevertheless, it is worth mentioning that it has influenced various qualitative spatial theories (Cohn and Hazarika 2001).

Most topological approaches to QSR describe relations between regions rather than points (Renz and Nebel 2007). Therefore, two approaches initially dealing with topological relations between two regions are described below: The Region Connection Calculus (RCC) (Randell et al. 1992b) and the Intersection Models (Egenhofer and Franzosa 1991; Egenhofer and Herring 1991). Both theories were developed independently at the beginning of the 90s. Although the Intersection Models originate from the domain of database theory and RCC is from the field of qualitative reasoning related to artificial intelligence, both have as draw the conclusion there are eight topological relations between two regions without holes in $\mathbb{R}^2$ (e.g the two-dimensional Euclidian space) (Van de Weghe 2004). An overview of the use of topology within GIS is given by Theobald (2001).

2.3.1.1 The Region Connection Calculus (RCC)

Based on the work of Clarke (Clarke 1981; Clarke 1985) and their own previous work (Randell 1991; Randell and Cohn 1989; Randell and Cohn 1992), Randell, Cui and Cohn introduced the Region Connection Calculus (Randell et al. 1992b). Reasoning in RCC is based on the primitive binary relation ‘x connects with y’, C(x,y). Using this primitive, further binary relations, shown in Table 2.3, can be defined on spatial non empty regions without holes.

Table 2.3 Relations defined in RCC
(based on Randell et al. (1992b, p.168))

Relation	Condition	Symbol
x is disconnected from y	$\neg \mathrm{C}(x,y)$	DC(x,y)
x is a part of y	$\forall z[\mathrm{C}(z,x) \rightarrow \mathrm{C}(z,y)]$	P(x,y)
x is a proper part of y	$\mathrm{P}(x,y) \wedge \neg \mathrm{P}(y,x)$	PP(x,y)
x is identical with y	$\mathrm{P}(x,y) = \mathrm{P}(y,x)$	EQ(x,y)
x overlaps y	$\exists \mathrm{z}[\mathrm{P}(z,x) \wedge \mathrm{P}(z,y)]$	O(x,y)
x is discrete from y	$\neg \mathrm{O}(x,y)$	DR(x,y)
x partially overlaps y	$\mathrm{O}(x,y) \wedge \neg \mathrm{P}(x,y) \wedge \neg \mathrm{O}(y,x)$	PO(x,y)
x is externally connected to y	$\mathrm{C}(x,y) \wedge \neg \mathrm{O}(x,y)$	EC(x,y)

x is a tangential proper part of *y*	$\mathrm{PP}(x,y) \wedge \exists z[\mathrm{EC}(z,x) \wedge \mathrm{EC}(z,y)]$	TPP(*x*,*y*)
x is a nontangential proper part of *y*	$\mathrm{PP}(x,y) \wedge \neg\exists z[\mathrm{EC}(z,x) \wedge \mathrm{EC}(z,y)]$	NTPP(*x*,*y*)

The relations P(*x*,*y*), PP(*x*,*y*), TPP(*x*,*y*) and NTPP(*x*,*y*) in Table 2.3 are not symmetrical and support an inverse relation, denoted respectively by Pi(*x*,*y*), PPi(*x*,*y*), TPPi(*x*,*y*) and NTPPi(*x*,*y*). It can be proven that the eight relations DC(*x*,*y*), EC(*x*,*y*), PO(*x*,*y*), EQ(*x*,*y*), TPP(*x*,*y*), TTPi(*x*,*y*), NTPP(*x*,*y*) and NTPPi(*x*,*y*), illustrated in Figure 2.1, form a JEPD set. This set is known as RCC-8 in order to distinguish from other sets of RCC relations: RCC-5, RCC-15 and RCC-23 (Cohn et al. 1997).

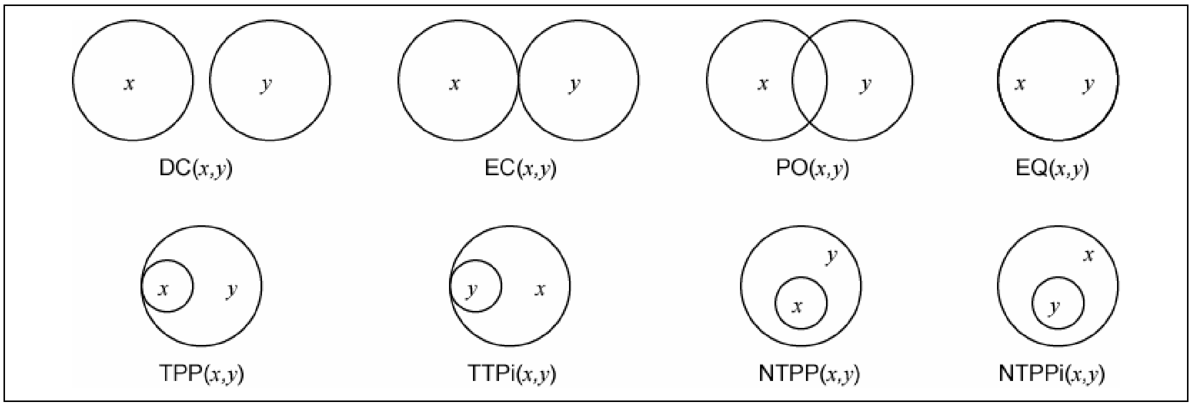


Figure 2.1 The RRC-8 relations
(based on Randell et al. (1992b, p.169))

In RCC-5, the differentiation made by two regions touching is neglected, as a result the RCC-8 relations DC(*x*,*y*) and EC(*x*,*y*) are combined into the DR(*x*,*y*) relation, as well as the relations TPP(i) and NTPP(i)(*x*,*y*) which are combined in the PP(i)(*x*,*y*) relation.

RCC-23 and RCC-15 extend the possible relations of respectively RCC-8 and RCC-5 by determining whether the primary region is inside, partially overlaps with, or is outside the convex hull of the other region involved in the relation.

As for the interval relations, operations for the composition of relations have been elaborated and the conceptual neighbours of the RCC relations have been examined (Cohn et al. 1997).

2.3.1.2 Topological Relations via n-Intersections

An alternative approach to representing and reasoning with topological relations arose from a series of papers by Egenhofer and co-authors. Originally, a 4-Intersection Model was proposed (Egenhofer 1989; Egenhofer and Franzosa 1991). This model represents a topological relation between two spatial entities by means of an intersection matrix which indicates if the intersections of the entities respective boundaries (δ) and interiors (°) are empty (∅) or non empty (¬∅) set of points. In theory this leads to a possible set of $2^4=16$ possible relations. By applying this model on regions without holes which are embedded in the two-dimensional Euclidean plane, eight relations remain. These eight relations correspond to the exact same set defined in RCC-8 but are named differently: disjoint = is disconnected from (DC), overlaps = partially overlap (PO), meets = is externally connected to (EC), equals = is identical with (EQ), inside = is a nontangential proper part of (NTPP), contains = is the inverse of a nontangential proper part of (NTPPi), covers = is a tangential proper part of (TPP), and covered-by = is the inverse of a tangential proper part of (TPPi). Figure 2.2 shows the 4-Intersection matrix of the overlap relation.

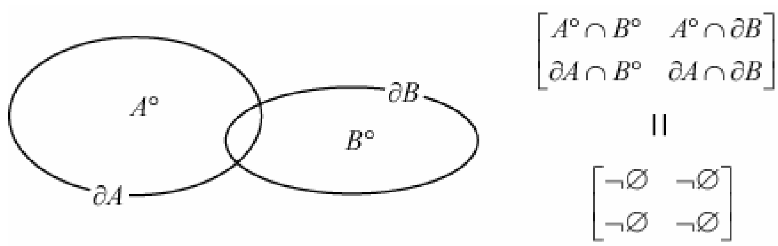


Figure 2.2 The overlap relation in the 4-Intersection Model (based on Egenhofer and Franzoza (1991))

The 4-intersection model is expressive enough to differentiate topological relations between n-dimensional entities embedded in an n-dimensional space (i.e. co-dimension = 0), e.g. relations between two regions which are two-dimensional embedded in $\mathbb{R}^2$ (Egenhofer et al. 1993). The situation is quite different if the dimension of at least one of the entities involved in a topological relation has a lower dimension than the space embedding it (i.e. co-dimension > 0). For example, there is no adequate representation of two equal lines, which are one-dimensional entities, embedded in $\mathbb{R}^2$ (Figure 2.3a) (Egenhofer et al. 1993). Extending the 4-Intersection model to a 9-Intersection Model (Egenhofer 1991; Egenhofer and Herring 1991), which adds in the exteriors (⁻) of spatial entities, gives a finer level of granularity and is able to cope with the problem induced by a co-dimension > 0 (Figure 2.3b).

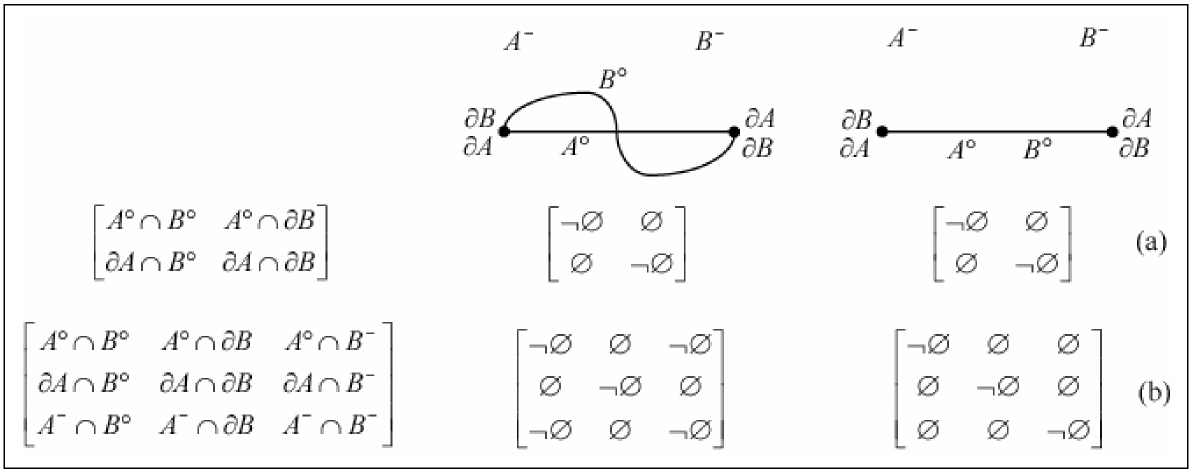


Figure 2.3 The 4 and 9-Intersection representation of two line-line relations in $\mathbb{R}^2$ (based on Egenhofer et al. (1993))

The 9-Intersection Model leads to the exact same eight relations between two regions without holes embedded in $\mathbb{R}^2$. Furthermore, there are 33 relations between simple lines, 20 relations between simple lines and regions without holes, 3 relations between points and regions without holes, 3 relations between points and simple lines and 2 relations between two points in $\mathbb{R}^2$ (Egenhofer and Herring 1991).

2.3.2 Direction

In everyday communication about directions, people tend to use qualitative descriptions such as “to the right of”, “on top of”, “behind”, “west of”, etc. instead of precise numerical descriptions such as “35 degrees”, which rather refer to professional communications such as in navigation. Therefore, directional information is very well suited for a qualitative approach (Renz and Nebel 2007). A directional relation of an object to another object can be defined in terms of three basic concepts: a primary object, a reference object and a certain frame of reference (Clementini et al. 1997). Thus, unlike topological relations, directional relations are not binary but ternary, since next to two objects, a frame of reference is required (Cohn and Renz 2007). Retz-Schmidt (1988) identifies three different kinds of frames of reference: extrinsic, intrinsic and deictic. An extrinsic reference system is imposed by external factors on the reference object (e.g. a north-south axis). On the other hand, an intrinsic reference system is given by some inherent property of the reference object itself (e.g. its front), while a deictic reference system is imposed by the point of view from which the reference object is seen (e.g. the viewer's left). When a direction calculus has a primary object with an intrinsic front, it is normally referred to as an orientation calculus (Cohn and Renz 2007). The primary and the reference object in direction calculi are usually points instead of regions or lines (Cohn and Renz 2007).

2.3.2.1 Cardinal Direction Calculi

A typical example of relations imposed by an extrinsic reference system is the set of cardinal directions. Frank (1991a; 1991b; 1992; 1996) suggests projected and cone based representations of the cardinal directions north, northeast, east, southeast, south, southwest, west and northwest. In some cases, the cardinal relations are extended by a qualitative value '0' for representing an undetermined direction between points that are too close to each other to be able to give a cardinal direction (Figure 2.4).

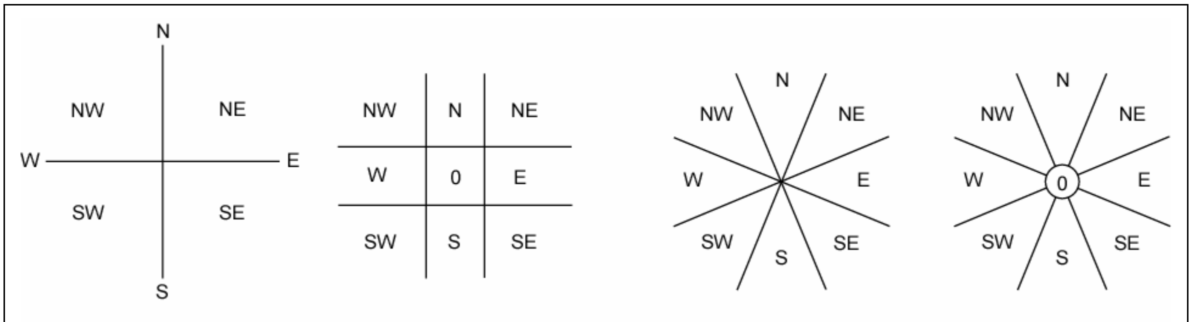


Figure 2.4 Cardinal direction relations
(based on Frank (1991a; 1991b; 1992; 1996))

With these sets of relations Frank reasons about inverse relations (e.g. if *A* is west of *B*, then *B* must be east of *A*) and combinations of the directions of two contiguous line segments (if *A* is north of *B*, and *B* is north of *C*, then *A* must be north of *C*). The complexity of reasoning with projection based cardinal relations was examined by Ligozat (1998) and is referred to as the Cardinal Direction Calculus.

In their Star Calculus Renz and Mitra (2004) generalise the Cardinal Direction Calculus over different granularities by allowing *n* different reference lines (instead of two or four explicit reference lines as defined by Frank) with arbitrary angles between them (instead of fixed equally large angles), resulting in a set of 2*n*+1 different directional relations.

Worth mentioning is the work of Hernandez (1994), which is very similar to the work of Frank. The main difference consists in the use of an extrinsic reference frame, instead of an intrinsic reference system, leading to relations such as front, front-right, right, back-right, back, back-left, left and front-left. Analogously, the Oriented Point Relation Algebra ($OPRA_m$) (Dylla and Wallgrun 2007; Moratz 2006; Moratz et al. 2005) is very well connected to the Star Calculus, by allowing multiple granularities over an intrinsic reference frame. Another difference with the Star Calculus is that in $OPRA_m$ both the primary and the reference object have an intrinsic reference frame and a relation is defined by the combination of the relation obtained by these two reference frames.

2.3.2.2 The Double Cross Calculus

Another approach individualising qualitative directional relations is given in the Double Cross Calculus originally introduced by Freksa (1992b) and further developed by Freksa and Zimmermann (1992; 1993; 1996). The central research question in the Double Cross Calculus is: "*Consider a person walking from some point a to point b. On his way, he is observing point c. He wants to relate point c to the vector ab*" (Zimmermann and Freksa 1996, p.51). The reference frame for deriving this relation consists of three reference lines (Figure 2.5a). A first reference line (*RL*) is instantiated and oriented by the intrinsic front/back reference system of the primary object. The other two reference lines are constructed perpendicular to the first one. These additional axes intersect the first reference line at the starting point (O) of the primary object ($RL\perp1$) and at the assumed end-point (D) of the primary object ($RL\perp2$). Using the reference frame, fifteen qualitative relations for an object with respect to a reference object can be deferred (Figure 2.5b): the co-location with origin or destination, a location on the line segment in between origin and destination, a location on one of the six half lines, or in one of the six half planes.

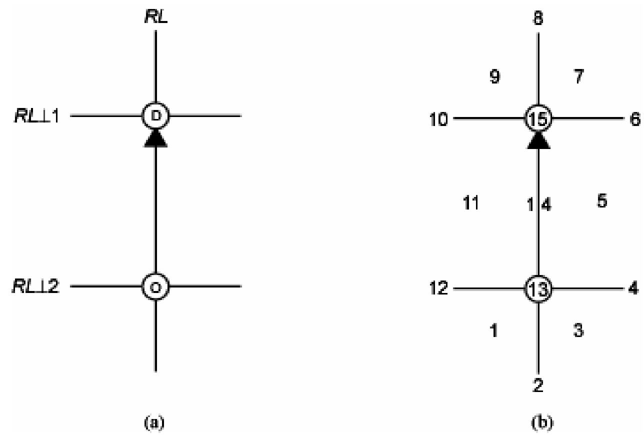


Figure 2.5 The Double Cross Calculus
(based on: Freksa (1992b))

2.3.3 Distance

Distance relations can be classified into two main categories. A distinction can be made between absolute distance relations which are obtained by the distance between two spatial entities, and relative distance relations which, in their turn, are obtained by the distance between two spatial entities as compared to the distance to a third entity (Hazarika 2005). In other words, a distinction has to be made in naming distances and comparing them (Hernandez et al. 1995). Relative distance relations are purely qualitative and result in ternary qualitative relations such as closer than, equidistant or further than (Figure 2.6b). Absolute distance relations, on the other hand, can be represented either qualitatively (e.g. "*A* is close to *B*") or quantitatively (e.g. "*A* is one metre away from *B*") (Renz and Nebel 2007). Qualitative absolute distance relations are commonly obtained by dividing the physical space into several regions of different sizes (Hernandez et al. 1995), for example when working in an isotropic two-dimensional space this leads to a set of concentric circles which stand for very close, close, commensurate, far, very far (Figure 2.1a) (Renz and Nebel 2007). The number of divisions depends on the level of granularity and the difference in size can for example be obtained in the order of magnitude (Mavrovouniotis and Stephanopoulos 1988; Raiman 1991). Naming distances is largely context dependent (Hernandez et al. 1995). According to Hernandez et al. (1995) this context depends on the frame of reference used in naming the relation. The frame of reference is defined by its type (intrinsic, extrinsic or deictic), its scale and the distance system used.

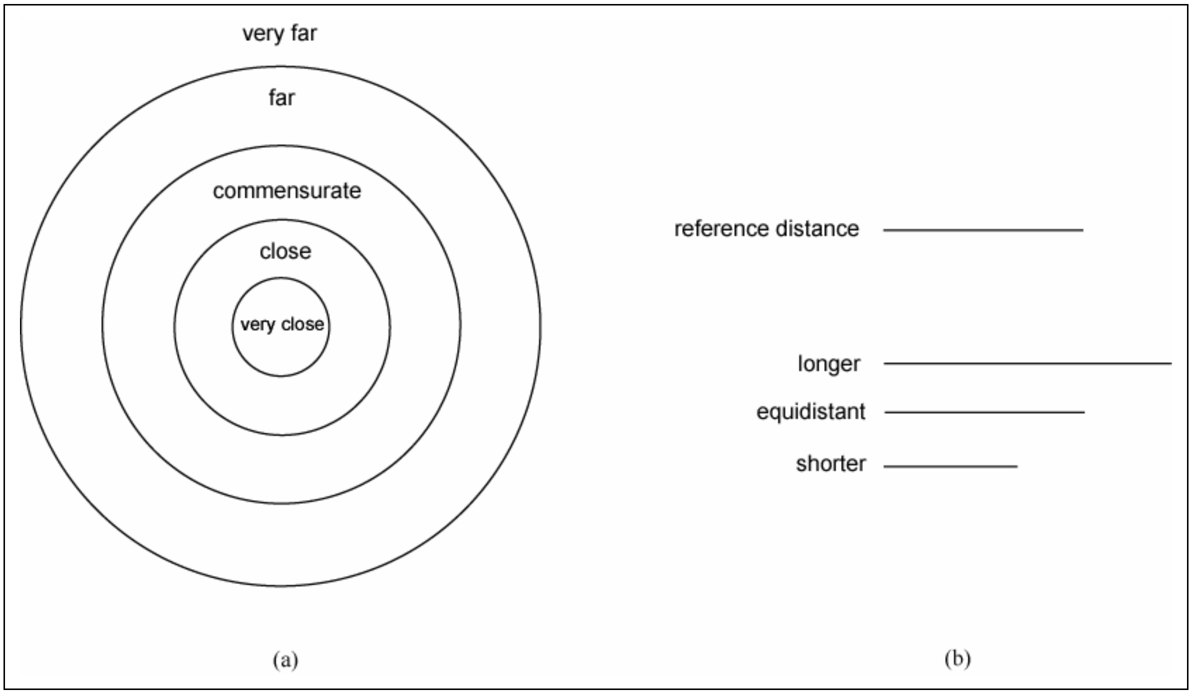


Figure 2.6 Absolute distance (a) and relative distance (b) relations

Like direction calculi, reasoning in distance calculi is often based on points rather than regions or lines. Reasoning with qualitative distance alone can lead to difficulties. An example illustrating the problem is given by Renz and Nebel (2007, p.18): "*…if point B is far from A and C is far from B, then C can be very far from A if A, B, and C are aligned and if B is between A and C; or C can be close to A if the angle between AB and BC is small*". Therefore, distance is often studied in combination with direction. The combination of directional and distance information is referred to as positional information (Hazarika 2005). Examples of positional Calculi are given by Frank (1992), who reasons about the combined information given by cardinal directions and two distance relations: close and far, Clementini et al. (1997), who reason about relations obtained by cone-based direction relations and absolute distance relations, and Isli and Moratz (1999), who combine relative direction with relative distance.

2.4 Qualitative Spatiotemporal Representation and Reasoning

So far, space and time have been treated separately. Yet, space and time are very closely connected (Peuquet 2002). Within QSR, time often enters the picture when dealing with the change of spatial representations (Muller 1998b) or as Puequet (1994) states: *"The passage of time is normally understood via changes we perceive occurring to objects in space".* Worboys (2001) pinpoints two definitions of the concept of change. A first definition, going back to the ancient Greek philosophers, refers to the verb change: "*An object o changes if and only if there exists a property P of o and distinct times t and t' such that o has property P at t and o does not have property P at t'"* (Worboys 2001, p.131) A second definition, introduced by Russel (1903) refers to change as a noun: "*A change occurs if and only if there exists a proposition Π and distinct times t and t' such that Π is true at t but false at t'"* (Worboys 2001, p.131).

Change can be continuous or discontinuous. A discontinuous change alters the value of a property of an object instantaneously from one value to another (e.g a parcel changing owner). On the other hand, when changes are continuous, a change in the value of a property of an object can be described by a continuous mathematical function, the property varies as a function of time (e.g. the change in temperature during the day) (Moreira et al. 1999).

According to Frank (2001), spatial entities can undergo two types of changes. The first type of change refers to the life of a spatial entity: it can appear, split, merge or disappear. The second type refers to the position and geometric form of a spatial entity: it can move or appear to move while or while not simultaneously changing its form. Qualitative Spatiotemporal Reasoning (QSTR), which is the subfield of QR dealing with representing and reasoning about combined spatial and temporal information (Egenhofer and Golledge 1998), has mainly focused on the second type of change. Galton (2000) gives an overview of qualitative changes in dimension, connectivity, location, orientation, size and shape. As stated in 2.1, qualitative spatial change is often assumed to be continuous. This assumption is widely integrated in different qualitative calculi (Cohn and Hazarika 2001). For example, most of these calculi define conceptual neighbours (see 5.1) of qualitative values or relations.

Next to reasoning about the change of qualitative spatial values and relations, only a few works integrate time and space in their formal definition of the representation of a qualitative value or relation (Gerevini and Nebel 2002). Most of these works concentrate on the combination of the spatial RCC-8 relations and a temporal formalism (Gerevini and Nebel 2002; Muller 1998a; Wolter and Zakharyaschev 2000). Muller (1998a), for example, combines the RCC-8 relations with the temporal relations $\asymp$ (temporal connection, which is the temporal equivalent of the spatial primitive C(x,y)) and < (before). Reasoning with these elements, a set of qualitative motion relations such as leave, reach, hit and cross can be represented. The combination of topological information and time aspects, can lead to interesting applications. Cole and Hornsby (2005; 2007), for example, try to identify noteworthy events by reasoning about the movement of point objects entering and leaving different regions. In spite of this, the qualitative relations offered by topology are not always sufficient, especially when reasoning about motion, since in the real world most moving objects have a disjoint (DC) relation. Neither the RCC Calculus nor the 9-Intersection Model can differentiate any further between disjoint objects, nor indeed could any purely topological representation. Moreover, when dealing with moving point objects there are, according to the 9-Intersection model, only two topological relations between points (i.e. disjoint and meet). Hence, these approaches fail to make explicit the level of disjointness of how two or more objects move with respect to each other. An obvious example, in which this type of information is of vital importance, is the case in which one tries to determine whether two airplanes are likely to stay in a disjoint relation, realising the consequences might be catastrophic. A Calculus able to describe a level of disjointness between two moving objects was introduced by Van de Weghe (2004): the Qualitative Trajectory Calculus (QTC). Since, in this thesis, QTC is extended for objects moving along networks, this calculus is described in more detail below.

2.4.1 The Qualitative Trajectory Calculus

In this section, the focus is limited to the formalisation and representational aspect of QTC. The different reasoning techniques applied on QTC relations will be handled in detail in the sections on composition (see 4.1), conceptual neighbours (see 5.2) and the transformation into a relative calculus (see 7.1). The linguistic and cognitive aspects are dealt with in Chapter 8.

QTC is a calculus used for the representation of and reasoning about movements of objects in a qualitative framework (Van de Weghe 2004). As stated by Galton (1995b, p.377): "*The phenomenon of movement arises whenever the same object occupies different positions in space at different times*". The movement or motion of an object used to derive a QTC relation is represented by a trajectory (Van de Weghe 2004). A trajectory is a connected, non-branching, continuous line having a certain shape and direction (Eschenbach et al. 1999). Between two points of a trajectory, one can always find, or at least imagine, an intermediate point. This implies that the movement of objects, for which a QTC relation can be derived, is assumed to be continuous.

In order to define and examine QTC relations, continuous movements of objects in the real world have been simplified in different levels. First of all, a QTC relation is defined at an exact moment in time which has no duration (i.e. a time point). Secondly, since QTC wants to describe a certain level of disjointness, only objects in a disjoint (DC(x,y)) relation are examined. Thirdly, QTC examines only the relations between two spatial entities with respect to a certain frame of reference. Finally, all objects are generalised into points. As stated by Van de Weghe (2004, p.25): *"This abstraction simplifies many complex motion problems without having significant disadvantages"*.

Depending on the level of detail and the number of spatial dimensions, different types of QTC are defined in Van de Weghe (2004) , all belonging to QTC-Basic (QTC_B) (Van de Weghe et al. 2006; Van de Weghe and De Maeyer 2005) or QTC-Double-Cross (QTC_C) (Van de Weghe et al. 2005a; Van de Weghe et al. 2005b).

2.4.1.1 The Qualitative Trajectory Calculus – Basic

QTC_B has been worked out for objects moving in one (QTC_{B1}) and objects moving in two dimensions (QTC_{B2}). In QTC_{B1}, it is assumed that the movement of the objects is restricted to a one-dimensional entity, such as a simple line (e.g. two trains moving on a single railroad track). In QTC_{B2}, two objects can move freely in a plane (e.g. a bird flying through the sky).

In both cases of QTC_B, qualitative relations are defined by comparing distances between the positions of the two objects involved in the relation at different moments in time. Assume that these two objects are denoted by *k*, the primary object and *l*, the secondary object. In order to define the relation of the object *k* with respect to the object *l*, object *l* is fixed in time, while the position of *k* varies over time (Figure 2.7). In QTC_B, object *k* is said to *'move towards'* object *l* at a specific moment in time *t*, if *k* reduces its distance over time with respect to the position of object *l* at time *t* (Figure 2.7a). Conversely, object *k* is said to *'move away from'* object *l* at a specific moment in time *t*, if *k* enlarges its distance over time with respect to the position of object *l* at time *t* (Figure 2.7b). In all other cases, object *k* is said to be *'stable'* with respect to object *l*.

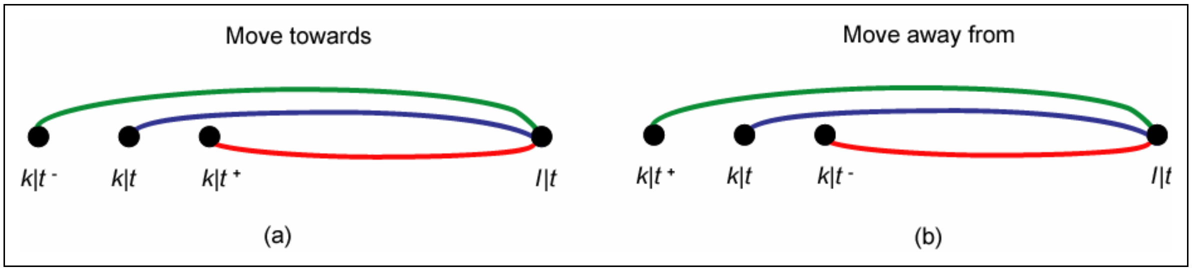


Figure 2.7 'Move towards' and 'move away from' as defined in QTC_B

To define the qualitative relations *'towards'*, *'away from'* and *'stable'* in a more formal way, the following notations are used:

- $x|t$ denotes the position of an object x at time t;
- $\mathrm{d}(u,v)$ denotes the distance between two positions u and v;
- $\mathrm{v}_x|t$ denotes the speed of x at time t;
- $t_1 \prec t_2$ denotes that t_1 is temporally before t_2;

Using these notations, the different possible movements of object *k* with respect to object *l* are formalised as follows:

- *k* is moving *'towards' l*:

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k\,|\,t^-, l\,|\,t) > \mathrm{d}(k\,|\,t, l\,|\,t))) \wedge \\ \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k\,|\,t, l\,|\,t) > \mathrm{d}(k\,|\,t^+, l\,|\,t))) \qquad (2\text{-}1)$$

- k is moving *'away from' l*:

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) < \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) < \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-2}$$

- k is '*stable'* with respect to l (all other cases):

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) = \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) = \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-3}$$

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) = \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) < \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-4}$$

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) = \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-5}$$

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) > \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) = \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-6}$$

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) > \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) < \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-7}$$

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) < \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) = \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-8}$$

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow \mathrm{d}(k \mid t^-, l \mid t) < \mathrm{d}(k \mid t, l \mid t))) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t^+, l \mid t))) \tag{2-9}$$

Since there are two objects involved in a QTC_B relation at level one (which is denoted by QTC_{B11} for objects moving in a one-dimensional space, and QTC_{B21} for objects moving in a two-dimensional space), a QTC_B relation is represented by a two character label. This label represents the following two qualitative relations:

1. The movement of object k, with respect to the position of object l at time point t:

   −: k is moving towards l (equation (2-1))

   +: k is moving away from l (equation (2-2))

   0: k is stable with respect to l (equation (2-3) to (2-9))

2. The movement of object l, with respect to the position of object k at time point t

   −: l is moving towards k (equation (2-1) with k and l interchanged)

   +: l is moving away from k (equation (2-2) with k and l interchanged)

   0: l is stable with respect to k (equation (2-3) to (2-9) with k and l interchanged)

QTC_B at level two (which is denoted by QTC_{B12} for objects moving in one dimension, and QTC_{B22} for objects moving in two dimensions), offers a finer level of granularity by also considering the relative speed of both objects k and l. Therefore, a third character is added to the label representing a QTC_B relation. Thus, a QTC_B relation at level two is represented by a three character label representing the following three qualitative relations:

1. The movement of object k, with respect to the position of object l at time point t:

   −: k is moving towards l (equation (2-1))

   +: k is moving away from l (equation (2-2))

   0: k is stable with respect to l (equation (2-3) to (2-9))

2. The movement of object l, with respect to the position of object k at time point t

   −: l is moving towards k (equation (2-1) with k and l interchanged)

   +: l is moving away from k (equation (2-2) with k and l interchanged)

   0: l is stable with respect to k (equation (2-3) to (2-9) with k and l interchanged)

3. The relative speed of object k at time point t, with respect to the speed of object l at time point t:

   −: k is moving *'slower'* than l:

$$v_k \mid t < v_l \mid t \tag{2-10}$$

   +: k is moving *'faster'* than l:

$$v_k \mid t > v_l \mid t \tag{2-11}$$

   0: k and l are moving *'equally fast'*:

$$v_k \mid t = v_l \mid t \tag{2-12}$$

By definition, in QTC_B at level 2, there should theoretically be 3^3 (27) different relations. However, in QTC_{B12} only 17 real-life possibilities remain (Figure 2.8). The reason, causing 10 relations to be inexistent, is a constraint imposed by the third character on the first two characters in the label representing the relation. An object moving in one dimension can only induce a ‘0’ in one of the first two characters of a QTC_{B1} relation if it is not moving with respect to the space embedding it. Therefore, in one dimension, a stable object can not move faster than a non-stable object (e.g. relation ‘– 0 –’). Also, it is impossible for one object to be faster than the other if both objects are stable (e.g. relation ‘0 0 +’ or ‘0 0 –’). This constraint does not hold when objects move in two dimensions, because an object moving in two dimensions can not only induce a ‘0’ in one of the first two characters of a QTC_B relation if it is not moving in the space embedding it, but also, by having a tangentional trajectory with respect to the other object involved in the relation (Van de Weghe et al. 2006). Thus, as shown in Figure 2.9, in QTC_{B22}, all 27 relations exist.

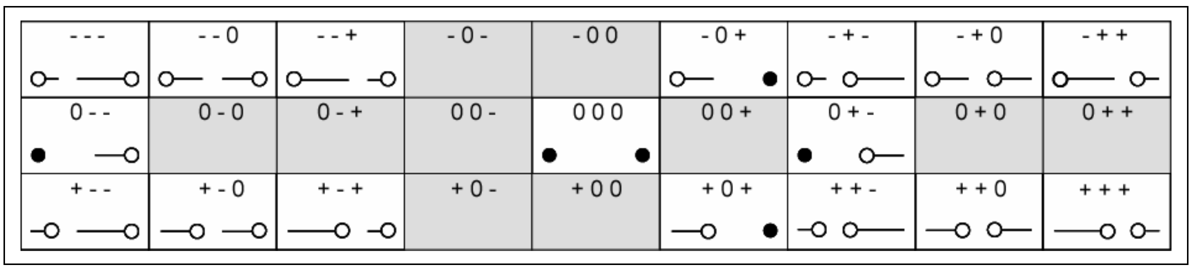


Figure 2.8 Iconic representation of QTC_{B12} relations (based on Van de Weghe et al. (2006, p.108))

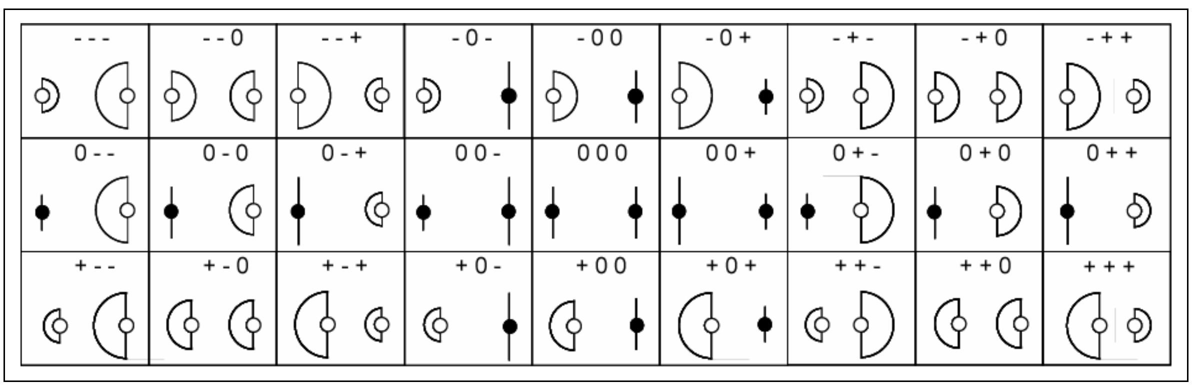


Figure 2.9 Iconic representation of QTC_{B22} relations (based on Van de Weghe et al. (2006, p.111))

In Figure 2.8 and Figure 2.9, the left and right dots represent the positions of k and l respectively. An open dot means that the object cannot be stationary. A dot is filled if the object can be stationary. In Figure 2.8, the line segments represent the potential object movements. Note that the lines can have different lengths giving the difference in relative speed. The line segments represent whether each object is moving towards or away from the other. The icons, in Figure 2.9, contain line segments with the point object positioned in the middle. The line segment denotes the possibility of movement to both sides of the point object. The icons also contain crescents with the point object in the middle of its vertical border. The crescent denotes an open polygon. If a crescent is used, then the movement starts in the dot and ends somewhere on the curved side of the crescent.

2.4.1.2 The Qualitative Trajectory Calculus – Double Cross

As stated above, a QTC_{B2} relation can not make a distinction between objects having a tangentional trajectory and objects which are not moving with respect to the space embedding them. The relations put forward by QTC_C are able to make this distinction. QTC_C offers a finer level of granularity by combining the distance based information of QTC_B together with orientation information. Therefore, QTC_C can be referred to as a positional calculus, while QTC_B is a pure distance calculus. QTC_C has only been worked out for objects moving in two dimensions (in order to be consistent with QTC_B, the notation QTC_{C2} is sometimes used).

QTC_{C2} is partly based on the Double Cross Calculus introduced by Freksa and Zimmermann (1992b; 1992; 1993; 1996). As stated in section 2.3.2.2, the reference frame for deriving a relation in the Double Cross Calculus consists of three reference lines (Figure 2.10a). A first reference line (*RL*) is instantiated and oriented by the intrinsic front/back reference system of the primary object. The other two reference lines are constructed perpendicular to the first one. These additional axes intersect the first reference line at the starting point of the primary object ($RL\perp1$) and at the assumed end-point of the primary object ($RL\perp2$). Thus, in the Double Cross Calculus, the reference frame is pinpointed on the (assumed) movement of the reference object. Using the reference frame, fifteen qualitative relations for an object with respect to a reference object can be deferred. The Double Cross Calculus only considers a single movement, in which one of both objects in the relation is moving. The reference frame in the Double Cross Calculus leads to a front/back (induced by $RL\perp1$ and $RL\perp2$) and left/right (induced by *RL*) dichotomy.

In QTC_{C2}, the double cross is oriented differently. The reference frame is pinpointed on the origin of both moving objects (Figure 2.10b). This allows capturing the movement of both objects in a single relation. Also, in contrast with the Double Cross Calculus, the reference lines $RL\perp1$ and $RL\perp2$ in QTC_{C2} lead to a towards/away from dichotomy instead of a front/back dichotomy.

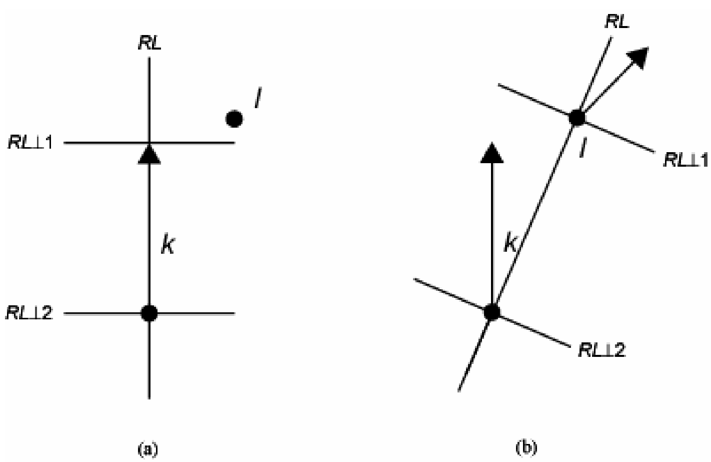


Figure 2.10 Difference between the Double Cross Calculus (a) and QTC_{C2} (b) (based on Van de Weghe et al. (2005b, p.63))

A relation in QTC_{C2} at level one (QTC_{C21}), is represented by a four character label representing the following qualitative relations:

1. The movement of object *k*, with respect to the first perpendicular reference line ($RL\perp1$) at time point *t*:
   −: *k* is moving towards *l* (equation (2-1))
   +: *k* is moving away from *l* (equation (2-2))
   0: *k* is stable with respect to *l* (equation (2-3) to (2-9))
2. The movement of object *l*, with respect to the second perpendicular reference line ($RL\perp2$) at time point *t*
   −: *l* is moving towards *k* (equation (2-1) with *k* and *l* interchanged)
   +: *l* is moving away from *k* (equation (2-2) with *k* and *l* interchanged)
   0: *l* is stable with respect to *k* (equation (2-3) to (2-9) with *k* and *l* interchanged)
3. Movement of object *k* with respect to the reference line through *k* and *l* (*RL*) at time point *t*:
   –: *k* is moving to the left side of *RL* (seen from *k* looking at *l*)

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \text{ is on the right side of } RL \text{ at } t)) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \text{ is on the left side of } RL \text{ at } t)) \tag{2-13}$$

+: k is moving to the right side of RL (seen from k looking at l)

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \text{ is on the right side of } RL \text{ at } t)) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \text{ is on the left side of } RL \text{ at } t)) \tag{2-14}$$

0: k is moving along RL

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \text{ is on } RL \text{ at } t)) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \text{ is on } RL \text{ at } t)) \tag{2-15}$$

4. Movement of object l with respect to the reference line through k and l (RL) at time point t:

   −: l is moving to the left side of RL (seen from l looking at k)

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow l \text{ is on the right side of } RL \text{ at } t)) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow l \text{ is on the left side of } RL \text{ at } t)) \tag{2-16}$$

   +: l is moving to the right side of RL (seen from l looking at k)

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow l \text{ is on the right side of } RL \text{ at } t)) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow l \text{ is on the left side of } RL \text{ at } t)) \tag{2-17}$$

   0: l is moving along RL

$$\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow l \text{ is on } RL \text{ at } t)) \wedge \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow l \text{ is on } RL \text{ at } t)) \tag{2-18}$$

All of the theoretically 81 (3^4) potential QTC_{C21}-relations exist for objects moving in two dimensions (Figure 2.11). The icons in Figure 2.11 are constructed identically to the icons representing a QTC_B relation (see p.27).

QTC_C at level two, (QTC_{C22}) adds two extra characters to the four character label of QTC_{C12}, in order to include additional information about the relative speed of both objects. These two additional characters stand for the following two qualitative relations:

5. The relative speed of object k at time point t, with respect to the speed of object l at time point t:

   −: k is moving *'slower'* than l: (equation (2-10))

   +: k is moving *'faster'* than l: (equation (2-11))

   0: k and l are moving *'equally fast'*: (equation (2-12))

6. The relative difference in angle of the velocity vector of objects k and l with respect to the reference line RL

$$-: \angle(\mapsto v_k \mid t, RL \mid t) < \angle(\mapsto v_l \mid t, RL \mid t) \tag{2-19}$$

$$+: \angle(\mapsto v_k \mid t, RL \mid t) > \angle(\mapsto v_l \mid t, RL \mid t) \tag{2-20}$$

$$0: \angle(\mapsto v_k \mid t, RL \mid t) = \angle(\mapsto v_l \mid t, RL \mid t) \tag{2-21}$$

The additional notations used to define the qualitative relations for the sixth character in a QTC_{C22} relation stand for:

- $\angle(x,y)$ denotes the angle between x and y;
- $\mapsto v_x$ denotes the velocity vector of object x.

From the 729 (3^6) potential QTC_{C22} relations, only 305 exist in the two-dimensional Euclidian plane. An iconic representation of these relations is given by Van de Weghe (2004, p.214-223).

