LONGITUDINALLY DIRECTED BANK EFFECTS

Evert Lataire, Ghent University, Belgium.

Marc Vantorre, Ghent University, Belgium.

Guillaume Delefortrie, Flanders Hydraulics Research, Belgium.

IN CONFINED WATERWAYS, THE WATER DISPLACED BY A SHIP IS SQUEEZED BOTH UNDER AND ALONG THE HULL. THE RESULTING PRESSURE DISTRIBUTION ON THE HULL CAUSES A COMBINATION OF FORCES AND MOMENTS ON THE VESSEL. IF THE PRESENCE OF ONE OR MORE BANKS GENERATE EXTRA HYDRODYNAMIC PRESSURES, THIS COMBINATION OF FORCES AND MOMENT IS KNOWN AS BANK EFFECTS. BASED UPON THE MODEL TESTS CARRIED OUT IN THE FULLY AUTOMATED TOWING TANK FOR MANOEUVRES IN SHALLOW WATER (COOPERATION FLANDERS HYDRAULICS RESEARCH - GHENT UNIVERSITY, AT FLANDERS HYDRAULICS RESEARCH IN ANTWERP, BELGIUM), A MATHEMATICAL MODEL FOR THE INCREASED RESISTANCE BECAUSE OF THE PRESENCE OF BANKS IS PROPOSED. THE INCREASE IN RESISTANCE IS NOT ONLY CAUSED BY A MORE CONFINED BLOCKAGE RATIO BUT ALSO BY THE RELATIVE LATERAL POSITION OF THE SHIP IN THE CROSS SECTION. THE PROPOSED MATHEMATICAL MODEL TAKES INTO ACCOUNT THE SHAPE OF THE CROSS SECTION WITHOUT OVERESTIMATING THE INFLUENCE OF THE BATHYMETRY AT A LARGER DISTANCE, NOR BY UNDERESTIMATING THE IMPACT OF THE BATHYMETRY CLOSER TO THE VESSEL. THIS IS OBTAINED BY DEFINING AN EQUIVALENT BLOCKAGE, AS WELL AS A TUCK NUMBER THAT TAKES INTO ACCOUNT THE INCREASED RETURN FLOW ALONG A SHIP IN CONFINED WATERS.

INTRODUCTION

Most ships are designed for sailing in open and deep water but the destinations they call at are often located in shallow, restricted and/or confined waters. This is the case for harbours as well as for the navigation areas in the approach to these harbours such as manmade canals, channels, dredged trenches and other fairways, both natural and artificial. Over the last decades the main dimensions of ships have increased dramatically. This is especially the case for container carriers, but also for LNG-carriers and bulk carriers. The approach channels however did not increase at the same rate (at all). Relatively more vessels will spend more time in shallow and/or restricted fairways.

Sailing in shallow and/or restricted waters results in a wide range of hydrodynamic influences on the ship as experienced by the navigator on board. Among other effects on the manoeuvrability, the resistance of the ship sailing through the water will increase if the water depth or channel width decreases. A mathematical model for this increase will be proposed.

MODEL TESTS

Over the last decade an enormous amount of model tests have been carried out in the Towing Tank for Manoeuvres in Shallow Water (cooperation Flanders Hydraulics Research – Ghent University) at Flanders Hydraulics Research in Antwerp, Belgium with the specific focus on bank effects. This systematic series consists of more than 12000 unique model tests
.

Tests have been carried out with eleven different ship models of about 4 m long (2 container carriers, 4 tankers, 3 RoRo-vessels, 1 inland vessel, 1 Wigley hull), some at different loading conditions. Most ships have been tested in three different water depths varying from 10% to 100% under keel clearance. The experiments were carried out at different combinations of forward speeds and (non-negative) propeller rotation.

Twenty-five different bank geometries were installed in the fully automated shallow water towing tank of Flanders Hydraulics Research (LxBxh 80x7x0.5m³) [1]. These banks varied from vertical quay walls and other steep sloped banks up to gently sloped surface piercing banks of 1/5 and 1/8. Some bank slopes had an underwater knuckle line resulting in semi-submerged bank geometries. The ship models were towed at about five different lateral positions from these installed banks.

