

XXXII International Seminar on Stability Problems for Stochastic Models

## Stability of retrial queueing system with constant retrial rate

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We study the stability of a single-server retrial queueing system with constant retrial rate and general input and service processes. In such system the external (primary) arrivals follow a renewal input with rate  $\lambda$ . The system also has service times with rate  $\mu$ . If a new customer finds all servers busy and the buffer full, it joins an infinite-capacity virtual buffer (or *orbit*). An orbital (secondary) customer attempts to rejoin the primary queue after an exponentially distributed time with rate  $\mu_0$ .

First, we present a review of some relevant recent results related to the stability criteria of similar systems. Sufficient stability conditions were obtained in Avrachenkov and Morozov [1] and have the following form:

$$(\lambda + \mu_0)P_{loss} < \mu_0, \quad (1)$$

where  $P_{loss}$  is a stationary loss probability in the majorant loss system. The presented statement holds for a rather general retrial system. However, only in case of Poisson input an explicit expression is provided; otherwise one has to rely on simulation.

On the other hand, the stability criteria derived in Lillo [2]

$$\frac{\lambda(\mu + \mu_0)^2}{\mu \left[ \lambda\mu[1 - C(\mu + \mu_0)] + \mu_0(\mu + \mu_0) \right]} < 1, \quad (2)$$

where

$$C(s) = \int_0^\infty e^{-xs} dF(x), \quad s > 0 \quad (3)$$

can be easily computed, but hold only for the case of exponential service times.

We present new sufficient stability conditions, which are less tight than the ones obtained in Avrachenkov and Morozov [1], but have an analytical expression under rather general assumptions. A key assumption is that the input intervals belong to the class of *new better than used* (NBU) distributions. The new condition is based on the connection between  $P_{loss}$  and  $P_{busy}$  (stationary

busy probability) in the majorant loss system. This statement was obtained in Morozov and Nekrasova [3] and can be expressed as:

$$P_{loss} = 1 - \frac{1}{\rho} P_{busy}. \quad (4)$$

We also illustrate the accuracy of these conditions (in comparison with known conditions when possible) for a number of non-exponential distributions.

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### References

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