Marjon Blondeel, Martine De Cock, Tommaso Flaminio
Vrije Universiteit Brussel (Belgium), Ghent University (Belgium), IIIA-CSIC Campus UAB (Spain)
marjon.blondeel@vub.ac.be, martine.decock@ugent.be, tommaso@iiia.csic.es

Complexity of finitely valued Lukasiewicz possibilistic modal logics

Consider the language L_k of Łukasiewicz logic with Var being the countable set of its propositional variables, the binary connective $\to$, and truth constants $\overline{c}$ for every rational $c \in S_k = \{0, \frac{1}{k}, \ldots, \frac{k-1}{k}, 1\}$. The set $Form$ of formulas in L_k is defined as usual. Let us denote by $L(\Box)$ the language obtained by adding a unary modality $\Box$ to L. The resulting set of formulas will be denoted by $Form(\Box)$.

We will henceforth consider as models triples $M = (W, e, \pi)$ with W a set of possible worlds, $e : W \times Var \to S_k$ is a mapping that naturally extends to a mapping $W \times Form \to S_k$ by the truth functionality of $\to$ and by requiring that, for every constant $\overline{c}$, $e(w, \overline{c}) = c$, and that $\pi : W \to S_k$ is a possibility distribution. A model $M = (W, e, \pi)$ is called *normalized* if there exists a $w \in W$ such that $\pi(w) = 1$.

Given a model $M = (W, e, \pi)$ and a world $w \in W$, the truth value of a formula $\Phi \in Form(\Box)$ is defined inductively as follows. If $\Phi \in Form$, then $\|\Phi\|_{M,w} = e(w, \Phi)$ and if $\Phi = \Box\Psi$, then $\|\Box\Psi\|_{M,w} = \inf\{\pi(w') \to \|\Psi\|_{M,w'} : w' \in M\}$. The truth value of compound formulas is defined a usual.

For any formula $\Phi \in Form(\Box)$, we will denote by $\#\Phi$ its complexity which is defined inductively as follows: $\#\overline{c} = 1$, $\#p = 1$ for $p \in Var$, $\#(\Phi \to \Psi) = 1 + \#\Phi + \#\Psi$, and $\#(\Box\Phi) = 1 + \#\Phi$.

We can then prove the following lemma.

Lemma 1. *For every formula $\Phi \in Form(\Box)$ and for every (not necessarily normalized) model $M = (W, e, \pi)$ and $w \in W$, there exists a model $M' = (W', e', \pi')$ and $w' \in W'$ such that $|W'| \leq \#\Phi$ and $\|\Phi\|_{M,w} = \|\Phi\|_{M',w'}$.*

The following result fixes the complexity for both the problem $\mathbf{Sat}^{=1}$ of deciding for a formula $\Phi \in Form(\Box)$ whether there exists a model $M = (W, e, \pi)$ and $w \in W$ such that $\|\Phi\|_{M,w} = 1$, and for the problem $\mathbf{Sat}^{>0}$ of deciding whether there exists a model $M = (W, e, \pi)$ and $w \in W$ such that $\|\Phi\|_{M,w} > 0$. It is worth noticing that in (Bou et al, 2011) the authors fixed a similar problem, but with respect to generic models, to PSPACE-complete.

Theorem 1. *The decision problems $\mathbf{Sat}^{=1}$ and $\mathbf{Sat}^{>0}$ are NP-complete, even if we only consider normalized models.*

References

1. F. Bou, M. Cerami, F. Esteva. Finite-Valued Lukasiewicz Modal Logic Is PSPACE-Complete. In Proceedings of the Twenty-Second International Joint Conference on Artificial Intelligence. pp. 774-779, 2011.

Acknowledgments Blondeel has been funded by a joint Research Foundation-Flanders (FWO) project. Flaminio acknowledges partial support of the Spanish projects ARINF (TIN2009-14704-C03-03) and TASSAT (TIN2010- 20967-C04-01). Flaminio also acknowledges partial support from the Juan de la Cierva Program of the Spanish MICINN.