During the captive manoeuvring tests the ship model was rigidly connected to the planar motion mechanism of the towing tank but was free to heave and trim. As such the forces and moments acting on the ship model (hull X, Y, N, K; rudder F_{NR}, F_{TR}, Q_R; propeller T_P, Q_P) are measured and registered, as well as (relative) positions of the hull (running sinkages z_{VF}, z_{VA}), rudder (angle δ) and propeller (rate of turn n).

The measured forces in the longitudinal direction of the ship model are thus: the overall longitudinal force X (connection between ship model and main carriage of the towing tank), the thrust delivered by the propeller T_P (measured in line with the propeller shaft) and the longitudinal force on the rudder decomposed of F_{NR} and F_{TR} depending of the rudder angle δ. If the rudder angle is fixed to zero (as during most of the model tests) then the longitudinal directed part of the rudder force equals F_{TR}.

BANK EFFECTS

A moving vessel continuously displaces an amount of water, which travels along the hull generating a return flow (in the opposite direction of the direction of movement of the ship). This return flow (δV in Figure 1) results in a pressure drop on the hull (Bernoulli principle). Because the pressure on the free water surface must be equal to the atmospheric pressure, the water level close to the hull will drop, and thus the hull itself will move downwards as well. This steady vertical motion is known as squat and can be decomposed in the running sinkage at the fore z_{VF} and at the aft perpendicular z_{VA}, or in a combination of a trim and a mean sinkage.
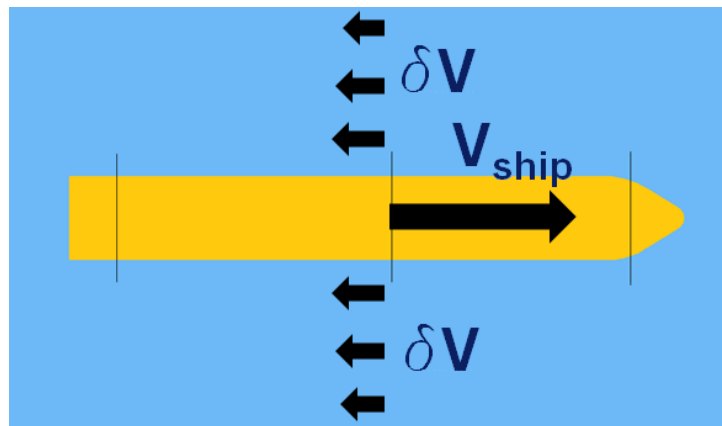
FIGURE 1 RETURN FLOW AND FORWARD SHIP SPEED IN OPEN WATER

LATERAL FORCE AND YAW MOMENT

In more shallow water the magnitude of the return flow will increase because there is less space under the ship for the evacuation of the same quantity of mass; as a consequence the velocity of the return flow must increase. This effect is even amplified when the return flow is also limited in combination with the presence of banks. At the side of the ship at the closest bank the return flow velocity will be larger than at the less restricted side (Figure 2). This larger return flow will result in a larger pressure drop (and the stream lines will be closer to each other); in most situations this results in an overall attraction force directed towards the closest bank in combination with a yaw moment directed bow-away from the bank.
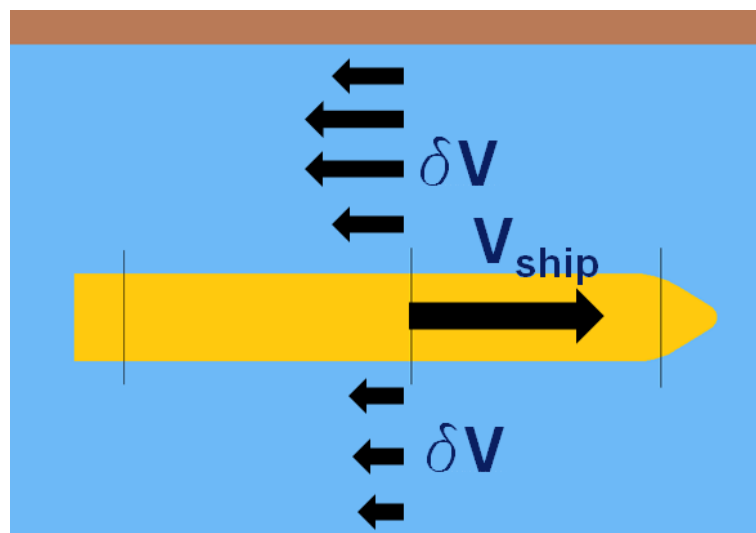

FIGURE 2 RETURN FLOW WITH A BANK PRESENT AT THE PORT SIDE OF THE SHIP

In general, the larger the ratio between the midship area of the ship A_M and the area of the cross section Ω, known as blockage ratio m, the larger the magnitude of the lateral force and the yaw moment will be.