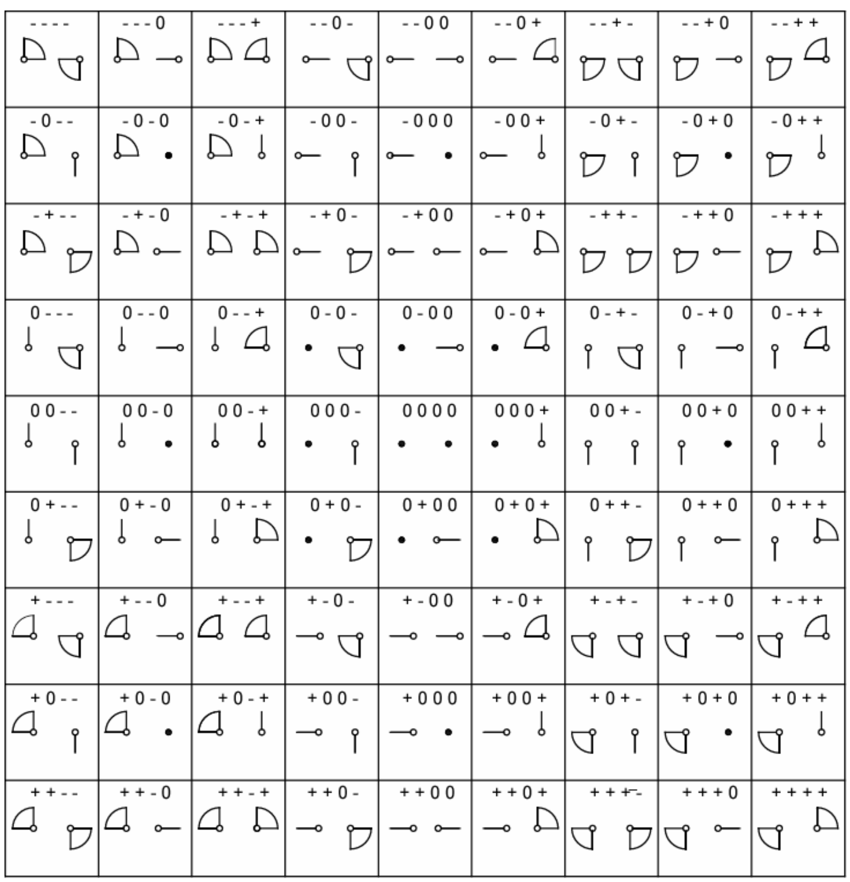


Figure 2.11 Iconic representation of QTC_{C21} relations (based on Van de Weghe et al. (2005b, p.64))

Chapter 3

A Qualitative Calculus on Networks

3.1 Introduction

Moreira et al. (1999) differentiate between two kinds of moving objects: objects that have a completely free trajectory, only constrained by the dynamics of the object itself (e.g. a bird flying through the sky) and objects that have a constrained trajectory (e.g. a train on a railway track). QTC_{B2}, for example, describes the movement of objects which have a free trajectory in two dimensions. QTC_{B1}, conversely, describes objects which have a constrained linear trajectory.

A large part of human movements can be generalised to movements which are constrained due to a network. For example, ignoring lane changing or lateral deviations within a lane, moving cars are restricted to evolve along the arcs of a road network, trains can only operate on railroad tracks, and canal boats are tied to navigable rivers and canals (Van de Weghe 2004). Hence, there is a need to develop a calculus that defines qualitative relations between two disjoint, moving objects on the constrained trajectory of a network.

A network, such as a road, rail or river network, is a set of interconnected linear features. In essence, a network is a co-dimensional structure. The concept of co-dimension can be used to express the difference in dimension between spatial entities (point: zero-dimensional; line: one-dimensional, region: two-dimensional, …) and the space they are embedded in (Galton 2000). In the case of a network, one-dimensional structures (a set of interconnected lines) are embedded in a two-dimensional (co-dimension 1) or three-dimensional space (co-dimension 2). A network is often represented by the mathematical concept of a graph. A graph is not a spatial structure itself. It needs to be embedded in a space or must be 'spatialised' (Galton and Worboys 2005). This can be done by a function which maps each node of the graph onto a location in the defined space, and maps each link of the graph onto a curve segment (Galton and Worboys 2005). Research has been done in modelling and querying (Erwig et al. 1999; Sistla et al. 1997), indexing (Agarwal et al. 2003; Saltenis et al. 2000), and generating (Brinkhoff 2002; Pfoser and Theodoridis 2003) network based moving objects in the field of spatiotemporal databases. However, there seems to be a lack in representing and reasoning about this specific type of constrained movement in a qualitative framework.(Cohn and Hazarika 2001; Van de Weghe 2004).

Directional and topological calculi are not very well suited for dealing with the description of movement relations between two point objects tied to a network. Direction, from its side, does not take the spatial structure of a network into account when describing a relation (Figure 3.1). While topology only allows, according to the 9-intersection model, two trivial topological relations between two point objects: equal and disjoint (Egenhofer and Herring 1991).

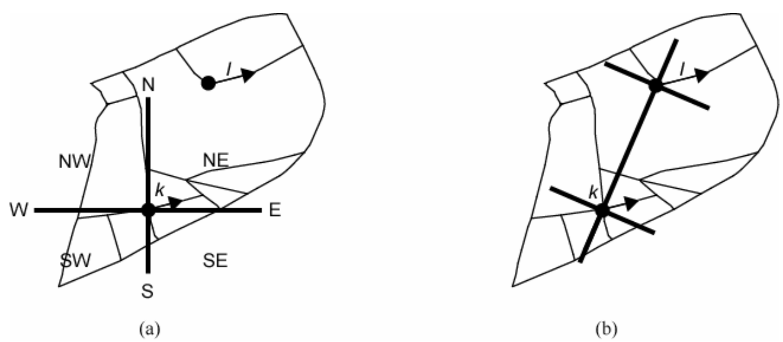


Figure 3.1 Example of a cardinal direction relation (a) and a QTC_{C2} relation (b) for objects moving along a network

Note that directional and topological calculi can be useful, when formulating a qualitative relation between an object and the network. Directional Calculi, for example, are very efficient for the use in route descriptions (Krieg-Bruckner and Shi 2006). A sequence of topological relations is useful to express qualitative motion on the network, and hence, can be used to define terms such as moving along, moving across, passing, etc. Nevertheless, a distance based calculus seems to be the best way to represent and reason about qualitative relations between two point objects moving along a network. Therefore, in this chapter, QTC_B is transformed into The Qualitative Trajectory Calculus on

Networks (QTC_N), a calculus able to represent and reason about qualitative relations between two disjoint moving point objects constrained in their movement due to a network. There are two main sections in this chapter. First of all, a definition concerning the network and the objects moving on it is stated. Afterwards, the focus on the formal definition of a relation in QTC_N and the different canonical cases are presented.

3.2 Definitions and Restrictions Concerning Networks and Moving Objects

As stated above, a network is a one-dimensional structure (a set of lines) embedded in a two-dimensional or three-dimensional space. Therefore, we assume an underlying spatial framework *S* for specifying locations. Typically this would be $\mathbb{R}^2$, but *S* could be any set with a metric distance function d(x,y) obeying the triangle inequality, and a notion of curve defined, such that curves(S) denotes the set of simple non closed curves in *S*.

In order to formally define a QTC_N relation for two moving point objects and the network they are moving on, which also serves the reference frame, three functions are defined on curves:

- For any curve, *c*, len(*c*) denotes its length;
- end(*c,x*) is true if *x* is an end point of a curve c;
- if *x* and *y* are two points incident in *c*, then subcurve(*c,x,y*) denotes the subcurve of *c,* between (and including) *x* and *y*.

The network on which objects move in QTC_N is defined as a set of linear features (edges) which are bounded by end-points (nodes) (Definition 3.1). A function loc embeds these nodes and edges in the spatial framework *S* (Definition 3.2 and Definition 3.4). As stated above, the linear features should represent simple non closed curves. To formally define this property, we do not allow two nodes to lie at the same location (Restriction 3.1), the edges should be bounded by exactly two different nodes (Definition 3.4) and two different edges only intersect at their respective end-points (Restriction 3.2). The number of edges intersecting at the same node denotes the degree of that node (Definition 3.3).

Definition 3.1 If *W* is a *network* then nodes(*W*) is its set of nodes and edges(*W*) is its set of edges.

Definition 3.2 If *n* is a *node* then loc(n)$\in S$ gives the spatial location of *n* in *S*.

Restriction 3.1 $\forall n_i \, \forall n_j \, [n_i \neq n_j \Rightarrow \text{loc}(n_i) \neq \text{loc}(n_j)]$

Definition 3.3 The *degree* of a node n, $\deg(n) = |\{e: e \in \text{edges}(W) \wedge \text{loc}(n) \in \text{loc}(e)\}|$

Definition 3.4 If e is an *edge* then $\text{loc}(e) \in \text{curves}(S)$ gives the spatial location of e in S, and $\exists\ (n_1,n_2) \in \text{nodes}(W)$ such that $[n_1 \neq n_2 \wedge \text{end}(\text{loc}(e),\text{loc}(n_1)) \wedge \text{end}(\text{loc}(e),\text{loc}(n_2))]$

Restriction 3.2 $\forall e_i \, \forall e_j \, [e_i \neq e_j \Rightarrow \text{loc}(e_i) \cap \text{loc}(e_j) \subseteq \{\text{loc}(x): x \in \text{nodes}(W)\}]$

The movement of objects in QTC_N is restricted by the network, which implies that the location of an object should at all times be situated on an edge (Definition 3.5). As stated in 2.4.1, QTC tries to relate disjoint objects, thus, two different objects cannot be at the same place at the same time (Restriction 3.3).

Definition 3.5 If o is an *object* and t a *time point*, then $o|t \in S$ gives the spatial location of o in S at t. An object o at time t is located in a network W if $\exists e \in \text{edges}(W)$ such that $o|t \in \text{loc}(e)$.

Restriction 3.3 For every pair of non identical objects, k and l,

$\forall t \, \forall k \, \forall l \, [k \neq l \Rightarrow k|t \neq l|t]$.

To relate two objects in QTC_N there needs to be at least one path between the two objects (see 3.3). A path is composed of a sequence of edges. Since the objects do not necessarily lie at one of the end-points of an edge, a function for defining edge segment is required (Definition 3.6). The function seg(e,x,y) defines that part of edge e between x and an endpoint of the edge y (including the points x and y). If x is the other end-point of e, then e' is just the whole edge e (as a special case). Thus, a path between two objects is composed of a sequence such that the first and last elements are edge segments on which the two objects are located (possibly the same segment), and the intermediate edges form a connected path between, such that no edge occurs more than once (Definition 3.7). The length of a path is defined as the sum of its edges and edge segments (Definition 3.8). A shortest path is defined as a path such that there is no path of a lesser length between the same two objects (Definition 3.9). There can be more than one shortest path between two objects at the same time. In the special case in which there are two or more shortest paths from object k to an object l and the first edge(-segment) of these paths is different for both paths we refer to these shortest paths as bifurcating shortest paths (Definition 3.10 and Figure 3.2).

Definition 3.6 If e is an edge and $x \in$ loc(e) and $\exists y$ such that end(e,y) $\wedge$. $x \neq y$ then e' = seg(e,x,y) is an *edge segment* such that loc(e') = loc(subcurve(e,x,y))

Definition 3.7 A *path p* in a network W between two different objects k and l located in W at time point t is a sequence $\langle e_1, \ldots e_m \rangle$ such that:

$$\text{end}(\text{loc}(e_1),k|t) \wedge \text{end}(\text{loc}(e_m),l|t) \wedge \{e_2, \ldots e_{m-1}\} \subseteq \text{edges}(W) \wedge$$
$$\exists(e_1', e_m', y, z) \subseteq \text{edges}(W)\ [e_1 = \text{seg}(e_1', k|t, y) \wedge e_m = \text{seg}(e_m', l|t, z)] \wedge$$
$$\forall 1 \leq i < j \leq m\ [\text{loc}(e_i) \cap \text{loc}(e_{i+1}) \neq \varnothing \Rightarrow |i-j| = 1]$$

Definition 3.8 $|p| = \sum_{e \in p} \text{len}(\text{loc}(e))$ is the *length of a path p*

Definition 3.9 A *shortest path* SP^t_{Wkl} in a network W between two different objects k and l at time t is defined as a path p such that there is no path of length less than $|p|$ between the same two objects. We may write SP^t_{Wkl} (p) when p is such a shortest path.

Definition 3.10 If there are at least two shortest paths, $<e_1, \ldots>$ and $<e_{1'}, \ldots>$ between object k and another object l at time t and $e_1 \neq e_{1'}$, then there is a *bifurcating shortest path* from k to l at t.

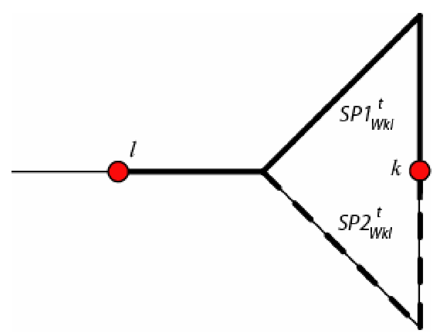


Figure 3.2 A bifurcating shortest path between objects *k* and *l*

It is obvious that objects moving on the network do not always move on the same edge. Objects can move from one edge to another. When doing so they pass a node (Definition 3.11). If k passes a node lying at the intersection of the edges e^- and e^+ at t, and neither of these edges are part of any shortest path between k and l at t, this event is referred to as a shortest path omitting node pass event (Definition 3.12 and Figure 3.3).

Definition 3.11 An object o is in *a node pass event* at time t in a network W, along edges, e^-, e^+, $NPE(o,t,e^-,e^+)$ iff :

$$\exists\, (t^-,t^+)\, [t^- < t \wedge t < t^+] \wedge \{ e^-, e^+ \} \subseteq \text{edges}(W) \;\wedge\; e^- \neq e^+ \wedge$$

$$\forall t_1\, [t^- \leq t_1 \leq t] \;\rightarrow \text{loc}(o) \in \text{loc}(e^-) \wedge$$

$$\forall t_2\, [t \leq t_2 \leq t^+] \;\rightarrow \text{loc}(o) \in \text{loc}(e^+)$$

Definition 3.12 An object k is in a *shortest path omitting node pass event* with respect to another object l at time t in a network W iff :

$$NPE(k,t,e^-,e^+) \wedge \forall\, p[\, SP^t_{Wkl}\ (p) \rightarrow [\text{loc}(e^-) \notin \text{loc}(p) \wedge \text{loc}(e^+) \notin \text{loc}(p)]]$$

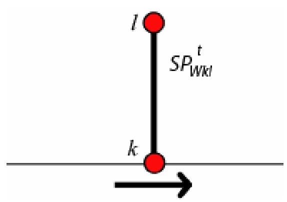


Figure 3.3 A shortest path omitting node pass event

3.3 Definition of QTC$_N$ Relations

The distance used to qualitatively compare the relation between two objects is measured along the shortest path. If there is no path between two objects, then there is no QTC$_N$ relation between these objects. Put differently, these objects will always be disjoint (since they occupy disjoint parts of a disconnected network). The shortest path is chosen because it seems to encode what it means for one object to approach or recede from another object in a network(Van de Weghe et al. 2004). In a network, an object can only approach another object, if and only if it moves along a shortest path between these two objects (Bogaert et al. 2007; Bogaert et al. 2004).

Theorem 3.1 A primary object k on a network can only decrease its distance to a reference object l on this network if and only if k moves towards l along a shortest path.

Proof: 1. Moving along a shortest path will decrease the distance.

Assume a shortest path between k and l is M, and therefore the shortest distance between k and l is $|M|$. If objects (k or l) move along the shortest path M over an infinitesimal unit of distance (ds), they will decrease their distance because ds is not negative.

$$|M| > |M| - ds \tag{3-1}$$

2. Moving along any other path (which is not a shortest path) will increase the distance.

Assume a shortest path between k and l is M, and therefore the shortest distance between k and l is $|M|$. Any other path N with a length of $|N|$ between k and l, which is not a shortest path, will be longer.

$$|M| < |N| \tag{3-2}$$

If k moves along N by a distance ds, and $ds < 0.5\ (|N|-|M|)$, then its distance from l will be $|M| + ds$, since N is not a shortest path. So, if k wants to approach l it must move along a shortest path;

Using this property, we can state that an object k can only approach another object l at time t in a network W if it does not lie on SP^t_{Wkl} immediately before t and lies on SP^t_{Wkl} immediately after t. An object moves away from another object if it is situated on SP^t_{Wkl} immediately before t and if it does not lie on SP^t_{Wkl} immediately after t. If an object lies on SP^t_{Wkl} only at t, but not immediately before and immediately after t, or if it lies on SP^t_{Wkl} immediately before and immediately after t, then the object will be stable with respect to the other object (although this relation may only last for an instantaneous moment in time). This property allows reformulating the conditions for the construction of the three character label for QTC_B to a QTC_N setting (Bogaert et al. 2006).

Definition 3.13 A relation in QTC_N is defined by a three character label. This label represents the following three relations between objects k and l:

1. Movement of the first object k, with respect to the position of the second object l at time t:

−: k is moving towards l:

$$\begin{aligned} &\exists t_1 (t_1 \prec t \wedge \forall t^- (t_1 \prec t^- \prec t \rightarrow k \,|\, t^- \notin SP^t_{Wkl})) \wedge \\ &\quad \exists t_2 (t \prec t_2 \wedge \forall t^+ (t \prec t^+ \prec t_2 \rightarrow k \,|\, t^+ \in SP^t_{Wkl})) \end{aligned} \tag{3-3}$$

+: k is moving away from l:

$$\begin{aligned}&\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow k \mid t^- \in SP^t_{Wkl})) \wedge \\ &\qquad \exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow k \mid t^+ \notin SP^t_{Wkl}))\end{aligned} \tag{3-4}$$

0: k is stable with respect to l (all other cases):

$$\begin{aligned}&\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow k \mid t^- \in SP^t_{Wkl})) \wedge \\ &\qquad \exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow k \mid t^+ \in SP^t_{Wkl}))\end{aligned} \tag{3-5}$$

$$\begin{aligned}&\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow k \mid t^- \notin SP^t_{Wkl})) \wedge \\ &\qquad \exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow k \mid t^+ \notin SP^t_{Wkl}))\end{aligned} \tag{3-6}$$

2. The movement of the second object l, with respect to the position of the first object k at time t can be described as in 1, with k and l interchanged, and hence:

−: l is moving towards k (equation (3-3) with k and l interchanged)

+: l is moving away from k

(equation (3-4) with k and l interchanged)

0: l is stable with respect to k

(equations (3-5) and (3-6) with k and l interchanged)

3. Relative speed of the first object k at time t, with respect to the second object l at time t:

−: k is moving *'slower'* than l: (equation (2-10))

+: k is moving *'faster'* than l: (equation (2-11))

0: k and l are moving *'equally fast'*: (equation (2-12))

Based on Definition 3.13, all canonical cases for QTC_N can be constructed. Let us analyse all possible movements for the first object in the relation. The object in the network can be stationary or not. If the object is not moving, it will automatically be an element of the shortest path around t, and so by definition lead to a '0' for the first character in the label. If the object is moving, then by definition there are four possibilities. The object can be an element of the shortest path immediately before t and not immediately after t, which leads to a '+' for the first character in the label. The object can be an element of the shortest path immediately after t but not just before t, which leads to a '−' for the first character in the label. When the object is in a shortest path omitting node pass event, it will not be an element of the shortest path either just before or just after t resulting in a '0' for the first character in the label. If there is a bifurcating shortest path between the object and another object in the network, then it will be an element of a shortest path both just before and just after t, which also leads to a '0' for the first character in the label. The same five movement cases exist for the second object in the relation. This means that there are 25 (5*5) different canonical cases looking at the first two characters of a QTC_N label. Adding the three different possibilities for the third character there should be, in theory, 75 (25*3) canonical cases in QTC_N. Due to the fact that a stationary object cannot be faster than or just as fast as a moving object, 18 of these relations can not physically occur, implying that 57 canonical cases remain. These cases are presented in Figure 3.4. The first column in the figure presents the QTC_N label. In the other columns, an icon is sketched for all canonical cases. A '0n' denotes whether a '0' label is due to a shortest path omitting node pass event. A '0b' denotes whether a '0' label is due to the existence of a bifurcating shortest path between the objects. The left and right dot, represent the position of k (the first object) and l (the second object), respectively. A dot is filled if the object can be stationary. The arrow symbols represent the potential object movements. Note that the arrows can have different lengths indicating the difference in relative speed.

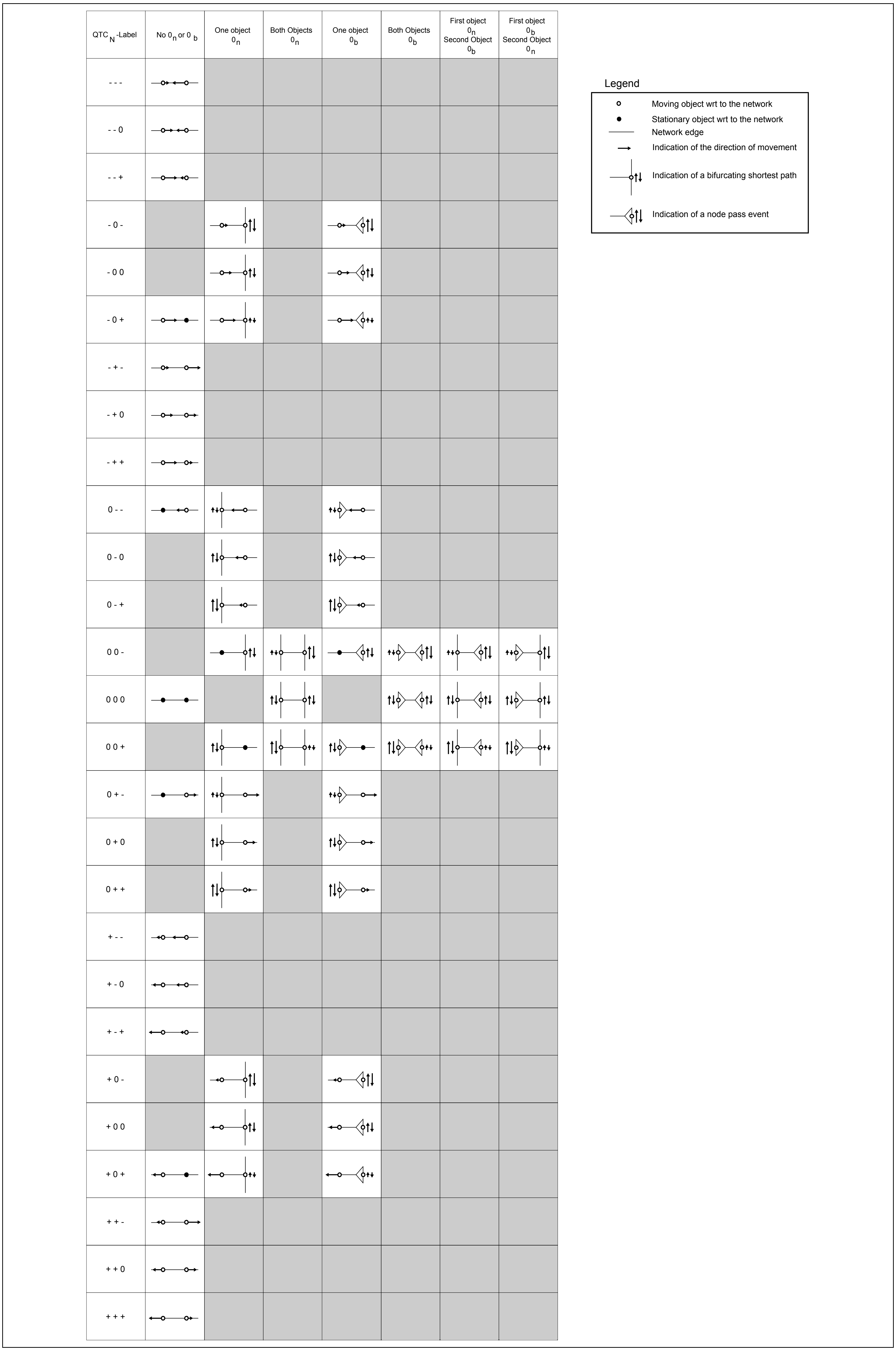


Figure 3.4 58 Canonical cases for QTC_N

Chapter 4

QTC_N and the Composition of its Relations

4.1 Composition and Composition Tables

People often make inferences of qualitative relations in daily life (Byrne and Johnson-Laird 1989). For example, if we know that Nico is taller than Philippe and Frank is taller than Nico, we infer that Frank is taller than Philippe. A specific type of inference mechanism, which is a fundamental part of a relational calculus, is the composition of its relations (Tarski 1941). The idea behind a composition of relations is to compose a finite set of new facts and rules from existing ones. Given two relations part of a particular set of relations, R_1 and R_2, between three objects k, l and m, sharing a common object ($R_1(k,l)$ and $R_2(l,m)$), then the composition of R_1 and R_2 infers the possible relations R_3 between the other two objects (k and m) part of the same set of relations (Cohn et al. 1997).

In scientific literature, different types of symbols have been used to denote the composition operator. Tarski (1941), Egenhofer (1994), and Bennett (1997), for example, make use of the ';' symbol, while Cohn and Renz. (2007) and Renz and Nebel (2007) employ '$\circ$' and Frank (1996) uses '∞'. In this thesis we will use the symbol '$\otimes$', which is coherent with Van de Weghe (2004), Freksa (1992a) and Isli et al. (2000). Thus, we can define the composition of two relations (Navarrete and Sciavicco 2006):

$$R_1 \otimes R_2 = \{R_3 \mid \exists k, \exists l, \exists m : R_1(k,m) \wedge R_2(l,m) \wedge R_3(k,m)\} \qquad (4\text{-}1)$$

It is worth mentioning that this definition corresponds to weak composition (Renz and Ligozat 2005), and not to the so-called strong composition, which in this thesis will be denoted by the '$\circ$' symbol, defined by:

$$R_1 \circ R_2 = \{(k,m) \mid \exists l : (k,l) \in R_1 \wedge (l,m) \in R_2\} \qquad (4\text{-}2)$$

A clear example of the difference between strong and weak composition is given by Düntsh et al. (2001) stated in Cohn and Renz (2007, p.24). *"Consider three regions k,l,m in two-dimensional space where k is a doughnut and l its hole. It is not possible to find a region m which is externally connected to k and l and therefore the tuple (k,l) which is contained in the relation EC is not contained in EC ◦ EC. So the composition of EC with EC does not contain EC even though this is specified in the RCC-8 composition table"*. As exemplified by Figure 4.1, the composition of EC(*k*,*l*)⊗EC(*l*,*m*) leads to the existence of EC(*k*,*m*), for a specific configuration of the regions *k* and *l*, but as stated in the above example it does not exist for all configurations of the regions *k* and *l*.

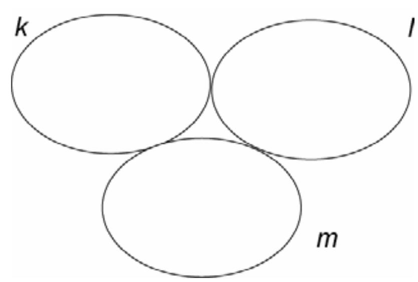


Figure 4.1 A composition of the RCC-8 relation EC(*k*,*l*)⊗EC(*l*,*m*) leading to EC(*k*,*m*)

As stated by Renz and Ligozat (2005, p.538): *"It is often very difficult to determine whether weak composition is equivalent to strong composition or not. Usually only non-equality can be shown by giving a counterexample, while it is very difficult to prove equality"*. In this thesis, the composition of two relations part of the same set of relations refers to weak composition.

When the (weak or strong) composition of every base relation with all base relations of a certain set of relations can be computed, they are usually stored in a (weak or strong) composition table (CT). Composition Tables (CT^s) originate from Allen's analysis of temporal relations (Allen 1983). The CT for the Interval Calculus is given in Table 4.1.

Usually, the left column contains R_1, the top row contains R_2, and the other cells in the table contain $R1 \otimes R_2$. Composition Tables (CT^s) make sense from a computational point of view (Bennett 1997), since a compositional inference can simply be looked up, instead of needing complex computations (Vieu 1997).

Table 4.1 The composition table for the Interval Calculus

. $R_1 \otimes R_2$	<	>	d	di	o	Oi	m	mi	s	si	f	fi	=
<	<	all	< o m s d	<	<	< o m s d	<	< o m s d	<	<	< o m s d	<	<
>	All	>	> oi mi d f	>	> oi mi d f	>	> oi mi d f	>	> oi mi d f	>	>	>	>
d	<	>	d	all	< o m s d	> oi mi d f	<	>	d	> oi mi d f	d	< o m s d	d
di	< o m di fi	> oi di mi si	d di o oi s si f fi =	di	o fi di	di si oi	o fi di	di si oi	o fi di	di	di si oi	di	di
o	<	> oi di mi si	o s d	< o m di fi	< m o	d di o oi s si f fi =	<	di si oi	o	o fi di	o s d	< m o	o
oi	< o m di fi	>	d f oi	> oi di mi si	d di o oi s si f fi =	oi mi >	o fi di	>	d f oi	oi mi >	oi	di si oi	oi
m	<	> oi di mi si	o s d	<	<	o s d	<	f fi =	m	m	o s d	<	m
mi	< o m di fi	>	d f oi	>	d f oi	>	s si =	>	d f oi	>	mi	mi	mi
s	<	>	d	< o m di fi	< m o	d f oi	<	mi	s	s si =	d	< m o	s
si	< o m di fi	>	d f oi	di	o fi di	oi	o fi di	mi	s si =	si	oi	di	si
f	<	>	d	> oi di mi si	o s d	oi mi >	m	>	d	oi mi >	f	f fi =	f
fi	<	> oi di mi si	o s d	di	o	di si oi	m	di si oi	o	di	f fi =	fi	fi
=	<	>	d	di	o	oi	m	mi	s	si	f	fi	=

Since their introduction, CT^s have been worked out for many different temporal (e.g. the semi interval calculus (Freksa 1992a)), spatial (e.g. topological calculi (Egenhofer 1994; Randell et al. 1992b), directional calculi (Frank 1991a; Freksa 1992b; Hernandez 1994) and distance calculi (Hernandez et al. 1995)) and spatiotemporal calculi (e.g. QTC (Van de Weghe 2004; Van de Weghe et al. 2005b)). The composition table for RCC-8, for example, is given in Table 4.2.

Table 4.2 The composition table for RCC-8

$R_1 \otimes R_2$	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	All	DC,EC,PO, TPP,NTPP	DC,EC,PO, TPP,NTPP	DC,EC,PO, TPP,NTPP	DC,EC,PO,TPP, NTPP	DC	DC	DC
EC	DC,EC,PO, TPPi,NTPPi	DC,EC,PO,T PP,TPPi,EQ	DC,EC,PO, TPP,NTPP	PO,TPP,NT PP	PO,TPP,NTPP	DC,EC	DC	EC
PO	DC,EC,PO, TPPi,NTPPi	DC,EC,PO,T PPi,NTPPi	All	PO,TPP,NT PP	PO,TPP,NTPP	DC,EC,PO,T PPi,NTPPi	DC,EC,PO, TPPi,NTPPi	PO
TPP	DC	DC,EC	DC,EC,PO, TPP,NTPP	TPP,NTPP	NTPP	DC,EC,PO,T PP,TPPi,EQ	DC,EC,PO, TPPi,NTPPi	TPP
NTPP	DC	DC	DC,EC,PO, TPP,NTPP	NTPP	NTPP	DC,EC,PO,T PP,NTPP	All	NTPP
TPPi	DC,EC,PO, TPPi,NTPPi	EC,PO,TPPi, NTPPi	PO,TPPi,NT PPi	PO,TPP,TP Pi,EQ	PO,TPP,NTPP	TPPi,NTPPi	NTPPi	TPPi
NTPPi	DC,EC,PO, TPPi,NTPPi	PO,TPPi,NT PPi	PO,TPPi,NT PPi	PO,TPPi,NT PPi	PO,TPP,NTPP,T PPi,NTPPi,EQ	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

4.2 A Composition Table for QTC$_N$

As stated in section 3.3, there are 27 JEPD relations in QTC$_N$. This implies that in order to construct a composition table, 729 (27x27) combinations of relations need to be examined. All combinations can lead to 27 possible relations available in each cell in the composition table. Thus, in order to construct a weak composition table for QTC$_N$, 19683 (27x27x27) possible combinations of three QTC$_N$ relations need to be examined for their existence or non-existence. To prove such a large number of possible combinations by hand is almost impossible and even with machine assistance it is a computationally intensive problem (Randell et al. 1992a). Therefore, in this thesis, the composition of two QTC$_N$ relations is split into two parts. The first part presented in 4.2.1, examines the composition of relative speed, which is presented by the third character in a QTC$_N$ label representing such a relation. The second part presented in 4.2.2, solely examines the composition of the qualitative movement of the objects, neglecting their relative speed. In other words, the composition of the first two characters in a QTC$_N$ label is investigated.

4.2.1 The Composition of Relative Speed

Since the relations greater than (>), equal to (=) and smaller than (<) are transitive, the composition of the relative speed represented in the third character of a label representing a QTC$_N$ is straightforward (Van de Weghe 2004). Assuming three moving point objects *k*, *l* and *m* the following compositions can be inferred concerning their relative speed.

$$v_k < v_l \wedge v_l < v_m \rightarrow v_k < v_m \qquad (4\text{-}3)$$
$$\leftrightarrow - \otimes - \rightarrow -$$

$$v_k < v_l \wedge v_l = v_m \rightarrow v_k < v_m \qquad (4\text{-}4)$$
$$\leftrightarrow - \otimes 0 \rightarrow -$$

$$v_k < v_l \wedge v_l > v_m \rightarrow v_k < v_m \vee v_k = v_m \vee v_k > v_m \qquad (4\text{-}5)$$
$$\leftrightarrow - \otimes + \rightarrow - \vee 0 \vee +$$

$$v_k = v_l \wedge v_l < v_m \rightarrow v_k < v_m \qquad (4\text{-}6)$$
$$\leftrightarrow 0 \otimes - \rightarrow -$$

$$v_k = v_l \wedge v_l = v_m \rightarrow v_k = v_m \qquad (4\text{-}7)$$
$$\Leftrightarrow 0 \otimes 0 \rightarrow 0$$

$$v_k = v_l \wedge v_l > v_m \rightarrow v_k > v_m \qquad (4\text{-}8)$$
$$\leftrightarrow 0 \otimes + \rightarrow +$$

$$v_k > v_l \wedge v_l < v_m \rightarrow v_k < v_m \vee v_k = v_m \vee v_k > v_m \qquad (4\text{-}9)$$
$$\leftrightarrow + \otimes - \rightarrow - \vee 0 \vee +$$

$$v_k > v_l \wedge v_l = v_m \rightarrow v_k > v_m \qquad (4\text{-}10)$$
$$\leftrightarrow + \otimes 0 \rightarrow +$$

$$v_k > v_l \wedge v_l > v_m \rightarrow v_k > v_m \qquad (4\text{-}11)$$
$$\leftrightarrow + \otimes + \rightarrow +$$

This leads to the following composition table

Table 4.3 The composition table for relative speed

$R_1 \otimes R_2$	$-$	0	$+$
$-$	$-$	$-$	$- \vee 0 \vee +$
0	$-$	0	$+$
$+$	$- \vee 0 \vee +$	$+$	$+$

4.2.2 The Composition of the first two Characters in a QTC$_N$ Relation

Nine different relations can be distinguished using only the first two characters of a QTC$_N$ label. Thus, the composition table of these relations contains 81 (9x9) cells, each potentially containing these exact same nine relations. As a consequence, 729 possible combinations of three relations need to be examined for their existence or non-existence. As can be seen in Appendix A, for each of these 729 possible combinations of three relations, an example can be drawn. As a result, each cell in the composition table contains all nine possible relations. This means that this composition table is not useful at all, since the composition of two relations does not generate new knowledge. For what it is worth, the composition table, of relations consisting of the first two characters in a QTC$_N$ label, is shown in Table 4.4. In Table 4.4, the letters A and B can take on any value part of the set {−, 0, +}.

Table 4.4 The composition of relations consisting of the first two characters of a QTC$_N$ label

$R_1 \otimes R_2$	- -	- 0	- +	0 -	0 0	0 +	+ -	+ 0	+ +
- -	A B	A B	A B	A B	A B	A B	A B	A B	A B
- 0	A B	A B	A B	A B	A B	A B	A B	A B	A B
- +	A B	A B	A B	A B	A B	A B	A B	A B	A B
0 -	A B	A B	A B	A B	A B	A B	A B	A B	A B
0 0	A B	A B	A B	A B	A B	A B	A B	A B	A B
0 +	A B	A B	A B	A B	A B	A B	A B	A B	A B
+ -	A B	A B	A B	A B	A B	A B	A B	A B	A B
+ 0	A B	A B	A B	A B	A B	A B	A B	A B	A B
+ +	A B	A B	A B	A B	A B	A B	A B	A B	A B

In order to have sparser composition tables extra knowledge about the network based moving objects is required. This extra knowledge can be converted into constraints limiting the possible entries for each cell in the composition table.

As can be deducted from section 3.3 a ‘0’ character for the first two characters in a QTC$_N$ label, caused by a bifurcating shortest path or a node path event, can only hold instantaneously, while a ‘0’ label caused by an object which is not moving with respect to the network can hold over an interval. This means that most of the time a ‘0’ character in the label is caused by an object which is not moving with respect to the network. Therefore, it is interesting to examine the composition of relations consisting of the first two characters of a QTC$_N$ label where the ‘0’ character in the label is restricted to objects which are not moving with respect to the network. An interesting consequence of this constraint is that an object which is stable in one relation always needs to be stable in any other relation containing that object. Applying this constraint to the composition table given in Table 4.4 leads to the composition table shown in Table 4.5, which is already a lot sparser and thus, more useful. In Table 4.5 A_0 and B_0 can take on any value part of the set {−, +}, the symbol ∅ represents an empty set of entries in a cell.

Table 4.5 The composition table for relations consisting of the first two characters of a QTC$_N$ label in which the '0' character is restricted to objects which are not moving with respect to the network

$R_1 \otimes R_2$	- -	- 0	- +	0 -	0 0	0 +	+ -	+ 0	+ +
- -	A_0 B_0	A_0 0	A_0 B_0	∅	∅	∅	A_0 B_0	A_0 0	A_0 B_0
- 0	∅	∅	∅	A_0 B_0	A_0 0	A_0 B_0	∅	∅	∅
- +	A_0 B_0	A_0 0	A_0 B_0	∅	∅	∅	A_0 B_0	A_0 B	A_0 B_0
0 -	0 B_0	0 0	0 B_0	∅	∅	∅	0 B_0	0 0	0 B_0
0 0	∅	∅	∅	0 B_0	0 0	0 B_0	∅	∅	∅
0 +	0 B_0	0 0	0 B_0	∅	∅	∅	0 B_0	0 0	0 B_0
+ -	A_0 B_0	A_0 0	A_0 B_0	∅	∅	∅	A_0 B_0	A_0 B	A_0 B_0
+ 0	∅	∅	∅	A_0 B_0	A_0 0	A_0 B_0	∅	∅	∅
+ +	A_0 B_0	A_0 0	A_0 B_0	∅	∅	∅	A_0 B_0	A_0 0	A_0 B_0

If on top of this restriction, it is known that one object lies on the shortest path between the other two objects; a simple line can be drawn containing all three objects. On this line, each object has three movement possibilities; it can be stable or move in two opposite directions. This implies that every possible configuration of these three objects can be generalised into 27 (3x3x3) different configuration classes. An example of a specific configuration of these configurations classes is shown in Figure 4.2, Figure 4.3 and Figure 4.4 respectively illustrating the case in which object *l* lies on the shortest path between *k* and *m*, *m* lies on the shortest path between *k* and *l* and *k* lies on the shortest path between *l* and *m*. The resulting composition tables are given in Table 4.6, Table 4.7 and Table 4.8. This kind of composition is very useful, since it always leads to exact knowledge.

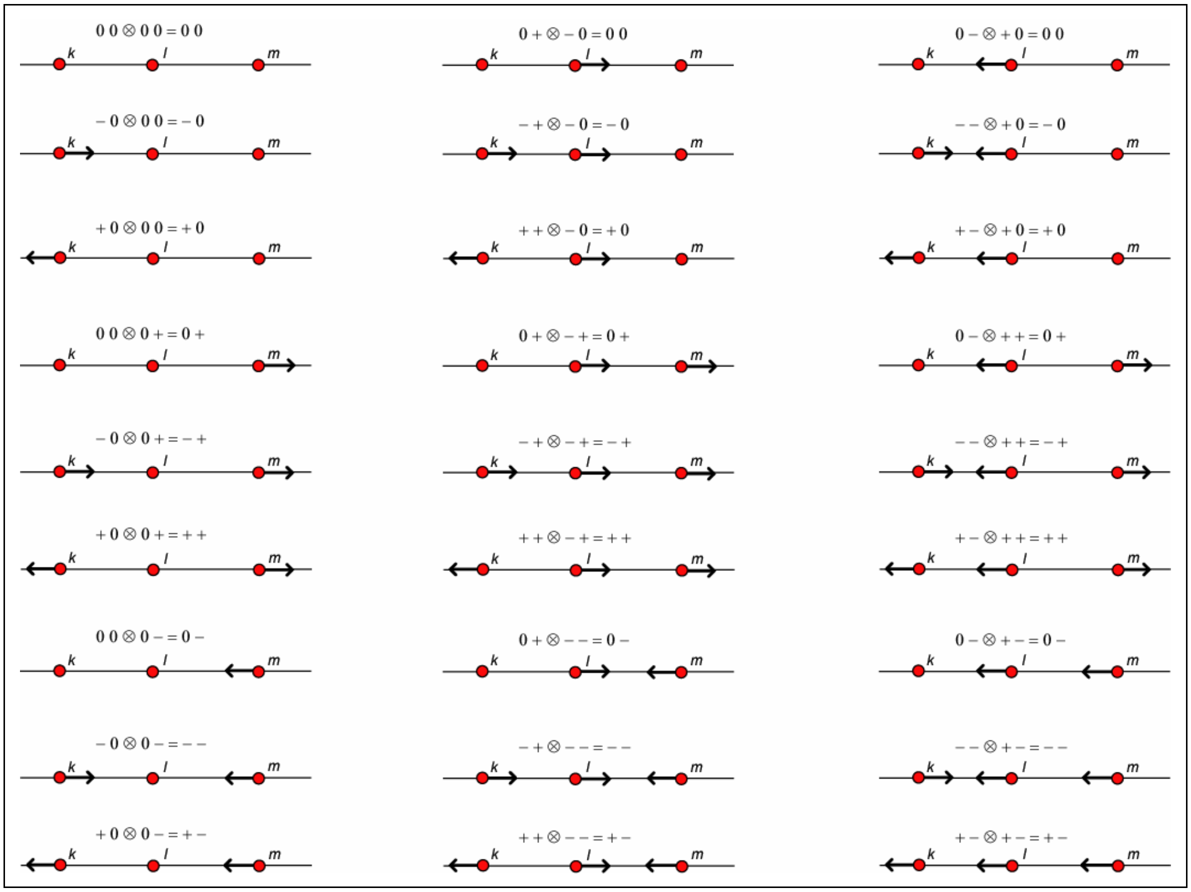


Figure 4.2 All possible combinations of relations consisting of the first two characters of a QTC$_N$ label in which the '0' character is restricted to objects which are not moving with respect to the network and object *l* lies on the shortest path between *k* and *m*.

Table 4.6 The composition table for relations consisting of the first two characters of a QTC$_N$ label in which the '0' character is restricted to objects which are not moving with respect to the network and object *l* lies on the shortest path between *k* and *m*.

$R_1 \otimes R_2$	- -	- 0	- +	0 -	0 0	0 +	+ -	+ 0	+ +
- -	∅	∅	∅	∅	∅	∅	- -	- 0	- +
- 0	∅	∅	∅	- -	- 0	- +	∅	∅	∅
- +	- -	- 0	- +	∅	∅	∅	∅	∅	∅
0 -	∅	∅	∅	∅	∅	∅	0 -	0 0	0 +
0 0	∅	∅	∅	0 -	0 0	0 +	∅	∅	∅
0 +	0 -	0 0	0 +	∅	∅	∅	∅	∅	∅
+ -	∅	∅	∅	∅	∅	∅	+ -	+ 0	+ +
+ 0	∅	∅	∅	+ -	+ 0	+ +	∅	∅	∅
+ +	+ -	+ 0	+ +	∅	∅	∅	∅	∅	∅

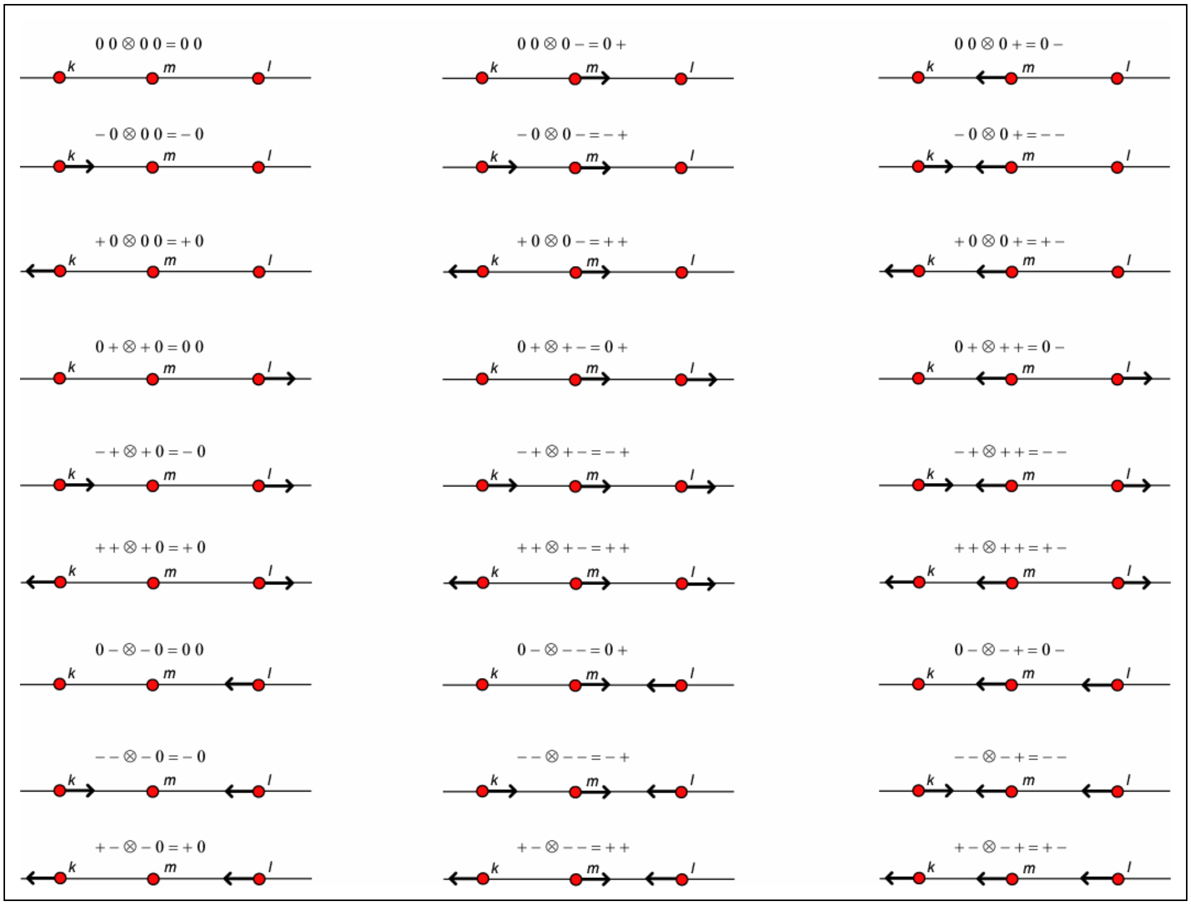


Figure 4.3 All possible combinations of relations consisting of the first two characters of a QTC_N label in which the '0' character is restricted to objects which are not moving with respect to the network and object *m* lies on the shortest path between *k* and *l*.

