# Flocks and locally hermitian 1-systems of $Q(6, q)$

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A flock of a quadratic cone $LQ(2, q)$, with line vertex $L$, of $\mathsf{PG}(4, q)$ is a partition of $LQ(2, q) \setminus L$ in $q^2$ mutually disjoint conics such that any two distinct conics generate $\mathsf{PG}(4, q)$. If $q$ is odd and the planes $\pi_1, \dots, \pi_{q^2}$ of the elements of the flock pairwise intersect in internal, resp. external, points of $LQ(2, q)$, then we speak of an i-flock, resp. e-flock. It is shown that to every i-flock of $LQ(2, q)$, a locally hermitian 1-system of $Q(6, q)$ is associated and conversely.

Next, the i-flock associated with the unique semi-classical non-hermitian spread $\mathcal{S}_{[9]}$ of the hexagon $\mathsf{H}(q)$, $q$ odd and $q \equiv 1 \bmod 3$ (which is locally hermitian at some line $L$) is studied. This yields a geometric construction of $\mathcal{S}_{[9]}$ starting from a rational normal cubic scroll $\mathcal{R}^3$ having $L$ as directrix line: the conics on $\mathcal{R}^3$ determine the $q^2$ conic planes of the i-flock and hence the i-flock can be reconstructed from the rational normal cubic scroll. Surprisingly this geometric construction not only yields the 1-system $\mathcal{S}_{[9]}$; it turns out that different cubic scrolls may give rise to non-isomorphic locally hermitian 1-systems of $Q(6, q)$. In particular, there are $\frac{q-3}{2}$ orbits in the set of all non-hermitian locally hermitian 1-systems of $Q(6, q)$ constructed from a cubic scroll, under the subgroup of $\mathsf{PGL}(7, q)$ fixing $Q(6, q)$.

Finally it is shown that a locally hermitian non-hermitian 1-system of $Q(6, q)$, $q$ odd, is semi-classical if and only if it arises from a rational normal cubic scroll $\mathcal{R}^3$ with directrix line $L \subseteq Q(6, q)$ and with the property that all points of $\mathcal{R}^3 \setminus L$ are internal points of $Q(6, q)$. As it is possible to determine all such cubic scrolls, this yields a complete characterization and determination of locally hermitian semi-classical 1-systems of $Q(6, q)$.

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