LONGITUDINAL FORCE

The changed pressure distribution induced on the hull because of the presence of a bank or banks does not only affect the lateral force and yaw moment but also initiates an extra longitudinal force. An increased ship resistance is observed when sailing in shallow water and in a more confined cross section. A mathematical model is proposed to be able to calculate this augmented resistance based upon the model tests carried out in the towing tank. The longitudinal bank force X_{BANK}, i.e. the increase of resistance due to the presence of (a) bank(s) relative to the ship's resistance in horizontally unrestricted but shallow water, is formulated based upon the superposition principle which is used in the mathematical models of the ship manoeuvring simulators at FHR. First the longitudinal bank force X_{BANK} must be extracted from the measured longitudinal forces during the tests. This is not that straightforward because the force X_{BANK} cannot be measured separately.

As mentioned before, three different types of longitudinal forces are measured during the model tests:

- the overall longitudinal force X (measured between ship model and carriage);
- the thrust T_P delivered by the propeller(s), measured on the propeller shaft(s);
- the force X_R on the rudder directed in the longitudinal direction of the vessel. If the rudder angle δ is set to zero then:

$$X_R(\delta = 0) = F_{TR}$$

<div align="right">EQUATION 1</div>

As a consequence the longitudinal force X_{BANK} must be derived from these three measured longitudinal forces X, X_R and T_P. In equation 2 these three longitudinal forces are combined in one equation (t being the thrust deduction fraction):

$$X = X_H + (1 - t)T_P + X_R + X_{BANK}$$

<div align="right">EQUATION 2</div>

X_H being the longitudinal force action on the hull (excluding rudder and propeller).

The open water resistance R_{OW} (including the effect of water depth, excluding the effect of banks) of the vessel is the sum of the longitudinal force on the hull and the longitudinal forces on the appendages:

$$R_{OW} = X_H + F_{TR}$$

<div align="right">EQUATION 3</div>

From Equation 2 and Equation 3 the force X_{BANK} can be written as:

$$X_{BANK} = X - (R_{OW} + (1 - t)T_P)$$

<div align="right">EQUATION 4</div>

Equation 4 is a simplification of reality since the presence of a bank very close to the appendages (rudder and propeller) will have an influence on both F_{TR} as well as T_P. The propeller-hull interaction through the thrust deduction fraction t can also be expected to be influenced by the surrounding bathymetry. For reasons of simplicity all these influences are neglected and only the overall longitudinally directed bank effects X_{BANK} are modelled.

Most ship models are tested in open water to define the self-propulsion point (equilibrium between propeller rate and forward speed). At the self-propulsion point, the overall longitudinal force X is zero by definition:

$$0 = R_{OW} + (1 - t)T_P$$

<div align="right">EQUATION 5</div>

The longitudinal hull force and thrust deduction fraction are derived from dedicated model tests [2] carried out in the towing tank at different water depths without banks installed. The propeller rate for self-propulsion is sought from these tests. As such, for each forward speed a propeller rate is defined so the delivered thrust on the propeller shaft T_P results in the absence of a longitudinal force (X=0) between the ship model and the carriage of the towing tank.

For determining the longitudinal bank induced force, model tests are carried out at different forward speeds with the same ship model along installed banks in the towing tank. These tests are (among others) carried out at propeller rates that match to the corresponding open water self-propulsion point (at the same forward speed). When sailing in the confined cross sections at these forward speeds and propeller rate combination, the measured longitudinal force X is no longer absent but a negative force or augmented resistance is measured. This longitudinal force is based upon previous simplifications assumed to be the longitudinal bank effect X_{BANK} sought for:

$$X_{BANK} = X - (R_{OW} + (1 - t)T_P) = X$$

<div align="right">EQUATION 6</div>

The longitudinal force is scaled from model scale to full scale according to [2]. This includes a part of the longitudinal force to be scaled according to Froude's law and a part scaled according to the Reynolds's law. The bank effects are mainly generated by inertial and gravitational forces and therefore, as a first step, the force X_{BANK} will be scaled (entirely) to full scale according to Froude's law.