Table 4.7 The composition table for relations consisting of the first two characters of a QTC_N label in which the '0' character is restricted to objects which are not moving with respect to the network and object *m* lies on the shortest path between *k* and *l*.

$R_1 \otimes R_2$	- -	- 0	- +	0 -	0 0	0 +	+ -	+ 0	+ +
- -	+ -	+ 0	+ +	∅	∅	∅	∅	∅	∅
- 0	∅	∅	∅	+ -	+ 0	+ +	∅	∅	∅
- +	∅	∅	∅	∅	∅	∅	+ -	+ 0	+ +
0 -	0 -	0 0	0+	∅	∅	∅	∅	∅	∅
0 0	∅	∅	∅	0 -	0 0	0 +	∅	∅	∅
0 +	∅	∅	∅	∅	∅	∅	0 -	0 0	0 +
+ -	- -	- 0	- +	∅	∅	∅	∅	∅	∅
+ 0	∅	∅	∅	- -	- 0	- +	∅	∅	∅
+ +	∅	∅	∅	∅	∅	∅	- -	- 0	- +

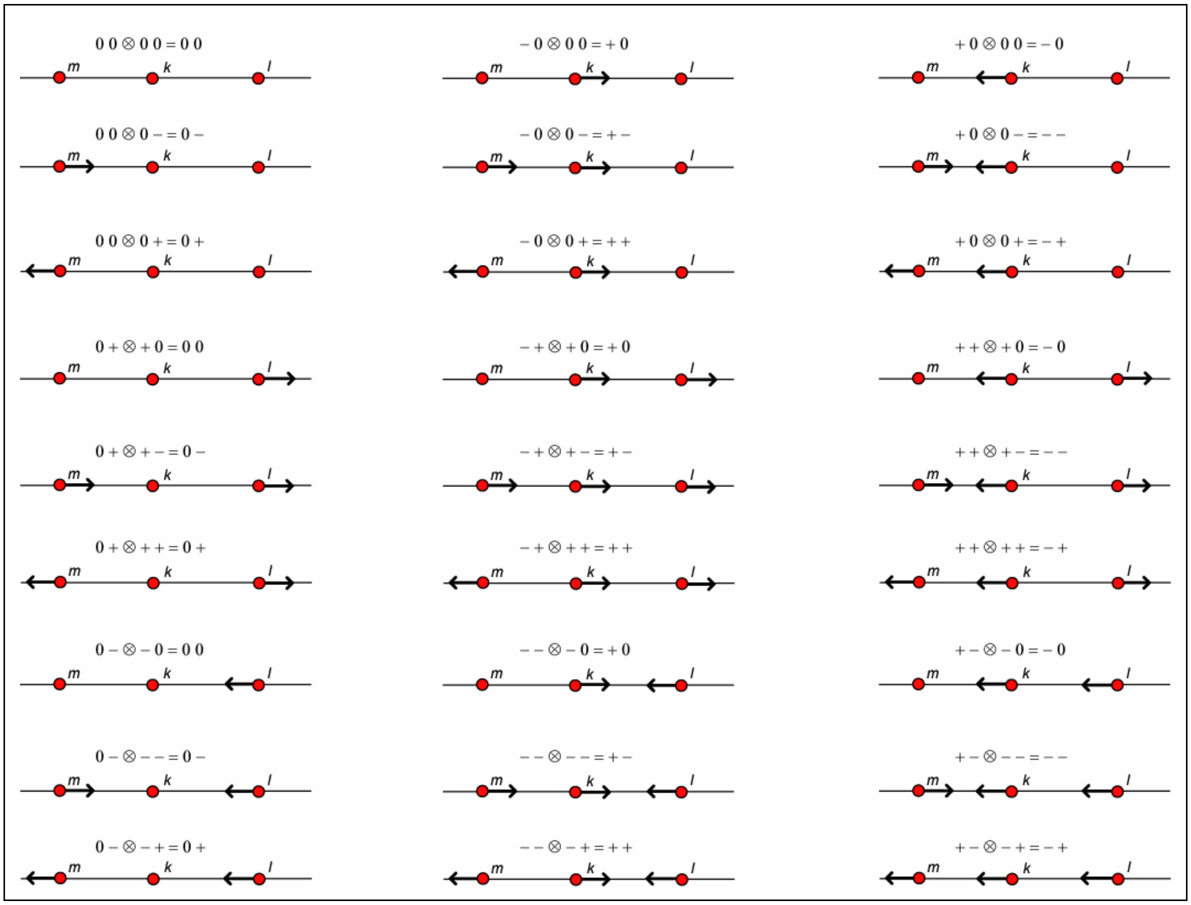


Figure 4.4 All possible combinations of relations consisting of the first two characters of a QTC$_N$ label in which the '0' character is restricted to objects which are not moving with respect to the network and object *k* lies on the shortest path between *l* and *m*.

Table 4.8 The composition table for relations consisting of the first two characters of a QTC$_N$ label in which the '0' character is restricted to objects which are not moving with respect to the network and object *k* lies on the shortest path between *l* and *m*.

$R_1 \otimes R_2$	- -	- 0	- +	0 -	0 0	0 +	+ -	+ 0	+ +
- -	- +	- 0	- -	∅	∅	∅	∅	∅	∅
- 0	∅	∅	∅	- +	- 0	- -	∅	∅	∅
- +	∅	∅	∅	∅	∅	∅	- +	- 0	- -
0 -	0 +	0 0	0 -	∅	∅	∅	∅	∅	∅
0 0	∅	∅	∅	0 +	0 0	0 -	∅	∅	∅
0 +	∅	∅	∅	∅	∅	∅	0 +	0 0	0 -
+ -	+ +	+ 0	+ -	∅	∅	∅	∅	∅	∅
+ 0	∅	∅	∅	+ +	+ 0	+ -	∅	∅	∅
+ +	∅	∅	∅	∅	∅	∅	+ +	+ 0	+ -

Chapter 5
A Conceptual Neighbourhood Diagram for QTC_N

As stated by Muller: *"Dealing with spatial representations is very often dealing with changing representations…"* (Muller 1998a, p.63). Therefore, it is important to analyse which changes between qualitative relations occur over time. In QTC, the movement of objects is assumed to be continuous. Hence, the transitions or changes between different states of QTC relations are examined under the assumption that change is continuous.

The central issue in this chapter is the construction of a conceptual neighbourhood diagram (CND) for QTC_N. First of all, the concepts of conceptual neighbours, conceptual neighbourhoods and conceptual neighbourhood diagrams are defined and their importance within qualitative reasoning is shown. Afterwards, the focus is on the construction of a CND for QTC_N. The construction of the CND^s for QTC_B and QTC_N are based on the theory of dominance introduced by Galton (1995a; 1995b; 2001).

5.1 Conceptual Neighbours, Conceptual Neighbourhood and Conceptual Neighbourhood Diagrams

The notion of conceptual neighbours and conceptual neighbourhood originates from the domain of qualitative temporal reasoning and was introduced by Freksa (1992a). Freksa's idea was to consider in which way Allen's interval relations (see 2.2.1) alter as the intervals are subject to continuous change. If two relations between intervals can directly transform into one another by continuously deforming (i.e. shortening, lengthening, moving) their end-points in a topological sense, then these relations are said to be conceptual neighbours (Freksa 1992a, p.204). A set of relations between intervals forms a conceptual neighbourhood if its elements are path-connected through conceptual neighbour relations (Freksa 1992a, p.205). Consequently, the visual representation of all possible conceptual neighbours available in a particular set of relations is defined as a conceptual neighbourhood diagram (CND) (Galton 2000). Note that the concept of a CND is sometimes denoted as a transition graph, conceptual neighbourhood graph, conceptual neighbourhood structure, and continuity network (Van de Weghe 2004). In Figure 5.1, the CND for the thirteen Allen relations is presented. The CND shows that *'meet'* (m) and *'overlap'* (o) are conceptual neighbours and *'before'* (<) and *'overlap'* are not. In other words, a relation between two intervals can be directly transformed from a *'meet'* relation into an '*overlap*' relation (and vice-versa) by continuously deforming the end-points of the intervals. A direct transformation from a *'before'* relation into an *'overlap'* relation is not possible without passing the intermediate *'meet'* relation. As a consequence, the relations *'before'*, *'meet'* and *'overlap'* form a conceptual neighbourhood.

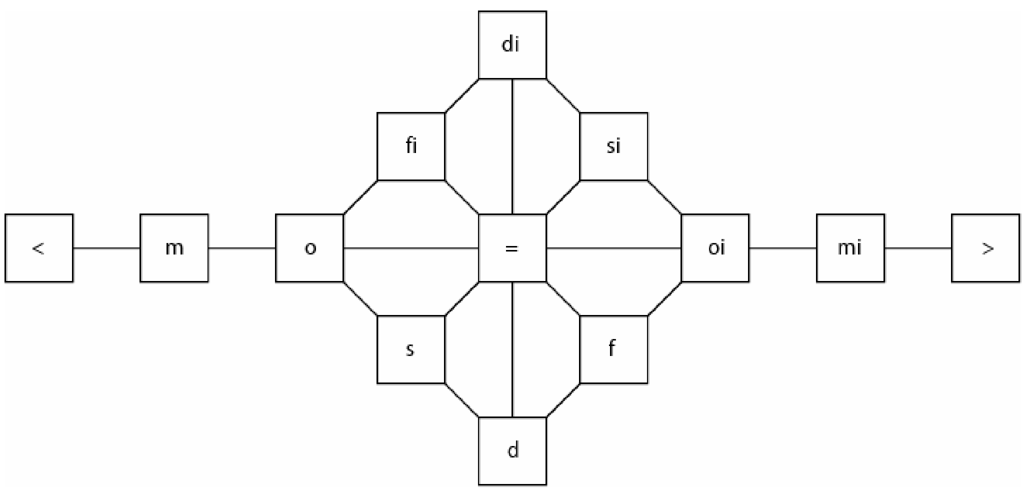


Figure 5.1 The CND for the thirteen interval relations (based on Freksa, (1992a p.211))

Freksa used the notion of conceptual neighbourhood to define the concept of coarse knowledge: *"Incomplete knowledge about relations is called coarse knowledge if the corresponding disjunction of at least two relations forms a conceptual neighborhood"* (Freksa 1992a, p.205). The opposite, a disjunction of at least two relations which does not form a conceptual neighbourhood, is denoted as scattered knowledge. Thus, using the above stated example, the disjunction of the relations, *'before'*, *'meet'* and *'overlap'* represents coarse knowledge, the disjunction of the relations *'before'* and *'overlap'* represent scattered knowledge. Coarse knowledge appears to be cognitively more adequate. Changes happen in steps rather than in jumps (Freksa 1992a). In addition, the composition of interval relations always leads to definite or coarse knowledge and never to scattered knowledge (Freksa 1992a).

As stated by Galton (2001), the deformation of time points and intervals is purely conceptual. Although they are used to model change, in nature they are not subject to change themselves. Various authors have used the notion of conceptual neighbours and conceptual neighbourhood for spatial entities instead of intervals (e.g. for topological relations (Egenhofer and Altaha 1992; Egenhofer and Mark 1995a; Egenhofer et al. 1993; Randell et al. 1992b), directional relations (Egenhofer 1997), positional information (Freksa 1992b; Pacheco et al. 2002), movement relations (Van de Weghe and De Maeyer 2005)). Two relations between spatial entities are conceptual neighbours if they can directly transform into one another by continuously deforming (i.e. shortening, lengthening, moving) them in a topological sense. A set of relations between spatial entities forms a conceptual neighbourhood if its elements are path-connected through conceptual neighbour relations. Figure 5.2 shows the CND for the RCC-8 relations.

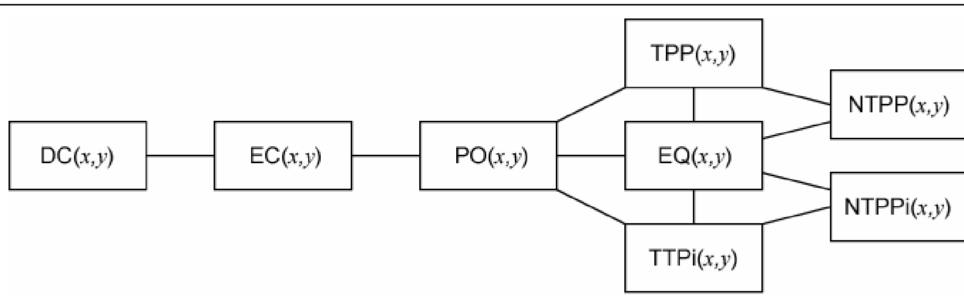


Figure 5.2 The CND for RCC-8
(Based on Randell et al. (1992b, p.169))

In addition to representing coarse knowledge, conceptual neighbourhoods for spatial relations can be used to analyse possible changes in space (Galton 2001), to model qualitative simulation (Cohn and Hazarika 2001), or to express conceptual animations (i.e. a sequence of qualitative relations following the constraints imposed by continuity) (Van de Weghe and De Maeyer 2005).

5.2 Theory of Dominance

The central idea behind the theory of dominance is that some relations dominate others. As defined by Galton: *"a qualitative state q_1 dominates a qualitative state q_2 if q_1 can hold at the beginning or end of an open interval over which q_2 holds"* (Galton 2001, p.59). To illustrate this definition, consider a qualitative relation representing a continuous variable in $\mathbb{R}$ capable of addressing the values part of the qualtitative set {–, 0, +}, consisting of the landmark value '0' and its neighbouring open intervals '−' and '+'. When this variable is subject to continuous change, it can change between the different qualitative values. However, as stated in 2.1, a direct change from '−' to '+' and vice versa is impossible, since such a change must always pass the qualitative value '0'. This landmark value '0' needs to hold for an instant at least. On the other hand, the '+' or '−'of a variable, when changing from '+' or '−' to '0', cannot hold instantaneously; they need to hold over an interval (Galton 1995a). The main reason for this statement is that between any two points of a continuous trajectory one can always find, or at least imagine, another intermediate point (Galton 1995b). Put differently, applied to the set of real numbers, between zero and any positive (or negative) real number, one can always find another positive (or negative) real number: $0 < 10 < 100$; $0 < 1 < 10$; $0 < 0.1 < 1$; $0 < 0.01 < 0.1$; $0 < 0.001 < 0.01$, etc. Hence, it is impossible that the qualitative value of '+' or '−' only holds instantaneously and when changing back and forward to the qualitative value '0' they should last over an open interval. To put it in Galton's words: "*When an object starts moving, there is a last moment when it is at rest, but no first moment when it is in motion*" (Galton 1996, p.101). Thus, in terms of dominance, '0' dominates '−' and '+', and '−' and '+' are dominated by '0' (Galton 1995a; Galton 1995b; Galton 2001). Based on the concept of dominance, a dominance space can be constructed. This is a space describing the dominance relations of a set of qualitative relations. Figure 5.3 presents the dominance diagram, which is the visual representation of a dominance space, of the above stated example. The connections between the different relations indicate a possible transition from one qualitative state to another (e.g. there is no direct connection between '−' and '+', which indicates that there is no direct transition from '−' into '+'). The arrows in the dominance diagrams are directed. The arrowheads point at a relation which is dominated by a relation at the start of the arrow.

This convention will be used for all dominance diagrams in this thesis.

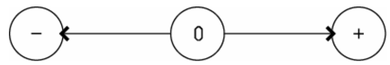


Figure 5.3 The dominance diagram for the qualitative values {–, 0, +} (based on Galton (2001, p.64))

It has been proven that a set of dominance spaces can be combined in order to build composite dominance spaces (Galton 1995a). Suppose we have n sets of qualitative relations $Q = \{q_1, q_2, \ldots, q_o\}$, $Q' = \{q'_1, q'_2, \ldots, q'_p\}, \ldots, Q'^n = \{q'^n_1, q'^n_2, \ldots, q'^n_q\}$, then a relation $(q_i, q'_i, \ldots, q'^n_i)$ in the composite dominance space $Q \cup Q' \cup \ldots \cup Q'^n$ dominates another relation $(q_j, q'_j, \ldots, q'^n_j)$ if and only if the respective qualitative values $q_i, q'_i, \ldots, q'^n_i$ dominate or are equal to $q_j, q'_j, \ldots, q'^n_j$ respectively for all $i,j = 1, 2, \ldots, n$ and $i \neq j$. Figure 5.4 illustrates the dominance diagram of two sets of qualitative relations both able to take on the qualitative values {–, 0, +}.

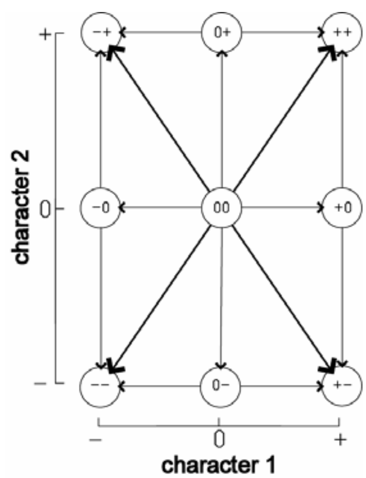


Figure 5.4 The dominance diagram for the composite dominance space consisting of two sets of qualitative relations Q_1 = {–, 0, +} and Q_2 = {–, 0, +} (based on Galton (2001, p.65)

Figure 5.4 clearly shows that the composite relation '0 0' dominates the composite relation '− +'. This is true because a '0' dominates a '−' for the first value of the composite relation and a '0' dominates a '+' for the second value of the composite relation. This means a direct transition between the composite qualitative relations '0 0' and '− +' and vice versa exists, and therefore these relations are conceptual neighbours. On the other hand, there is no direct connection between the relations '0 +' and '− 0'.

Although a '0' dominates a '−' for the first value of the composite relation, a '+' does not dominate a '0' for the second value of the composite relation. This means that '0 +' does not dominate '− 0', nor does '− 0' dominate '0 +'. Therefore, a direct transition is impossible for these relations and hence they are not conceptual neighbours.

The composite dominance diagram shown in Figure 5.4 can be used to construct the CND for QTC_B at level 1, since this calculus combines two sets of qualitative relations, both able to take on the qualitative values {–, 0, +}. Van de Weghe (2004) has proven that all possible transitions between the composite qualitative relations, as can be deducted from the composite dominance diagram, in both QTC_{B11} and QTC_{B21} physically can occur, and hence the CND^s for both sets of QTC relations are similar to this composite dominance diagram. The CND for QTC_{B11} and QTC_{B21} is shown in Figure 5.5.

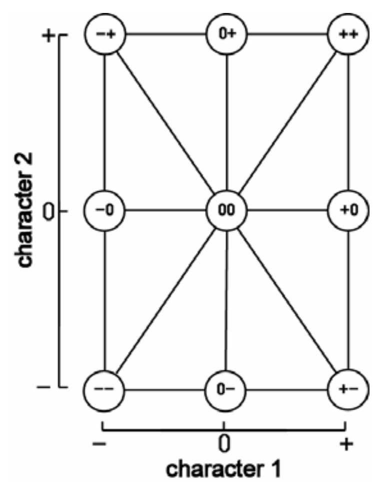


Figure 5.5 The CND for QTC_{B11} and QTC_{B21}
(based on Van de Weghe (2004, p.226))

QTC_{B12} and QTC_{B22} combine three qualitative relations able to take on the qualitative values {–, 0, +}. In order to create a CND for these calculi a composite dominance space for these three qualitative relations needs to be constructed. This can be done analogous to the above constructed composite dominance space for two qualitative relations, using Galton's (1995a) rule for the construction of composite dominance spaces. The composite dominance diagram representing this composite dominance space is shown in Figure 5.6.

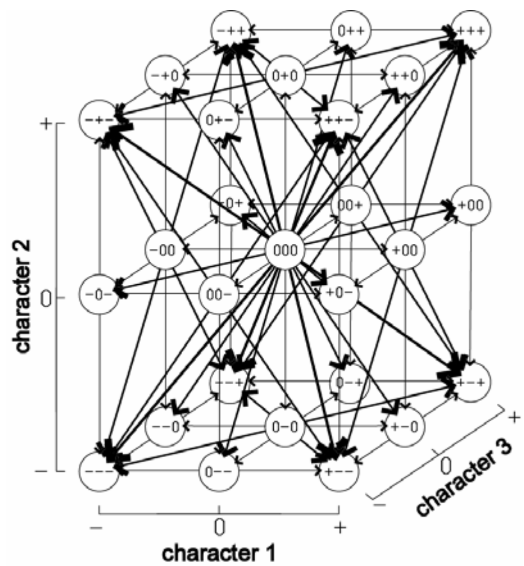


Figure 5.6 The dominance diagram for the composite dominance space consisting of three sets of qualitative relations Q_1 = {–, 0, +}, Q_2 = {–, 0, +} and Q_3 = {–, 0, +} (based on Van de Weghe and De Maeyer (2005, p.233))

Because of the 27 different relations and 98 dominance relations, the visualisation and interpretation of this dominance diagram becomes rather difficult. In order to give a clearer overview on the dominance diagram, it will be split into different dominance diagrams based on the conceptual distance between qualitative relations. The notion of conceptual distance was introduced by Van de Weghe (2004) based on the concepts of topology distance (Egenhofer and Altaha 1992) and the distance between two cardinal directions (Goyal 2000). The conceptual distance between two relations is defined as the sum of the minimum number of transitions for every individual qualitative relation in a composite relation needed to have a transition between both relations (Van de Weghe and De Maeyer 2005). For example, the conceptual distance between the two composite relations ‘– 0 +’ and ‘+ + +’, consisting of three single qualitative relations, is equal to three, being the sum of two transitions to transform a ‘−’ into a ‘+’ for the first set of relations, one transition to transform a ‘0’ into a ‘+’ for the second set of relations and zero transitions between a ‘+’ and ‘+’ for the third set of relations.

An *n*-dominance space is defined as a dominance space where the conceptual distance between the relations is equal to *n* (Van de Weghe and De Maeyer 2005). Figure 5.7 represents three dominance diagrams of the one-dominance space for the combined sets of qualitative relations Q_1 = {–, 0, +}, Q_2 = {–, 0, +} and Q_3 = {–, 0, +}, allowing only transitions for the first, second and third relation respectively. Figure 5.8 represents three dominance diagrams of the two-dominance space for the same combined sets of qualitative relations, keeping respectively the first, second and third relation fixed. Figure 5.9 gives the dominance diagram for the three-dominance space for the same set of composite relations. The combination of all dominance diagrams leads to the dominance diagram in Figure 5.6.

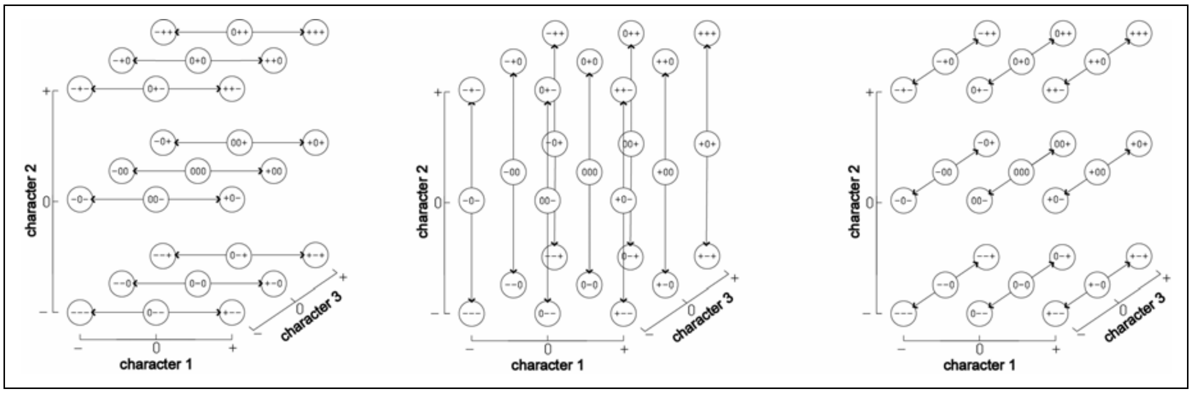


Figure 5.7 One-dominance diagrams
(based on Van de Weghe and De Maeyer (2005, p.232))

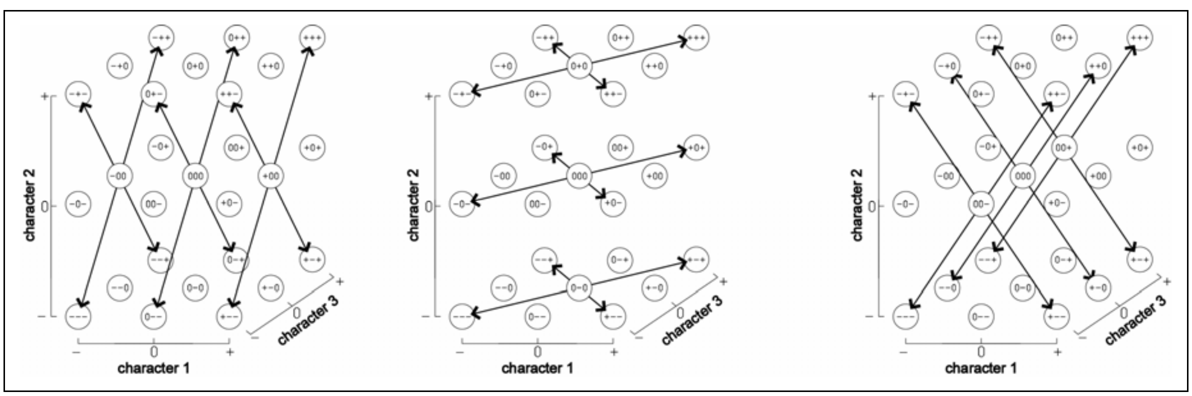


Figure 5.8 Two-dominance diagrams
(based on Van de Weghe and De Maeyer (2005, p.233))

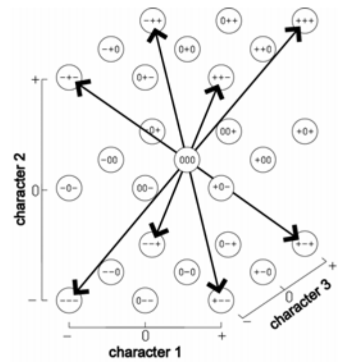


Figure 5.9 The three-dominance diagram
(based on Van de Weghe and De Maeyer (2005, p.233))

As stated above, the dominance diagram can be used to deduct all theoretically possible conceptual neighbours. The conceptual neighbourhood diagram for QTC_B at level 2 can be created by examining all physically possible transitions and relations. Deleting all 'nonexistent' transitions between relations (edges in the CND) and deleting all 'nonexistent' relations (nodes in the CND) gives the CND for a specific calculus. For QTC_{B22}, Van de Weghe (2004) has proven that all possible transitions between the composite qualitative relations, as can be deducted from the composite dominance diagram, can physically occur, and hence the CND for QTC_{B22} is similar to this composite dominance diagram (Figure 5.10).

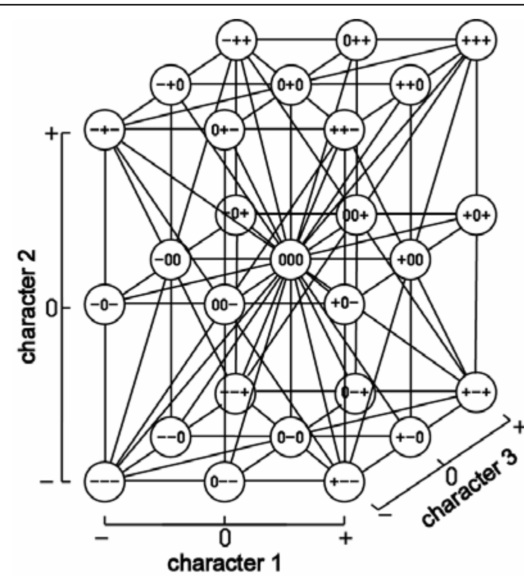


Figure 5.10 The CND for QTC_{B22}
(based on Van de Weghe (2004, p.232))

As stated in section 2.4.1.1, for QTC_{B12}, ten relations can physically not occur. Thus, the ten impossible relations can be deleted from the CND as well as all transitions between impossible and (im)possible relations. Van de Weghe (2004) has shown that all remaining transitions between the 17 remaining relations exist. As a consequence, the CND for QTC_{B12} is shown in Figure 5.11.

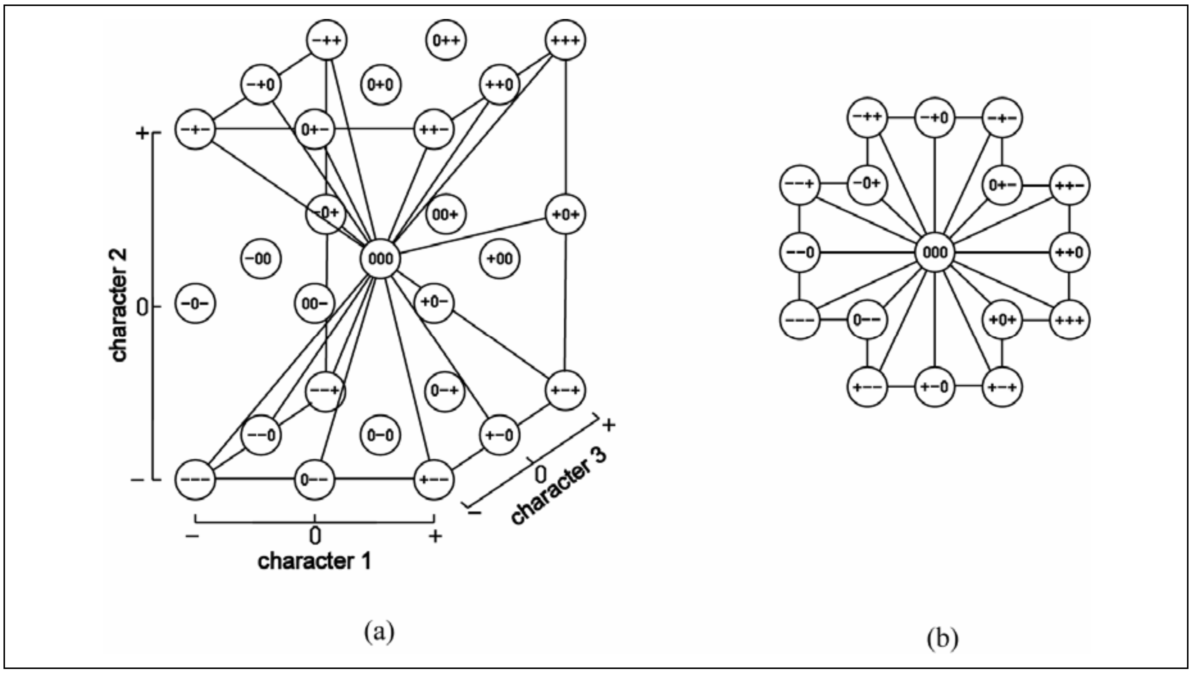


Figure 5.11 The CND for QTC_{B12}
(based on Van de Weghe and De Maeyer (2005, p.234))

5.3 A Conceptual Neighbourhood Diagram for QTC_N

5.3.1 Possible Relations and Conceptual Neighbours

In order to construct a CND for QTC_N, its relations, which can physically occur, and the transitions between them need to be examined. As shown in section 0, all 27 theoretically possible relations can physically occur. Figure 3.4 clearly shows that the set of QTC_N relations {'− 0 −', '− 0 0', '0 0 −', '0 − +', '0 0 −', '0 0 +', '0 + 0', '0 + +', '+ 0 −', '+ 0 0'} can only exist due to a shortest path omitting node pass event or the existence of a bifurcation shortest path. Note that ten relations correspond to the ten nonexistent relations in QTC_{B12}. In general, it can be stated that relations existing due to a shortest path event or the existence of a bifurcating shortest path, are caused by moving objects and can only occur at an exact moment in time. In other words, they can only hold instantaneously. As a result, these ten relations can not be dominated by other relations, and consequently, a direct transition between one of these QTC_N relations and another QTC_N relation by which it is dominated, is by definition not possible.

Furthermore, there are some conditions for relations caused by a bifurcating shortest path or a shortest path omitting node pass event to physically exist. In order to have a QTC_N relation which is caused by a shortest path omitting node pass event, the object causing this event should, first of all, move towards the other object in the relation immediately before it passes the node (i.e. raising a qualitative value '−' in the QTC_N label). Suppose the object causing the event, moves away from the other object (i.e. raising a '+' in the QTC_N label) just before it passes a node, then the only option for the object is to continue its way along an arc which does not belong to the shortest path, after it passes a node. Thus, by definition this object does not cause a shortest path omitting node pass event. Since an object has to move away from another object in order to have a shortest path omitting node pass event, the object will move away from the other object (i.e. raising a '+' in the QTC_N label) just after passing the node. In conclusion, an object involved in a shortest path omitting node pass event always leads to a conceptual animation from a qualitative value '−', over the intermediate value '0' into a qualitative value '+' for the first or second character in a QTC_N relation. Due to the above stated restrictions imposed by continuity, the '−' and '+' value should hold over an interval. As a second condition, the node that the object passes should have a degree of at least three. If the degree of the node, which is passed, is less than three and the object moves along the shortest path, then an object with positive speed can only continue its way along this shortest path (degree = 2) or needs to stop (degree = 1).

In order to have a relation which is caused by a bifurcating shortest path, there are also two conditions. First of all, at least one of the objects in the relation, causing the occurrence of a bifurcating shortest path, needs to move away (i.e. raising a '+' in the QTC_N label) from the other object, just before it induces a bifurcating shortest path. Suppose both objects move towards each other, or at least one of the objects is moving towards the other stationary object, then the shortest path between these two objects will shorten over time. In this situation, any other path which is not a shortest path can only shorten by at most the exact same amount as the shortest path. This means that it is physically impossible for these objects to generate two equally short shortest paths and thus, by definition, they can never induce a bifurcating shortest path. Objects have to move in order to have a relation caused by a bifurcating shortest path. When an object, inducing a bifurcating shortest path, is moving towards another object just before the occurrence of this path, it needs to move away from the other object (i.e. raising a '+' in the QTC_N label) just after that occurrence. Analogously, an object, inducing a bifurcating shortest path, which is moving away from another object, just before the occurrence of this path needs to move towards the other object (i.e. raising a '−' in the QTC_N label) just after that occurrence. Due to the above stated restrictions imposed by continuity, the '−' and '+' value should hold over an interval. As a second condition, at least one of the objects needs to lie on a cycle in the network. When both objects do not lie on a cycle in the network, they can only be reached by paths using the same immediately proceeding node, and thus, by definition, there can never be a bifurcation shortest path between these two objects.

Deleting all transitions from the CND representing all theoretically possible transitions, which do not respect the above stated restrictions, leads to the CND shown in Figure 5.12. Below, it will be shown that each one of these transitions in this CND exist in QTC_N and therefore the CND in Figure 5.12 represents the CND for QTC_N.

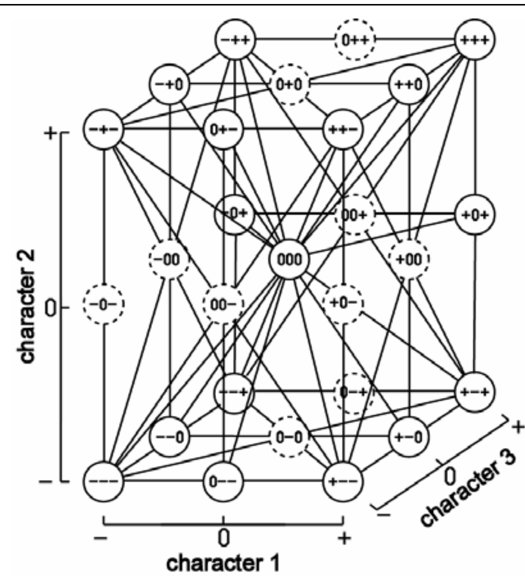


Figure 5.12 The CND for QTC_N

The CND clearly shows that all of the 27 (3^3) theoretically possible relations exist, but not all of them last over an interval. The ten dashed nodes represent QTC_N relations which can only hold instantaneously. These relations are equal to the ten nonexistent relations in QTC_{B12}. The CND also reveals that in contrast to the CND for QTC_{B22}, not all theoretically possible transitions between relations for QTC_N exist. Out of a possible 98 transitions, 76 remain feasible.

These transitions in the CND can be caused by three possible events:

- a speed change event : one or both objects change their speed;
- a shortest path omitting node pass event;
- a shortest path change event: a transition caused by objects inducing bifurcating shortest paths between the objects.

First, all transitions will be pointed out for the unique occurrence of one of these events, afterwards the transitions for a combination of two or more events will be shown.

5.3.1.1 A Speed Change Event

If a network is connected, and none of the objects are involved in a shortest path omitting node pass or shortest path change event, all shortest paths between two objects, involved in a QTC_N relation at *t*, have a simple linear structure with no junctions. Thus, they can be considered to have a movement in one dimension. Since QTC_{B12} describes such a movement, every relation and every transition between these relations stated in QTC_{B12} exists in QTC_N. Every relation in QTC_{B12} can be reached by only changing the speed of the objects. Consequently, a transition between relations is triggered by a speed change event. Thus, a single speed change event leads to the transitions shown in Figure 5.11.

5.3.1.2 A Single Shortest Path Omitting Node Pass Event

Suppose object *k* moves towards object *l* (Figure 5.13a). By definition, object *k* will invoke a '−' in the first character of the QTC_N label. If *k* reaches a node in the network with a minimum degree of three, it can either continue its way along a shortest path or it can continue its way on an arc that does not belong to a shortest path. The latter implies that there will be a change in the relation between *k* and *l*, because an object can only move towards another object if it moves along a shortest path. At the exact moment in time when *k* passes the node, it will not move towards nor move away from the other object l (Figure 5.13b).Thus, by definition, k will invoke a ‘0’ in the first character of the QTC_N label. A fraction of time later, k will increase its distance with regard to l, invoking a ‘+’ in the first character of the QTC_N label defining the relation between objects k and l (Figure 5.13c).

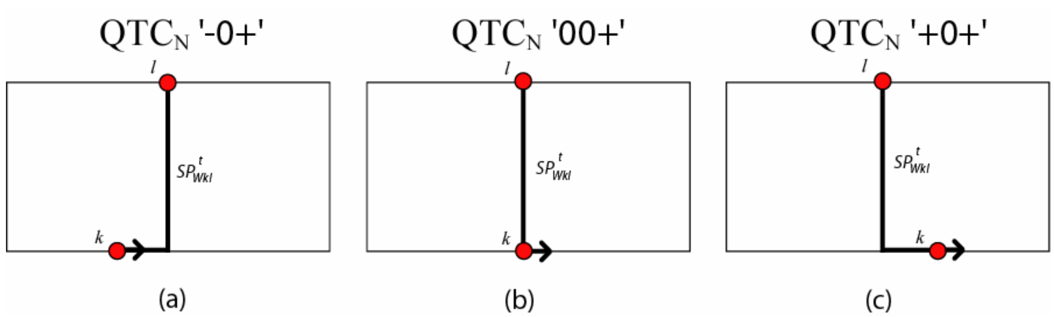


Figure 5.13 A transition due to a shortest path omitting node pass event

In general (meaning for k or l), a single shortest path omitting node pass event always results in a conceptual animation in which one of the first two characters in the label changes from ‘−’ to ‘0’ to ‘+’.

Given this condition, the transitions caused by this event can be visualised in a CND. Figure 5.14 gives an overview of all possible transitions between relations due to a single shortest path omitting node pass event.

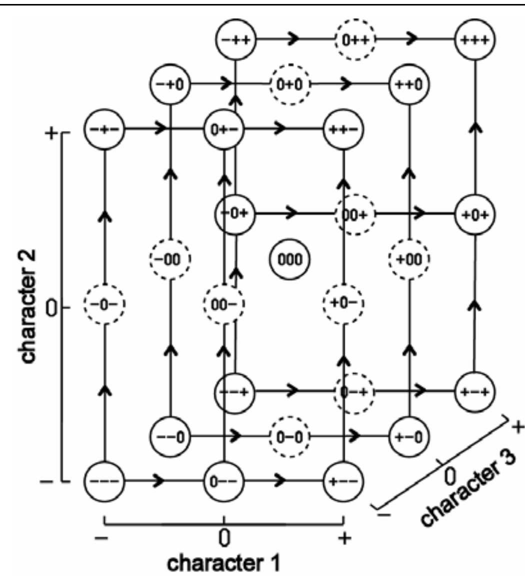


Figure 5.14 Possible transitions due to a single shortest path omitting node pass event

The lines connecting two QTC_N relations in Figure 5.14 indicate a possible transition. Sometimes, these lines are directed by means of an arrow symbol. The arrow indicates the direction of a transition (from-to). The absence of an arrow indicates that a transition is possible in both ways. This convention will be used for all CND^s in this thesis.

5.3.1.3 A Single Shortest Path Change Event

Assume object *k* lies in between nodes B and C, and *k*, B and C lie on a cycle (Figure 5.15). In Figure 5.15a, there is a shorter path via node B (*k*,B,A,*l*) and a longer path via node C (*k*,C,A,*l*). When *k* moves away from this shorter path, and therefore moves away from the other object *l*, *k* will, according to the definition, invoke a '+' in the first character of the QTC_N label. While *k* moves towards *l*, the shorter path extends and the longer path shortens. At some moment in time, these two paths will become equally long (Figure 5.15b) and thus *k* induces a bifurcating shortest path. At that instantaneous moment, *k* will not approach nor move away from *l*. As a result, *k* will invoke a '0' in the first character of the QTC_N label. A fraction of time later, *k* will move along the newly defined shortest path, and, as a consequence, its distance compared to the other object will decrease, invoking in a '−' in the first character QTC_N label (Figure 5.15c).

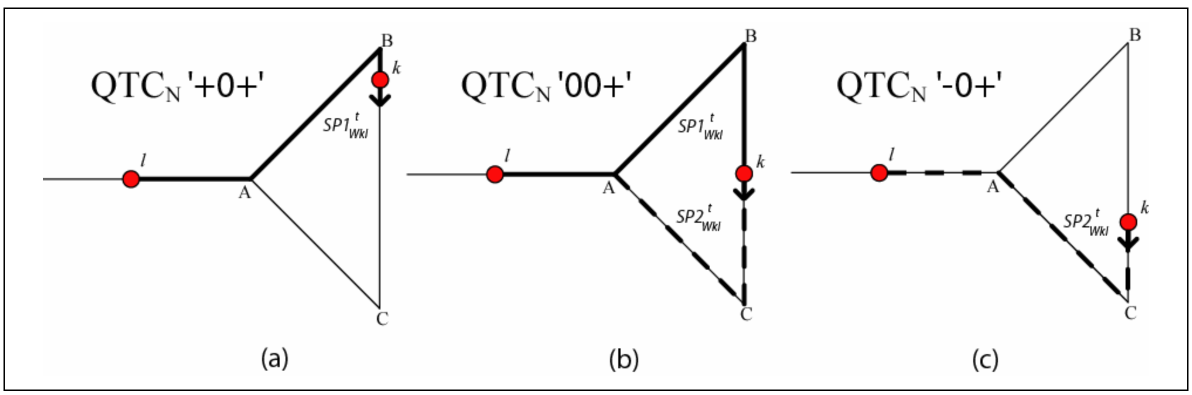


Figure 5.15 A transition due to a shortest path change event

In general, a single shortest path change event results in a conceptual animation in which one of the first two characters in the label changes from '+' over '0' into '−'. Figure 5.16 gives an overview of all possible transitions between relations due to a single shortest path change event.

