On the smallest minimal blocking sets of $\mathrm{Q}(2n, q)$

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Using results on the size of the smallest minimal blocking sets of $\mathrm{Q}(4, q)$, q even, of Eisfeld, Storme, Szőnyi and Sziklai [2], and results concerning the number of internal nuclei of $(q+2)$-sets in $\mathrm{PG}(2, q)$, q even, of Bichara and Korchmáros [1], together with projection arguments, we obtain the following characterization of the smallest minimal blocking sets of $\mathrm{Q}(6, q)$, q even and $q \geqslant 32$:

Theorem 1 *Let $\mathcal{K}$ be a minimal blocking set of $\mathrm{Q}(6, q)$, q even, $|\mathcal{K}| \leqslant q^3 + q$, $q \geqslant 32$. Then there is a point $p \in \mathrm{Q}(6, q) \setminus \mathcal{K}$ with the following property: $T_p(\mathrm{Q}(6, q)) \cap \mathrm{Q}(6, q) = p\mathrm{Q}(4, q)$ and $\mathcal{K}$ consists of all the points of the lines L on p meeting $\mathrm{Q}(4, q)$ in an ovoid $\mathcal{O}$, minus the point p itself, and $|\mathcal{K}| = q^3 + q$.*

Replacing the used results on $\mathrm{Q}(4, q)$ by a computer result on $\mathrm{Q}(4, 5)$, we can apply the projection arguments to prove the theorem for the case $q = 5$. Generalizing the proof for $\mathrm{Q}(6, 5)$ we find the characterization of the smallest minimal blocking sets of $\mathrm{Q}(8, 5)$:

Theorem 2 *Let $\mathcal{K}$ be a minimal blocking set of $\mathrm{Q}(8, 5)$, $|\mathcal{K}| \leqslant q^4 + q^2$. Then there is a line $L \subset \mathrm{Q}(8, 5)$, $L \cap \mathcal{K} = \emptyset$, with the following property: $T_L(\mathrm{Q}(8, 5)) \cap \mathrm{Q}(8, 5) = L\mathrm{Q}(4, 5)$ and $\mathcal{K}$ consists of all the points of the lines M on p_i, $p_i \in L$, meeting $\mathrm{Q}(4, q)$ in an ovoid $\mathcal{O}$, minus the points p_i themselves, and $|\mathcal{K}| = q^4 + q^2$.*

We will discuss the use of inductive arguments the obtain the characterization of the smallest minimal blocking sets of $\mathrm{Q}(2n, 5)$.

References

1. A. Bichara and G. Korchmáros, *Note on $(q + 2)$-sets in a Galois plane of order q*, Combinatorial and geometric structures and their applications (Trento, 1980), pages 117–121. North-Holland, Amsterdam, 1982.
2. J. Eisfeld, L. Storme, T. Szőnyi, and P. Sziklai, *Covers and blocking sets of classical generalized quadrangles*, Discrete Math., 238(1-3):35–51, 2001.