The behaviour of the force X_{BANK} on the lateral position in a cross section is different from the lateral forces and yaw moment. When sailing on the centre line of a symmetric cross section the force Y and moment N will be zero but the longitudinal force X_{BANK} will not. At this symmetric position there will be an influence of the banks on the longitudinal force of the vessel. A new type of blockage ratio will be suggested which takes into account the entire geometry of the cross section as well as the relative position of the vessel in the cross section.
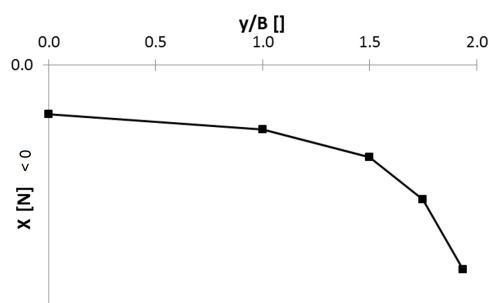

FIGURE 3 INFLUENCE OF THE LATERAL POSITION ON THE LONGITUDINAL BANK EFFECT X_{BANK} IN A RECTANGULAR CROSS SECTION

Figure 3 shows the influence of the lateral position on the longitudinal force X_{BANK}. In this example the VLCC ship model is towed according to a velocity of 8 knots full scale in a rectangular cross section ($W_0 = W_h = 3.865$) with a water depth of 1.50 T. The closer the ship sails to the bank the larger the magnitude of the negative force X with a non-zero value at y = 0. These tests were carried out at a propeller rate according to self-propulsion in open, shallow water at this forward speed. The measured force X is adopted as the longitudinal force of the bank effect X_{BANK}. The negative sign of this force indicates an increase of resistance.

EQUIVALENT BLOCKAGE

The (classic) definition of blockage m indicates the amount of space a vessel utilizes in the entire cross section of a fairway. The blockage ratio m does not change if the same vessel is located at a different lateral position in the same cross section. To overcome this constraint, the equivalent blockage m_{eq} is introduced. This new equivalent blockage should meet some conditions:

- The equivalent blockage must take into account the area of the cross section Ω but also be sensible for the relative position of the ship in the cross section.
- The equivalent blockage must be zero when sailing in deep and unrestricted areas and have the unit value one as maximal theoretic value.

For defining the equivalent blockage in a random shaped cross section, use is made of a weight factor w, which was introduced by [3]. The formulation of this weight factor was published in [4] for modelling lateral oriented bank effects. For reasons of consistency and readability relevant parts of this paper will be partly repeated.

The weight factor w is a value between 0 and 1 which indicates the importance of a water particle for the bank effects induced on a ship. A water particle closer to the hull will have a value closer to 1. The weight factor will tend to zero once the water particle is far away from the ship (in all directions). The closer the water particle is located to the free surface, the larger the weight factor of the water particle. The maximal value for the weight factor (=1) is located at the cross section of the centre line of the ship and the free surface (at rest). The weight factor is graphically shown in Figure 4.
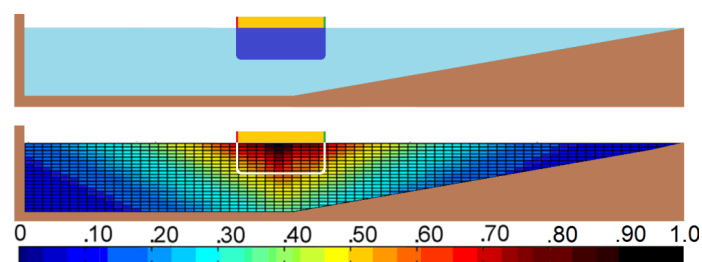

FIGURE 4 THE SHIP IN A CROSS SECTION AND A GRAPHICAL REPRESENTATION OF THE WEIGHT DISTRIBUTION IN THE SAME CROSS SECTION [4]

Analogous to Norrbin's factor [5] the weight factor w is a decreasing exponential function and its expression in the ship bound coordinate system is:

$$w = e^{-\left(\xi_y \frac{|y|}{y_{infl}} + \xi_z \frac{|z|}{T}\right)}$$

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EQUATION 7

The influence distance y_{infl} can be described as the boundary between open and confined water. If the ship-bank distance exceeds this value, no (significant) influence of the bank on the forces and moments on the ship will be observed [3].