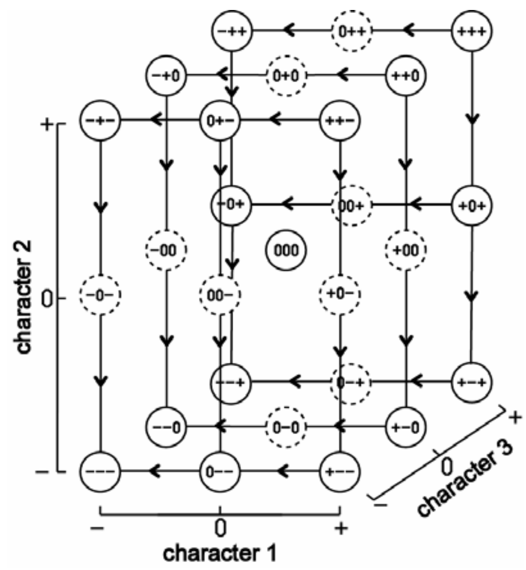


Figure 5.16 Possible transitions due to a single shortest path change event

5.3.1.4 Combination of Events

A transition between the QTC$_N$ relations can be caused by three events. Since these three events occur independently, and, in addition, a single shortest path omitting node pass event or a single shortest path change event is caused by only one object, two or more events can occur simultaneously.

5.3.1.4.1 A Combined Shortest Path Omitting Node Pass Event

When object *k* and object *l* approach each other, it can occur that both objects simultaneously pass a node and therefore create the possibility of a combined shortest path omitting node pass event (Figure 5.17). This combined event leads to the possibility of six transitions, shown in Figure 5.18.

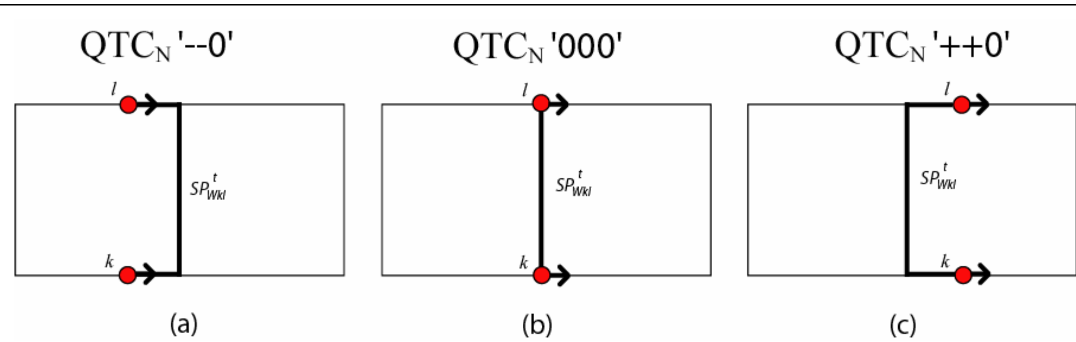


Figure 5.17 A transition due to a combined shortest path omitting node pass event

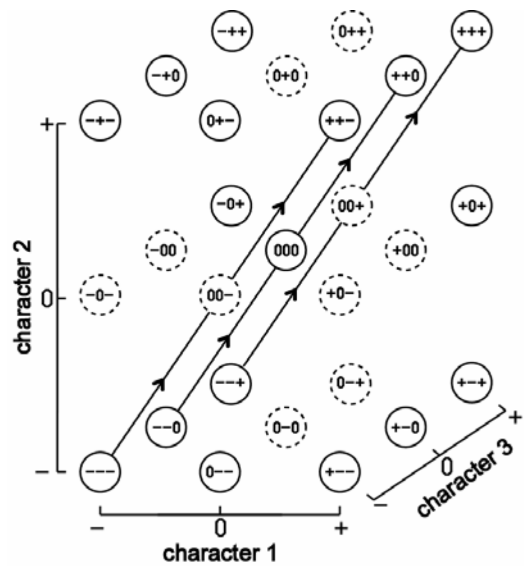


Figure 5.18 Possible transitions due to a combined shortest path omitting node pass event

5.3.1.4.2 A Combined Shortest Path Change Event

Assume that object *k* and object *l* both lie on a cycle within the network. Both objects can lie on two different cycles (Figure 5.19) or the same cycle (Figure 5.20). When both objects are moving away from each other, there is a possibility that both *k* and *l* induce a bifurcating shortest path simultaneously. This leads to a combined shortest path change event. This event leads to six transitions (Figure 5.22a).

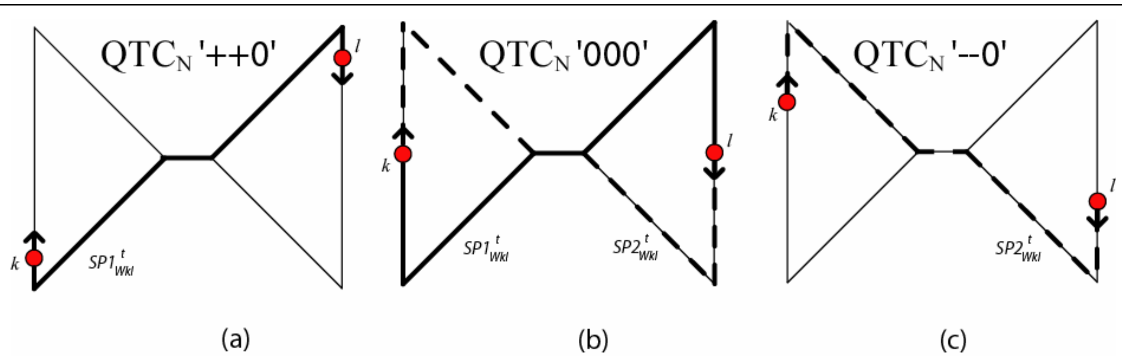


Figure 5.19 A transition due to a combined shortest path change event when both objects lie on a different cycle

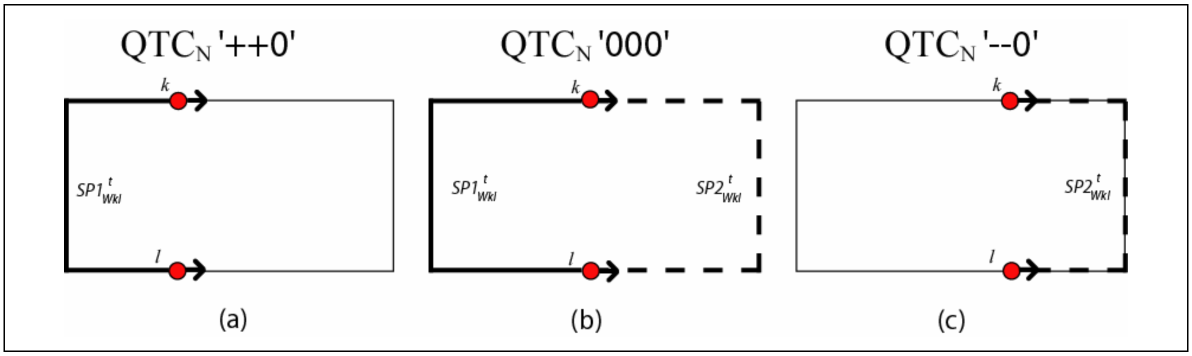


Figure 5.20 A transition due to a combined shortest path change event when both objects lie on the same cycle

A combined shortest path change event can also occur when only one object is moving away from the other object which is also moving. This transition is exemplified via the conceptual animation in Figure 5.21. In Figure 5.21a both objects lie on the same cycle. This means that there are two paths between object *k* and object *l*. There is one shorter path (*k*,A,B,), and one longer path (*k*,D,C,*l*). When *l* is moving away from *k*, *k* is moving towards *l* and *l* is moving faster than *k*, the shorter path will be extended and the longer path will get shorter. At some moment in time, these two paths will become equally long and thus *k* and *l* induce a bifurcating shortest path (Figure 5.21b). At that instantaneous moment, neither object will approach nor move away from each other. As a result, both objects will invoke a ‘0’ in the three character label. A fraction of time later, both objects will move along the newly defined shortest path and *l* will decrease its distance compared to *k* and *k* will increase its distance compared to *l* (Figure 5.21c). This type of animation leads to four possible transitions (Figure 5.22b).

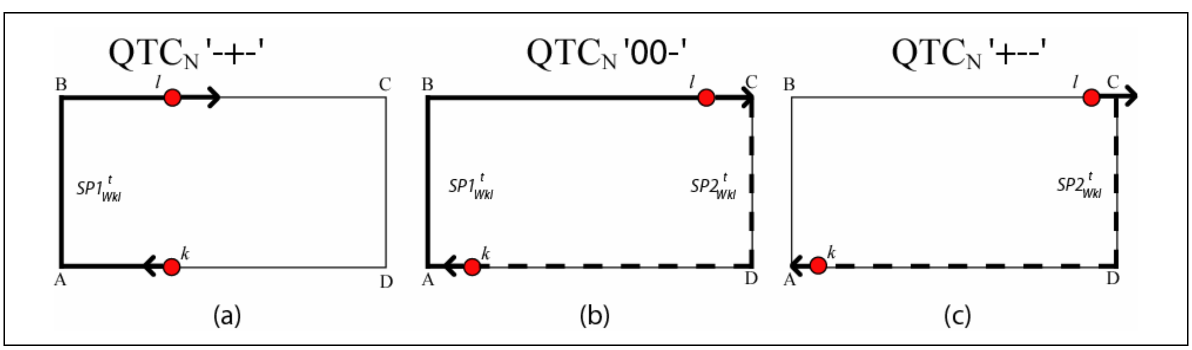


Figure 5.21 A transition due to a combined shortest path change event

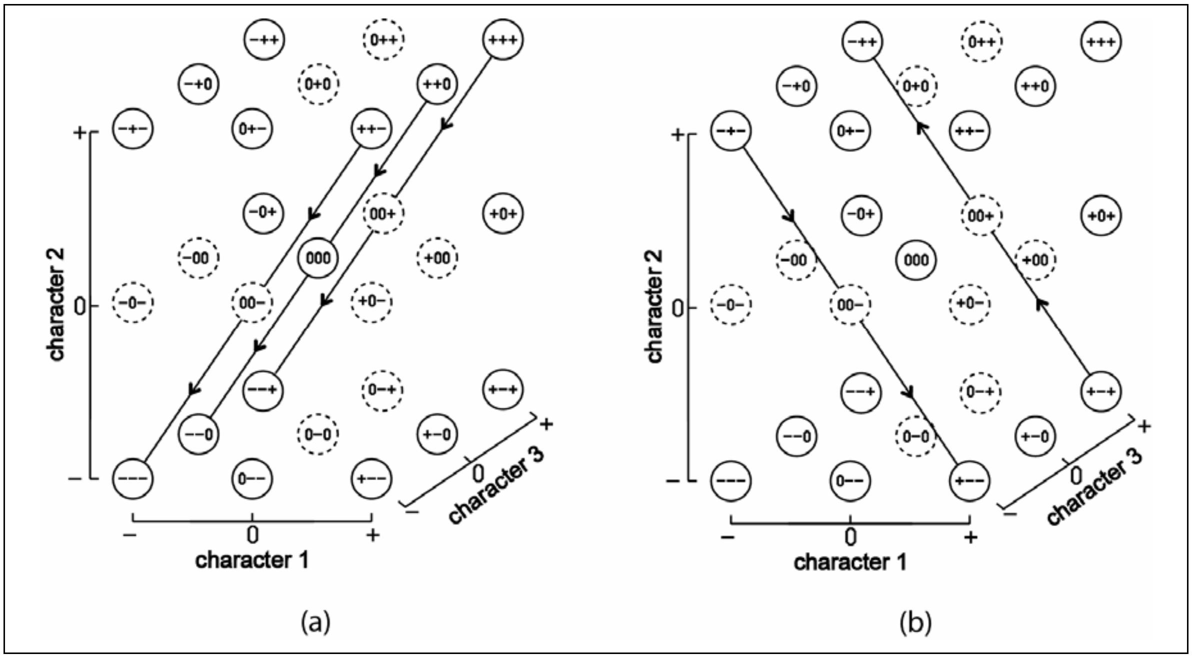


Figure 5.22 Possible transitions due to a combined shortest path change event

5.3.1.4.3 A Combination of a 'Node Pass' Event and a 'Shortest Path Change' Event

Figure 5.23 illustrates a transition caused by a combination of a 'Node Pass' event and a 'Shortest Path Change' event. This transition occurs when one object passes a node and simultaneously the shortest path changes due to the other. This transition can only occur if the object that passes a node approaches the other object. The other object must then move away from this object and lie on a cycle of the network. A combination of a 'Node Pass' event and a 'Shortest Path Change' event allows six additional conceptual animations resulting in twelve new transitions as shown in Figure 5.24.

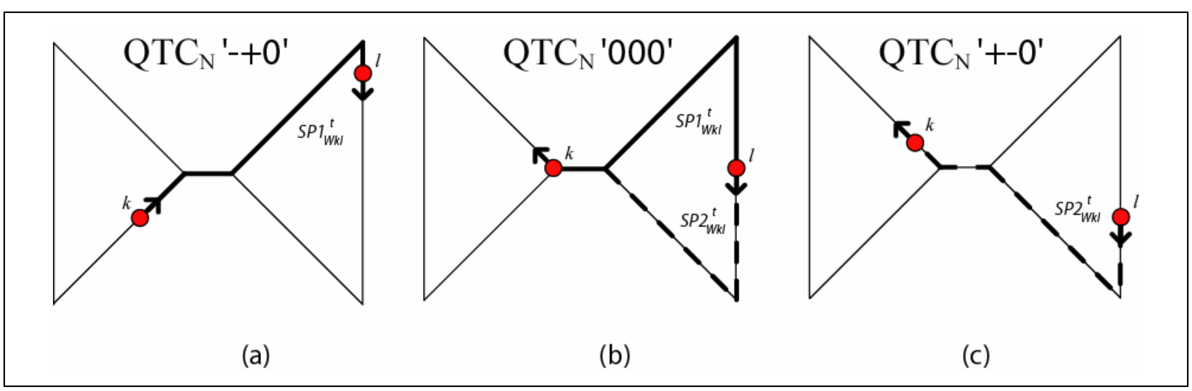


Figure 5.23 A transition due to a combined shortest path omitting node pass and shortest path change event

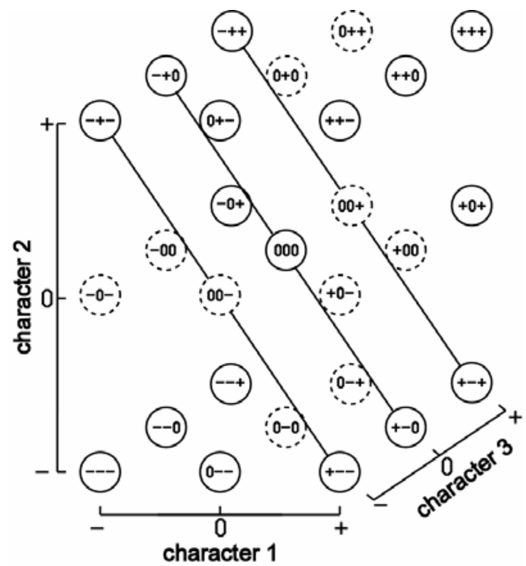


Figure 5.24 Possible transitions due to a combined shortest path omitting node pass and shortest path change event

5.3.1.4.4 A Combination of a ‘Speed Change’ Event and/or a (Combined) ’Node Pass’ Event and/or a (Combined) ‘Shortest Path Change’ Event

Apart from the fact that objects need to move in order for a shortest path omitting node pass event or a shortest path change event to occur, speed is independent of these two events. Therefore, a speed change event is also independent of these two events. This means that a speed change event can occur simultaneously with a single or a combination of shortest path omitting node pass events and/or a single or a combination of shortest path change events. An example of a combination of such events is shown in Figure 5.25. The transitions caused by a combination of a speed change event and/or a shortest path omitting node pass event and/or a shortest path change event are visualised in Figure 5.26.

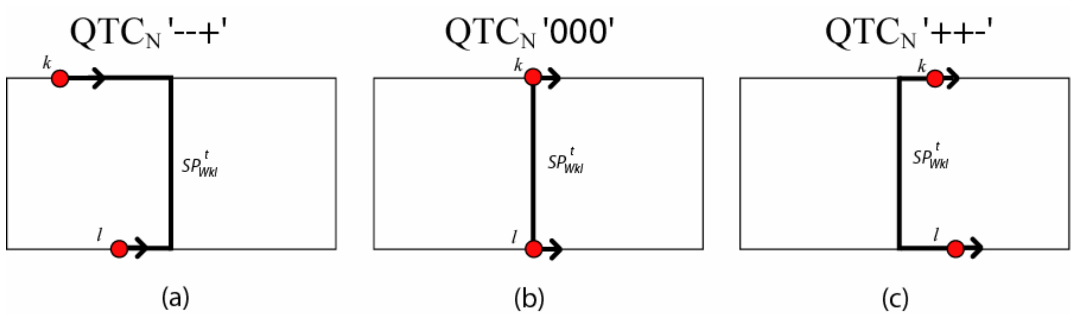


Figure 5.25 A transition due to a combination of a speed change event and/or a (combined) shortest path omitting node pass event and/or a (combined) shortest path change event

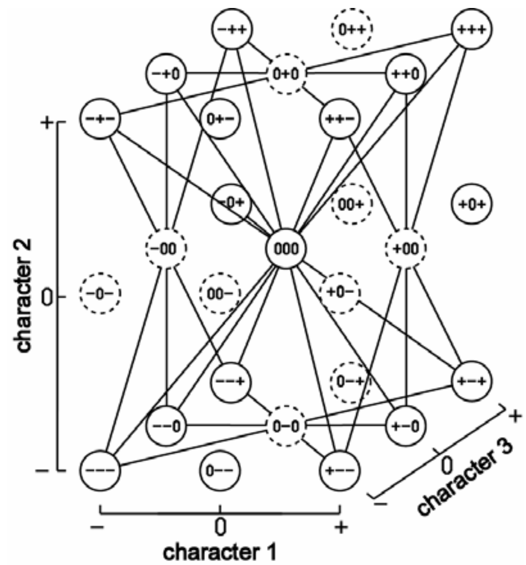


Figure 5.26 Possible Transitions due to a combination of a speed change event and/or a (combined) shortest path omitting node pass event and/or a (combined) shortest path change event

Chapter 6
The Influence of Changing Networks on QTC_N Relations

6.1 Changes in a Network Affecting QTC_N Relations

So far, the network on which objects move, is assumed to be invariable. However, in many lifelike situations the network itself is often the subject of change. Since the frame of reference used to represent QTC_N relations is the shortest path, network changes that affect the length of possible paths between two objects need to be studied (Delafontaine 2006). Changes in the network can be caused by a change in the geographic location of its edges and nodes (e.g. the construction of a new road) or by a change of its non-spatial characteristics (e.g. the travelling time to pass an edge decreases). Given that *S*, the space embedding the network, does not necessarily have to be a physical space, but can be any space with a metric distance function d(x,y) obeying the triangular inequality, and a notion of curve defined (see 3.2), such as the conceptual cost and time spaces (spaces respectively representing distance in terms of an economic cost and a travelling time), both types of changes can have an effect on a QTC_N relation (Delafontaine 2006). For the sake of clarity, the notation $QTC_{N'}$ will be used for QTC_N relations between objects moving along changing networks. Changes to the network can be continuous or discontinuous. The notation $QTC_{CN'}$ will be used for $QTC_{N'}$ relations only affected by continuously changing networks. Analogously, $QTC_{DN'}$ will be used for $QTC_{N'}$ relations exclusively affected by discontinuous changes to the network.

Many changes in the network lead to a change in its topology. Given that the nodes in a network do not have a length, only topological changes which come down to additions or deletions of an edge in the network can have an effect on a $QTC_{N'}$ relation. These changes always come down to a discontinuous change, since an edge has a non negative length and a topological change always occurs at a specific moment (Delafontaine et al. to appear).

Below, the effect of continuous and discontinuous network changes will, first of all, be dealt with separately. Afterwards, the combination of these two network changes will be examined.

6.2 The Effect of Continuous Network Changes on QTC_N Relations ($QTC_{CN'}$)

Continuous network changes in a physical (geographical) space do not occur very often. The most applicable kinds of these changes occur in more conceptual spaces such as time spaces or cost spaces (Delafontaine 2006). For example, weather (rainfall, wind, snow, fog etc.), road or traffic conditions may cause the time to pass a road in a transport network to change. These changes can in some cases be assumed to be continuous. For instance, the travelling time of a boat navigating from harbour A to harbour B can be a function of the current of a river, in other words, if the current increases in the opposite navigation direction, the travelling time will increase as well.

QTC_N can be seen as a special case of $QTC_{CN'}$, i.e. the case where there are no changes to the network. This means that all relations and transitions between relations existing in QTC_N are present in $QTC_{CN'}$ as well. On top of this, a continuous network change can have additional effects on $QTC_{CN'}$ relations. An example of such a change is given in Figure 6.1. In Figure 6.1, the length of the edges on which objects *k* and *l* move grows continuously. If the increase in length of these edges extends the distance travelled by objects *k* and *l* during the same time period, then the shortest path between objects *k* and *l* increases as well. This means that there is a possibility this growth induces a bifurcating shortest path, and thus, causes a change in the $QTC_{CN'}$ relation.

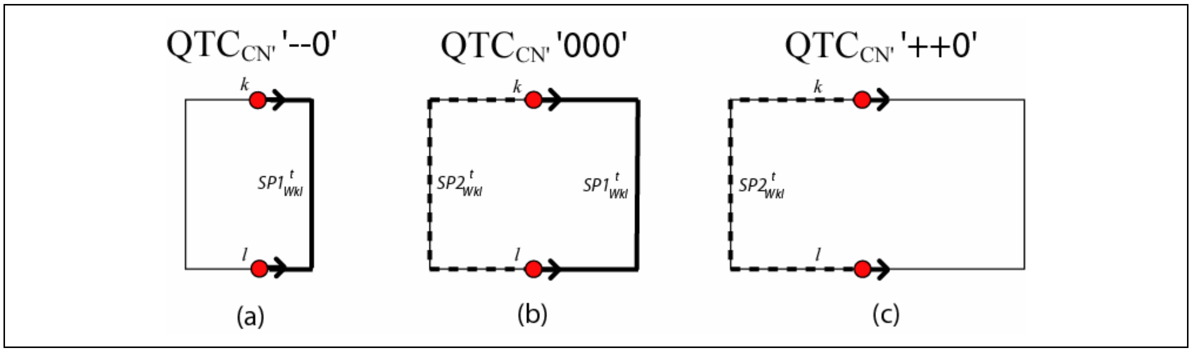


Figure 6.1 A transition in $QTC_{CN'}$ relation caused by a continuously changing network

In view of the fact that the network changes continuously, the changes caused by a continuous network change need to obey the constraints imposed by continuity. For example, if a path *M* is longer than path *N* between the same two objects, it can not change its length in being shorter than path *N* without being equally short at first.

As stated in 5.3.1, objects need to move in order to cause a bifurcating shortest path or a shortest path omitting node pass event. If a continuous change in the network changes the length of a path equally fast as the distance travelled by the object bounding the path, during the same time period, then these paths remain equally long over that period. Thus, in contrast to relations in QTC_N, relations in $QTC_{CN'}$ which are caused by a bifurcating shortest path (Figure 6.2) or by a shortest path omitting node path event (Figure 6.3) can hold over an interval (Delafontaine 2006). An important consequence of this statement is that these relations, also in contrast to relations in QTC_N, can be dominated by other relations. In addition, the other conditions for relations caused by a bifurcating shortest path or a shortest path omitting node pass event to physically exist, stated in 5.3.1, do not apply in $QTC_{CN'}$.

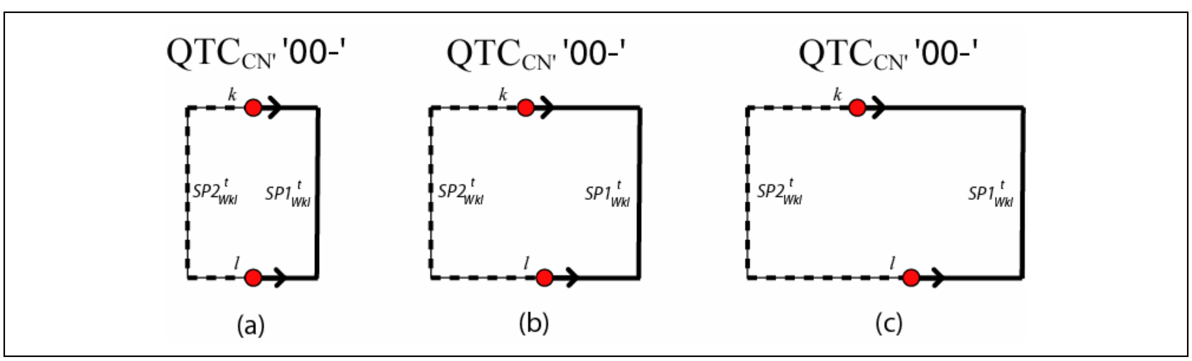


Figure 6.2 A bifurcating shortest path lasting over an interval

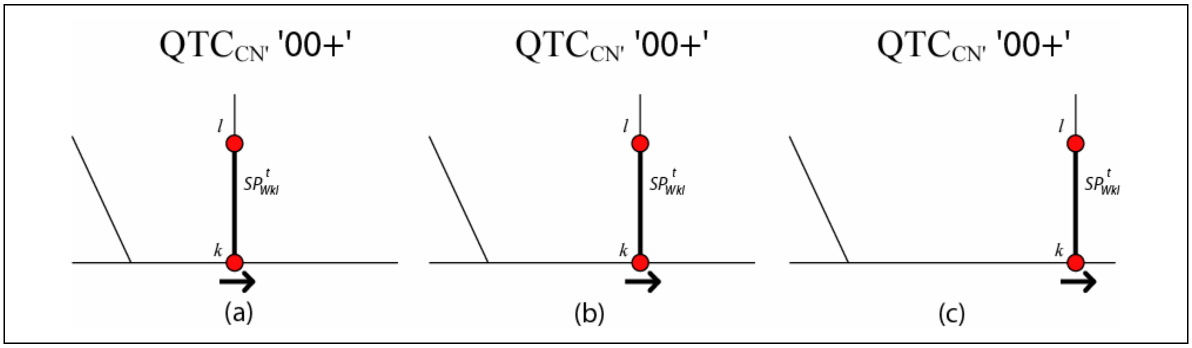


Figure 6.3 A shortest path omitting node pass event lasting over an interval

As a result, the CND for relations in $QTC_{CN'}$ is equal to the CND for objects moving freely in a two-dimensional space. This is not surprising since the network is able to change continuously in the space it is embedded in. This CND is visualised in Figure 6.4a. Figure 6.4b gives an overview of the transitions existing in $QTC_{CN'}$ but not in QTC_N. In this thesis, these transitions will be denoted by transitions in $QTC_{CN'/N}$, meaning transitions which can occur in $QTC_{CN'}$ but not in QTC_N.

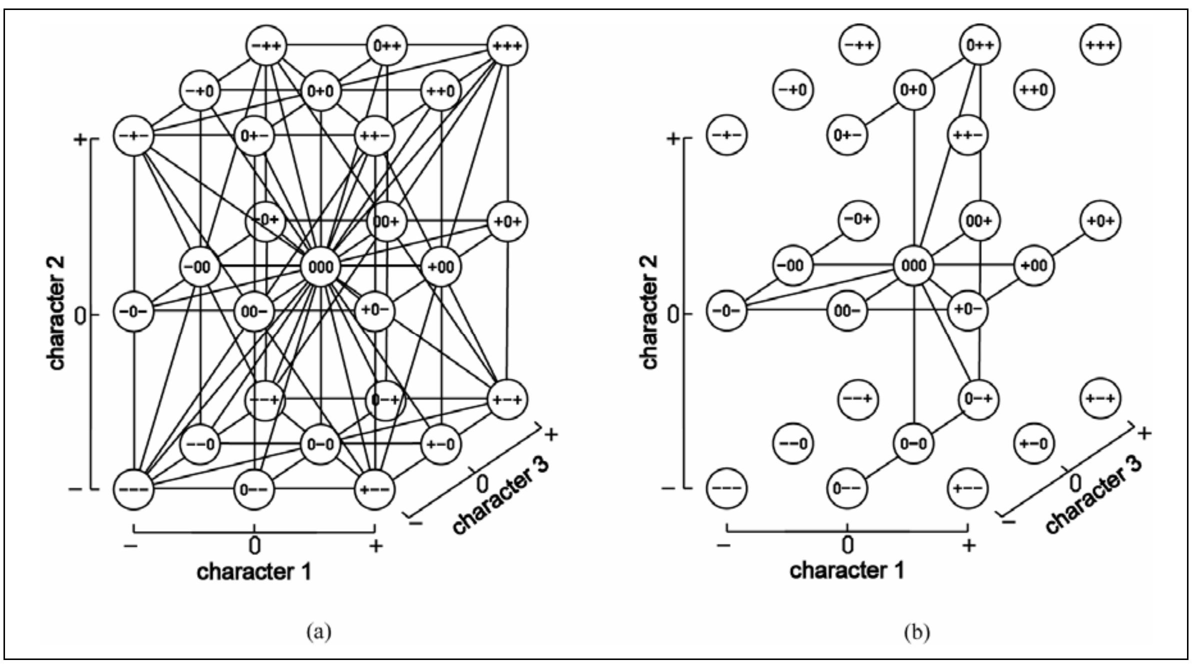


Figure 6.4 The CND for $QTC_{CN'}$

6.3 The Effect of Discontinuous Network Changes on QTC_N Relations ($QTC_{DN'}$)

Discontinuous changes to a network are manifold, both in the physical (geographical) space as well as in more conceptual (time, cost) spaces (Delafontaine 2006). For example, the closure of a road in a transportation network implies a topological change in the physical space (the deletion of an edge), while this implies a discontinuous length change in the time space (the time needed to follow a deviation or for the road to be opened again).

As for $QTC_{CN'}$, QTC_N can be seen as a special case of $QTC_{DN'}$, i.e. the case in which there are no changes to the network. This means that all relations and all events causing transitions between relations present in QTC_N exist $QTC_{DN'}$ as well. In contrast to $QTC_{CN'}$, relations caused by a bifurcating shortest path or a shortest path omitting node pass event can only hold instantaneously, due to the discontinuous nature of a change in the network (Delafontaine et al. to appear). In other words, unlike network changes in $QTC_{CN'}$, an edge or a node in the network can not change its length or location continuously with the movement of an object. A direct consequence is that these relations can not be dominated by other relations. Another important difference with respect to $QTC_{CN'}$, is that due to discontinuous network changes the constraints imposed by continuity do not apply for changes in $QTC_{DN'}$ relation triggered by such change in the network (Delafontaine et al. to appear). This implies that if a path *M* is longer than path *N* at time *t*, it can change its length into being shorter than path *N* just after or before *t*, without being of equal length first. A direct consequence is that an object in a $QTC_{DN'}$ relation can directly change its relation from moving towards into moving away from another object. Put in QTC terms, a qualitative value for the first two characters in a $QTC_{DN'}$ relation can directly change from a '−' value into a '+' value and vice versa (Delafontaine et al. to appear). An example of such a transition is given in Figure 6.5.

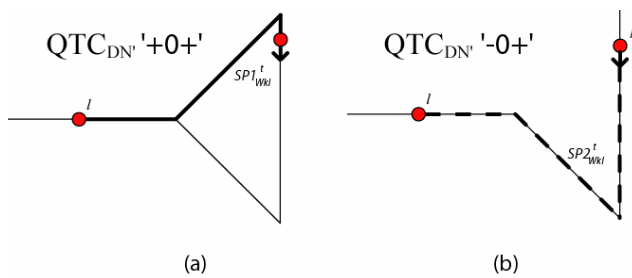


Figure 6.5 A transition in $QTC_{DN'}$ relation caused by a discontinuously changing network

Note that a discontinuous network change does not necessarily change one or both of the first two characters of a QTC_{DN} relation from a '−' value into a '+' value or vice versa. There can be no change at all or these value can be transformed in to a '0' value when the discontinuous network change at time *t* induces a bifurcating shortest path at *t*. These last two changes in $QTC_{DN'}$ relation can not be distinguished from equal changes in QTC_N or $QTC_{CN'}$ (Delafontaine et al. to appear). Below, we will only examine changes occurring in $QTC_{DN'}$ and not in $QTC_{CN'}$ or QTC_N. These changes belong to the set denoted by $QTC_{DN'/N}$.

As stated above, a discontinuous network change is able to change a qualitative value for the first two characters in a $QTC_{DN'}$ relation directly from a '−' value into a '+' value and vice versa. A change in $QTC_{DN'}$ relation due to this event is referred to as a discontinuous shortest path change event (Delafontaine et al. to appear). A discontinuous shortest path change event can cause a change in one (a single discontinuous shortest path change event) (Figure 6.5) or both (a combined discontinuous shortest path change event) (Figure 6.6) of these values in a $QTC_{DN'}$.

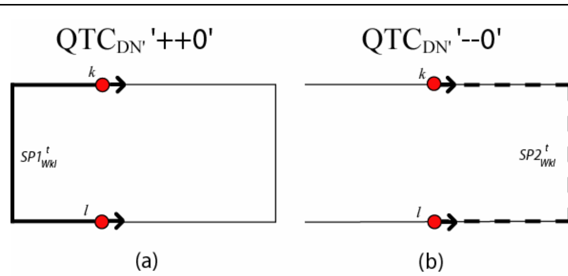


Figure 6.6 A transition in $QTC_{DN'}$ relation caused by a combined discontinuous shortest path change event

Relations caused by a bifurcating shortest path or a shortest path omitting node pass event can only hold instantaneously, and thus, can not be dominated by other relations. This implies that a transition caused by discontinuous shortest path change can never start from one of these relations.

In conclusion, transitions caused by a (combined) discontinuous shortest path change event are visualised in Figure 6.7.

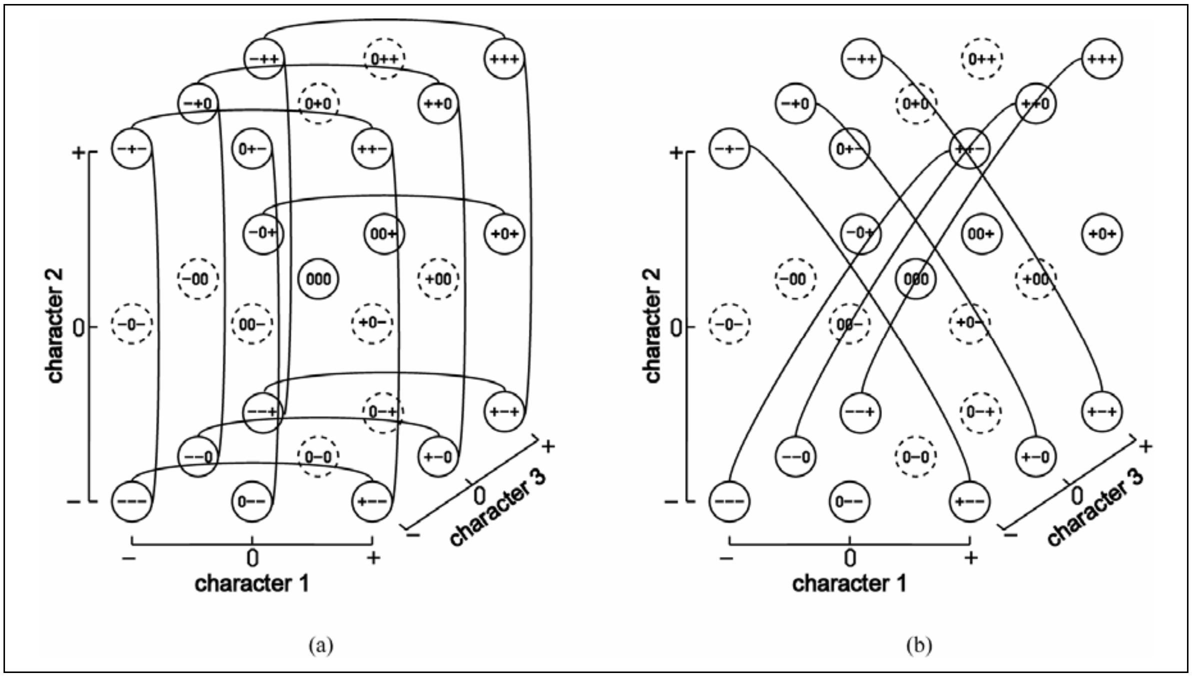


Figure 6.7 Visualisation of transitions caused by (a) a single or (b) combined discontinuous shortest path change event

Given that objects can change their speed independent of changes occurring in a network, a single or combined discontinuous shortest path change event can be combined with a speed change event (Delafontaine et al. to appear). An example of a combination of a speed change event and a single discontinuous shortest path change event is given in Figure 6.8a. The combination of a combined discontinuous shortest path change event and a speed change event is given in Figure 6.8b.

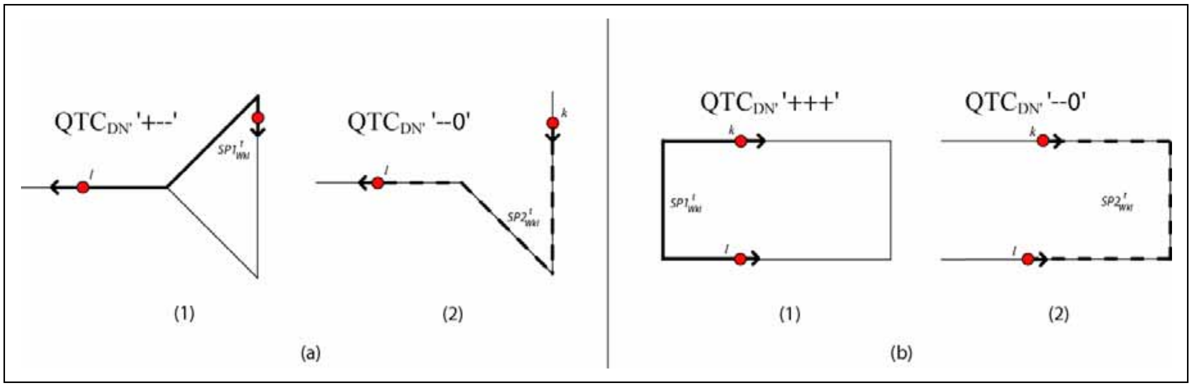


Figure 6.8 A transition in $QTC_{DN'}$ relation caused by the combination of a speed change event and (a) a single or (b) a combined discontinuous shortest path change event

Since the objects in $QTC_{DN'}$ are assumed to move continuously, a change in the relative speed of objects needs to obey the constraints imposed by continuity. This implies that a change in a character representing the relative speed caused by a speed change event can not transform directly from ‘−’ to ‘+’ and vice versa, since such a change must always pass the qualitative value ‘0’. Secondly, as stated in 5.2, a qualitative value ‘−’ or ‘+’ changing into a qualitative value ‘0’, for a character representing the relative speed of two objects, holds over an open interval, while the ‘0’ value can hold instantaneously or over a closed interval. Therefore, it is impossible to determine the exact first moment a ‘+’ or ‘−’ value holds. In the view of the fact that a discontinuous shortest path change always occurs at an exact moment in time, a combination of a speed change event and a discontinuous shortest path change event can only occur simultaneously at the exact moment when the value representing the relative speed is equal to ‘0’. As a direct consequence, a direct change from a ‘+’ or ‘−’ (or vice versa) for the first two characters in a $QTC_{DN'}$ relation can never coincide with a change from ‘0’ into a ‘−’ or ‘+’ representing the third character in a $QTC_{DN'}$ relation (Delafontaine et al. to appear).

Taken into account the two above stated restrictions imposed by continuity, the possible transitions caused by the combination of a speed change event and a single or combined discontinuous shortest path change event are visualised respectively in Figure 6.9a and Figure 6.9b. Note that the transitions are directed due to the above stated restrictions.

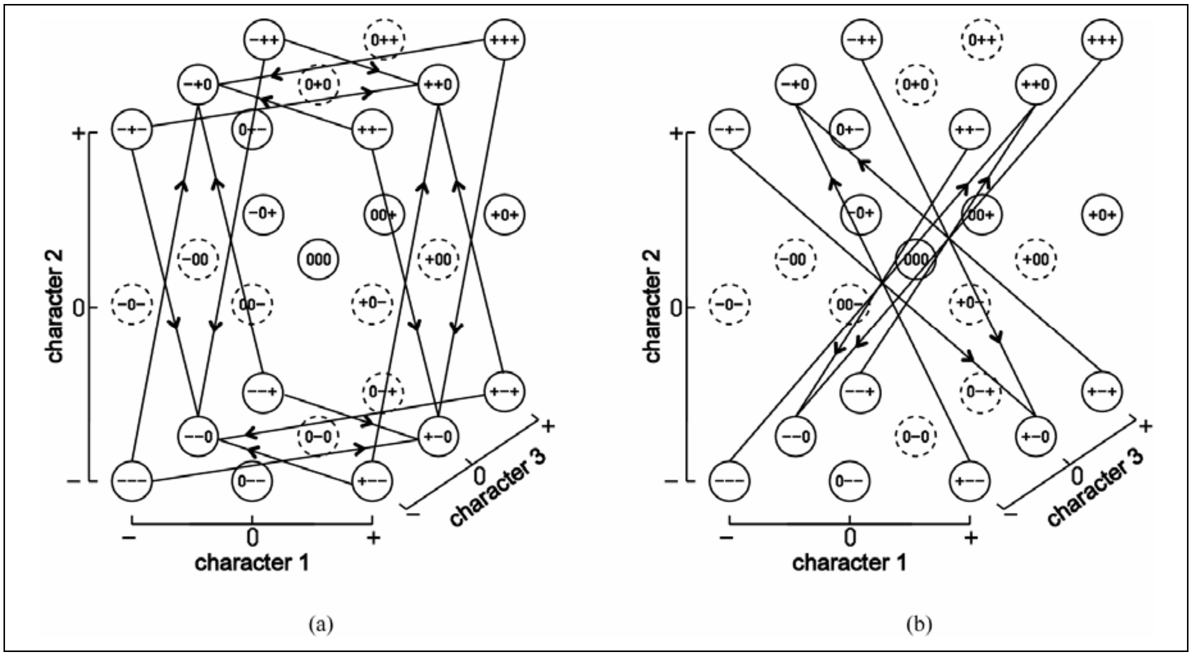


Figure 6.9 Visualisation of transitions caused by a combination of a speed change event and (a) a single or (b) combined discontinuous shortest path change event

If only one of the first two characters in a $QTC_{DN'}$ label undergoes a transition due to a discontinuous shortest path change event, the other object can still be affected by other events existing in QTC_N. Hence, a combination of a single discontinuous shortest path change event in combination with a shortest path change event or a node pass event is possible (Delafontaine et al. to appear). Examples of such transitions are respectively shown in Figure 6.10a and Figure 6.10b. Given that objects can change their speed independently from these three events, a combination of a single discontinuous shortest path change event in combination with a shortest path change event or a node pass event can additionally be combined with a speed change event (Delafontaine et al. to appear). An example of a transition in $QTC_{DN'}$ relation caused by a speed change event in combination with a single discontinuous shortest path change event and a shortest path change event is given in Figure 6.11a, an example of a transition in $QTC_{DN'}$ relation caused a speed change event in combination with a single discontinuous shortest path change event and a node pass event Figure 6.11b.

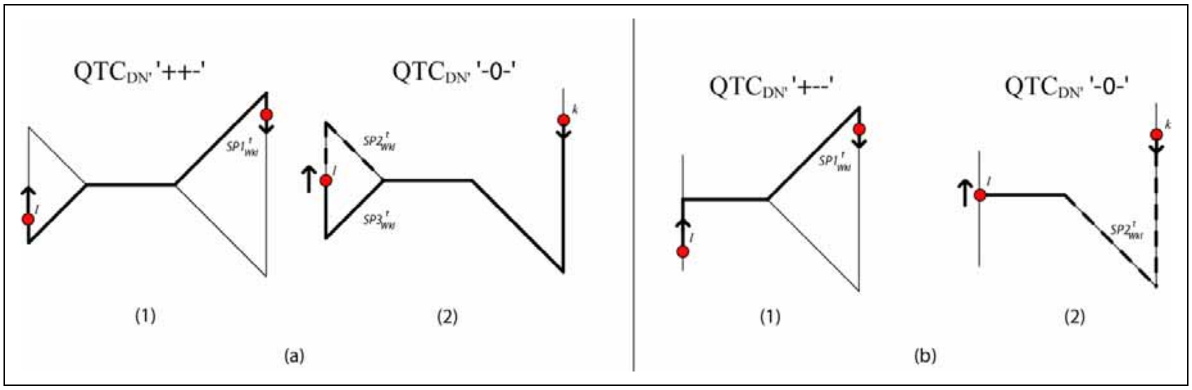


Figure 6.10 A transition in $QTC_{DN'}$ relation caused by the combination of a single discontinuous shortest path change event and (a) a shortest path change event or (b) a node pass event

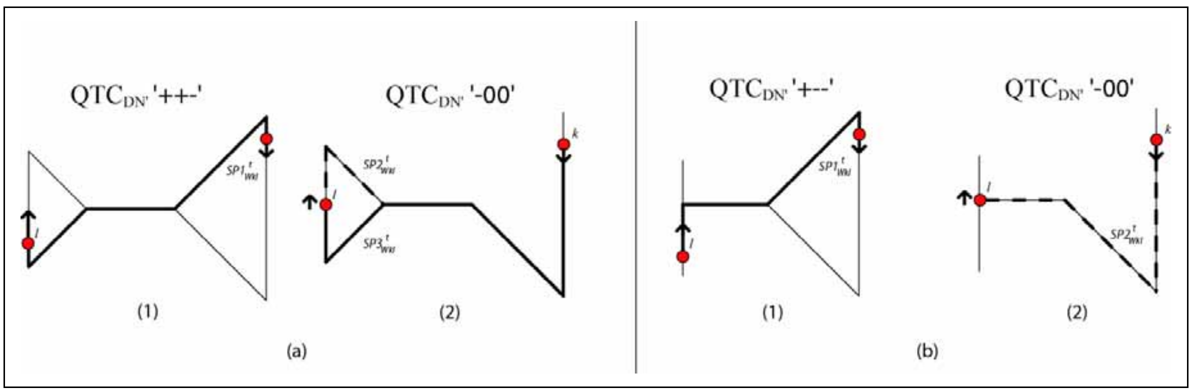


Figure 6.11 A transition in $QTC_{DN'}$ relation caused a speed change event in combination with a single discontinuous shortest path change event and a shortest path change event (a) or a node pass event (b)

As stated in 5.3.1 speed change events, shortest path change events and node pass events alter a relation continuously. This means that they need to obey the constraints imposed by continuity. This implies first of all that a change in a character of the label representing a $QTC_{DN'}$ caused by one of these events can not transform directly from '–' to '+' and vice versa, since such a change must always pass the qualitative value '0'. Secondly, a direct change from a '+' or '–' (or vice versa) in one of the first two characters in a $QTC_{DN'}$ relation caused by a discontinuous shortest path change event, can never coincide with a change from '0' into a '–' or '+' for the other characters in the label representing a $QTC_{DN'}$ relation (Delafontaine et al. to appear). Taking into account the two above stated restrictions imposed by continuity, the possible transitions caused by the combination of a discontinuous shortest path change event and a node pass event or a shortest path change event are visualised in Figure 6.12. The possible transitions caused by a speed change event in combination with a single discontinuous shortest path change event and a shortest path change event or a node pass event are shown in Figure 6.13. Note that all transitions are directed due to the above stated restrictions.

