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A constrained optimization problem under uncertainty

Keivan Shariatmadar, Erik Quaeghebeur & Gert de Cooman
Ghent University, SYSTeMS Research Group
{Keivan.Shariatmadar,Erik.Quaeghebeur,Gert.deCooman}@UGent.be

Abstract

Consider the following optimization problem: maximize a bounded real-valued function f – defined on a set $\mathscr{X}$ – over all x in $\mathscr{X}$ that satisfy the constraint xRY, where Y is a random variable taking values in a set $\mathscr{Y}$ and R is a relation on $\mathscr{X}\times\mathscr{Y}$. The aim of our active research is to reduce this problem to a (constrained) optimization problem from which the uncertainties present in the description of the constraint are eliminated.

We investigate what results can be obtained for different types of uncertainty models for the random variable Y – linear previsions, vacuous previsions, possibility distributions, p-boxes, etc. [see, e.g., 1, 2, 4] – and for two different optimality criteria – maximinity and maximality [see, e.g., 3]. We work with general $\mathscr{X}$, $\mathscr{Y}$, and R for the combinations of uncertainty model and optimality criterion that allow it and restrict our attention to more concrete situations otherwise.

In our poster, we will present the problem statement, give illustrated solutions for the most interesting cases we have investigated, as well as discuss the strengths and weaknesses of our approach. In the remainder of this abstract, we sketch this approach and give the solutions of two of the cases we have already fully investigated.

The first thing we do is reformulate the optimization problem as a decision problem. To wit, with every x in $\mathscr{X}$ we associate a utility function G_x on $\mathscr{Y}$ that gives the constant value $f(x)$ for y in $\mathscr{Y}$ such that xRy and that gives a penalty-value L otherwise (i.e., when $x\not R y$). As the name suggest, it penalizes the fact that x such that $x\not R y$ are considered; we therefore choose $L<\inf f$. The uncertainty model associated to Y is formulated as a coherent lower prevision $\underline{P}$ (and its conjugate upper prevision $\overline{P}$) for a sufficiently rich set of gambles on $\mathscr{Y}$ [see, e.g., 4, for terminology]. Using the maximinity criterion, the optimal values for $\mathscr{X}$ are those that maximize $\underline{P}(G_x)$. Using the more conservative maximality criterion, the optimal values for $\mathscr{X}$ are those for which $\min_{z\in\mathscr{X}}\overline{P}(G_x-G_z)\geq 0$. Both finding the prevision on the sufficiently rich set of gambles $\{G_x : x\in\mathscr{X}\}$ or $\{G_x-G_z : x,z\in\mathscr{X}\}$ and checking the optimality criteria are, in general, nontrivial steps.

For the general case, when Y is described by a linear prevision P, we find that under both maximinity and maximality the optimal x are those that maximize the function on $\mathscr{X}$ that takes the value $(f(x)-L)P(\{y\in\mathscr{Y} : xRy\})$ in x.

For the concrete case $\mathscr{X}=\mathscr{Y}:=\mathbb{R}$ and $R:=\leq$, and when Y is described by a triangular possibility distribution with basis $[a,b]\subset\mathbb{R}$ and top $c\in(a,b)$, we find that under maximinity the optimal x are those that maximize the function on $(-\infty,c]$ that takes the value $f(x)$ for $x\leq a$ and $\frac{x-a}{c-a}L+\frac{c-x}{c-a}f(x)$ for x in $(a,c]$.

Keywords. Constrained optimization, maximinity, maximality, linear prevision, vacuous prevision, possibility distribution, p-box.

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