The integration of the cross section at both sides of the vessel can be calculated with equations 8 and 9. Here the weight factor can be seen as a (ship dependent) overlay sheet which is placed on the cross section under consideration. All 'water particles' are taken into account, also the particles at a distance far away from the vessel but the weight value for these particles will be insignificantly small.

$$\chi_s = \int_0^h \int_0^{y_s} e^{-\left(\xi_y \frac{|y|}{y_{infl}} + \xi_z \frac{|z|}{T}\right)} \, dy \, dz$$

EQUATION 8

$$\chi_p = \int_0^h \int_0^{y_p} e^{-\left(\xi_y \frac{|y|}{y_{infl}} + \xi_z \frac{|z|}{T}\right)} \, dy \, dz$$

EQUATION 9

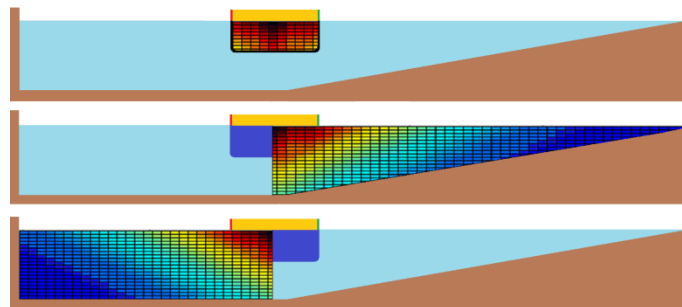A graphical interpretation of χ_p and χ_s is shown in Figure 5.

FIGURE 5 GRAPHICAL INTERPRETATION (TOP DOWN) OF X$_{SHIP}$, X$_S$ (THE INTEGRATED AREA AT STARBOARD) AND X$_P$ (THE INTEGRATED AND WEIGHTED AREA AT PORT) [4]

The values of coefficients ξ_y and ξ_z occurring in Equations 8 and 9 have been determined with the regression program "R" [7] making use of the previously mentioned model tests.

The equivalent blockage m_{eq} takes into account the weight distribution w (and the integration χ) as expressed in previous section. The equivalent blockage is defined as:

$$m_{eq} = \frac{1}{2}\left(\frac{\chi_{ship}}{2\chi_s} + \frac{\chi_{ship}}{2\chi_p}\right) - \frac{\chi_{ship}}{\chi_{ocean}}$$

EQUATION 10

The ratio $\frac{\chi_{ship}}{\chi_{ocean}}$ is subtracted from the ratio $\frac{1}{2}\left(\frac{\chi_{ship}}{2\chi_s} + \frac{\chi_{ship}}{2\chi_p}\right)$ to have a zero m_{eq} value when sailing in open and deep water. Both χ_{ship} and χ_{ocean} can be analytically calculated and do not depend on the geometry or position in the cross section.

$$\chi_{ship} = 2\frac{y_{infl}\,T}{\xi_y\xi_z}\left(1 - e^{-\frac{\xi_y B}{2y_{infl}}}\right)\left(1 - e^{-\xi_z}\right)$$

EQUATION 11

$$\chi_{ocean} = 2\frac{y_{infl}\,T}{\xi_y\xi_z}$$
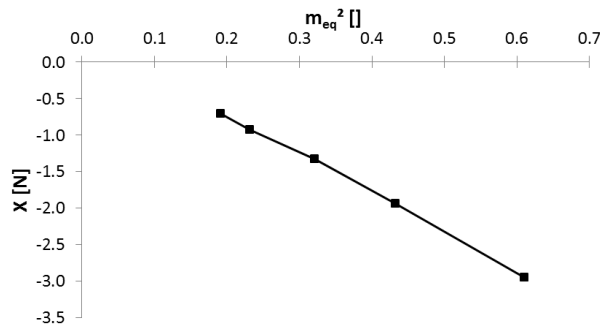
EQUATION 12

FIGURE 6 RELATION BETWEEN X_{BANK} AND THE SQUARE OF M_{EQ} FOR THE SAME TESTS AS PLOTTED IN FIGURE 3

Figure 6 shows the relation between the longitudinal force X and the square of m_{eq} for the same model tests as plotted in Figure 3.