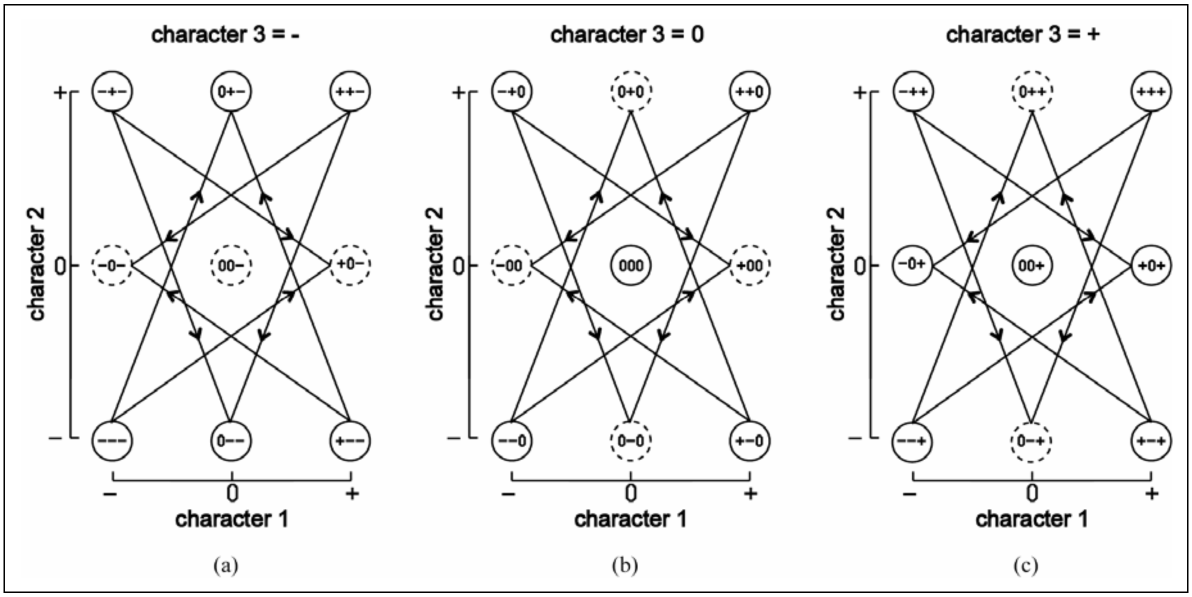


Figure 6.12 Visualisation of transitions caused by a combination of a discontinuous shortest path change event and a node pass event or a shortest path change event

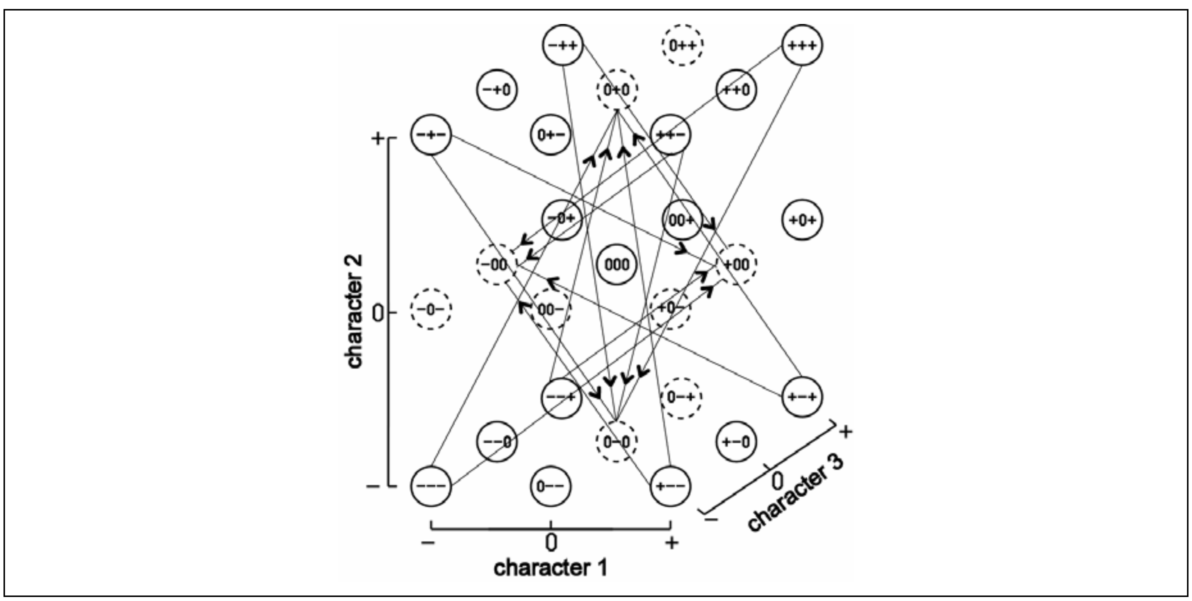


Figure 6.13 Visualisation of transitions caused by a speed change event in combination with a single discontinuous shortest path change event and a shortest path change event or a node pass event

6.4 Combination of Continuous and Discontinuous Changes ($QTC_{N''}$)

There is no reason why a network can not be affected by both continuous and discontinuous changes. Therefore, in this section the additional changes on $QTC_{N'}$ relations induced by combinations of both changes will be examined. The set of relations and additional transitions caused by such a combination will be denoted by $QTC_{N''}$ (Delafontaine 2006).

QTC_N, $QTC_{CN'}$ and $QTC_{DN'}$ can be regarded as special cases of $QTC_{N''}$, i.e. respectively the case when there are no changes or only continuous or discontinuous changes to the network. As stated in 6.2, all relations in $QTC_{CN'}$ can physically hold over an interval. Suppose a qualitative value '0' for the first or second character in a $QTC_{N'}$ relation, induced by a node pass event or a bifurcating shortest path, holds over an interval due to continuous changes in the network, then the other character representing the movement of the other object can change due to a discontinuous shortest path change during that interval (Delafontaine 2006). An example of such a transition is given in Figure 6.14a. Furthermore, this change in relation can coincide with a change in the relative speed between the objects (Delafontaine 2006), as exemplified in Figure 6.14b.

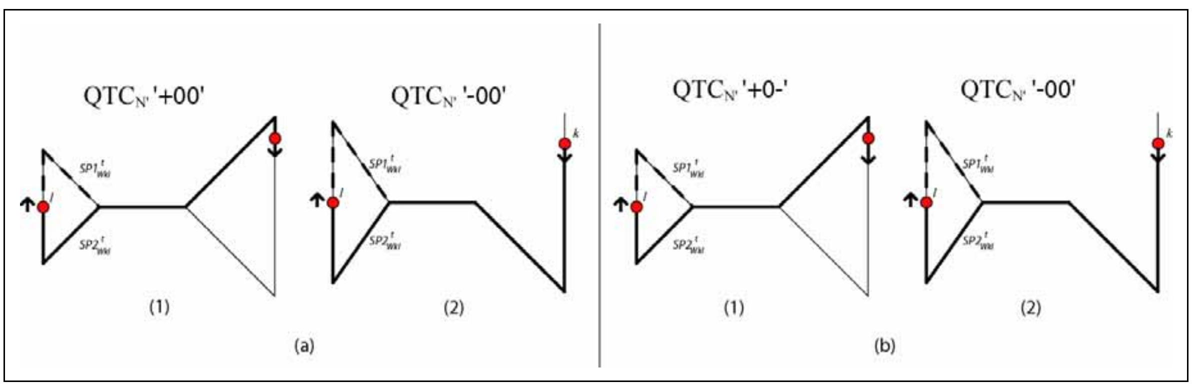


Figure 6.14 A transition in $QTC_{N'}$ relation caused by (a) a discontinuous shortest path change and (b) a combination of a discontinuous shortest path change and a speed change event

The additional transitions in $QTC_{N''}$ caused by a mutual occurrence of such events is visualised in Figure 6.15a and b. Note that due to the restrictions on the combination of a discontinuous shortest path change event and a speed change event stated in 6.3 the transitions in Figure 6.15b are directed.

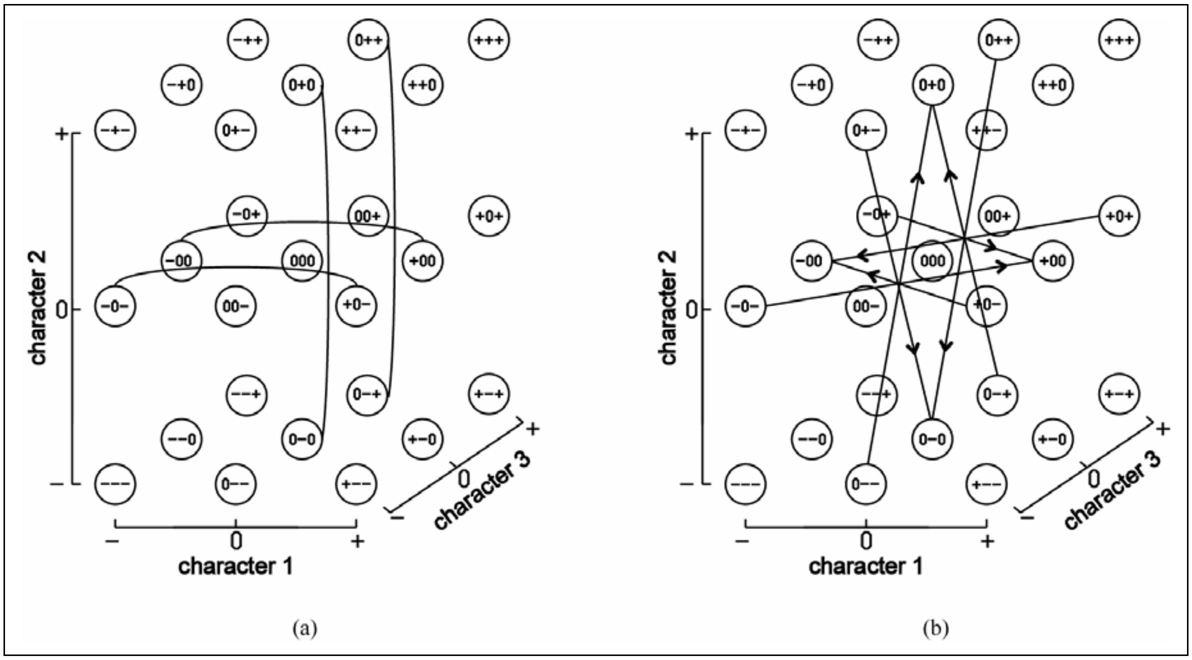


Figure 6.15 Visualisation of transitions in $QTC_{N''}$ caused by (a) a discontinuous shortest path change and (b) a combination of a discontinuous shortest path change and a speed change event

A qualitative value '0' for the first and/or second character in a $QTC_{N'}$ relation, induced by a node pass event or a bifurcating shortest path, holding over an interval due to continuous changes in the network, can itself be affected by a discontinuous shortest path change at some moment in time (Delafontaine 2006). As stated in section 5.2, if a qualitative value '0' part of the qualitative set {–, 0, +} is a landmark, separating the qualitative values '−' and a '+', then the '0' value can hold instantaneously or over a closed interval, when subject to continuous change, while the '−' and '+' value holds over an open interval. On the other hand, as stated in section 6.3 discontinuous changes occur at an exact moment in time, meaning that there is no last moment in time before the discontinuous change but there is a first moment the change takes place. Hence, if a qualitative value '0' holds over an interval and at the end of this interval it changes into a qualitative value '−' or '+' due to a discontinuous shortest path change event, this interval will not be closed but open, and the '−' or '+' value holds at the end of the interval. Using the above stated definition on dominance (see 5.2) this implies that in contrast to continuous changes the '0' value is dominated by a '−' or '+' value. A direct consequence is that a change from a '0' value into a '−' or a '+' value for the first or second character in a $QTC_{N'}$ relation, caused by a discontinuous shortest path change event on a relation caused by a bifurcating shortest path or a node pass event holding over an interval, can coincide with a change from a qualitative value '−' or a '+' into a qualitative value '0' for the third character in a $QTC_{N'}$ relation representing the relative speed. An example of such a transition is given in Figure 6.16. All possible transitions caused by the combination of these two events are visualised in Figure 6.17. Once again the transitions in Figure 6.17 are directed due to the restrictions stated in 6.3 on the combination of a discontinuous shortest path change event and a speed change event.

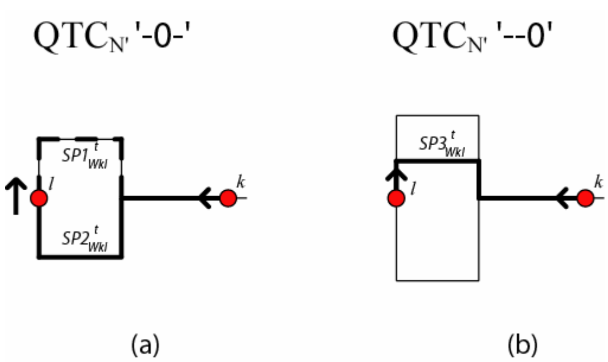


Figure 6.16 A transition in $QTC_{N''}$ relation caused by a discontinuous shortest path in combination with a speed change event

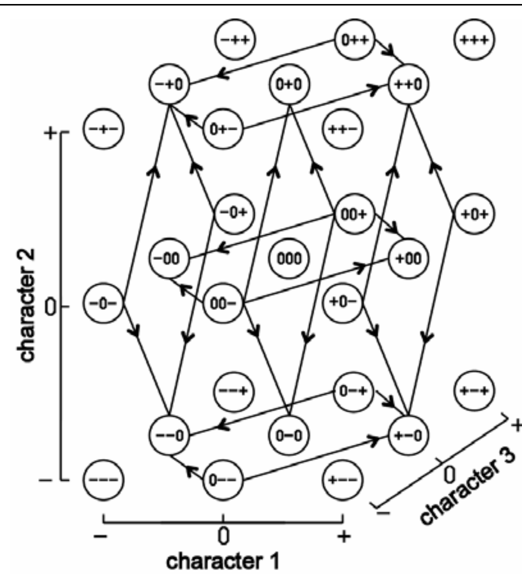


Figure 6.17 Visualisation of transitions in $QTC_{N''}$ caused by a discontinuous shortest path in combination with a speed change event

Analogously, when a relation caused by a bifurcating shortest path or a node pass event holding over an interval, is affected by a discontinuous shortest path change, this change can coincide with a node pass event (Figure 6.18a), a shortest path change event or a discontinuous shortest path change (Figure 6.19a). These changes can additionally happen simultaneously with a speed change event (Figure 6.18b, Figure 6.19b). All possible transitions caused by the combination of such events in $QTC_{N''}$ are visualised respectively in Figure 6.20, Figure 6.21 and Figure 6.22. Once more some transitions are directed due to the restrictions stated in 6.3 on the combination of a discontinuous shortest path change event and a speed change event.

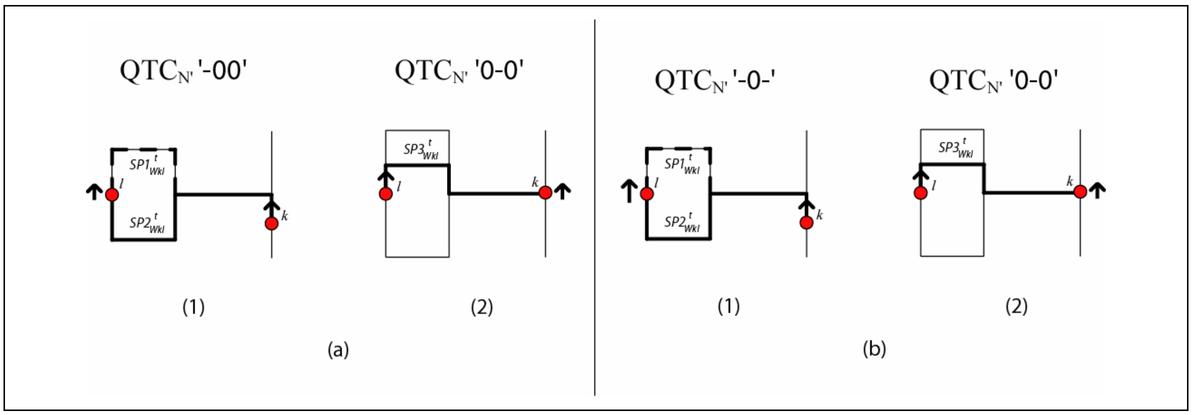


Figure 6.18 A transition in $QTC_{N''}$ relation caused by a discontinuous shortest path in combination with (a) a node pass event or a shortest path change event and (b) a combination of a node pass event or a shortest path change event and a speed change event

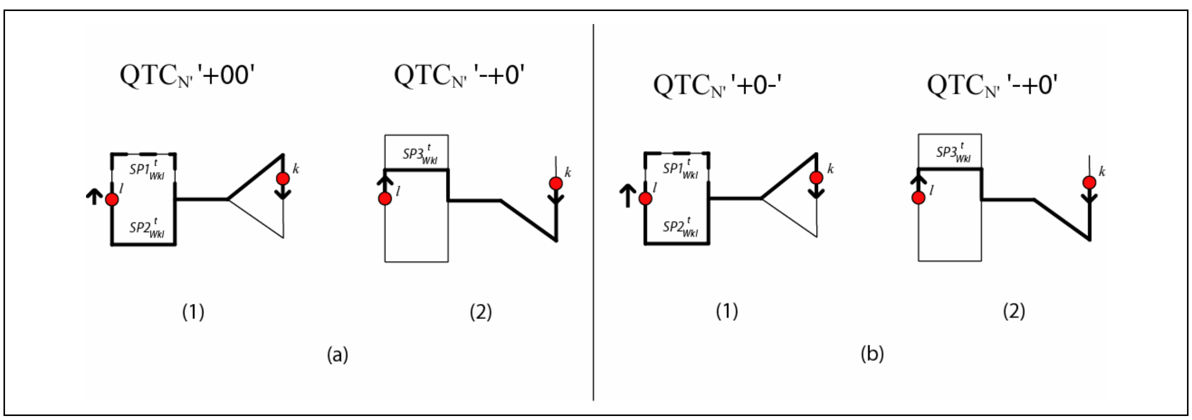


Figure 6.19 A transition in $QTC_{N''}$ relation caused by a discontinuous shortest path in combination with (a) a discontinuous shortest path change and (b) a combination of a discontinuous shortest path change and a speed change event

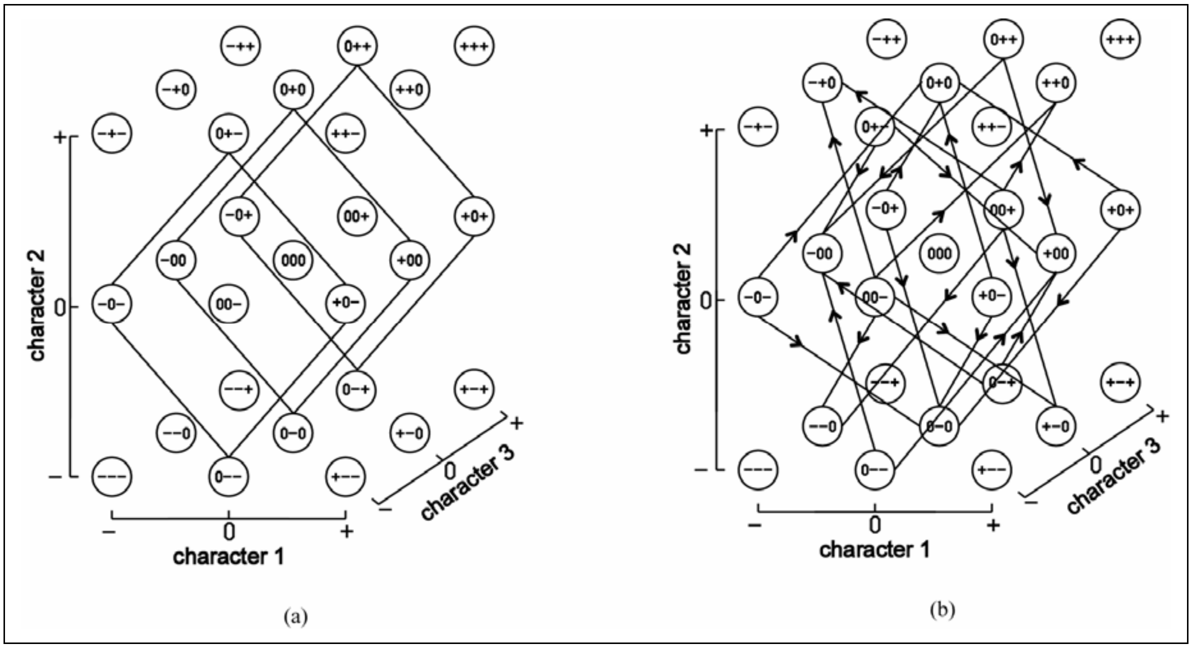


Figure 6.20 Visualisation of transitions in $QTC_{N''}$ caused by a discontinuous shortest path in combination with (a) a node pass event or a shortest path change event and (b) a combination of a node pass event or a shortest path change event and a speed change event

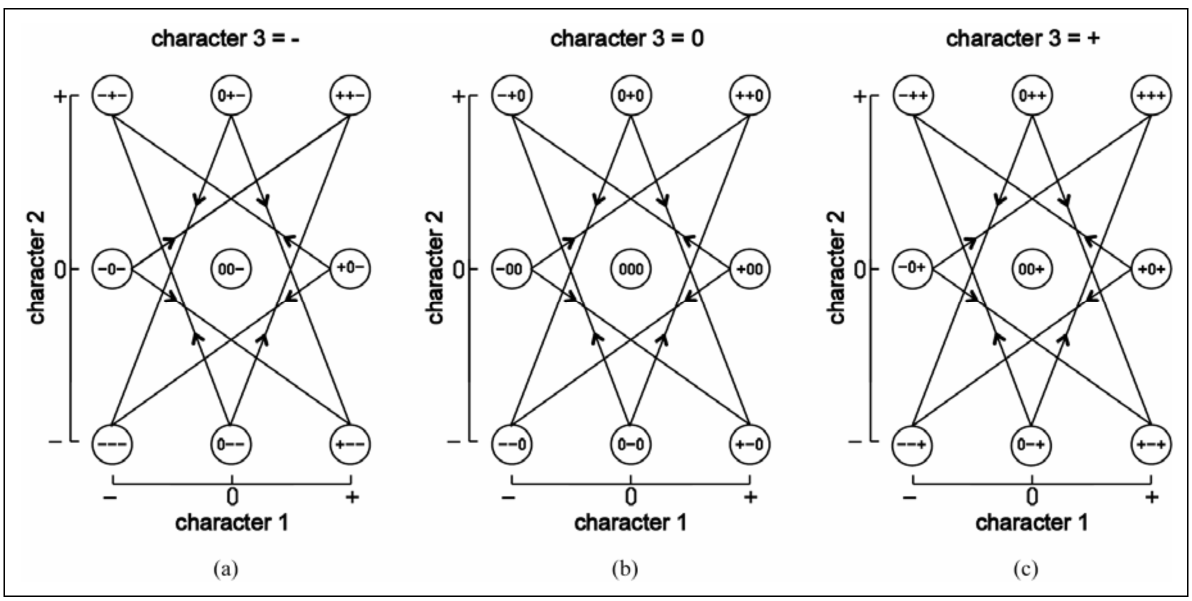


Figure 6.21 Visualisation of transitions in $QTC_{N''}$ caused by a discontinuous shortest path in combination with a discontinuous shortest path change

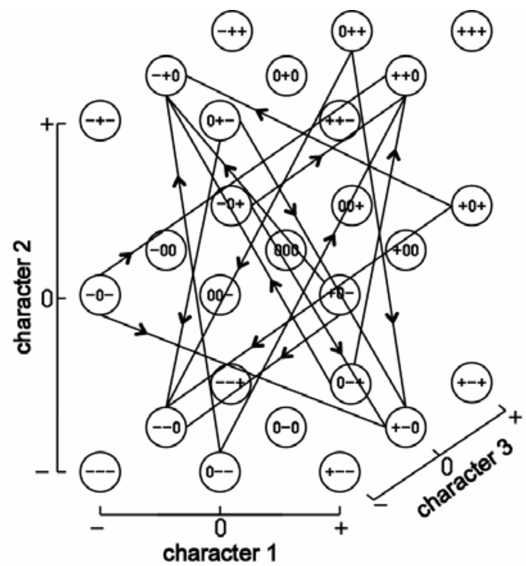


Figure 6.22 Visualisation of transitions in $QTC_{N'}$ caused by a discontinuous shortest path in combination with a combination of a discontinuous shortest path change and a speed change event

6.5 An Overview

Figure 6.23 gives a schematic overview of different sets containing transitions in $QTC_{N'}$.

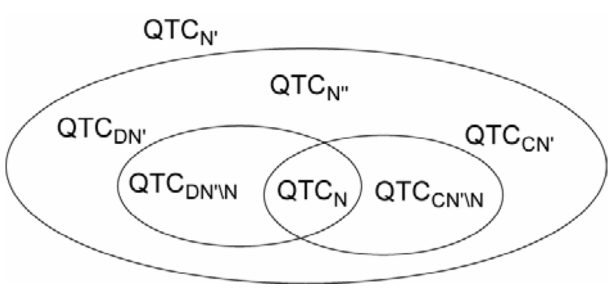


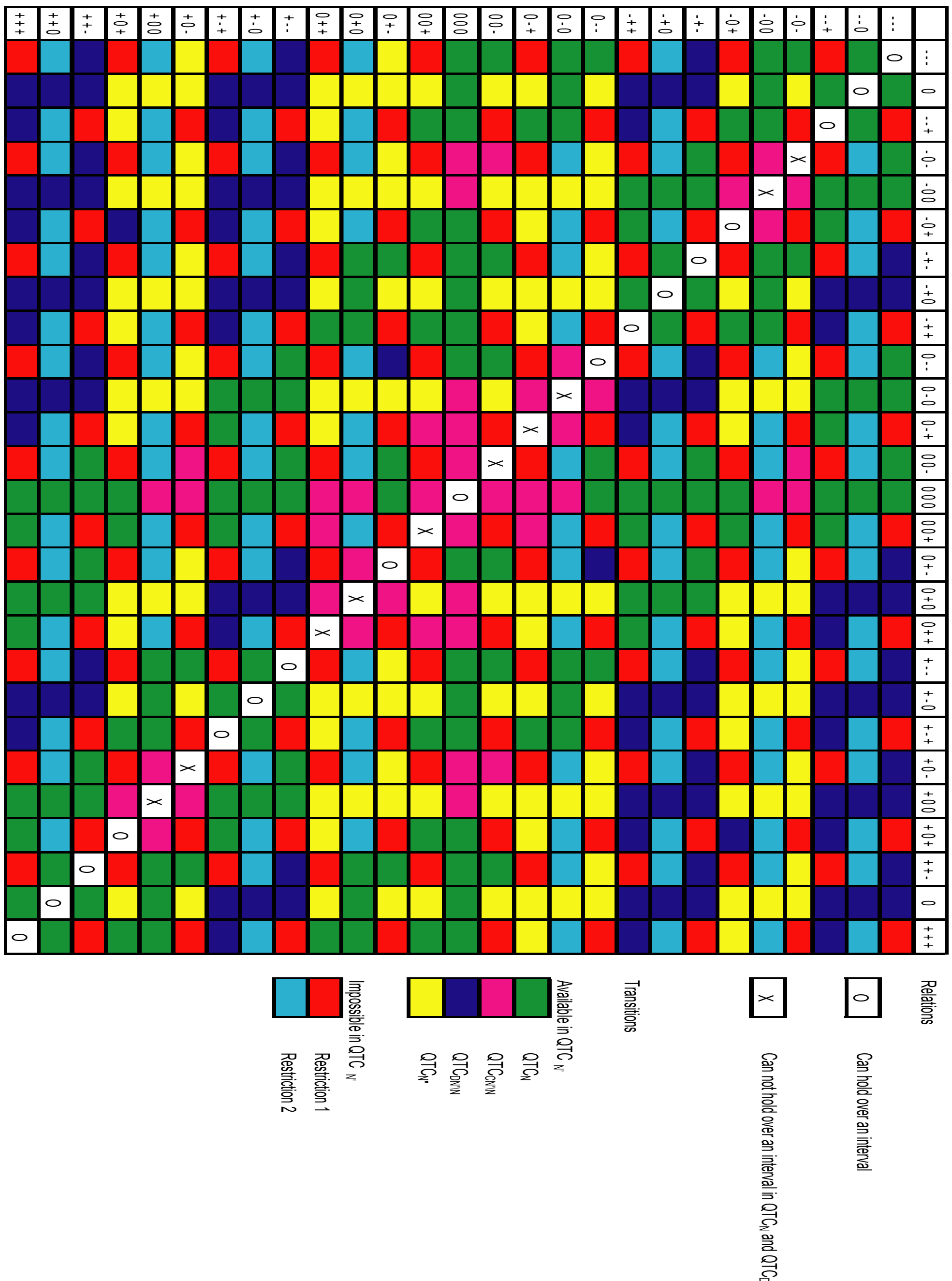
Figure 6.23 Schematic overview of the different sets containing transitions in $QTC_{N'}$

The different transitions available in each set are shown in Table 6.1. There are two restrictions leading to impossible transitions in $QTC_{N'}$ (Delafontaine 2006).

1. The relative speed can only change continuously, meaning that a change from a qualitative value '−' into '+' or vice versa always needs to pass the intermediate qualitative value '0'
2. A change for one or both of the first two characters in a $QTC_{N'}$ relation from a qualitative value '+' or '−' into a '−', '0' or a '+' can never coincide with a change for the third character representing the relative speed from a qualitative

value ‘0’ into a ‘+’ or ‘–’, because the qualitative values ‘+’ or ‘–’ representing the movement of the objects always hold over a open or half open (ending open) interval while the qualitative value ‘0’ representing the relative speed always holds over a closed interval.

Table 6.1 An overview of the transitions between relations in $QTC_{N'}$



Chapter 7 Transforming QTC_N into a Relative Calculus

7.1 Transforming QTC_N into the Relative Trajectory Calculus on Networks (RTC_N)

Having defined the QTC_N relation between two moving objects, a set of trivial qualitative questions can be answered. For example, by looking at the third character of the label, one can identify which object is moving the fastest. Looking at the first two characters of the QTC_N label, queries such as whether an object is moving towards or away from another object can be resolved. In addition to these trivial questions, QTC_N has the power to answer additional questions using the information contained by all three characters in the label. This information can be obtained by transforming QTC relations into relations defined by the Relative Trajectory Calculus (Van de Weghe 2004).

In contrast to QTC, which computes distances between objects at different times (e.g. computing the distance between object k at time point t_1 and object l at time point t_2), the Relative Trajectory Calculus (RTC) defines relations based on the relative motion of an object k in comparison with an object l at the same moment in time (Van de Weghe 2004).

Definition 7.1 A relation in RTC is defined by a single label. This label represents the relation between two point objects (k and l) by comparing the distance between these two objects during the period immediately before the current time point with the distance between these objects during the period immediately after the current time point. This results in three possibilities:

−: the distance between both objects decreases:

$$\exists t_1, t_2 (t_1 \prec t \prec t_2 \wedge \forall t^-, t^+ (t_1 \prec t^- \prec t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t^-, l \mid t^-) > \mathrm{d}(k \mid t^+, l \mid t^+))) \tag{7-1}$$

0: the distance between both objects remains the same:

$$\exists t_1, t_2 (t_1 \prec t \prec t_2 \wedge \forall t^-, t^+ (t_1 \prec t^- \prec t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t^-, l \mid t^-) = \mathrm{d}(k \mid t^+, l \mid t^+))) \tag{7-2}$$

+: the distance between both objects increases:

$$\exists t_1, t_2 (t_1 \prec t \prec t_2 \wedge \forall t^-, t^+ (t_1 \prec t^- \prec t \prec t^+ \prec t_2 \rightarrow \mathrm{d}(k \mid t^-, l \mid t^-) < \mathrm{d}(k \mid t^+, l \mid t^+))) \tag{7-3}$$

RTC_N examines RTC relations on networks. As has been stated by Van de Weghe (2004), a direct mapping between QTC_{B12} and RTC exists, for objects moving in one dimension. On the other hand, there is no direct mapping between QTC_{B22} and RTC, for objects able to move in a two-dimensional space. In what follows, it will be shown that every QTC_N relation can be mapped onto an RTC_N relation. This allows QTC_N questions such as whether two objects are getting closer to each other or whether they are getting further away from each other to be answered easily.

Let us first consider the relations in which none of the objects are involved in a shortest path omitting node pass event and in which there is no bifurcating shortest path between the two objects, *k* and *l*. In this situation, all shortest paths between two objects, involved in a QTC_N relation at *t*, have a simple linear structure with no junctions. Thus, they can be considered to have a movement in one dimension, allowing us to state the following equations:

- a label '−' in the first character of a QTC_N relation label leads to:

$$\begin{aligned} &\mathrm{d}(k \mid t^+, x) + \mathrm{d}(k \mid t, k \mid t^+) = \mathrm{d}(k \mid t, x) \\ &\mathrm{d}(k \mid t^-, x) - \mathrm{d}(k \mid t, k \mid t^-) = \mathrm{d}(k \mid t, x) \end{aligned} \tag{7-4}$$

- a label '−' in the second character of a QTC_N relation label leads to:

$$\begin{aligned} &\mathrm{d}(x, l \mid t^+) + \mathrm{d}(l \mid t, l \mid t^+) = \mathrm{d}(x, l \mid t) \\ &\mathrm{d}(x, l \mid t^-) - \mathrm{d}(l \mid t, l \mid t^-) = \mathrm{d}(x, l \mid t) \end{aligned} \tag{7-5}$$

- a label '+' in the first character of a QTC_N relation label leads to:

$$\begin{aligned} &\mathrm{d}(x, l \mid t^+) - \mathrm{d}(l \mid t, l \mid t^+) = \mathrm{d}(x, l \mid t) \\ &\mathrm{d}(x, l \mid t^-) + \mathrm{d}(l \mid t, l \mid t^-) = \mathrm{d}(x, l \mid t) \end{aligned} \tag{7-6}$$

- a label '+' in the second character of a QTC_N relation label leads to:

$$\begin{aligned} &\mathrm{d}(x,l \mid t^{+}) - \mathrm{d}(l \mid t, l \mid t^{+}) = \mathrm{d}(x,l \mid t) \\ &\mathrm{d}(x,l \mid t^{-}) + \mathrm{d}(l \mid t, l \mid t^{-}) = \mathrm{d}(x,l \mid t) \end{aligned} \tag{7-7}$$

- regardless of the label of the QTC_N relation it can be stated that:

$$\begin{aligned} &\mathrm{d}(k \mid t^{+}, k \mid t^{-}) = \mathrm{d}(k \mid t, k \mid t^{+}) + \mathrm{d}(k \mid t, k \mid t^{-}) \\ &\mathrm{d}(l \mid t^{+}, l \mid t^{-}) = \mathrm{d}(l \mid t, l \mid t^{+}) + \mathrm{d}(l \mid t, l \mid t^{-}) \end{aligned} \tag{7-8}$$

Theorem 7.1: A QTC_N relation '− − −' can be transformed into an RTC_N relation '−'

Proof: By definition, the first two characters in the QTC_N relation '− − −' stand for:

$$\mathrm{d}(k \mid t^{-}, l \mid t) > \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t^{+}, l \mid t) \tag{7-9}$$

$$\mathrm{d}(k \mid t, l \mid t^{-}) > \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t, l \mid t^{+}) \tag{7-10}$$

From (7-9) and (7-10) follows that:

$$\mathrm{d}(k \mid t^{-}, l \mid t) > \mathrm{d}(k \mid t, l \mid t^{+}) \tag{7-11}$$

$$\leftrightarrow \mathrm{d}(k \mid t^{-}, l \mid t) + (l \mid t, l \mid t^{-}) > \mathrm{d}(k \mid t, l \mid t^{+}) - \mathrm{d}(k \mid t, k \mid t^{+}) \tag{7-12}$$

$$\rightarrow \mathrm{d}(k \mid t^{-}, l \mid t^{-}) > \mathrm{d}(k \mid t^{+}, l \mid t^{+}) \tag{7-13}$$

Which is by definition equal to the RTC_N relation '−';

Analogously, it can be proven that QTC_N relations {'− − 0', '− − +', '− 0 +', '0 − −'} can be converted into an RTC_N relation '−'.

Theorem 7.2: A QTC_N relation '+ + +' can be transformed into an RTC_N relation '+'

Proof: By definition, the first two characters in the QTC_N relation '+ + +' stand for:

$$\mathrm{d}(k \mid t^{+}, l \mid t) > \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t^{-}, l \mid t) \tag{7-14}$$

$$\mathrm{d}(k \mid t, l \mid t^{+}) > \mathrm{d}(k \mid t, l \mid t) > \mathrm{d}(k \mid t, l \mid t^{-}) \tag{7-15}$$

From (7-14) and (7-15) follows that:

$$\mathrm{d}(k \mid t^{+}, l \mid t) > \mathrm{d}(k \mid t, l \mid t^{-}) \tag{7-16}$$

$$\leftrightarrow \mathrm{d}(k \mid t^{+}, l \mid t) + \mathrm{d}(l \mid t, l \mid t^{+}) > \mathrm{d}(k \mid t, l \mid t^{-}) - \mathrm{d}(k \mid t, k \mid t^{-}) \tag{7-17}$$

$$\rightarrow \mathrm{d}(k \mid t^{+}, l \mid t^{+}) > \mathrm{d}(k \mid t^{-}, l \mid t^{-}) \tag{7-18}$$

Which is by definition equal to the RTC_N relation '+';

Analogously, it can be proven that QTC_N relations {'+ + 0', '+ + −', '+ 0 +', '0 + −'} can be converted into an RTC_N relation '+'.

Theorem 7.3: A QTC_N relation '− + −' can be transformed into an RTC_N relation '+'

Proof: By definition, the third character in the QTC_N relation '− + −' stands for:

$$v_k < v_l \tag{7-19}$$

$$\leftrightarrow \frac{\partial x_k}{\partial t} < \frac{\partial x_l}{\partial t} \tag{7-20}$$

$$\leftrightarrow \frac{\mathrm{d}(k \mid t^-, k \mid t^+)}{\partial t} < \frac{\mathrm{d}(l \mid t^-, l \mid t^+)}{\partial t} \tag{7-21}$$

$$\rightarrow \mathrm{d}(k \mid t^-, k \mid t) + \mathrm{d}(k \mid t, k \mid t^+) < \mathrm{d}(l \mid t^-, l \mid t) + \mathrm{d}(l \mid t, l \mid t^+) \tag{7-22}$$

$$\begin{aligned} \leftrightarrow\ & \mathrm{d}(k \mid t^-, k \mid t) + \mathrm{d}(k \mid t, l \mid t) + \mathrm{d}(k \mid t, k \mid t^+) \\ & < \mathrm{d}(l \mid t^-, l \mid t) + \mathrm{d}(k \mid t, l \mid t) + \mathrm{d}(l \mid t, l \mid t^+) \end{aligned} \tag{7-23}$$

$$\begin{aligned} \leftrightarrow\ & \mathrm{d}(k \mid t^-, k \mid t) + \mathrm{d}(k \mid t, l \mid t) - \mathrm{d}(l \mid t^-, l \mid t) \\ & < \mathrm{d}(l \mid t^+, l \mid t) + \mathrm{d}(k \mid t, l \mid t) - \mathrm{d}(k \mid t, k \mid t^+) \end{aligned} \tag{7-24}$$

$$\rightarrow \mathrm{d}(k \mid t^-, l \mid t^-) < \mathrm{d}(k \mid t^+, l \mid t^+) \tag{7-25}$$

Which is by definition equal to the RTC_N relation '+';

Analogously, it can be proven that the QTC_N relation '− + +' can be converted into an RTC_N relation '+', QTC_N relations {'+ − −', '+ − +'} can be converted into an RTC_N relation '−', and QTC_N relations {'− + 0', '+ − 0', '0 0 0'} can be converted into an RTC_N relation '0'.

Note that the reasoning above is not valid for relations in which at least one of the objects is involved in a shortest path omitting node pass event or when there is a bifurcating shortest path between two objects. In these cases, all shortest paths between two objects, involved in a QTC_N relation at t, cannot be described by a simple line. Therefore, equations (7-4) to (7-8) are not valid. Based on restrictions imposed by continuity, it can be shown that, in these cases, there is also a unique transformation from a QTC_N relation into a single RTC_N relation. Consider the qualitative distinction between '−', '0' and '+', then a variable capable of assuming any of these three descriptions may change between them. However, a direct change from '−' to '+' and vice versa is impossible, since such a change must always pass the qualitative value '0' (Galton 1995a). Consider the case in Figure 7.1. In Figure 7.1a there is a QTC_N relation '− 0 +'. As has been proven above, this relation can be transformed into an RTC$_N$ relation '–'. In Figure 7.1c, there is a QTC$_N$ relation '+ 0 +'. Again, as proven above, this relation can be transformed into an RTC$_N$ relation '+'. Using the above stated restrictions imposed by continuity, the QTC$_N$ relation '0 0 +' in Figure 7.1b must be an RTC$_N$ relation '0'.

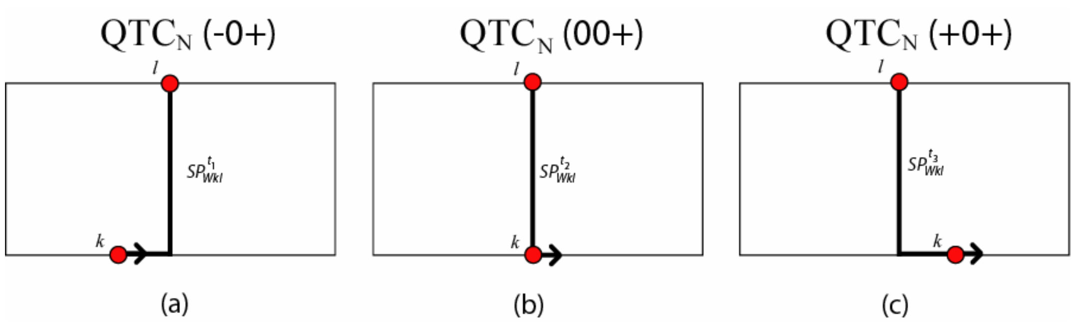


Figure 7.1 A transition between three different QTC$_N$ relations.

A similar transformation can be applied for all QTC$_N$ relations that are in a shortest path omitting node pass event or when there is a bidirectional shortest path between two objects. Table 7.1gives an overview of the transformations from each canonical case of a QTC$_N$ relation into the respective RTC$_N$ relation. A '0n' denotes that a '0' label is due to a shortest path omitting node pass event. A '0b' denotes that a '0' label is due to the existence of a bifurcating shortest path between the objects. A '0s' denotes a '0' label due to the fact that an object is stationary on the network. The cells in black in the RTC$_N$ label column indicate that the corresponding QTC$_N$ relation does not physically occur.

