

## Solving Billions of Unknowns using the Parallel MLFMA and a Tier 1 Supercomputer

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Algorithmic improvements to the parallel, distributed-memory Multilevel Fast Multipole Algorithm (MLFMA) have resulted in implementations with favorable weak scaling properties. This allows for the simulation of increasingly larger electromagnetic problems, provided that sufficient computational resources are available (B. Michiels *et al.*, “Weak Scalability Analysis of the Distributed-Memory Parallel MLFMA”, *IEEE Trans. Antennas Propag.*, **61**(11), 2013). Recently, we were able to benchmark our implementation on the Flemish Supercomputing Centre's (VSC) Tier 1 supercomputer. This cluster consists of 512 nodes interconnected by an FDR Infiniband network. Each node contains two 8-core Intel Xeon E5-2670 processors and 64 GByte of RAM. The complete system hence provides 8192 CPU cores and 32 TByte of RAM in total.

On this system, full-wave simulations were performed of both an extremely large perfectly electrically conducting (PEC) sphere and an aircraft-shaped 'Thunderbird' geometry. Both problems were formulated using the combined field integral equation (CFIE), discretized in over respectively 3 and 2.5 billion unknowns and solved using 4096 CPU cores and 25 TByte of RAM (B. Michiels *et al.*, “Full-wave Simulations of Electromagnetic Scattering Problems with Billions of Unknowns”, *IEEE Trans. Antennas Propag.*, in press). To the best of our knowledge, this is the largest number of unknowns and the highest amount of parallel processes reported to date, for this type of simulation. The time for one matrix-vector product was 4m 53s for the sphere and 4m 8s for the Thunderbird geometry.

Even though the accuracy of the bistatic radar cross section (RCS) of the sphere was found to be in excellent concordance with the analytic Mie series solution, the iterative TFQMR solver failed to reduce the relative residual error below approximately 3-4% after about hundred iterations. This was observed for both the sphere and the Thunderbird geometries. Remarkably, slightly smaller problems, e.g., the same Thunderbird geometry discretized in around 1 billion unknowns, converge below the 1% threshold without problem with otherwise identical settings. These emerging convergence problems could be caused by a high condition number of the system matrix, even though the CFIE was used. Alternatively, a possible cause is the numerical round-off error arising from the summation of billions of floating point numbers, as happens in the TFMQR algorithm when computing inner products of vectors with dimension equal to the number of unknowns. Unless special summation techniques such as Kahan summation are used, the relative errors caused by this follow a random-walk and are of the order of  $\epsilon N^{0.5}$  (around 0.006 in this case, as single precision computations were used). In any case, these convergence problems could put a limit on the size of the problems that can be handled using single precision computations, as is currently commonly used in parallel MLFMA simulations.

At the time of conference, we will highlight not only the results and successes, but also the difficulties and challenges of dealing with such large-scale compute infrastructure.