ADAPTED TUCK $TU_M(V_{EQ})$*

*parts of this section have been published in [4] but for reasons of consistency and readability this is here partly repeated and adapted to the present topic of the longitudinal force.

The same model tests as in Figure 6 are plotted together with similar model test conditions with a more shallow water depth to draft ratio of 1.35 in Figure 7. The decreasing water depth results in an increasing resistance of the longitudinal force X (or if the previous assumption still stands X_{BANK}). This increase in magnitude of the force X is more prominent the higher the forward speed.
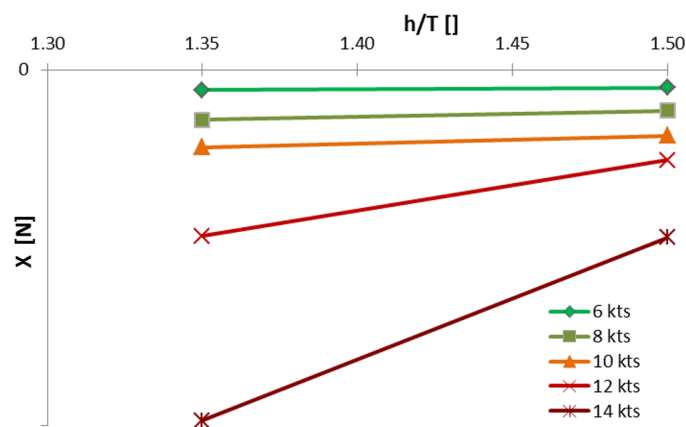

FIGURE 7 WATER DEPTH TO DRAFT RATIO PLOTTED TO THE LONGITUDINAL FORCE X

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TUCK NUMBER

In [8] a non-dimensional ship speed parameter was introduced, based on the water depth dependent Froude number Fr_h (i.e. the ratio between the forward velocity of the ship and the critical velocity in open water $\sqrt{gh}$):

$$Tu(V) = \frac{Fr_h{}^2}{\sqrt{\left|1 - Fr_h{}^2\right|}}$$

Analogous to [4], this non-dimensional number will be referred to as "Tuck number". This dimensionless number rapidly increases when the vessel sails close to the critical speed in open water ($Fr_h = 1$).
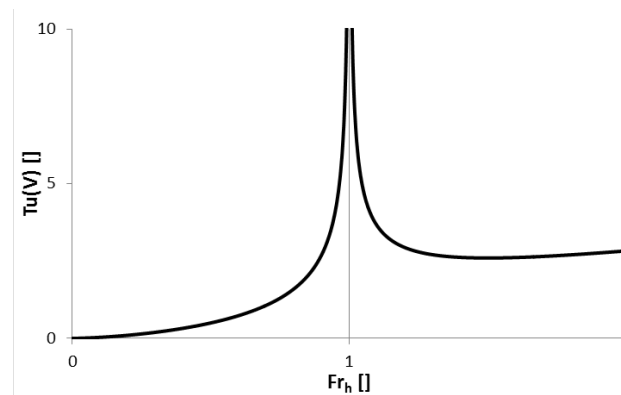

FIGURE 8 THE TUCK NUMBER TU(V) IN THE SUB (FR$_H$<1) AND SUPER CRITICAL (FR$_H$>1) SPEED REGION

This Tuck number Tu(V) does not take into account the lateral restrictions of the fairway, although these restrictions will decrease the critical speed. The critical velocity decreases in confined waters and will be smaller than $\sqrt{gh}$. In [6] the critical velocity V_{crit} is calculated taking into account the (classic) blockage m.

In infinitely wide cross sections the blockage factor tends to zero and for this reason the bathymetry at a lateral distance y_{infl}, or larger, from the vessel is not taken into account:

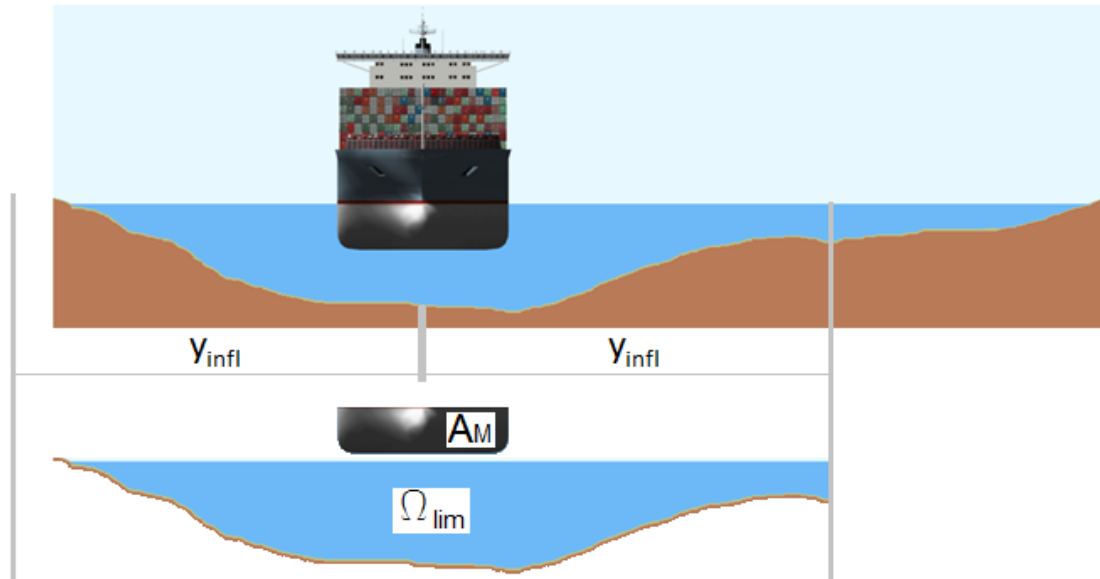$$\Omega_{lim} = \int_0^h \int_{-y_{infl}}^{y_{infl}} d\Omega$$

FIGURE 9 THE SHIP IN THE CROSS SECTION WITH THE INFLUENCE WIDTH Y_{INFL}, THE BLOCKAGE RATIO M IS THE RATIO OF THE MIDSHIP AREA A_M AND THE CROSS SECTION OF THE FAIRWAY WITHIN THE INFLUENCE WIDTH Ω_{LIM}

Now the blockage is the ratio between A_M and Ω_{lim} (Figure 9). A disadvantage of limiting the cross section to the influence width is that the minimal value for the blockage in shallow unrestricted waters is no longer zero but will have a minor value. The dimensionless critical speed Fr_{crit} according to [9] is calculated with the cross section within the influence width Ω_{lim}. As such, the critical speed $Fr_{crit,lim}$ is obtained:

$$Fr_{crit,lim} = \left(2 \, sin \left(\frac{Arcsin(1 - m_{lim})}{3} \right) \right)^{\frac{3}{2}}$$

EQUATION 15

The Tuck number is now adapted to Tu_m causing a shift to the left of the vertical asymptote in Figure 8, which is now located at the critical speed which takes into account the limited blockage:

$$Tu_m(V) = \frac{\left(\frac{Fr_h}{Fr_{crit,lim}} \right)^2}{\sqrt{1 - \left(\frac{Fr_h}{Fr_{crit,lim}} \right)^2}}$$

EQUATION 16

An active propeller accelerates the water flow passing the propeller disk and therefore increases the velocity of the water between bank and ship and thus influences the pressure on that area of the hull. The influence of the propeller action on the longitudinal force will be modelled as a partial increase of the forward speed of the vessel (V_{eq}):

$$V_{eq} = V + \xi_{VT} V_T$$

EQUATION 17

The coefficient ξ_{VT} takes a value between 0 and 1 and is calculated with a regression model but based upon the lateral force at the aft perpendicular and not on X_{BANK}. This is because of the difficulty of extracting X_{BANK} when the propeller does not run at self-propulsion. As published before, the thrust velocity V_T is calculated based upon the thrust T_P (as measured on the propeller shaft):

$$V_T = \frac{T_P}{|T_P|} \sqrt{\frac{|T_P|}{\frac{1}{2}\rho\pi\frac{D^2}{4}}}$$

<div align="right">EQUATION 18</div>

The Tuck number $Tu_m(V_{eq})$ from equation 18 takes into account:

- ☞ V: the forward speed through the water of the vessel;
- ☞ V_T: the propeller action;
- ☞ m_{lim}: the dimensions of the fairway's cross section (Ω_{lim}) and the midship area (A_M).