Table 7.1 Overview of transformations from each canonical case in QTC$_N$ into RTC$_N$ relations

QTC$_N$-label		RTC$_N$-label	QTC$_N$-label		RTC$_N$-label	QTC$_N$-label		RTC$_N$-label
– – –	⇒	–	0s 0s 0	⇒	0	0n 0n +	⇒	0
– – 0	⇒	–	0s 0s +	⇒		0s + –	⇒	+
– – +	⇒	–	0b 0s –	⇒		0s + 0	⇒	
– 0s –	⇒		0b 0s 0	⇒		0s + +	⇒	
– 0s 0	⇒		0b 0s +	⇒	0	0b + –	⇒	+
– 0s +	⇒	–	0n 0s –	⇒		0b + 0	⇒	0
– 0b –	⇒	0	0n 0s 0	⇒		0b + +	⇒	0
– 0b 0	⇒	0	0n 0s +	⇒	0	0n + –	⇒	+
– 0b +	⇒	–	0s 0b –	⇒	0	0n + 0	⇒	0
– 0n –	⇒	0	0s 0b 0	⇒		0n + +	⇒	0

− 0n 0	⇒	0	0s 0b +	⇒		+ − −	⇒	−
− 0n +	⇒	−	0b 0b −	⇒	0	+ − 0	⇒	0
− + −	⇒	+	0b 0b 0	⇒	0	+ − +	⇒	+
− + 0	⇒	0	0b 0b +	⇒	0	+ 0s −	⇒	
− + +	⇒	−	0n 0b −	⇒	0	+ 0s 0	⇒	
0s − −	⇒	−	0n 0b 0	⇒	0	+ 0s +	⇒	+
0s − 0	⇒		0n 0b +	⇒	0	+ 0b −	⇒	0
0s − +	⇒		0s 0n −	⇒	0	+ 0b 0	⇒	0
0b − −	⇒	−	0s 0n 0	⇒		+ 0b +	⇒	+
0b − 0	⇒	0	0s 0n +	⇒		+ 0n −	⇒	0
0b − +	⇒	0	0b 0n −	⇒	0	+ 0n 0	⇒	0
0n − −	⇒	−	0b 0n 0	⇒	0	+ 0n +	⇒	+
0n − 0	⇒	0	0b 0n +	⇒	0	+ + −	⇒	+
0n − +	⇒	0	0n 0n −	⇒	0	+ + +	⇒	+
0s 0s −	⇒		0n 0n 0	⇒	0	+ + +	⇒	+

Table 7.1 clearly shows that a label ‘0s’, a label ‘0n’ or a label ‘0b’ does not influence transformation from a QTC_N relation into an RTC_N relation. Therefore, Table 7.1 can be compressed into Table 7.2.

Table 7.2 Overview of transformations from QTC_N into RTC_N relations

QTC_N-label		RTC_N-label	QTC_N-label		RTC_N-label	QTC_N-label		RTC_N-label
− − −	⇒	−	0 − −	⇒	−	+ − −	⇒	−
− − 0	⇒	−	0 − 0	⇒	0	+ − 0	⇒	0
− − +	⇒	−	0 − +	⇒	0	+ − +	⇒	+
− 0 −	⇒	0	0 0 −	⇒	0	+ 0 −	⇒	0
− 0 0	⇒	0	0 0 0	⇒	0	+ 0 0	⇒	0
− 0 +	⇒	−	0 0 +	⇒	0	+ 0 +	⇒	+
− + −	⇒	+	0 + −	⇒	+	+ + −	⇒	+
− + 0	⇒	0	0 + 0	⇒	0	+ + +	⇒	+
− + +	⇒	−	0 + +	⇒	0	+ + +	⇒	+

Thus, each canonical case of movements defined in QTC_N can be transformed into exactly one of the three RTC_N relations. This is notable since, as stated above, for objects having a free trajectory in $\mathbb{R}^2$, this is not the case (Van de Weghe 2004). This non-unique transformation is illustrated by the example given in Figure 7.2. Knowing that the dotted line has a constant length, the figure clearly shows that the QTC_{B22} relation '− + 0' can be transformed into all possible RTC_{B22} relations.

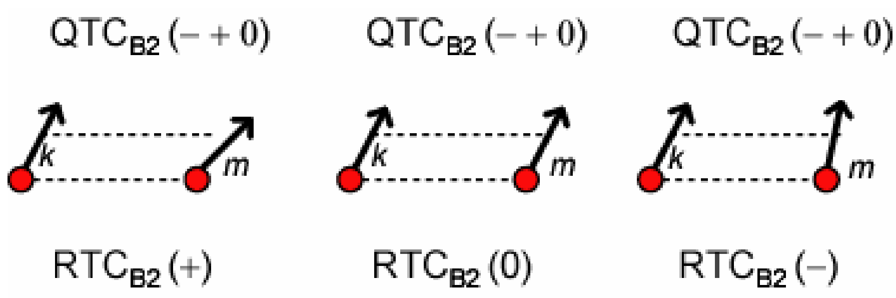


Figure 7.2 Examples of transformations from QTC_{B22} relations into RTC_{B22} relations

7.2 An Example Application

An application in which QTC_N can be useful is in collision avoidance systems. If one wants to know if two objects are going to collide, then, as a first step, it is only interesting to examine the objects which might meet. In other words, only the objects which are getting closer to each other (objects in an RTC_N relation '−') are relevant, because objects not getting closer to each other (objects in an RTC_N relation '0' or '+') can not collide. Thus, QTC_N relations eliminate many movements from further examination, greatly reducing calculation times. Further examining the QTC_N relation between two objects gives information on the type of collision. QTC_N relations which are part of the set {'− + +', '+ − −'} indicate a "rear-end collision", QTC_N relations part of the set {'− − −', '− − 0', '− − +'} indicate "head-on" collision and QTC_N relation part of the set {'− 0 +', '0 − −'} could indicate a collision with a stationary object. Note that these QTC_N relations only indicate a potential collision; this indication does not necessarily lead to a collision. Related work on collision avoidance has, on the one hand, focused on detecting possible collision between objects which have a completely free trajectory in a two-dimensional space (Dylla et al. 2007; Gottfried 2005; Schlieder 1995). These approaches mainly focus on the direction of movement. Although they have all shown their usefulness when the movement of objects is not constrained, directional methods can not directly be transformed to networks, since they do not take into account the spatial structure of a network. The movement in Figure 7.3a, for example, would indicate a possible collision in all the above mentioned directional approaches, while in QTC_N it is clear that the objects move away from each other and therefore cannot collide. Furthermore, none of the methods above incorporate the relative speed between two moving objects. However, the notion of relative speed is important for detecting possible collision in cases in which the objects move in the same direction. Consider the movement in Figure 7.3b. Using only directional information, this movement would indicate a possible collision, but since *l* is moving faster than *k*, the distance between these objects grows, and, by consequence, there is no danger of a collision. For these two reasons they over-predict possible collisions, while QTC_N does not.

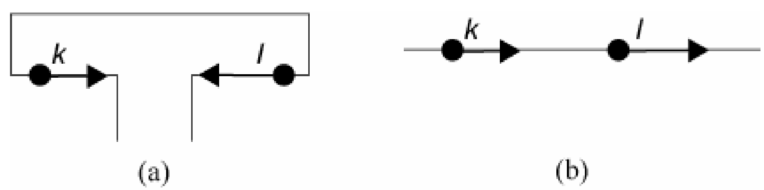


Figure 7.3 Two scenes of two moving objects in which there is no possibility of collision

On the other hand, techniques for collision avoidance when objects have a constrained trajectory mainly focus on train networks. Collisions in these systems are avoided by not allowing two trains to evolve on the same track segment (Hansen 1998; Haxthausen and Peleska 2000). First of all, this method also over-predicts possible collisions, since two trains can run on the same track without colliding (i.e. when at least one train is moving away from the other and its speed is equal to or greater than the speed of the other train, which is moving towards the former (Figure 7.3b)). Secondly, this constraint does not capture every possible collision situation. If two trains are at different segments, they can still be close and move towards each other. Hence, not all possible collisions can be predicted in real time collision avoidance systems using only this constraint (especially for objects colliding at the intersection of two edges).

Chapter 8
Linguistic and Cognitive Aspects of QTC_{B12}

8.1 Introduction

As stated in Chapter 2, one of the reasons to conduct research in qualitative representation and reasoning, is the fact that human beings are more likely to communicate in qualitative categories, supporting their intuition, rather than using quantitative measures (Freksa 1992b). This implies that QR is, among other, very well suited for use in the domain of Human Computer Interaction (HCI) (Schultz et al. 2006). On the one hand, specific information available in information systems (such as GISs), can be communicated back to the user by transforming that information into functional language (Egenhofer and Shariff 1998). As stated by Schultz et al. (2006, p.43): "*People find numerical methods non-intuitive, for example, statements such as 'The café is at latitude 23 minutes, 8 degrees, and longitude.' (using attribute data) or 'The café is within 46m of the art gallery, and intersects Symonds St' (using spatial data), are far less natural than 'The café is opposite the art gallery on Symonds St'*". In this way, QR can be used to overcome information overload. Information overload occurs whenever more information has to be handled than can be used efficiently (O'Reilly 1980). For example, it is easier to communicate a certain slope characteristic of a region (e.g. flat, steep, and accidental) than to provide over a thousand height points (Donlon and Forbus 1999).

The other way around, qualitative information given by text or speech can be stored in information systems for further processing, analysis or to infer additional knowledge (Frank 1996). Note that expressions in natural language often cause a certain level of uncertainty (e.g. the library is located in the centre of the town; he is moving towards the cinema) (Guesgen and Albrecht 2000). They usually do not provide enough information to identify the exact geographical location of an object or event (Kalashnikov et al. 2006). However, as stated in Chapter 2, although reasoning with qualitative information can sometimes lead to a partial answer, this answer is often better than having no answer at all (Freksa 1992a).

These ideas completely fit within the scope of Naive Geography. The theory of Naive Geography originates from the landmark paper of Egenhofer and Mark (Egenhofer and Mark 1995b) and is based on Hayes' ideas on Naive Physics (Hayes 1978): *"Naive Geography is the body of knowledge that people have about the surrounding geographic world. Naive Geography captures and reflects the way people think and reason about geographic space and time, both consciously and subconsciously. Naive stands for instinctive or spontaneous"* (Egenhofer and Mark 1995b, p.4). The authors argue that Naive Geography comprises a set of theories upon which next generation GISs can be built or as Renz *et al.* wrote (2000, p.184): *"... new approaches to GIS try to come closer to the way spatial information is communicated by natural language and, thus, to the way human cognition is considered to represent spatial information , …"*. Much of Naive Geography should employ qualitative reasoning methods (Egenhofer and Mark 1995b).

As can be deducted from Chapter 2, over the last few decades a variety of qualitative calculi have been developed in the domain of temporal, spatial and spatiotemporal reasoning. Most of these works focus on the formalization and usefulness of the calculus and concentrate on well-known reasoning techniques like composition tables and conceptual neighbourhood diagrams. However, very little attention has been paid to the cognitive and linguistic adequacy of these qualitative calculi (Cohn and Hazarika 2001). This adequacy is mostly based on the intuition of researchers rather than on empirical data (Renz et al. 2000). Nonetheless, if qualitative calculi are to be used in terms of Naive Geography e.g. as a means to overcome information overload or in the domain of HCI, empirical evidence is mandatory in order to express usefulness or strength of a qualitative calculus in these domains. Conversely, from the domain of linguistics and cognitive linguistics, there is a substantial amount of literature dealing with the link between language and space (e.g. Byrne and Johnson-Laird 1989; Landau and Jackendoff 1993; Levinson 2003; Talmy 2000; Tversky and Lee 1998). But once again, the link with spatial calculi is most of the times absent.

Therefore, in this Chapter, the first steps to reveal the cognitive and linguistic semantics of QTC_B are set. The focus is on QTC_B, since this calculus is (intuitively) assumed to describe the prepositions *'towards'* and *'away from'*. At this initial stage, the empirical tests are limited movements defined in QTC_{B12}, thus, objects which have a constrained linear trajectory. For the remainder of this Chapter, first of all, a brief overview of different descriptions of *'towards'* and *'away from'* in the (cognitive) linguistic literature will be given (section 8.2). In section 8.3, three research questions are stated and the basic experiments to tackle them are described. The results of these tests are given in section 8.4 and 8.5, leading to a discussion in section 8.6.

8.2 'Towards' and 'away from' as Described in (Cognitive) Linguistics

In linguistics, the prepositions *'towards'* and *'away from'* are discussed in terms of paths (Eschenbach et al. 2000; Jackendoff 1990). A path is the equivalent of a trajectory along which an object moves. It can geometrically be represented as a directed curve with a starting point, an end-point and points in between, on which the path imposes an order (Zwarts 2005). Frequently, *'towards'* and *'away from'* are considered in combination with three other prepositions: *'to'*, *'from'* and *'via'*. According to Jackendoff (1983), the prepositions *'to'* and *'from'* are paths which end or start at a reference object respectively. In other words, a movement *'to the market'* indicates that a trajectory ends at the market, while a movement *'from the market'* denotes a trajectory that leaves the market, and started there (Figure 8.1). The prepositions *'towards'* and *'away from'* are described as progressive and a successive part of a trajectory specifying '*to*' or '*from*' path respectively (Figure 8.1) or as stated by Jackendoff (1983, p.165): *"the reference object does not fall on the path, but would if the path were extended by some unspecified distance"*. Put differently, *'towards'* and *'away from'* do not end or start at the object of reference. '*Via*', in its turn, describes a path which passes the reference object (Figure 8.1). For example, a movement via the library means that the trajectory passes the library somewhere in between the start and the end of the movement. Consequently, *'to'* is often labelled as a goal preposition, *'from'* as a source preposition, and *'via'* as a course (intermediate place) preposition (Eschenbach et al. 2000).

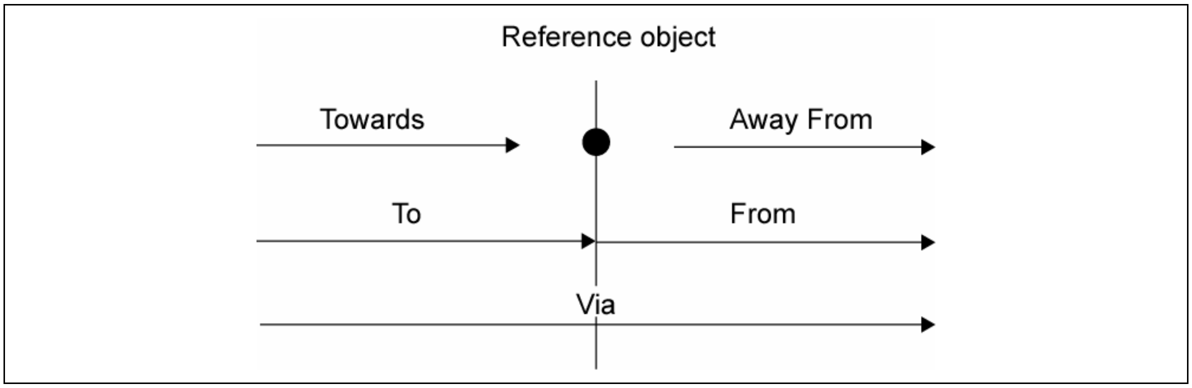


Figure 8.1 The prepositions *'towards'*, *'to'*, *'via'*, *'from'*, *'away from'* as descibed by Jackendoff (1983)

A distance based definition, given by Nam (2000), identifies a movement *'towards'* a reference object as a path of which the end-point is nearer to the reference object than the starting point of the path. Conversely, a movement *'away from'* a reference object ends further away from the reference object than its starting point.

Zwarts (2005) argues that neither of the above definitions are correct. The definition of *'towards'* and *'away from'* as a progressive or successive part of *'to'* and *'from'* seems to work fine when dealing with moving objects constrained by a linear trajectory, but not when objects can move freely in the plane, for instance, parts of the *'to'* movement in Figure 8.2a, intuitively indicate a movement *'away from'* instead of *'towards'* the reference object. Furthermore, the trajectory in Figure 8.2b naturally leads to a movement *'via'* instead of '*towards'* the reference object, although the end-point of the trajectory is nearer to the reference object than the starting point of the trajectory.

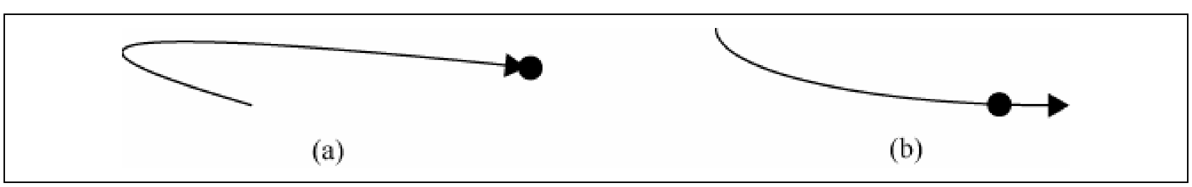


Figure 8.2 Two paths (arrows) in the plane with respect to a reference object (dot) (based on Zwarts (2005, p.765))

In order to correctly define *'towards'* and *'away from'*, Zwarts (2005) suggests to slightly alter Nam's distance based definitions. A movement *'towards'* is defined as a movement in which the distance to the reference objects decreases monotonically. In other words, every consecutive point on the path is nearer to the reference object. Consequently, *'away from'* is defined as a movement in which the distance to the reference object increases monotonically.

All the above mentioned authors agree that *'to'* and *'from'* are telic prepositions and that *'towards'* and *'away from'* are atelic prepositions. A telic movement refers to completed movement (e.g. a movement which ends at the reference object), while atelic movements are said to be incomplete.

8.3 Research Questions and Basic Experiments

Based on the overview of the prepositions *'towards'* and *'away from'* in the (cognitive) linguistic literature, first of all, two research questions are set in this thesis:

1. Does every '−' in the first two characters of a QTC_{B12} label express a movement *'towards'* another object, and the other way around, does every '+' refer to a movement *'away from'* another object?
2. Are there trajectories expressing a *'towards'* movement which end at the reference object and conversely, can trajectories expressed as an *'away from'* movement start at the reference object?

The second research question is related to Jackendoff's definition on *'towards'* and *'away from'* as being a subpath of *'to'* or *'from',* while the first research question arises from Zwarts' definition on *'towards'* and *'away from'*. Intuitively, this definition seems correct when the reference object is stationary (e.g. a house, a classroom, a crossing). The question remains if this definition also applies when the reference object also moves (e.g. a person, a car, an animal). According to Zwarts' definition, only objects which move in the direction of the reference object and decrease their distance with respect to that reference object can be labelled as moving *'towards'* the reference object. In terms of QTC_{B12}, this means that a '−' in one of the first two characters of a QTC_{B12} label only expresses a movement *'towards'* if the QTC_{B12} relation can be transformed in an RTC relation labelled by a '−'. Analogously, a '+' in one of the first two characters of a QTC_{B12} label only expresses a movement *'away from'* if the QTC_{B12} relation can be transformed in an RTC relation labelled by a '+'.

In order to tackle both research questions at the same time, a rating experiment was set up. Participants to the experiment were asked to evaluate on a scale from 1 to 7 (where 1 is 'does apply well', and 7 'does not apply at all') whether *'away from'*, *'towards'*, *'via'*, *'to'*, or *'from'* describes where a red dot moves in relation to a blue dot by means of screen animations. Each of these five prepositions were evaluated for 58 different screen animations, leading to a total of 290 (5x58) animations each participant had to score. These 58 different screen animations are shown in Figure 8.3 and consist of 14 QTC_{B12} animations which, according to Jackendoff's definition, express a movement *'towards'* the blue dot (there are a total of 14 distinct '−' in the first two characters of a QTC_{B12} label), 14 QTC_{B12} animations which express a movement *'away from'* the blue dot (there are a total of 14 distinct '+' in the first two characters of a QTC_{B12} label), twice 10 movements, can lead to a movement expressing a *'to'* or a *'via'* the blue dot (there are 10 '−' in the first two characters of a QTC_{B12} label available for QTC_{B12} relations which can be transformed in a RTC_{B12} relation labelled by a '−', i.e. they can possibly meet or pass each other), and finally, there are 10 animations which can express a movement 'from' the blue dot (there are 10 '+' in the first two characters of a QTC_{B12} label available for QTC_{B12} relations which can be transformed in a RTC_{B12} relation labelled by a '+', i.e. a red object can move away from the blue dot starting at the same location).

In Figure 8.3, the dots respectively represent the starting position of the red and blue dot for all screen animations, except for those listed under *'from'*, for which the dots have to start at the same place. The line segments represent the direction of movement of the objects. These line segments can have different lengths giving the difference in relative speed. An animation classified as *'towards'* stops before the red dot reaches the blue dot, while an animation representing *'to'* stops when blue and red meet, and for the *'via'* animations the dots stop after they have crossed.

In order to make a clear difference between the different categories of screen animations shown in Figure 8.3 and the different prepositions the test persons had to rate, the different animation categories are referred to as *'towards'*, *'to'*, *'via'*, *'from'* and *'away from'* stimuli, while the words people had to rate are referred to as *'towards'*, *'to'*, *'via'*, *'from'* and *'away from'* prepositions.

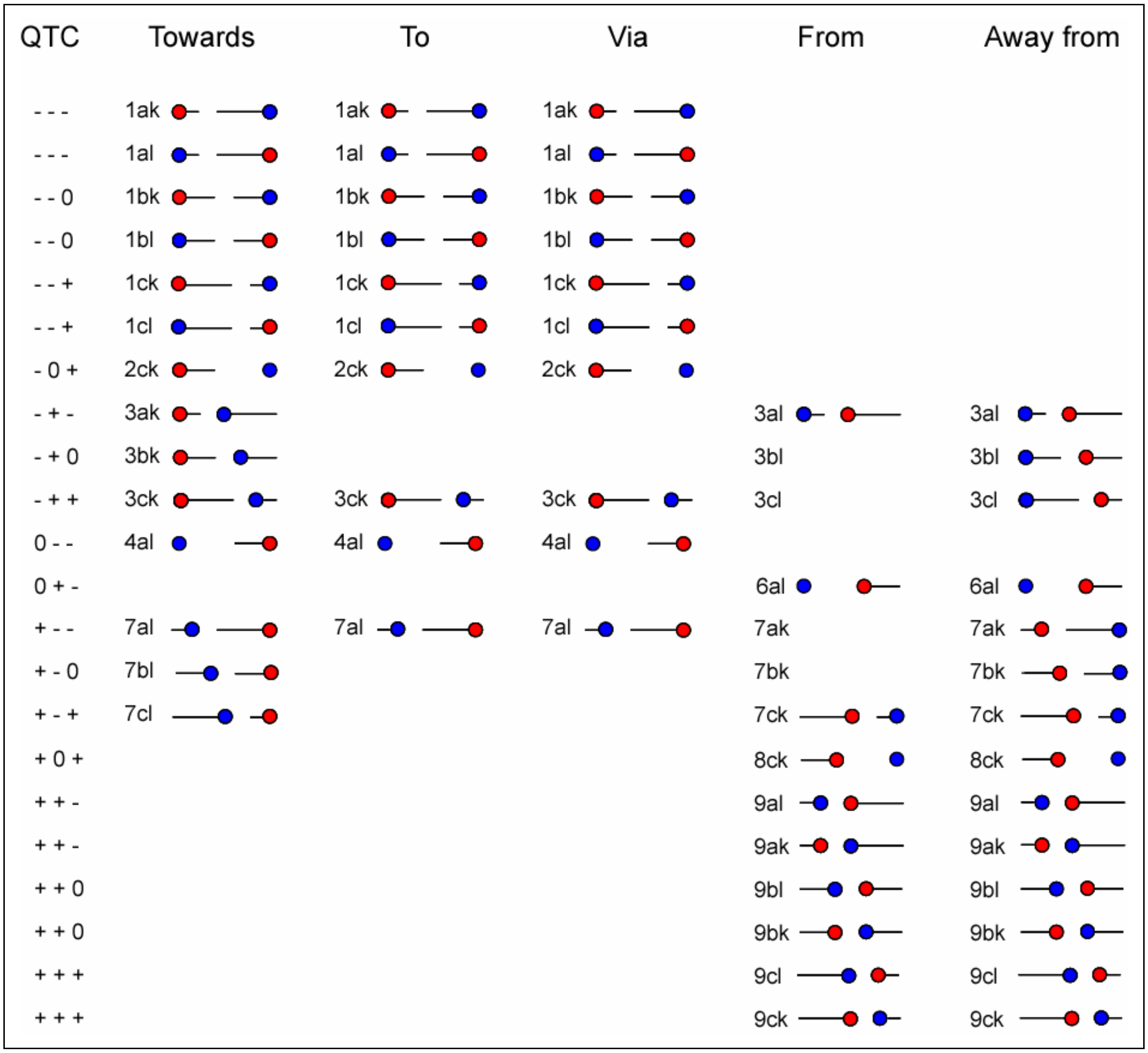


Figure 8.3 58 different screen animations for the rating experiment

People with different cultural and linguistic backgrounds seem to perceive and think very differently about spatial concepts (Montello 1995). As a consequence, Anglo-Saxon concepts (towards, away from, …) are difficult to translate for a community with a different background (Campari 1991). Thus, if QTC_{B12} is to be used as a means to overcome information overload or in the domain of HCI, for other languages than English, additional tests are required. Therefore, in this chapter, a third research question is set:

3. Which Dutch prepositions are equivalent for the English prepositions *'towards'*, *'to'*, *'via'*, *'from'*, *'away from'*, and is the answer to the first two research questions the same for English prepositions as for their Dutch counterparts.

To deal with this research question, first of all, a free response experiment was set up. Native Dutch speakers were asked to describe where the red dot moved in relation to the blue dot for each of the 58 different screen animations given in Figure 8.3. Based on the resulting descriptions, five Dutch prepositions which come closest to Jackendoff's concepts of *'towards'*, *'to'*, *'via'*, *'from'*, *'away from'* will be chosen. For the five Dutch prepositions the same two research questions as for their English counterpart will be examined via the same rating experiment.

8.4 Results for English prepositions

For the rating experiment concerning the prepositions in English, 23 psychology undergraduate students of the University of Lincoln were tested, some of which were rewarded with credits. All participants were English native speakers. Tables 8.1 to 8.5 show the average ratings (and its standard deviations) awarded to each of the prepositions for the different screen animations, subdived by stimulus. The codes in the animations column are equal to the codes given in Figure 8.3.

Table 8.1 Average ratings for the *'towards'* stimuli

Animation/Preposition	Via	Away From	From	To	Towards
1AKTOWARDS	5.68 (1.81)	6.23 (1.63)	5.68 (1.96)	2.64 (1.33)	2.36 (1.99)
1ALTOWARDS	5.55 (1.79)	6.14 (1.67)	5.55 (2.09)	2.82 (1.89)	1.55 (1.14)
1BKTOWARDS	5.45 (1.84)	6.23 (1.66)	5.86 (1.96)	2.50 (1.57)	1.86 (0.94)
1BLTOWARDS	5.18 (1.87)	6.41 (1.62)	5.59 (2.22)	2.36 (1.68)	2.23 (1.85)
1CKTOWARDS	5.73 (1.45)	6.00 (1.90)	5.77 (2.05)	2.45 (1.77)	1.95 (1.40)
1CLTOWARDS	5.95 (1.40)	6.27 (1.64)	6.09 (1.41)	3.00 (2.00)	2.00 (1.51)
2CKTOWARDS	5.14 (1.91)	6.32 (1.62)	5.68 (2.03)	2.36 (1.65)	1.68 (1.21)
3AKTOWARDS	5.82 (1.82)	3.95 (2.40)	4.86 (2.10)	4.41 (1.87)	**4.50 (1.99)**
3BKTOWARDS	5.68 (2.01)	5.41 (2.09)	5.14 (2.05)	3.73 (2.14)	**3.77 (2.27)**
3CKTOWARDS	6.18 (1.62)	5.36 (2.15)	5.27 (2.07)	3.73 (1.78)	2.18 (1.33)
4ALTOWARDS	6.00 (1.45)	6.41 (1.65)	6.09 (1.95)	2.55 (1.65)	1.82 (1.26)
7ALTOWARDS	5.55 (1.95)	5.73 (1.98)	5.32 (2.03)	3.59 (2.11)	2.82 (1.50)
7BLTOWARDS	6.09 (1.34)	4.55 (2.46)	5.64 (2.19)	3.82 (1.87)	**3.27 (1.91)**
7CLTOWARDS	5.41 (2.09)	4.32 (2.30)	5.00 (2.12)	4.09 (2.09)	**3.59 (2.15)**

Table 8.2 Average ratings for the *'to'* stimuli

Animation/Preposition	Via	Away From	From	To	Towards
1AKTO	5.18 (1.84)	6.41 (1.50)	6.23 (1.63)	1.50 (0.80)	1.82 (1.26)
1ALTO	4.82 (1.76)	6.45 (1.63)	6.00 (1.88)	2.14 (2.01)	1.73 (1.64)
1BKTO	5.18 (1.71)	6.36 (1.50)	5.68 (2.08)	1.59 (1.50)	1.91 (1.63)
1BLTO	5.23 (1.77)	6.23 (1.85)	5.86 (1.91)	1.59 (1.37)	1.91 (1.66)
1CKTO	5.36 (1.59)	6.41 (1.62)	5.45 (2.30)	1.91 (2.11)	1.77 (1.77)
1CLTO	4.91 (1.60)	5.91 (2.11)	5.95 (1.81)	1.95 (1.79)	2.23 (2.22)
2CKTO	5.27 (1.80)	6.09 (1.97)	6.05 (1.73)	1.64 (1.50)	1.68 (1.04)
3CKTO	4.27 (1.83)	6.09 (1.72)	5.59 (2.11)	1.86 (1.98)	1.91 (1.60)
4ALTO	5.23 (1.74)	6.23 (1.77)	5.82 (2.06)	1.50 (1.63)	2.05 (1.65)
7ALTO	5.27 (1.83)	5.50 (2.13)	5.73 (1.80)	1.73 (1.20)	2.09 (1.60)

Table 8.3 Average ratings the *'via'* stimuli

Animation/Preposition	Via	Away From	From	To	Towards
1AKVIA	2.45 (2.02)	4.55 (1.87)	4.45 (1.65)	3.55 (1.14)	3.45 (1.87)
1ALVIA	1.91 (1.90)	4.64 (1.71)	4.64 (1.68)	4.41 (1.71)	3.64 (1.59)
1BKVIA	2.05 (1.76)	4.23 (1.82)	4.55 (1.95)	4.00 (1.95)	3.36 (1.76)
1BLVIA	1.86 (1.70)	4.36 (1.99)	4.50 (1.65)	4.05 (2.01)	3.55 (1.47)
1CKVIA	2.18 (1.87)	4.00 (1.63)	3.91 (2.02)	3.59 (1.74)	4.14 (1.70)
1CLVIA	2.18 (1.87)	4.77 (1.63)	3.95 (1.62)	3.82 (1.62)	3.55 (1.53)
2CKVIA	1.77 (1.38)	4.50 (1.71)	4.55 (1.71)	4.27 (1.55)	3.64 (1.81)
3CKVIA	2.00 (1.75)	4.18 (1.87)	4.32 (1.73)	3.82 (1.65)	3.82 (1.84)
4ALVIA	1.82 (1.74)	4.68 (1.76)	4.86 (1.49)	3.68 (1.64)	3.27 (1.61)
7ALVIA	1.91 (1.54)	4.36 (1.56)	4.23 (1.45)	3.64 (1.56)	3.64 (1.53)

Table 8.4 Average ratings the *'from'* stimuli

Animation/Preposition	Via	Away From	From	To	Towards
3ALFROM	4.95 (1.73)	2.45 (1.97)	2.14 (1.55)	5.55 (2.02)	6.00 (1.27)
6ALFROM	4.77 (2.31)	1.95 (1.89)	2.55 (2.13)	5.95 (1.99)	6.05 (1.89)
7CKFROM	4.68 (1.73)	2.18 (1.79)	2.41 (1.92)	5.45 (2.02)	5.50 (2.09)
8CKFROM	4.50 (1.99)	2.27 (2.14)	2.23 (2.18)	5.73 (2.21)	6.14 (1.88)
9AKFROM	5.32 (2.23)	2.05 (1.79)	2.77 (2.18)	6.09 (1.69)	6.14 (1.83)
9ALFROM	6.00 (1.51)	1.95 (1.84)	2.32 (1.91)	6.00 (1.93)	6.45 (1.63)
9BKFROM	5.50 (2.24)	1.68 (1.76)	2.50 (2.04)	6.14 (1.88)	6.50 (1.34)
9BLFROM	5.55 (1.57)	2.09 (2.11)	2.05 (1.84)	5.95 (2.06)	6.23 (1.66)
9CKFROM	5.73 (1.83)	1.50 (1.41)	2.32 (2.01)	5.64 (2.08)	5.95 (2.08)
9CLFROM	5.05 (2.03)	1.95 (1.76)	1.77 (1.27)	6.41 (1.76)	6.18 (1.74)

Table 8.5 Average ratings for the *'away from'* stimuli

Animation/Preposition	Via	Away From	From	To	Towards
3ALAWAY FROM	5.91 (1.63)	2.59 (2.13)	3.50 (2.13)	5.64 (2.06)	5.41 (2.11)
3BLAWAY FROM	5.82 (1.74)	**3.91 (2.00)**	4.50 (2.04)	5.73 (1.67)	5.23 (2.31)
3CLAWAY FROM	5.82 (1.94)	**3.73 (2.16)**	4.50 (1.90)	4.77 (2.22)	4.41 (2.44)
6ALAWAY FROM	6.00 (1.66)	1.86 (1.78)	2.45 (2.15)	6.14 (1.55)	6.36 (1.33)
7AKAWAY FROM	5.91 (1.57)	**3.45 (2.15)**	4.55 (1.95)	4.45 (2.39)	5.00 (2.41)
7BKAWAY FROM	5.50 (2.02)	**3.64 (2.34)**	3.59 (1.82)	5.14 (2.27)	5.14 (2.19)
7CKAWAY FROM	6.09 (1.77)	2.64 (1.79)	2.82 (2.13)	5.41 (2.13)	5.32 (1.94)
8CKAWAY FROM	5.86 (2.01)	2.18 (2.26)	2.91 (2.27)	5.86 (2.05)	6.36 (1.62)
9AKAWAY FROM	6.00 (1.69)	2.27 (2.05)	2.14 (1.39)	5.95 (1.79)	5.95 (1.94)
9ALAWAY FROM	6.09 (1.82)	1.95 (2.08)	3.32 (2.40)	6.27 (1.61)	5.91 (2.11)
9BKAWAY FROM	5.55 (2.15)	2.36 (2.17)	3.00 (2.39)	6.00 (1.88)	6.41 (1.65)
9BLAWAY FROM	5.36 (2.17)	1.73 (1.52)	2.82 (2.08)	6.45 (1.41)	6.45 (1.77)
9CKAWAY FROM	6.14 (1.58)	2.09 (2.04)	3.23 (2.33)	6.00 (1.98)	6.45 (1.63)
9CLAWAY FROM	5.91 (1.87)	2.59 (2.13)	2.86 (2.14)	6.05 (1.91)	6.27 (1.67)

The question whether every '−' in the first two characters of a QTC_{B12} label expresses a movement *'towards'* another object, can be answered by looking at the average ratings in Table 8.1. In Table 8.1 the average ratings for the preposition *'towards'* awarded to the stimuli 3AKTOWARDS, 3BKTOWARDS, 7BKTOWARDS, and 7CKTOWARDS (cells marked in grey) are systematically higher and differ rather largely from the average ratings awarded to the other stimuli. In terms of QTC, these four stimuli represent a QTC_{B12} relation which can uniquely be transformed into an RTC_{B12} relation '0' (3BKTOWARDS, 7BKTOWARDS) or '+' (3AKTOWARDS, 7CKTOWARDS). The other animations represent a QTC_{B12} relation which is only transformable in an RTC_{B12} relation '−'. The differences between the average ratings become clearer if we aggregate the different average ratings for the stimuli based on their RTC relation (Table 8.6).

Table 8.6 Average ratings grouped by RTC relation for the *'towards'* stimuli

RTC_{B12}	Relative distance	Average Ratings	St. Dev.
'−'	decrease	2.05	1.04
'0'	equal	3.52	1.87
'+'	increase	4.05	1.90

This means that there is a strong suspicion that not every '−' in the first two characters expresses a movement *'towards'* a reference object and that Zwarts' intuition seems to be correct (at least for objects moving in one-dimension); i.e. only objects which move in the direction of the reference object and decrease their distance with respect to that reference object can be labelled as moving *'towards'* the reference object. In terms of QTC_{B12}, this means that '−' in one of the first two characters of a QTC_{B12} label only expresses a movement *'towards'* if the QTC_{B12} relation can be transformed in an RTC_{B12} relation labelled by a '−'. Paired samples t-tests on the rating data give an indication whether or not these differences in average ratings are caused by random effects or not. The null-hypothesis in these tests is that there is no significant difference between the respective average ratings. The null-hypothesis is rejected if its probability (p-value) is lower than 5 percent (=0.05). In this section, six tests (see below) will be executed on the same English rating data, therefore the p-value needs to be corrected according to the Bonferroni correction. Hence, the p-value leading to a rejection or acceptance of the different null hypothesises needs to be divided by six (=0.008) for all tests conducted on the English rating data.

Thus, in order to test if the difference in average ratings for the *'towards'* stimuli which have an RTC_{B12} relation '−' and the average ratings for the *'towards'* stimuli which have an RTC_{B12} relation '0' or '+' is significant, the following two null-hypothesis are set:

1. There is no significant difference in average ratings between *'towards'* stimuli representing a decrease in relative distance and *'towards'* stimuli for which the relative distance remains equal (i.e. $mean_{decrease}$-$mean_{equal} = 0$)
2. There is no significant difference in average ratings between *'towards'* stimuli representing a decrease in relative distance and *'towards'* stimuli for which the relative distance increases (i.e. $mean_{decrease}$-$mean_{increase} = 0$)

The results of these two tests are given in Table 8.7. For both tests the null-hypothesis can be rejected (<0.008), meaning that there is a significant difference between the average.

Table 8.7 Results of paired samples t-test for tests 1 and 2

Test	Mean Difference	St. Dev	St. Error Mean	95% Confidence Interval of the Difference		T	df	p
				Lower	Upper			
1	-1.48	2.29	0.49	-2.49	-0.46	-3.021	21	0.007
2	-2.00	2.25	0.48	-3.00	-1.00	-4.172	21	0.000

The question whether every '+' in the first two characters of a QTC_{B12} label expresses a movement *'away from'* another object, can be answered analogously by looking at the average ratings in Table 8.5. In Table 8.5 the average ratings for the preposition *'away from'* awarded to the stimuli 3BKTOWARDS, 3CKTOWARDS, 7AKTOWARDS, and 7BKTOWARDS (cells marked in grey) are systematically higher and differ rather largely from the average ratings awarded to the other stimuli. In terms of QTC, these four stimuli represent a QTC_{B12} relation which can uniquely be transformed into an RTC_{B12} relation '0' (3BKTOWARDS, 7BKTOWARDS) or '−' (3AKTOWARDS, 7CKTOWARDS). The other animations represent a QTC_{B12} relation which is only transformable in an RTC_{B12} relation '+'. The differences between the average ratings become clearer if we aggregate the different average rating for the screen animations based on their RTC relation (Table 8.8).

Table 8.8 Average ratings grouped by RTC relation for the *'away from'* stimuli

RTC_{B12}	Relative distance	Average Ratings	St. Dev.
'−'	decrease	3.41	1.80
'0'	equal	3.73	1.92
'+'	increase	2.20	1.23

Once again, there is a strong suspicion that not every '+' in the first two characters expresses a movement *'away from'* a reference object and that Zwarts' intuition seems to be correct (once again only for objects moving in one-dimension); i.e. only objects which move in the opposite direction of the reference object and increase their distance with respect to that reference object can be labelled as moving *'away from'* the reference object. In terms of QTC_{B12}, this means that '+' in one of the first two characters of a QTC_{B12} label only expresses a movement *'away from'* if the QTC_{B12} relation can be transformed in an RTC_{B12} relation labelled by a '+'. Two additional paired samples t-tests on the English rating data give an indication whether or not these differences in average ratings are caused by random effects or not. The null-hypothesises for these two tests are:

3. There is no significant difference in average ratings between *'away from'* stimuli representing an increase in relative distance and *'away from'* stimuli for which the relative distance remains equal (i.e. $mean_{increase}$-$mean_{equal}$ = 0)
4. There is no significant difference in average ratings between *'away from'* stimuli representing an increase in relative distance and *'away from'* stimuli for which the relative distance decreases (i.e. $mean_{increase}$-$mean_{decrease}$ = 0)

The results of these two tests are given in Table 8.9. The null-hypothesis can only be rejected (<0.008) for test 3, and not for test 4. This result is rather strange, since the RTC_{B12} can only change from a '+' over a '0' into '–' if relative distance between two objects changes continuously. Thus, one would suspect an acceptance of the null-hypothesis in test 3 rather than in test 4. Therefore, there is a strong belief that this acceptance, given that it is quite small, is caused by the limited number of test persons, because a mistake made by one test person has a big influence on the rejection or acceptation of the null-hypothesises.

Table 8.9 Results of paired samples t-test for tests 3 and 4

Test	Mean Difference	St. Dev	St. Error Mean	95% Confidence Interval of the Difference		T	df	p
				Lower	Upper			
3	-1.53	2.05	0.44	-2.44	-0.62	-3.505	21	0.002
4	-1.21	2.24	0.48	-2.21	-0.22	-2.539	21	0.019

The overall conclusion concerning research question one, is that there is a strong belief that Zwarts' definition for the prepositions *'towards'* and *'away from'* is correct for object which have a constrained linear trajectory (this means Jackendoff's definition applies as well), but additional tests should strengthen this belief and the power of the statistical test.

In order to tackle the second research question whether a *'towards'* preposition can correspond to a movement ending in the reference object, and an *'away from'* preposition can express a movement starting at the reference object, the attention is turned to Table

8.2 and Table 8.4, giving the average ratings of the *'to'* and *'from'* stimuli respectively. The tables show that the difference in average ratings awarded to the *'to'* and *'towards'* prepositions for *'to'* stimuli and the *'from'* and *'away from'* prepositions for *'from'* stimuli differ very little and are all acceptable for the test persons. The similarities between the average ratings become clearer if we aggregate the different average rating for the different stimuli (Table 8.10).

Table 8.10 Average ratings for the preposition *'to'* and *'towards'* grouped by *'to'* stimuli and the prepositions *'from'* and *'away from'* grouped by *'from'* stimuli

Preposition	Average Ratings	St. Dev.
To	1.74	1.28
Towards	1.91	1.45
From	2.30	1.21
Away From	2.01	1.41

Paired samples t-tests on the rating data give an indication whether or not these differences in average ratings are caused by random effects or not. The two null-hypothesises are:

5. There is no significant difference in average ratings awarded to the *'to'* and *'towards'* prepositions for 'to' stimuli (i.e. $mean_{to}$-$mean_{towards} = 0$)
6. There is no significant difference in average ratings awarded to the *'from'* and *'away from'* prepositions for *'from'* stimuli (i.e. $mean_{from}$-$mean_{awayfrom} = 0$)

The results of these two tests are given in Table 8.11. For both tests the null-hypothesis is accepted (>0.008). Thus, the overall conclusion concerning research question two is that the prepositions *'to'* and *'towards'* are equally acceptable to communicate *'to'* stimuli, i.e. movements ending in the reference object, and the prepositions *'from'* and *'away from'* are equally suitable to express *'from'* stimuli, i.e. movements starting at the reference object.

Table 8.11 Results of paired samples t-test for tests 5 and 6

Test	Mean Difference	St. Dev	St. Error Mean	95% Confidence Interval of the Difference		T	df	p
				Lower	Upper			
5	-0.17	0.58	0.12	-0.42	0.09	-1.369	21	0.185
6	0.30	0.89	0.19	-0.10	0.69	1.558	21	0.134

8.5 Results for Dutch prepositions

24 persons undertook the free response test, part of them were students attending the course 'Applied Informatics II' at Ghent University, part of them were staff members of the Department of Geography of Ghent University. All participants were Dutch (Flemish) native speakers.

According to the responses given by the participants, the Dutch preposition *'naar'* occurred most often for both the *'to'* and *'towards'* stimuli, the Dutch preposition *'weg van'* was the most frequent preposition responded to both the *'from'* and *'away from'* stimuli. Intuitively, it can be stated, that based on these results, the Dutch preposition *'naar'* en *'weg van'* are respectively very closely related to the English preposition *'towards'* and *'away from'* in English, since it seems that, like the *'towards'* preposition, the preposition *'naar'* can be used to express movements in the direction of the reference objects which can end either before or at the reference object, and analogous to the preposition *'away from'*, the preposition *'weg van'* can be used to express movements in the opposite direction of the reference object which can either start at or not at the reference object. Therefore, *'naar'* and *'weg van'* are chosen as the Dutch counterpart of the English prepositions *'towards'* and *'away from'*. In order to have an equivalent of the English prepositions *'from'* and *'to'*, the second most frequent preposition responded to the *'to'* and *'from'* stimuli are selected. For the *'to'* stimuli, prepositions starting with *'tot'* (*'tot bij'*, *'tot op'*, *'tot aan'*) had the second highest occurrence, for the *'from'* stimuli these were prepositions starting with '*van'* (*'vanuit'*, '*vanaf'*, *'vanop'*). The prepositions *'tot bij'* and *'vanuit'* are chosen as the Dutch counterpart of the respective English prepositions *'to'* and *'from'*. For the *'via'* stimuli, the participants did not responded very often with prepositions (*'door'*, *'over'*), the frequency of verbs (*'kruisen'*, *'inhalen'*) and adverbs (*'voorbij'*) was much higher. Since the focus in this chapter is on prepositions, the literal translation of *'via'* (i.e. also *'via'*) is selected for the rating experiment. In order to see if the answer to the two research questions is different when using the Dutch prepositions instead of their English counterparts, the same research questions were transformed into a Dutch equivalent and the exact same rating experiments were conducted.

Thus, the research questions are:

1. Does every '−' in the first two characters of a QTC_{B12} label express a movement *'naar'* another object, and the other way around, does every '+' refer to a movement *'weg van'* another object?
2. Are there trajectories expressing a *'naar'* movement which end at the reference object and conversely, can trajectories expressed as an *'weg'* movement start at the reference object?

Thirty one students, all attending the course 'Introduction to Geographical Information Systems' at Ghent University, were tested. All participants were Dutch (Flemish) native speakers. Tables 8.12 to 8.16, give the average rating (and its standard deviations) awarded to each of the prepositions for the different screen animations, subdivided by stimulus. The codes in the animations column are equal to the codes given in Figure 8.3.

Table 8.12 Average ratings for the *'towards'* stimuli

Animation/Preposition	Naar	Tot bij	Via	Vanuit	Weg van
1AKTOWARDS	2.32 (2.07)	3.74 (2.08)	6.61 (0.88)	6.87 (0.43)	6.74 (1.03)
1ALTOWARDS	1.45 (0.81)	3.71 (2.18)	6.65 (0.80)	6.74 (1.09)	6.90 (0.30)
1BKTOWARDS	1.71 (1.49)	4.19 (2.20)	6.55 (0.96)	6.97 (0.18)	6.74 (1.09)
1BLTOWARDS	1.74 (1.61)	3.74 (2.10)	6.26 (1.50)	6.74 (1.09)	6.97 (0.18)
1CKTOWARDS	1.74 (1.32)	3.87 (2.20)	6.61 (0.92)	6.84 (0.58)	6.68 (1.19)
1CLTOWARDS	1.77 (1.48)	3.97 (2.12)	6.55 (0.99)	6.94 (0.25)	6.90 (0.40)
2CKTOWARDS	1.48 (1.18)	3.97 (2.06)	6.16 (1.46)	6.87 (0.56)	6.97 (0.18)
3AKTOWARDS	**4.16 (2.57)**	6.39 (1.33)	6.71 (0.69)	6.97 (0.18)	6.13 (1.80)
3BKTOWARDS	**4.35 (2.30)**	6.48 (1.21)	6.48 (1.06)	6.68 (1.19)	6.61 (1.05)
3CKTOWARDS	1.94 (1.63)	4.52 (2.23)	6.42 (1.09)	6.77 (0.62)	6.71 (1.10)
4ALTOWARDS	1.58 (0.99)	4.00 (2.32)	6.55 (1.06)	6.94 (0.25)	6.58 (1.50)
7ALTOWARDS	1.65 (1.33)	5.42 (2.03)	6.13 (1.59)	6.94 (0.25)	6.87 (0.34)
7BLTOWARDS	**3.68 (2.34)**	6.61 (0.72)	6.48 (1.18)	6.84 (0.58)	6.61 (0.95)
7CLTOWARDS	**4.26 (2.63)**	6.10 (1.60)	6.71 (0.78)	6.77 (0.67)	5.84 (1.57)

Table 8.13 Average ratings for the *'to'* stimuli

Animation/Preposition	Naar	Tot bij	Via	Vanuit	Weg van
1AKTO	1.74 (1.39)	1.48 (1.29)	5.52 (1.86)	6.87 (0.56)	6.68 (0.91)
1ALTO	1.29 (0.53)	1.16 (0.73)	5.06 (2.08)	6.97 (0.18)	6.97 (0.18)
1BKTO	1.61 (1.31)	1.52 (1.36)	5.84 (1.77)	6.87 (0.34)	6.77 (1.09)
1BLTO	1.61 (1.50)	1.32 (1.11)	5.65 (1.92)	6.97 (0.18)	6.94 (0.25)
1CKTO	1.52 (1.15)	1.23 (1.09)	5.65 (1.78)	6.84 (0.58)	6.87 (0.56)
1CLTO	1.58 (1.31)	1.71 (1.68)	5.65 (1.80)	6.90 (0.40)	6.94 (0.25)
2CKTO	1.77 (1.52)	1.35 (1.23)	5.74 (1.86)	6.94 (0.25)	6.87 (0.56)
3CKTO	1.42 (0.76)	1.23 (1.09)	5.52 (1.75)	6.87 (0.56)	6.94 (0.25)
4ALTO	1.35 (0.66)	1.23 (1.09)	5.32 (1.87)	6.90 (0.30)	6.97 (0.18)
7ALTO	1.32 (0.54)	1.10 (0.30)	5.29 (1.75)	6.87 (0.56)	6.55 (1.50)

Table 8.14 Average ratings for the *'via'* stimuli

Animation/Preposition	Naar	Tot bij	Via	Vanuit	Weg van
1AKVIA	4.71 (1.99)	5.71 (1.55)	1.45 (0.89)	6.19 (1.17)	5.39 (1.54)
1ALVIA	4.97 (1.74)	5.71 (1.51)	1.45 (1.12)	6.03 (1.64)	5.10 (1.89)
1BKVIA	4.84 (1.49)	5.74 (1.59)	1.32 (0.75)	6.29 (1.24)	5.23 (1.69)
1BLVIA	4.94 (1.75)	6.03 (1.14)	1.48 (1.23)	6.35 (1.05)	5.19 (1.60)
1CKVIA	4.90 (1.66)	6.10 (1.27)	1.45 (1.12)	6.16 (1.44)	5.29 (1.72)
1CLVIA	4.87 (1.94)	6.06 (1.39)	1.58 (1.43)	6.42 (1.09)	5.26 (1.61)
2CKVIA	4.26 (2.03)	5.61 (1.56)	1.00 (0.00)	6.55 (0.85)	5.23 (2.00)
3CKVIA	4.84 (1.71)	5.39 (1.84)	1.19 (0.40)	6.32 (1.25)	5.32 (1.70)
4ALVIA	4.94 (1.91)	5.23 (1.76)	1.26 (1.12)	6.23 (1.33)	5.26 (1.75)
7ALVIA	4.65 (1.99)	5.65 (1.47)	1.45 (1.29)	6.16 (1.21)	4.87 (1.75)

Table 8.15 Average ratings for the *'from'* stimuli

Animation/Preposition	Naar	Tot bij	Via	Vanuit	Weg van
3ALFROM	6.94 (0.25)	6.97 (0.18)	4.68 (2.18)	1.58 (1.57)	1.61 (1.52)
6ALFROM	6.97 (0.18)	6.87 (0.56)	5.58 (1.95)	1.55 (1.71)	1.26 (1.12)
7CKFROM	6.77 (0.67)	6.87 (0.56)	5.55 (1.71)	1.48 (1.23)	1.97 (1.94)
8CKFROM	6.87 (0.56)	6.90 (0.40)	5.71 (1.55)	1.74 (1.97)	1.26 (1.09)
9AKFROM	6.77 (1.09)	6.61 (1.38)	5.55 (1.86)	1.94 (1.79)	1.29 (0.69)
9ALFROM	6.97 (0.18)	6.97 (0.18)	5.94 (1.57)	2.03 (2.09)	1.55 (1.39)
9BKFROM	6.97 (0.18)	6.84 (0.58)	6.23 (1.50)	1.52 (1.46)	1.71 (1.72)
9BLFROM	6.77 (1.09)	6.97 (0.18)	5.74 (1.83)	1.58 (1.71)	1.58 (1.52)
9CKFROM	6.94 (0.36)	6.94 (0.25)	5.97 (1.74)	1.55 (1.48)	1.58 (1.57)
9CLFROM	6.77 (1.09)	6.97 (0.18)	5.42 (1.78)	1.81 (1.62)	1.84 (1.53)

Table 8.16 Average ratings for the *'away from'* stimuli

Animation/Preposition	Naar	Tot bij	Via	Vanuit	Weg van
3ALAWAY FROM	6.84 (0.58)	6.94 (0.25)	6.74 (0.73)	5.45 (1.93)	1.77 (1.71)
3BLAWAY FROM	6.81 (0.75)	6.90 (0.30)	6.90 (0.40)	6.65 (0.95)	**3.55 (2.46)**
3CLAWAY FROM	6.13 (1.65)	6.71 (1.04)	6.81 (0.48)	6.68 (0.79)	**4.19 (2.56)**
6ALAWAY FROM	6.97 (0.18)	7.00 (0.00)	6.39 (1.41)	4.71 (2.45)	1.23 (0.62)
7AKAWAY FROM	6.39 (1.31)	6.65 (0.88)	6.71 (0.69)	6.61 (1.02)	**3.35 (2.46)**
7BKAWAY FROM	6.74 (1.09)	6.77 (0.76)	6.68 (1.05)	6.39 (1.38)	**3.23 (2.26)**
7CKAWAY FROM	6.81 (0.65)	6.68 (1.11)	6.71 (0.74)	6.19 (1.62)	1.55 (1.41)
8CKAWAY FROM	6.74 (1.09)	6.97 (0.18)	6.45 (1.31)	4.94 (2.34)	1.16 (0.45)
9AKAWAY FROM	6.77 (1.09)	6.97 (0.18)	6.77 (0.76)	5.77 (1.80)	1.84 (1.81)
9ALAWAY FROM	6.97 (0.18)	6.77 (1.09)	6.81 (0.54)	5.39 (2.20)	1.26 (0.63)
9BKAWAY FROM	6.87 (0.56)	6.87 (0.56)	6.61 (1.17)	5.77 (1.80)	1.29 (1.10)
9BLAWAY FROM	6.77 (1.09)	6.74 (0.77)	6.81 (0.65)	5.65 (2.01)	1.61 (1.56)
9CKAWAY FROM	6.97 (0.18)	6.77 (1.09)	6.71 (0.74)	5.61 (1.87)	1.58 (1.54)
9CLAWAY FROM	6.87 (0.56)	6.94 (0.25)	6.94 (0.25)	5.35 (2.03)	1.87 (1.71)

The question whether every '−' in the first two characters of a QTC_{B12} label expresses a movement *'naar'* another object, can once again be answered by looking at the average ratings in Table 8.1. As for the preposition *'towards'*, the average ratings for the preposition *'naar'* awarded to the stimuli 3AKTOWARDS, 3BKTOWARDS, 7BKTOWARDS, and 7CKTOWARDS (cells marked in grey) are systematically higher and differ rather largely from the average ratings awarded to the other stimuli. As stated in section 8.4, in terms of QTC, these four stimuli represent a QTC_{B12} relation which can uniquely be transformed into an RTC_{B12} relation '0' (3BKTOWARDS, 7BKTOWARDS) or '+' (3AKTOWARDS, 7CKTOWARDS). The other animations represent a QTC_{B12} relation which is only transformable in an RTC_{B12} relation '−'. The differences between the average ratings become clearer if we aggregate the different average ratings for the stimuli based on their RTC relation (Table 8.17).

Table 8.17 Average ratings grouped by RTC relation for the *'towards'* stimuli

RTC_{B12}	Relative distance	Average Ratings	St. Dev.
'−'	decrease	1.74	0.87
'0'	equal	4.02	2.00
'+'	increase	4.23	2.48

Paired samples t-tests on the rating data give an indication whether or not these differences in average ratings are caused by random effects. The null-hypothesises for these tests are:

1. There is no significant difference in average ratings between *'towards'* stimuli representing a decrease in relative distance and *'towards'* stimuli for which the relative distance remains equal (i.e. $mean_{decrease}$-$mean_{equal} = 0$)
2. There is no significant difference in average ratings between *'towards'* stimuli representing a decrease in relative distance and *'towards'* stimuli for which the relative distance increases (i.e. $mean_{decrease}$-$mean_{increase} = 0$)

The results of these two tests are given in Table 8.18. In contrast to the English test, the null-hypothesis for both tests are strongly rejected (<0.008). This means, that people prefer to use the preposition *'naar'* for objects which move in the direction of the reference object and decrease their distance with respect to that reference object over movements in the direction of the reference object which increase or remain at an equal distance with respect to that reference object. In terms of QTC_{B12}, this means that a '–' in one of the first two characters of a QTC_{B12} label only expresses a movement *'naar'* if the QTC_{B12} relation can be transformed in an RTC_{B12} relation labelled by a '–'.

Table 8.18 Results of paired samples t-test for tests 1 and 2

Test	Mean Difference	St. Dev	St. Error Mean	95% Confidence Interval of the Difference		T	df	p
				Lower	Upper			
1	-2.28	1.82	0.33	-2.95	-1.61	-6.966	30	0.000
2	-2.49	2.34	0.42	-3.34	-1.63	-5.928	30	0.000

The question whether every '+' in the first two characters of a QTC_{B12} label expresses a movement *'weg van'* another object, can be answered analogously by looking at the average ratings in Table 8.5. As for the preposition *'away from'*, the average ratings for the preposition *'weg van'* awarded to the stimuli 3BKTOWARDS, 3CKTOWARDS, 7AKTOWARDS, and 7BKTOWARDS (cells marked in grey) are systematically higher and differ rather largely from the average ratings awarded to the other stimuli. As stated in section 8.4, in terms of QTC these four stimuli represent a QTC_{B12} relation which can uniquely be transformed into an RTC_{B12} relation '0' (3BKTOWARDS, 7BKTOWARDS) or '−' (3AKTOWARDS, 7CKTOWARDS). The other animations represent a QTC_{B12} relation which is only transformable in an RTC_{B12} relation '+'. The differences between the average ratings become clearer if we aggregate the different average rating for the screen animations based on their RTC relation (Table 8.19).

Table 8.19 Average ratings grouped by RTC relation for the *'away from'* stimuli

RTC_{B12}	Relative distance	Average Ratings	St. Dev.
'−'	Decrease	3.77	2.16
'0'	Equal	3.39	2.02
'+'	Increase	1.51	0.43

Paired samples t-tests on the rating data give an indication whether or not these differences in average ratings are caused by random effects or not. The null-hypothesises for these tests are:

3. There is no significant difference in average ratings between *'away from'* stimuli representing an increase in relative distance and *'away from'* stimuli for which the relative distance remains equal (i.e. $mean_{increase}$-$mean_{equal}$ = 0)
4. There is no significant difference in average ratings between *'away from'* stimuli representing an increase in relative distance and *'away from'* stimuli for which the relative distance decreases (i.e. $mean_{increase}$-$mean_{decrease}$ = 0)

The results of these two tests are given in Table 8.20. While the null-hypothesises for the tests on the English data was rejected for test 3 and accepted for test 4, the null-hypothesises for both test on the Dutch data are clearly rejected (<0.008). This means that people prefer to use the preposition *'weg van'* for objects which move in the opposite direction of the reference object and increase their distance with respect to that reference object over movements in the direction of the reference object which decrease or remain at an equal distance with respect to that reference object. In terms of QTC_{B12}, this means that a '+' in one of the first two characters of a QTC_{B12} label only expresses a movement *'weg van'* if the QTC_{B12} relation can be transformed in an RTC_{B12} relation labelled by a '+'.

Table 8.20 Results of paired samples t-test for tests 3 and 4

Test	Mean Difference	St. Dev	St. Error Mean	95% Confidence Interval of the Difference		T	df	p
				Lower	Upper			
3	-1.87	2.07	0.37	-2.63	-1.11	-5.042	30	0.000
4	-2.26	2.27	0.41	-3.09	-1.43	-5.544	30	0.000

The clear results concerning research question 1 on the Dutch prepositions *'naar'* and *'weg van'*, strengthens the belief that, although the rather fuzzy results on the English rating data, Zwarts' definition for the English prepositions *'towards'* and '*away from*' is correct for objects having a constrained linear trajectory, since the average rating awarded to the Dutch prepositions does not differ much from the average rating awarded to their English counterpart. The major difference is in the number of people that were tested in the rating experiments (31 vs. 22).

The equivalent of research question 2 concerning the Dutch prepositions is whether a *'naar'* preposition can correspond to a movement ending in the reference object, and a *'weg van'* preposition can express a movement starting at the reference object. As for the English data, the tables 8.13 and 8.15 show that the difference in average ratings awarded to the *'tot bij'* and *'naar'* prepositions for *'to'* stimuli and the *'vanuit'* and *'weg van'* prepositions for *'from'* stimuli differ very little and are all acceptable for the test persons. The similarities between the average ratings become clearer if we aggregate the different average rating for the different stimuli (Table 8.21).

Table 8.21 Average ratings for the preposition *'tot bij'* and *'naar'* grouped by *'to'* stimuli and the prepositions *'vanuit'* and *'weg van'* grouped by *'from'* stimuli

Preposition	Average Ratings	St. Dev.
Tot bij	1.33	1.01
Naar	1.52	1.28
Vanuit	1.68	0.80
Weg van	1.56	0.54

Paired samples t-tests on the rating data give an indication whether or not these differences in average ratings are caused by random effects or not. The two null-hypothesises are:

5. There is no significant difference in average ratings awarded to the *'tot bij'* and *'naar'* prepositions for *'to'* stimuli (i.e. $mean_{to}$-$mean_{towards}$ = 0)
6. There is no significant difference in average ratings awarded to the *'vanuit'* and *'weg van'* prepositions for *'from'* stimuli (i.e. $mean_{from}$-$mean_{awayfrom}$ = 0)

The results of these two tests are given in Table 8.22. As for the tests on the English data, the null-hypothesises for both tests are accepted (>0.008), but this time with a much higher p-value. Thus, the overall conclusion concerning research question two is that the prepositions *'tot bij'* and *'naar'* are equally acceptable to communicate *'to'* stimuli, i.e. movements ending in the reference object and the prepositions *'vanuit'* and *'weg van'* are equally suitable to express *'from'* stimuli, i.e. movements starting at the reference object.

Table 8.22 Results of paired samples t-test for tests 5 and 6

Test	Mean Difference	St. Dev	St. Error Mean	95% Confidence Interval of the Difference		T	df	p
				Lower	Upper			
5	-0.19	1.03	0.19	-0.57	0.19	-1.028	30	0.312
6	0.11	0.83	0.15	-0.19	0.42	0.753	30	0.457

8.6 Discussion

The conclusions derived from the above stated research questions have an effect on the use of QTC_{B12} in the domain of Human Computer Interaction (HCI). To communicate a certain movement representing a constrained linear trajectory at a specific moment in time, it is only preferable to transform a '−' represented in one or both of the first two characters of a QTC_{B12} into a movement *'towards'* or *'naar'* a reference object if the QTC_{B12} relation can be transformed into an RTC_{B12} relation labelled by a '−', analogously it is preferred to only express a '+' represented in one or both of the first two characters of a QTC_{B12} as a movement *'away from'* or *'weg van'* if the QTC_{B12} relation can be transformed into an RTC_{B12} relation labelled by a '+'.

Additionally, when communicating trajectories of moving point objects (lasting over an interval), it is only preferable to label them as a movements *'towards'* or *'naar'* if these trajectories represent a conceptual animation consisting of one or more relations part of the set {'− − −', '− − 0', '− − +', '− 0 +', '− + +', '0 − −', '+ − −'}. This set needs to be extended with the QTC_{B12} relation '0 0 0' (since an object can pause for a while somewhere along or at the end of the trajectory) and the topological relation 'equal' (as can be deducted from the empirical test, *'towards'* and *'naar'* movements can end at the reference object). Note that the conceptual animation can only contain these two additional animations, if it consists of more than one relation. In the same way, it is only preferable to label them as a movements *'away from'* or *'weg van'* if these trajectories represent a conceptual animation consisting of one or more relations part of the set {'− + −', '0 + −', '+ − +', '+ 0 +', '+ + −', '+ + 0', '+ + +'}. This set also needs to be extended with the QTC_{B12} relation '0 0 0' (since an object can pause for a while somewhere along or at the end of the trajectory) and the topologic relation 'equal' (as can be deducted from the empirical test, *'away from'* and *'weg van'* movements start at the reference object). As for *'towards'* movements, the conceptual animation can only contain these two additional animations if it consists of more than one relation. The third character in a QTC_N label can be used to correspond about a 'slower', 'faster' or 'equally fast' movement. Depending on the granularity of information a user requires, a movement in one dimension can be communicated in detail by giving back each character in the QTC_N relation or in a coarser way by communicating only whether the objects are moving further away from or getting closer to each other by means of their RTC relation.

The other way around, text or speech based information can be translated into a spatial setting which can be used to process, analyse or infer additional knowledge. For example, the transformation of a QTC_{B12} relation into a unique RTC_{B12} relation can be used to infer additional knowledge. In other words, if we know k and l have a constrained one-dimensional trajectory and k moves towards l and l moves towards k, we know that they are approaching each other, even without knowledge of their relative speed. Conversely, if we know that k and l are getting further away from each other, we have definite knowledge about the RTC_{B12} relations between k and l, and the set of relations

{'− + −', '0 + −', '+ − +', '+ 0 +', '+ + −', '+ + 0', '+ + +'} represents coarse knowledge of the QTC_{B12} relations (since these relations form a conceptual neighbourhood).

Chapter 9
Conclusions and Directions for Further Research

9.1 Conclusions

In this thesis, a distance based qualitative spatiotemporal calculus for representing and reasoning about moving point objects constrained by networks is presented: QTC_N. For various reasons, defining and examining its properties clearly extends the theoretical understanding of the QTC calculus. Due to the co-dimensional structure of a network, QTC_N can, in a certain sense, be positioned somewhere in between QTC_{B1} and QTC_{B2}. As can be deducted from Chapter 3, QTC_N is able to distinguish more JEPD relations than QT_{B12} (17 vs. 27), but in contrast to QTC_{B22} not all relations can hold over an interval. These relations correspond to the ten nonexistent relations in QTC_{B12} and only exist due to the occurrence of node pass events and bifurcating shortest paths.

QTC_N can be extended from 27 to 57 JEPD relations when splitting the '0'-character into a '0'-character caused by a node pass event ('0n'), a bifurcating shortest path ('0b') or an object not moving with respect to the network ('0s'). This extension is of particular interest for inferring new knowledge via the composition of relations. As shown in Chapter 4, neglecting this distinction leads to an utterly useless composition table (see Table 4.4), since each cell in the composition table contains all possible relations and therefore the composition of relations does not generate new knowledge. Allowing a '0' character in one of the first two characters of a QTC_N label, only for objects which are stable with respect to the network ('0s'-character) will already reduce the number of entries in the composition table dramatically (see Table 4.5). Adding additional knowledge, such as knowing that one object lies on the shortest path between the other two objects, leads to even sparser and quite functional composition tables (see Tables 4.6 to 4.8).

Another interesting feature of QTC_N is that, in contrast to QTC_{B22}, the relation between two moving objects can be derived by means of the topological relations between a moving object and the network instead of comparing distances between the moving object and the reference object. When deriving a QTC_N relation from quantitative information, this considerably reduces calculation times, since instead of five shortest paths between the two objects, only one needs to be computed (based on the distance based definition of QTC_B, five distances need to be calculated in order to derive a QTC_B relation).

The fact that some relations can only hold instantaneously has its repercussions on QTC_N's CND, since according to Galton's (2001) theory of dominance a qualitative relation q_1 can only dominate another qualitative relation q_2, if it can hold at the end of an open interval (i.e. not an instant) over which q_2 holds. Therefore, the ten instantaneous QTC_N relations can not be dominated and this eliminates quite a number of transitions that exist in QTC_{B22}. Still the CND for QTC_N contains more transitions than the CND for QTC_{B12}, positioning QTC_N once again somewhere in between QTC_{B12} and QTC_{B22}.

In contrast to objects moving freely in the plane, it is realistic that the network space in which objects move can be subject to both continuous and discontinuous change as exemplified in Chapter 5. When the network is solely affected by continuous deformations ($QTC_{CN'}$), not surprisingly, the QTC relations and conceptual neighbours are equal to the ones defined when objects can move freely in the plane (QTC_{B22}). As can be deducted from section 6.3 and 6.4, additionally allowing discontinuous changes to the network still does not lead to all-to-all transitions between QTC relations for network based moving objects, since in the whole QTC theory, objects are assumed to move continuously and not teleport from one location to another.

The three characters representing a QTC_N relation provide more information than each individual character itself. As shown in Chapter 7, a QTC_N relation can be uniquely transformed into a purely relative RTC_N relation, just as the relations defined in QTC_{B11}. This is noteworthy, because this is not the case when objects have a complete free trajectory in the plane (QTB_{B22}). This unique transformation clearly extends the power of

QTC_N, since from a QTC_N relation it is directly possible to infer whether the objects are getting closer to each other or whether they are getting further away from each other.

A unique transformation from QTC to RTC relations becomes particularly handy when using the QTC calculus in terms of HCI. In Chapter 8, it is shown that if objects are restricted to move on a straight line, there is a strong belief that only QTC_{B12} relations which can be directly transformed into a RTC_{B12} labelled by a '−' can be communicated back as a movement *'towards'* or its Dutch counterpart *'naar'*. Analogously, it is preferred to only express a '+' represented in one or both of the first two characters of a QTC_{B12} as a movement *'away from'* or *'weg van'* if the QTC_{B12} relation can be transformed into an RTC_{B12} relation labelled by a '+'. The other way around, text or speech based information which states that the distance between two objects enlarges or recedes (an RTC_{B12} relation labelled by a '+' or a '−' respectively) can be represented by coarse knowledge in QTC_{B12}.

9.2 Future Work

In this thesis, the focus was rather on the theoretical aspects of QTC_N. Potential applications in which QTC_N can be interesting were only addressed as a side issue. Thus, from the application point of view, there clearly is still a lot of work to be done. Given that nearly all traffic movements are tied to a network, QTC_N seems to offer great potential within the field of Geographical Information Systems for Transportation (GIS-T). Potential domains might include Traffic Management Systems (TMS), Accessibility measures, Advanced Traveller Information Systems (ATIS), etc.

Since qualitative calculi only introduce a distinction if it is relevant to the current research context, and computing using qualitative techniques is often easier than using quantitative methods, sequences of QTC_N relations (conceptual animations) could be very interesting to analyse both human and animal movement patterns. Mining large moving object databases could reveal specific motion patterns, e.g. consistent (an equal motion over a time period), concurrent (equal motion patterns for many objects at a specific moment in time), and trend-setter motion (one object anticipates the motion of *n* other objects) (Laube et al. 2005).

At the present time, the application in which not only QTC_N but the whole range of QTC calculi offers great potential is in the domain of HCI. The mapping of words in terms of QTC and RTC relations could be a very interesting start point to represent knowledge about linguistic motion terms in information systems, not only for Anglo-Saxon terms (since these terms are used most of the time in information systems, such as Geographical Information Systems (Campari 1991)), but also for other languages. As Campari (1991) argued, Anglo-Saxon concepts are difficult to translate for users from a different cultural or linguistic background. Intuitively, the QTC/RTC mapping of motion words in different languages can be used to translate these concepts and catch subtle cross-linguistic and cross-cultural differences between them. As motion relations are most often communicated by prepositions or verbs (Aurnague and Vieu 1993), further research could first of all try to match the motion verbs or prepositions revealed in a free response test by means of similar linguistic and cognitive tests equal to those in Chapter 8. Furthermore, when objects move freely in the plane, there is still discussion about the correct definition of '*towards*' and '*away from*'. Should objects moving '*towards*' a reference object decrease there distance monotonically with respect to the reference object as defined by Zwarts (2005), or is a path '*towards*' a reference object a progressive part of a path '*to*' that object as defined by Jackendoff (1983), or should this be defined in an other way. If instead of words, drawings are used to query moving object data, the QTC theory could be used in terms of query-by-sketch as suggested Van de Weghe (2004). For QTC_N in particular, the test conducted on one-dimensional movements should first of all be extended to a network like setting. Secondly, further free response tests could also give also give an indication how people communicate about particular QTC_N events such as node pass events or shortest path change events. Thirdly, the influence of representing networks at different scales and grain levels on motion verbs or prepositions should be examined. From the theoretical point of view, there is still some work on the composition tables of QTC_N. The construction of the composition tables would surely benefit from a more mathematical proof instead of the rather tedious but easy proof by means of sketches. This mathematical proof should ease up introducing realistic additional constraints leading to fewer entries in the tables. Furthermore, constructing a composition table based on the extended version of QTC_N consisting out of

57 JEPD relations is almost impossible by means of sketches, since in this case 185193 (57 x 57 x 57) entries need to be examined.

For the sake of completeness of the general QTC theory, it would be interesting, as Van de Weghe (2004) already stated, to extend the calculus to three or more dimensions. Intuitively, a purely distance based calculus (QTC_B) representing relations between two objects in a three-dimensional space will not be expressive enough in most cases. A triple cross structure (extending the double cross) dividing the plane into three dichotomies (front-back, up-down and left-right) could be an interesting case to examine, but then again this would theoretically lead to 729 (3^6) JEPD relations which might be difficult to reason with. This approach is quite similar to the Pacheco et al. (2002) three dimensional extension of the Double Cross Calculus.

I would like to end with Muhlberger's quote stated in Van de Weghe (2004): *"You never finish a PhD; you just stop working on it"*. I sincerely hope that after this, I can continue the work on the untackled issues within this vast and multidisciplinary domain.

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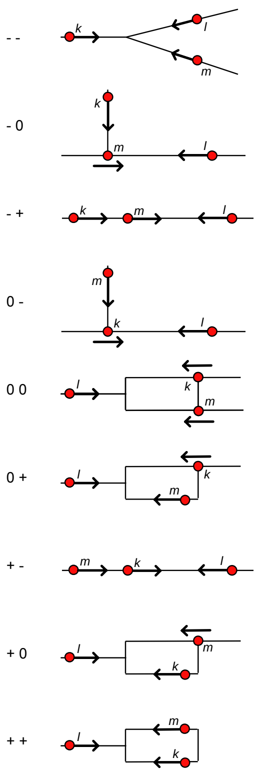
Appendix A
The Composition of Relations Consisting out of the first two Characters in a QTC_N Label

The lines in the drawings in this appendix represent the network restricting the movement of objects. The dots represent the objects *k*, *l* and *m*. A dot can be filled or not. If a dot is not filled, this indicates that the object represented by the dot has a bifurcating shortest path with respect to at least one other object. The arrow symbols indicate the movement of an object. When there is no arrow near an object this indicates that the object is not moving. When the arrow symbol is placed next to an object (above, below, left, right) this indicates that this object is involved in a node pass event.

R_1 (k,l): - - ⊗ R_2 (l,m): - - = R_3 (k,m): ?



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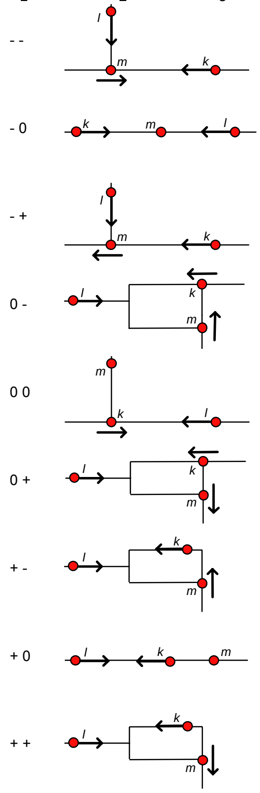
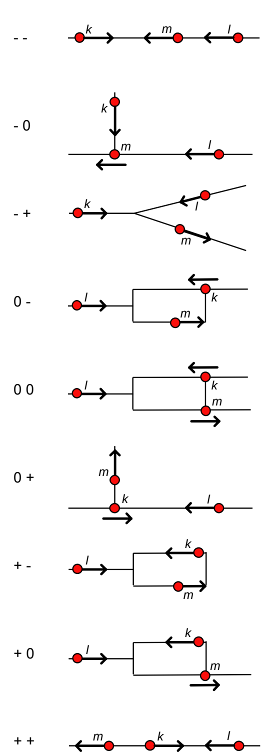
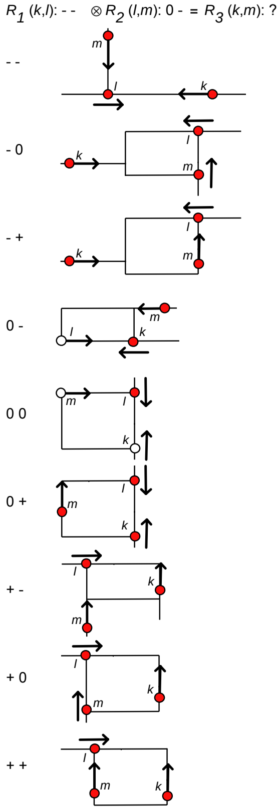


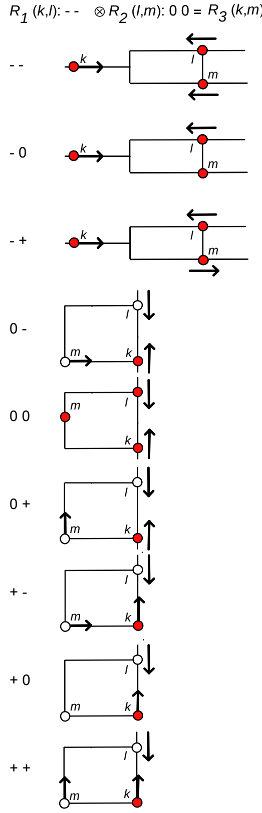
R_1 (k,l): - - ⊗ R_2 (l,m): - + = R_3 (k,m): ?



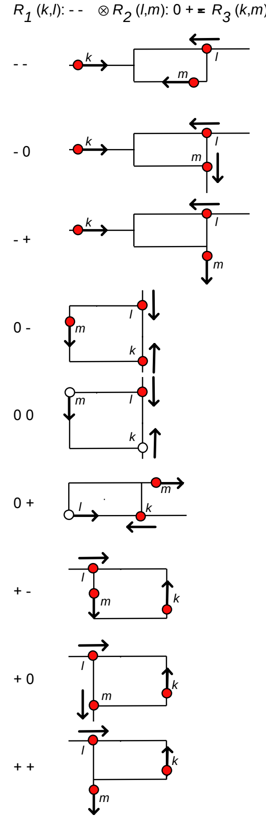
R_1 (k,l): - - ⊗ R_2 (l,m): 0 - = R_3 (k,m): ?



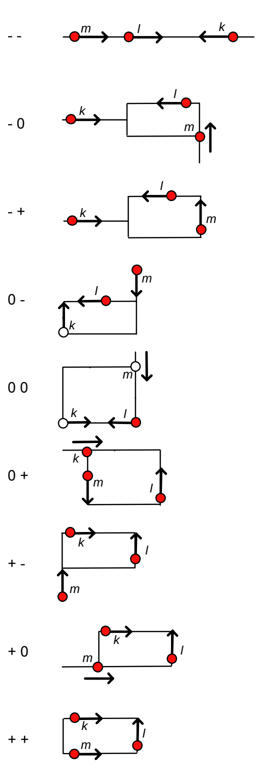
R_1 (k,l): - - ⊗ R_2 (l,m): 0 0 = R_3 (k,m): ?



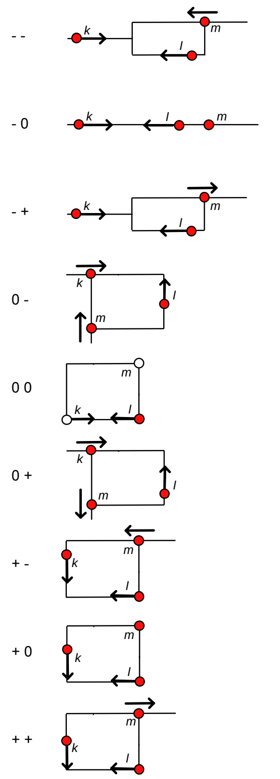
R_1 (k,l): - - ⊗ R_2 (l,m): 0 + = R_3 (k,m): ?



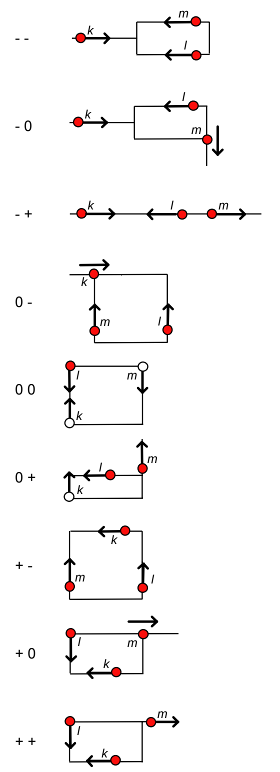
R_1 (k,l): - - ⊗ R_2 (l,m): + - = R_3 (k,m): ?



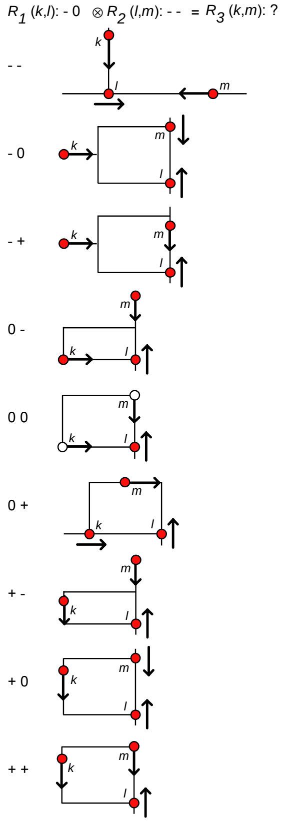
R_1 (k,l): - - ⊗ R_2 (l,m): + 0 = R_3 (k,m): ?



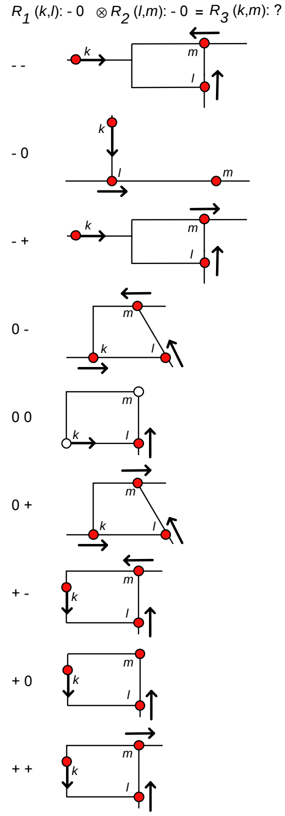
R_1 (k,l): - - ⊗ R_2 (l,m):+ + = R_3 (k,m): ?



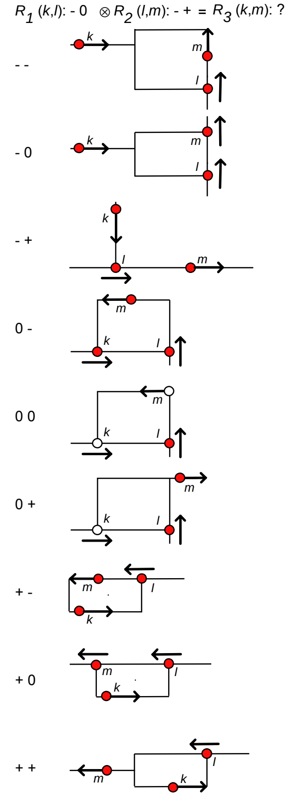
R_1 (k,l): - 0 ⊗ R_2 (l,m): - - = R_3 (k,m): ?



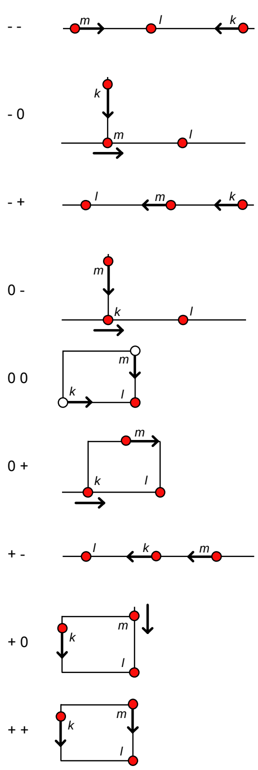
R_1 (k,l): - 0 ⊗ R_2 (l,m): - 0 = R_3 (k,m): ?



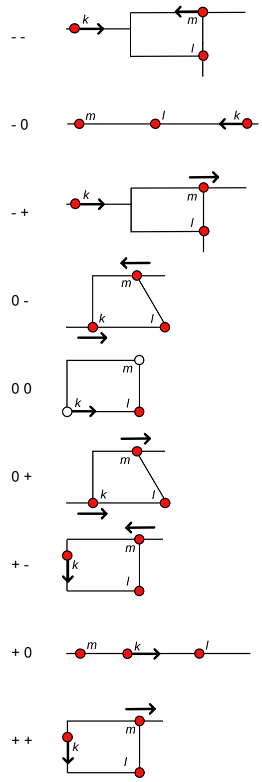
R_1 (k,l): - 0 ⊗ R_2 (l,m): - + = R_3 (k,m): ?



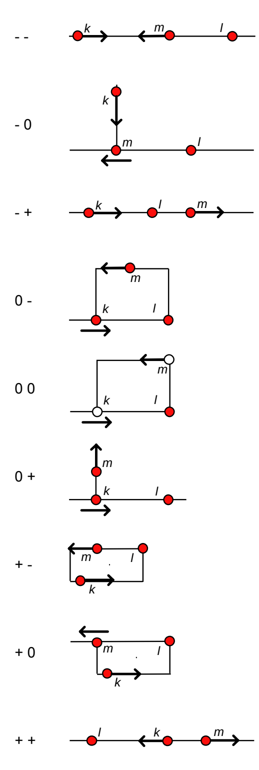
R_1 (k,l): - 0 $\otimes R_2$ (l,m): 0 - = R_3 (k,m): ?



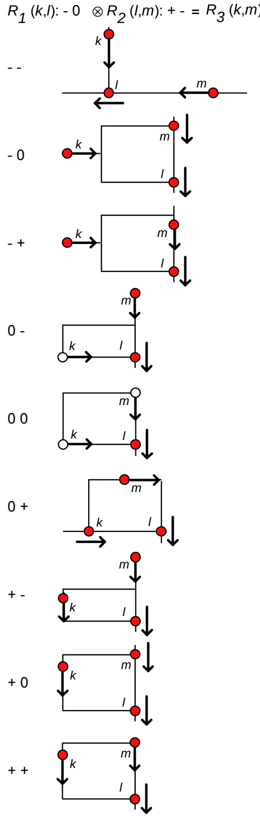
R_1 (k,l): - 0 $\otimes R_2$ (l,m): 0 0 = R_3 (k,m): ?



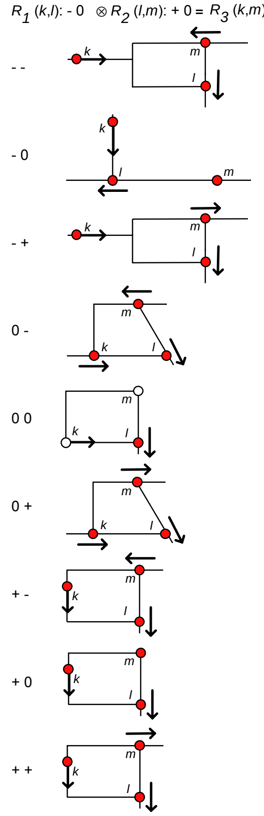
R_1 (k,l): - 0 $\otimes R_2$ (l,m): 0 + = R_3 (k,m): ?



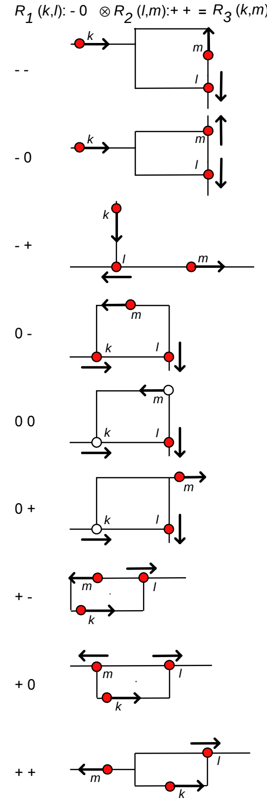
R_1 (k,l): - 0 $\otimes R_2$ (l,m): + - = R_3 (k,m): ?



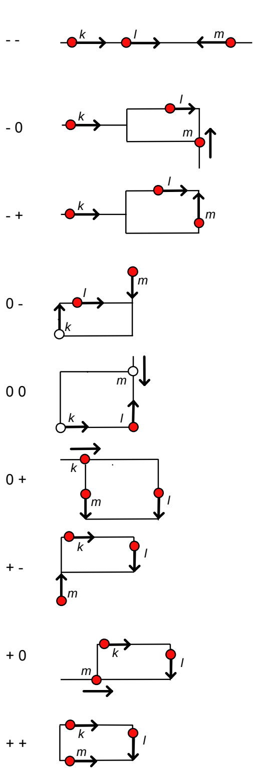
R_1 (k,l): - 0 $\otimes R_2$ (l,m): + 0 = R_3 (k,m): ?



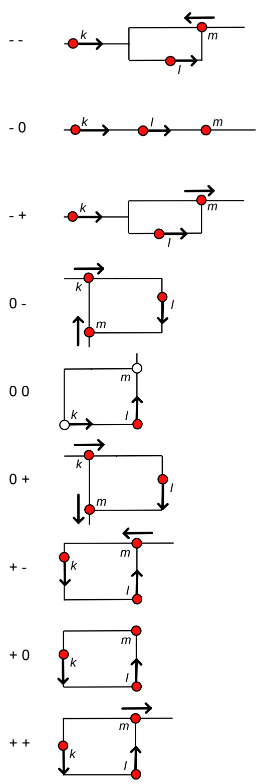
R_1 (k,l): - 0 $\otimes R_2$ (l,m):+ + = R_3 (k,m): ?



R_1 (k,l): - + $\otimes$ R_2 (l,m): - - = R_3 (k,m): ?



R_1 (k,l): - + $\otimes$ R_2 (l,m): - 0 = R_3 (k,m): ?



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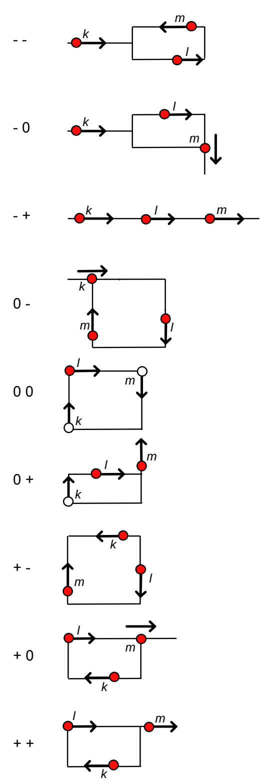

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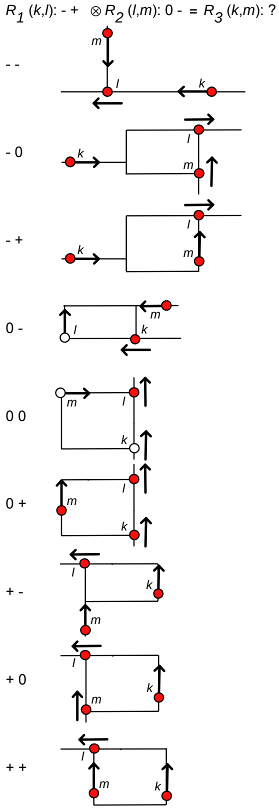

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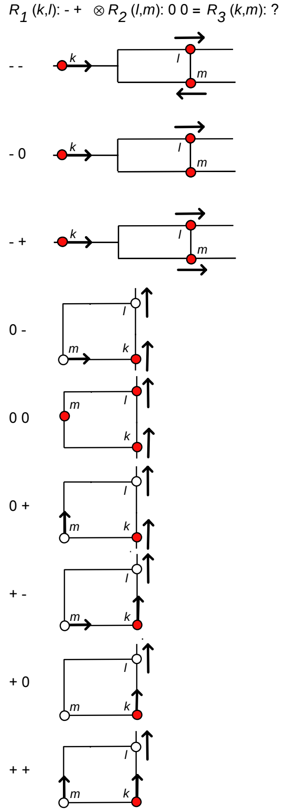

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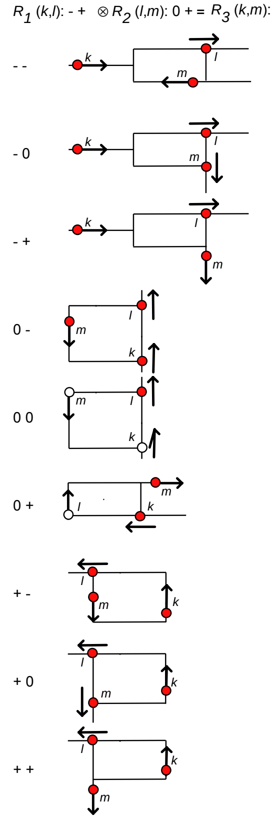

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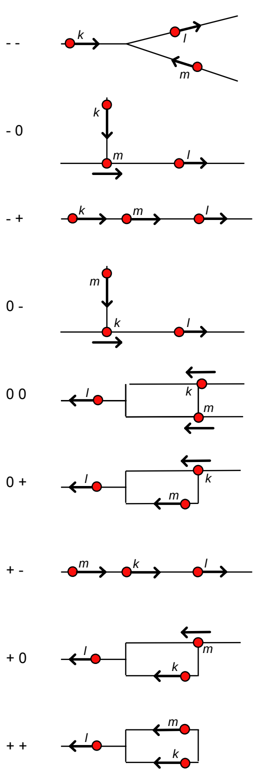

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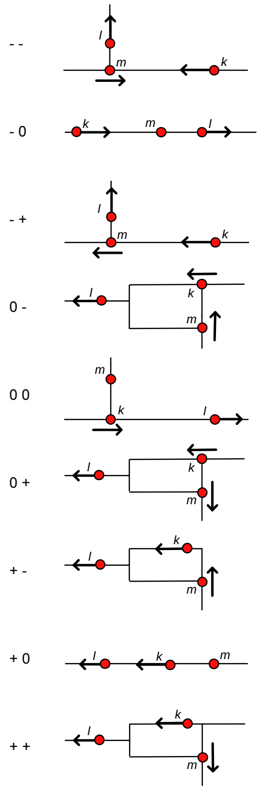

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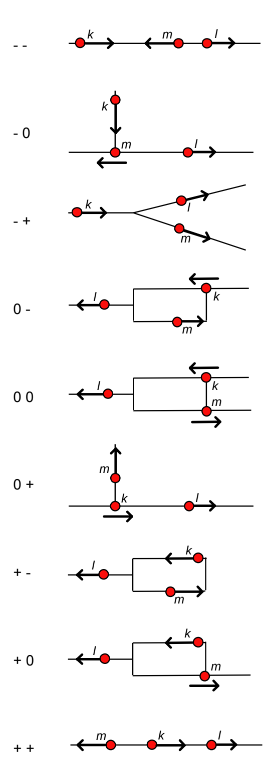

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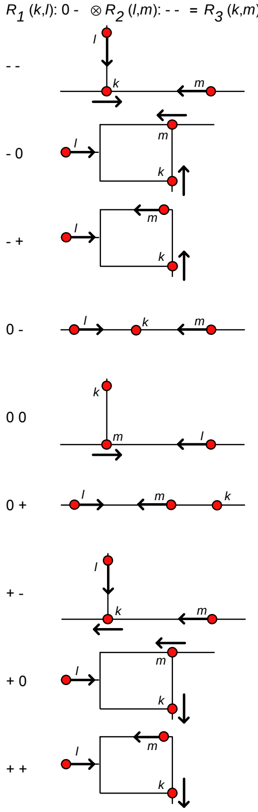

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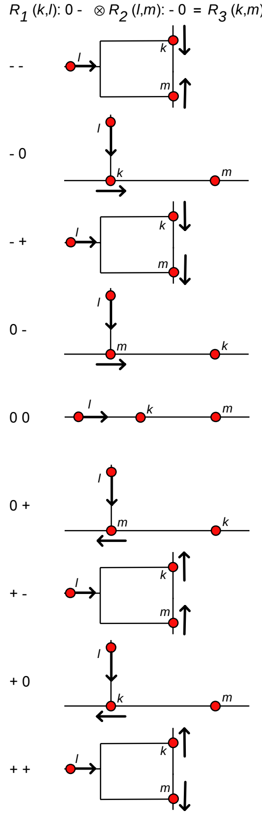

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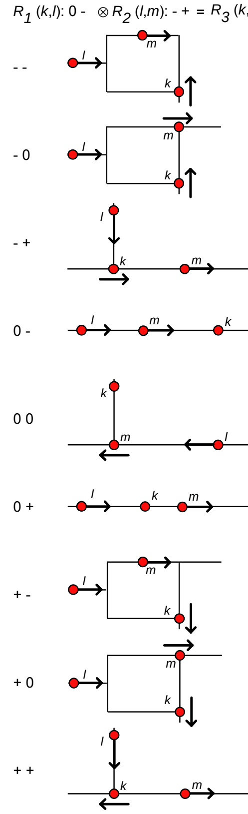

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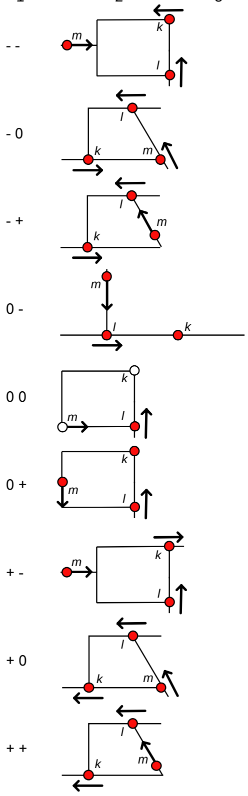

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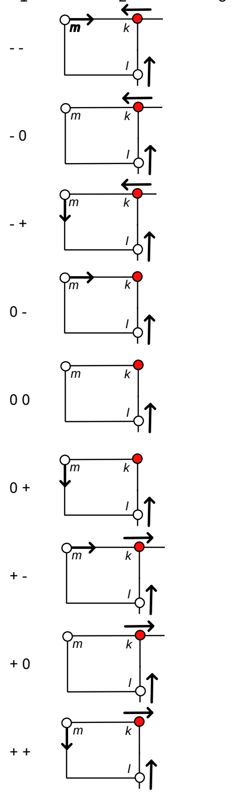

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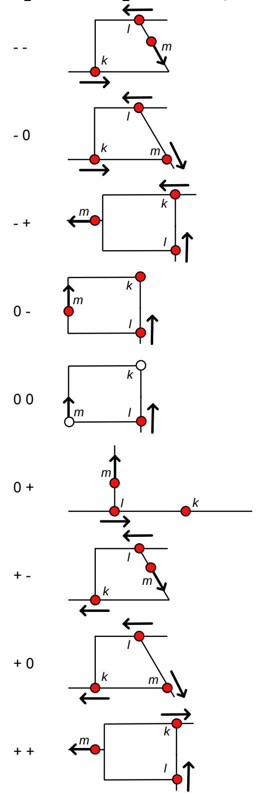

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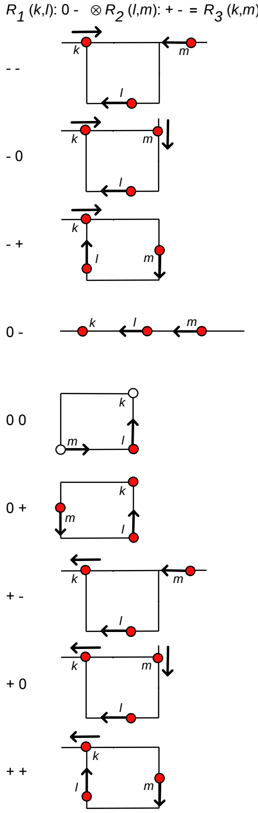

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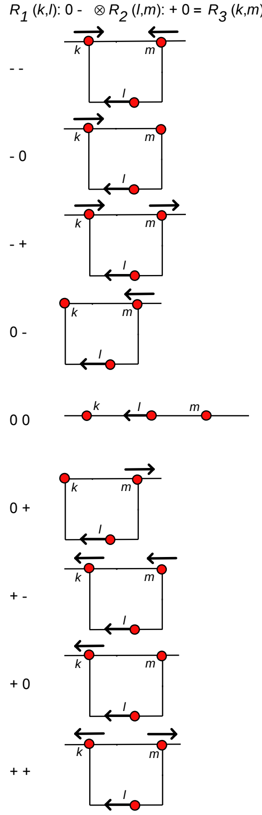

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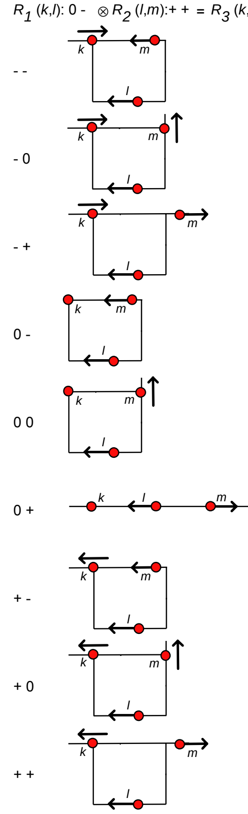

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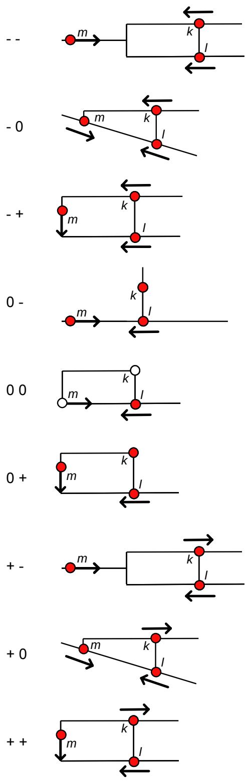

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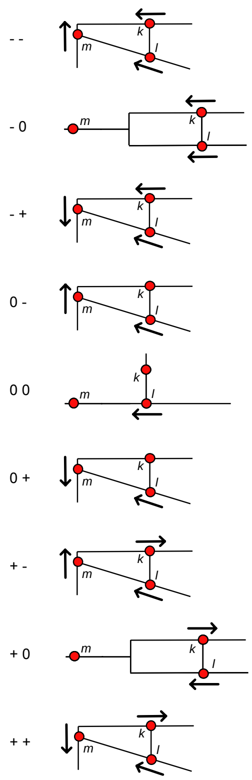

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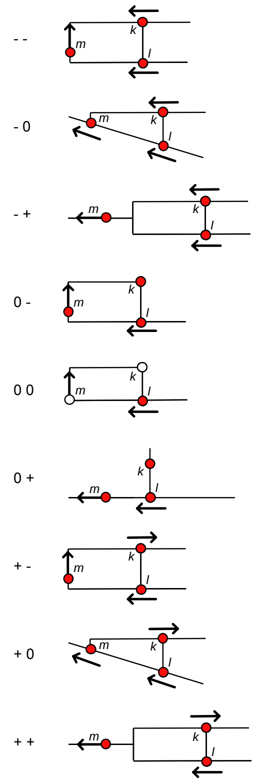

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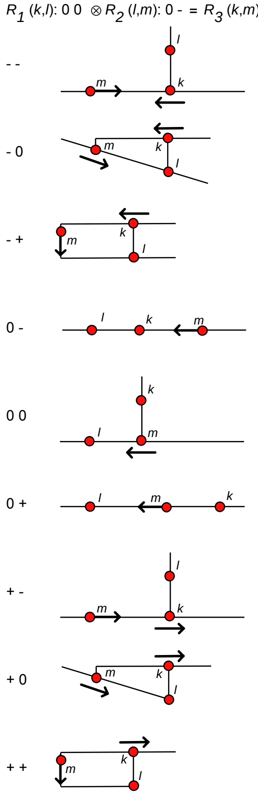
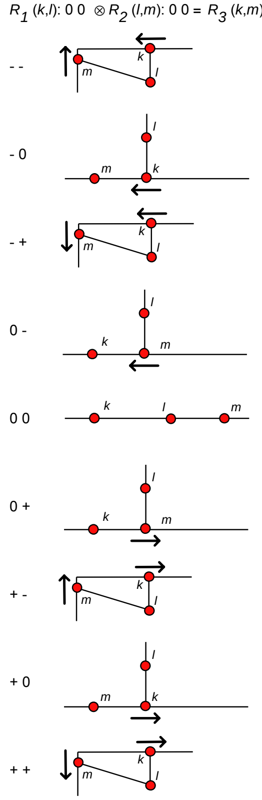
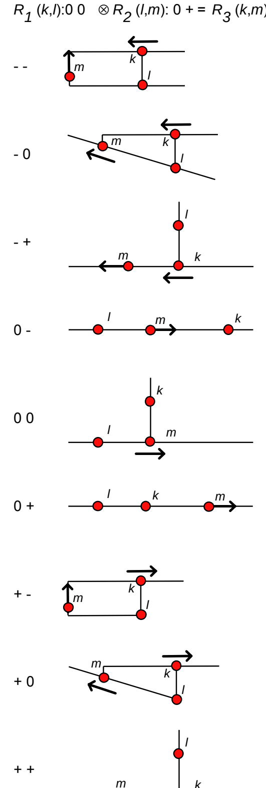


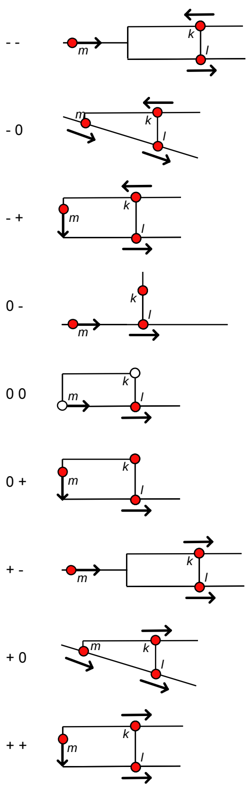
R_1 (k,l): 0 0 $\otimes$ R_2 (l,m): 0 0 = R_3 (k,m): ?



R_1 (k,l):0 0 $\otimes$ R_2 (l,m): 0 + = R_3 (k,m): ?



$R_1\,(k,l)$: 0 0 $\otimes\, R_2\,(l,m)$: + - = $R_3\,(k,m)$: ?



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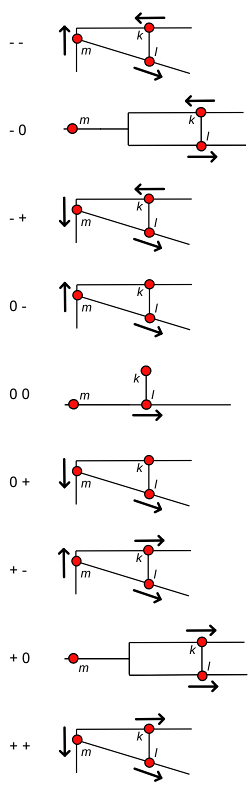
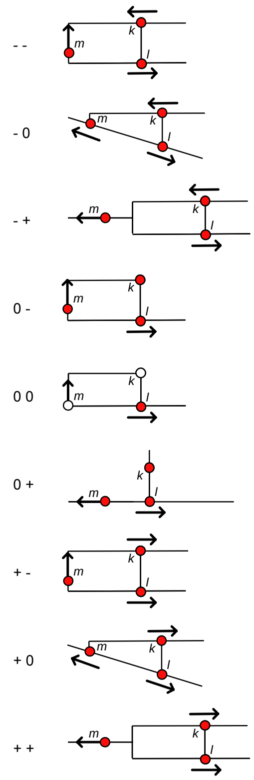


$R_1\,(k,l)$: 0 0 $\otimes\, R_2\,(l,m)$:+ + = $R_3\,(k,m)$: ?



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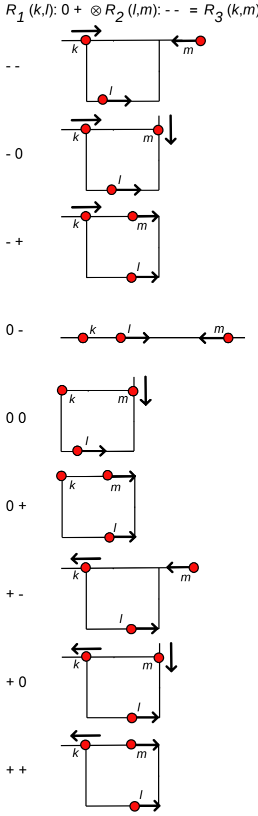

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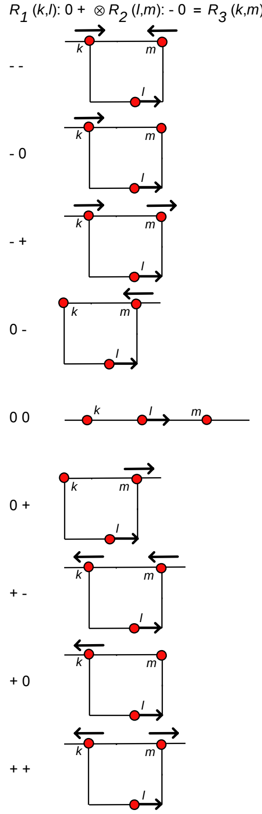

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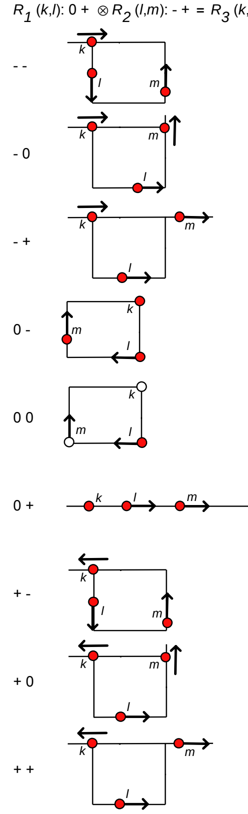
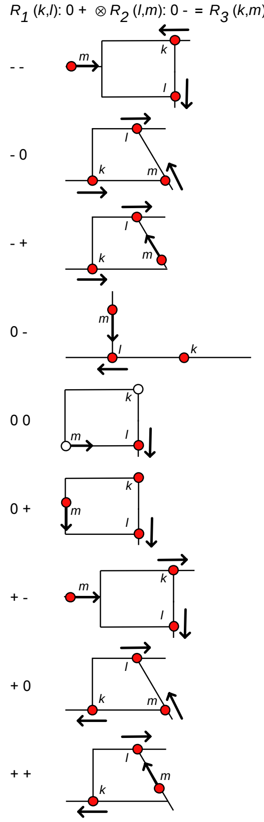
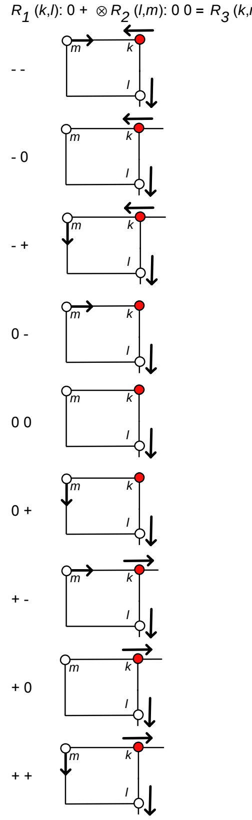


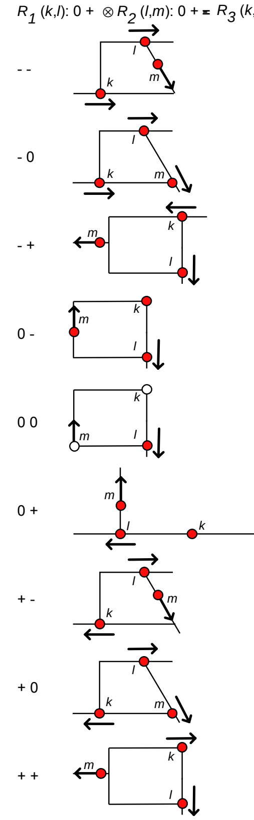
R_1 (k,l): 0 + ⊗ R_2 (l,m): 0 - = R_3 (k,m): ?



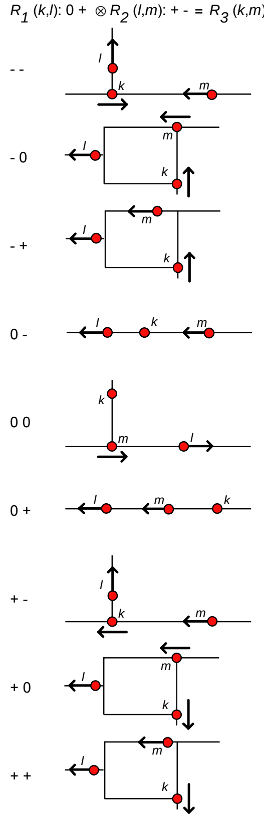
R_1 (k,l): 0 + ⊗ R_2 (l,m): 0 0 = R_3 (k,m): ?



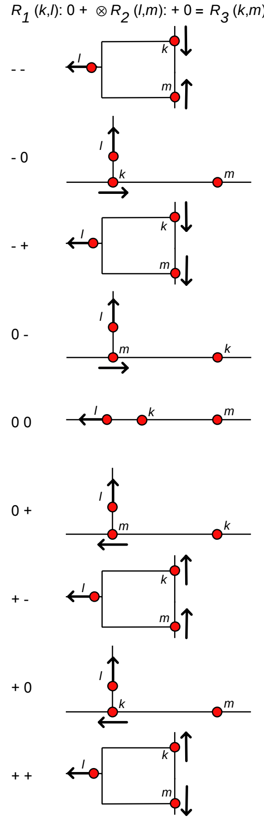
R_1 (k,l): 0 + ⊗ R_2 (l,m): 0 + = R_3 (k,m): ?



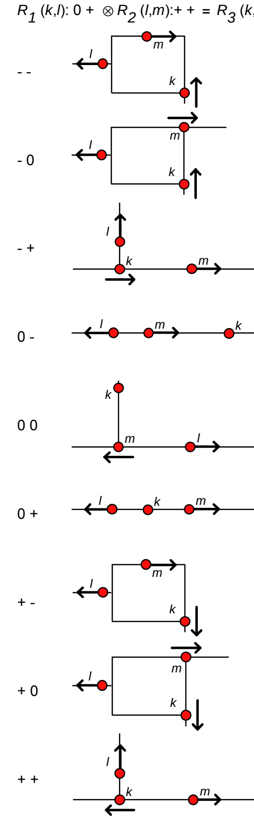
R_1 (k,l): 0 + ⊗ R_2 (l,m): + - = R_3 (k,m): ?



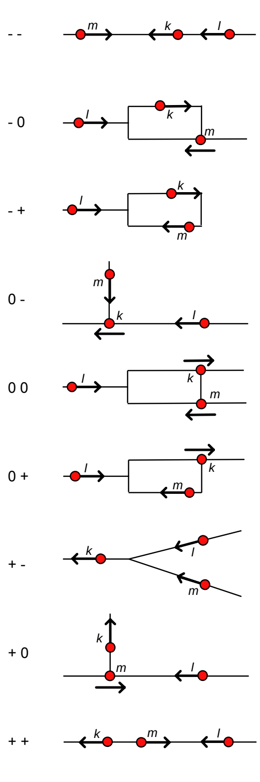
R_1 (k,l): 0 + ⊗ R_2 (l,m): + 0 = R_3 (k,m): ?



R_1 (k,l): 0 + ⊗ R_2 (l,m):+ + = R_3 (k,m): ?



$R_1\,(k,l)$: + - $\otimes\, R_2\,(l,m)$: - - = $R_3\,(k,m)$: ?



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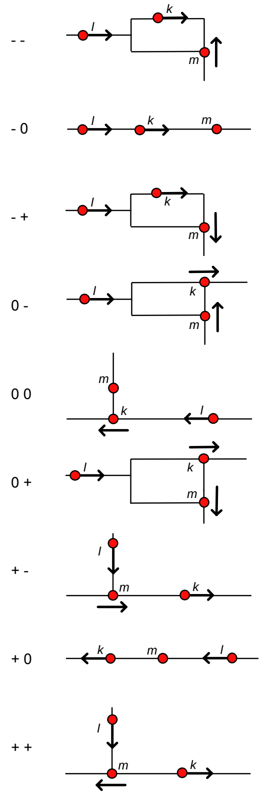
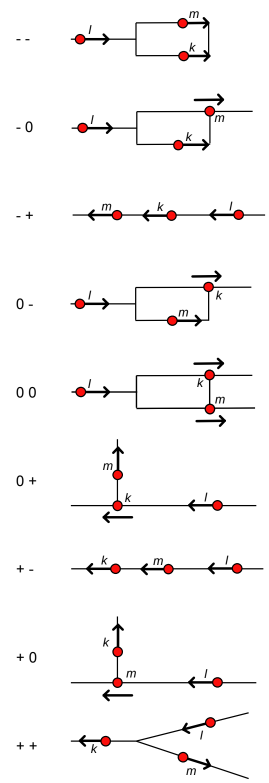
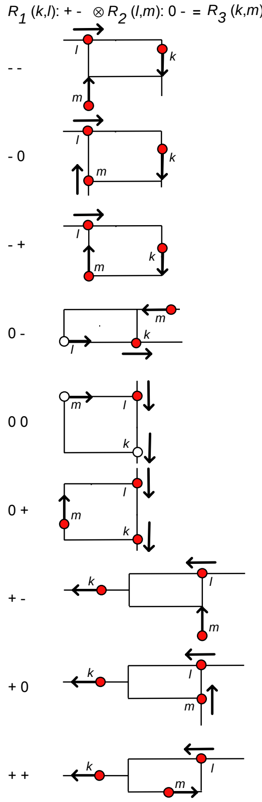


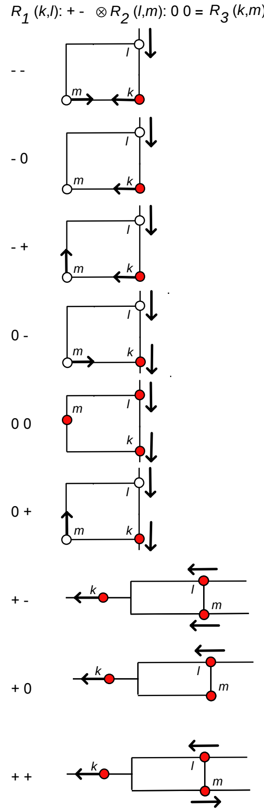
$R_1\,(k,l)$: + - $\otimes\, R_2\,(l,m)$: - + = $R_3\,(k,m)$: ?



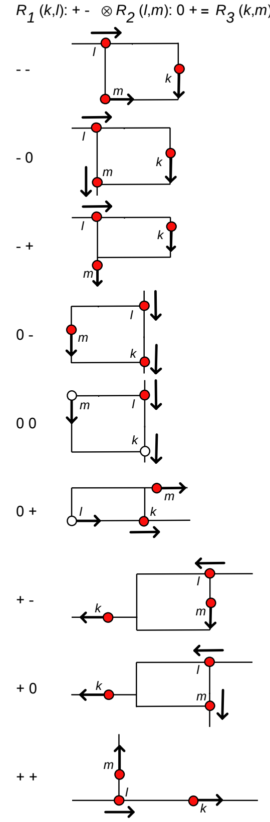
$R_1\,(k,l)$: + - $\otimes\, R_2\,(l,m)$: 0 - = $R_3\,(k,m)$: ?



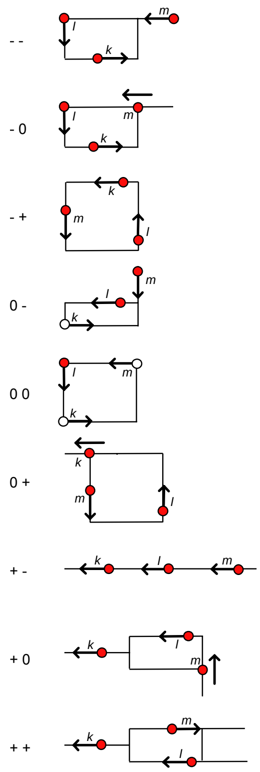
$R_1\,(k,l)$: + - $\otimes\, R_2\,(l,m)$: 0 0 = $R_3\,(k,m)$: ?



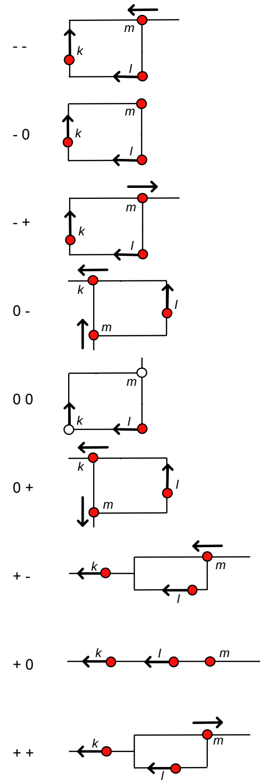
$R_1\,(k,l)$: + - $\otimes\, R_2\,(l,m)$: 0 + = $R_3\,(k,m)$: ?



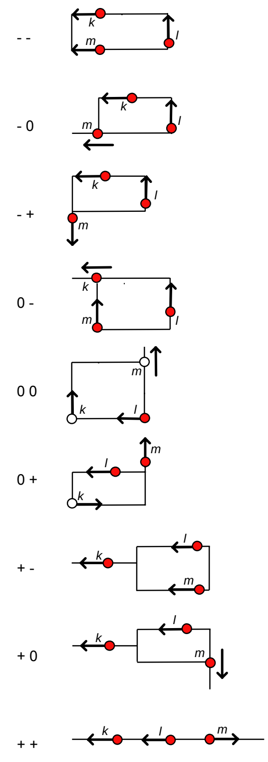
R_1 (k,l): + - $\otimes R_2$ (l,m): + - = R_3 (k,m): ?



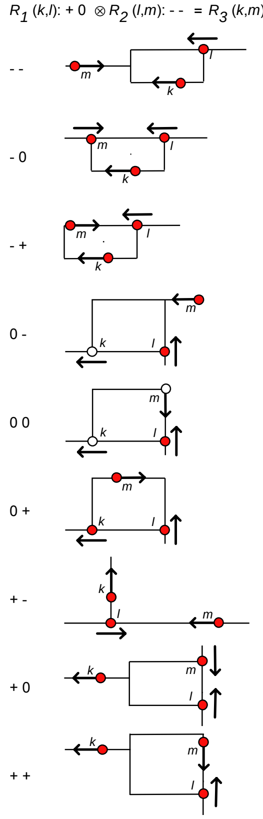
R_1 (k,l): + - $\otimes R_2$ (l,m): + 0 = R_3 (k,m): ?



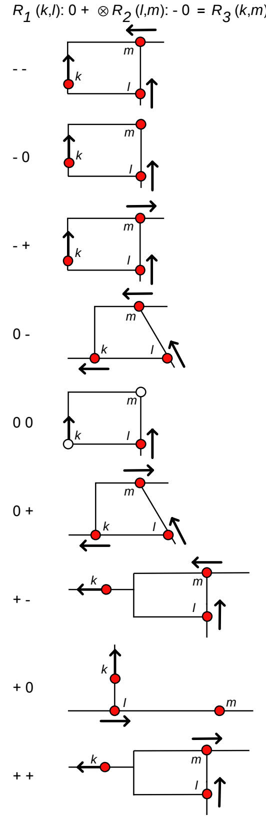
R_1 (k,l): + - $\otimes R_2$ (l,m):+ + = R_3 (k,m): ?



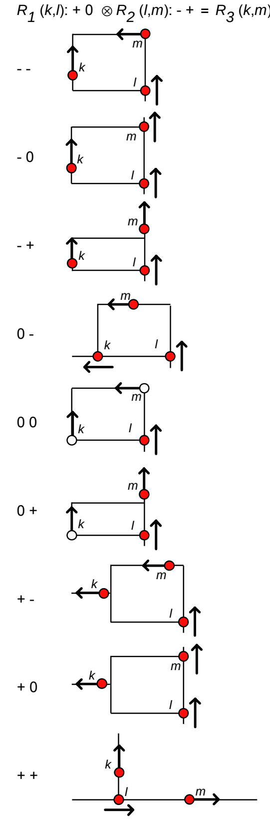
R_1 (k,l): + 0 $\otimes R_2$ (l,m): - - = R_3 (k,m): ?



R_1 (k,l): 0 + $\otimes R_2$ (l,m): - 0 = R_3 (k,m): ?



R_1 (k,l): + 0 $\otimes R_2$ (l,m): - + = R_3 (k,m): ?



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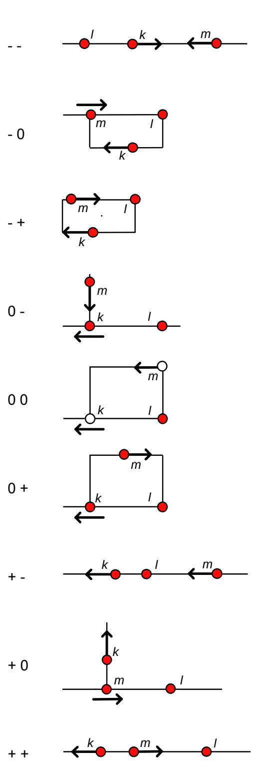

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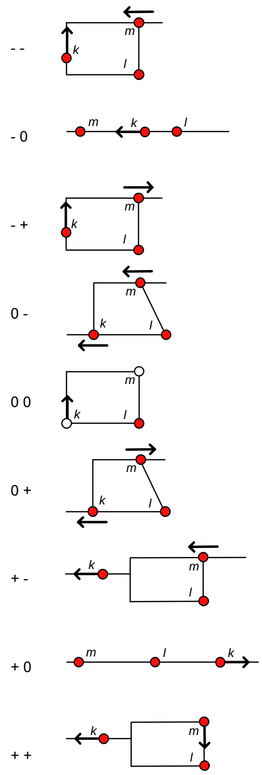

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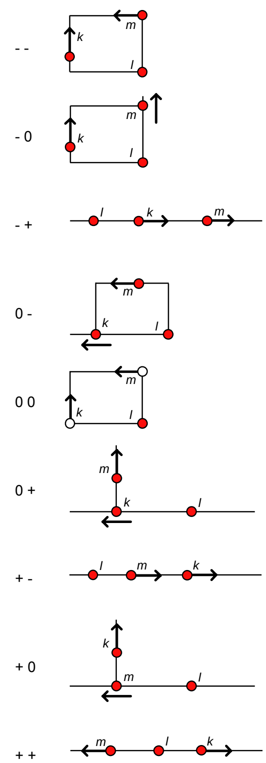
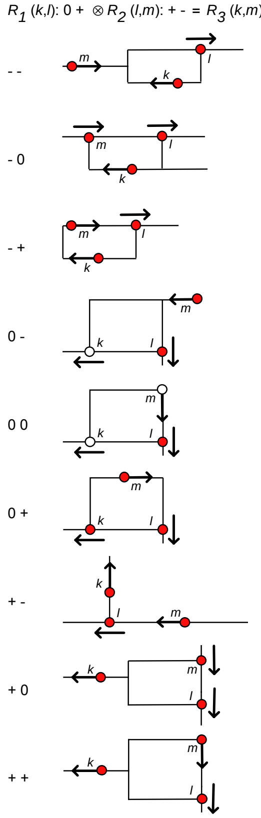
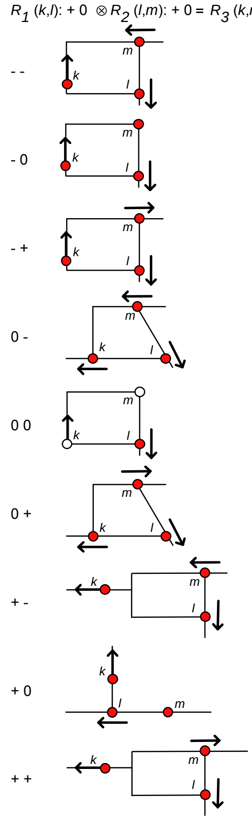


$R_1\,(k,l)$: 0 + $\otimes\, R_2\,(l,m)$: + - $=\, R_3\,(k,m)$: ?



$R_1\,(k,l)$: + 0 $\otimes\, R_2\,(l,m)$: + 0 $=\, R_3\,(k,m)$: ?



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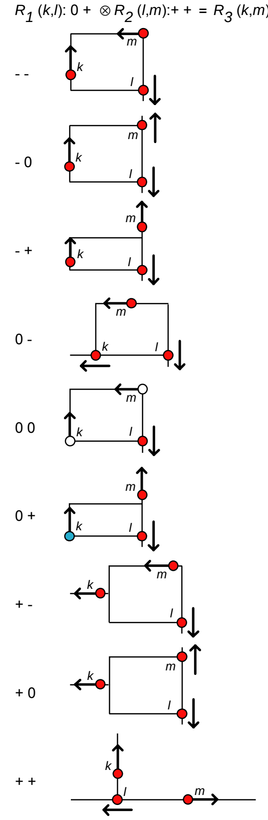
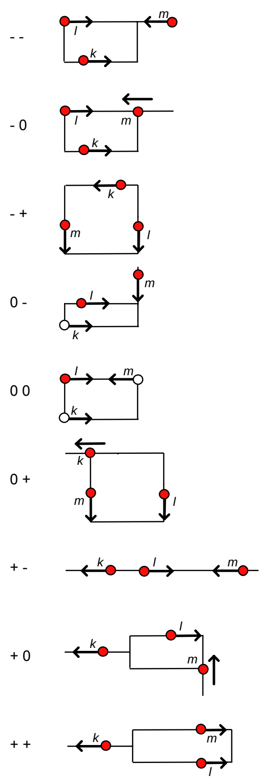
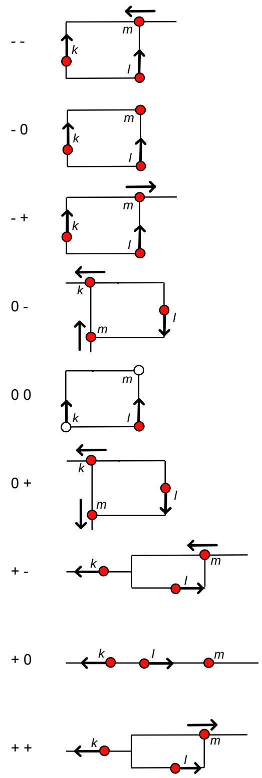


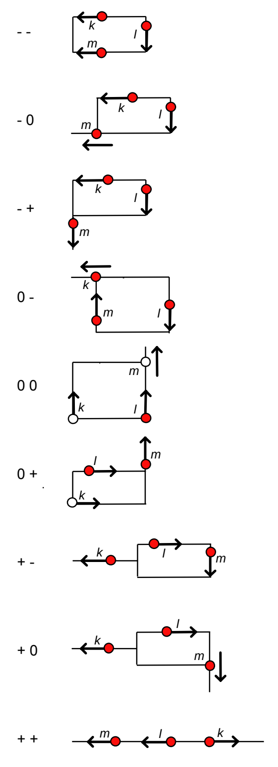
R_1 (k,l): + + $\otimes$ R_2 (l,m): - - = R_3 (k,m): ?



R_1 (k,l): + + $\otimes$ R_2 (l,m): - 0 = R_3 (k,m): ?



R_1 (k,l): + + $\otimes$ R_2 (l,m): - + = R_3 (k,m): ?



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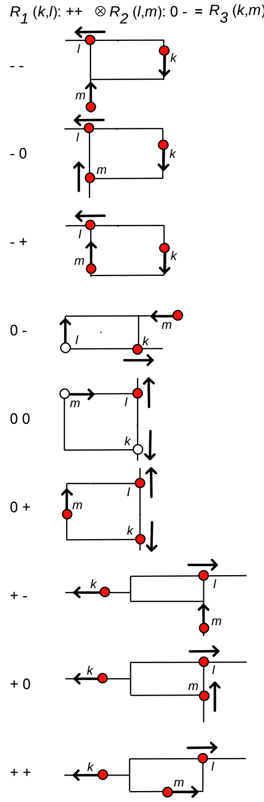

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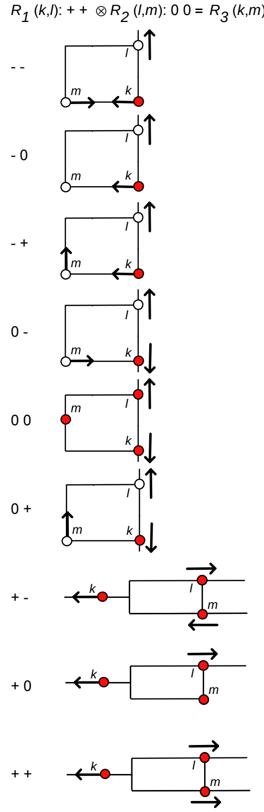
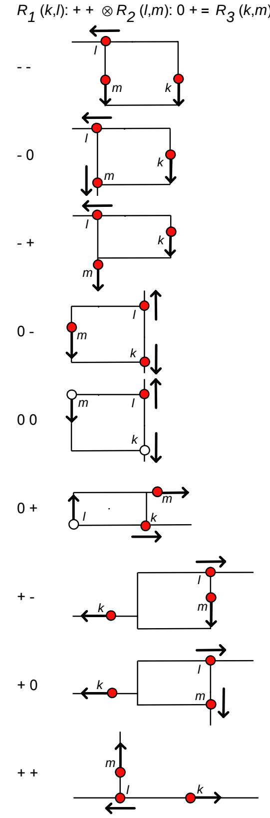
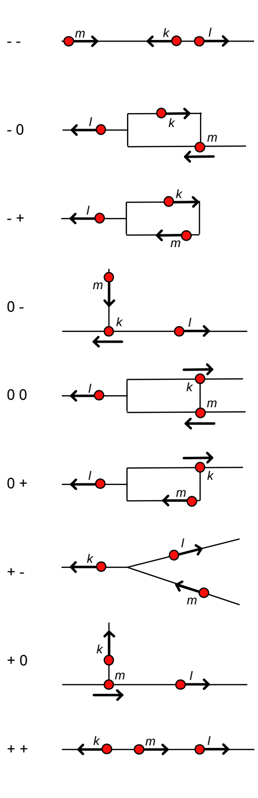


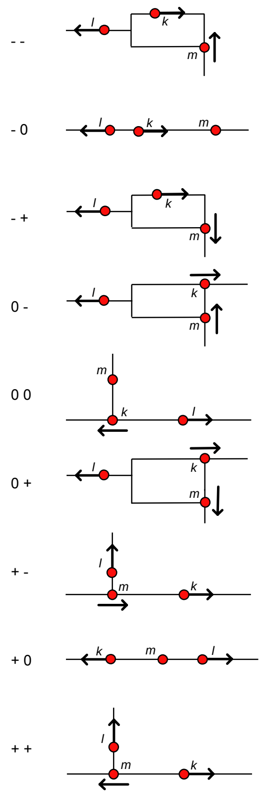
R_1 (k,l): + + $\otimes$ R_2 (l,m): 0 + = R_3 (k,m): ?



R_1 (k,l): + + $\otimes$ R_2 (l,m): + - = $R_3(k,m)$: ?



R_1 (k,l): + + $\otimes$ R_2 (l,m): + 0 = R_3 (k,m): ?



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