MATHEMATICAL MODEL

Similar as for the lateral forces at the fore and aft perpendiculars [4] the longitudinal bank effect force X_{BANK} appears to be proportional to the Tuck number $Tu_m(V_{eq})$.

$$X_{BANK} \propto Tu_m(V_{eq})$$

<div align="right">EQUATION 19</div>

In Figure 10 the correlation between the adapted Tuck number Tu_m and the longitudinal force X_{BANK} is visualised and the value m_{eq} is added as a label to the data points. Beware, the relation between the Tuck number and X_{BANK} cannot be visualised exactly because the blockage m_{eq} is not the same constant value for the ten tests plotted. As a consequence, the impact of m_{eq} is not excluded entirely in Figure 10 to support the relation 19.
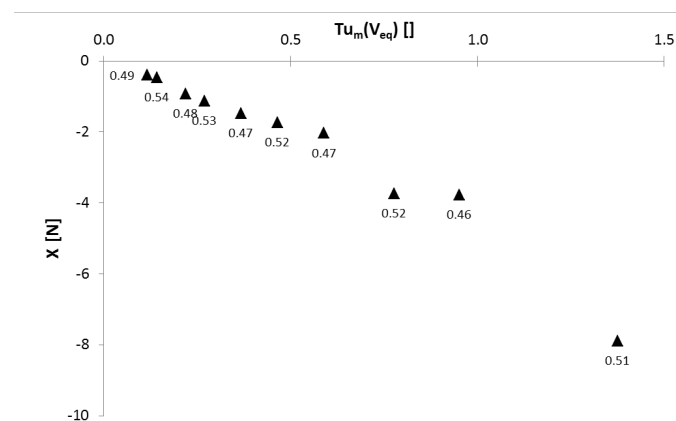

FIGURE 10 RELATION BETWEEN ADAPTED TUCK NUMBER TU$_M$ AND THE LONGITUDINAL FORCE FOR WITH A VARIATION OF M$_{EQ}$ FROM 0.46 UP TO 0.54 (ADDED AS LABEL TO THE DATA POINTS)

This Tuck number $Tu_m(V_{eq})$ is calculated with the equivalent velocity as defined and derived with the lateral force at the aft perpendicular. The coefficient ξ_{VT} is incorporated from the regression based on the lateral force at the aft perpendicular Y_A and thus not based upon the longitudinal force.

MATHEMATICAL MODEL X_{BANK}

The product of the square of the equivalent blockage m_{eq} and the adapted Tuck number Tu_m are proportional to the longitudinal bank force X_{BANK}. Multiplied with the displacement force Δ to introduce a force dimension and with constant, but ship dependent coefficient ξ_X to cope with the proportionality, the formula for X_{BANK} becomes:

$$X_{BANK} = \xi_X \Delta \, m_{eq}^2 \, Tu_m(V_{eq})$$

EQUATION 20

The longitudinal force X (assumed to be equal to X_{BANK}) for all model tests with a VLCC ship model at a propeller rate according to self-propulsion in open water are plotted in Figure 11 (171 model tests). The relation between the modelled force and force derived from model tests is satisfying although some deviation is observed. Some reasons for this deviation are ascribed to the error introduced in defining the force X_{BANK}, since the mathematical model for X_{BANK} runs on only one dedicated coefficient ξ_X because the other coefficients ξ are copied from the mathematical model for the lateral force Y_A. This model is therefore seen as a very robust mathematical model.
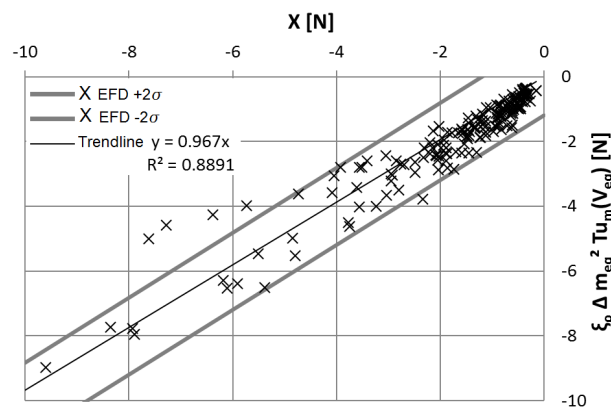

FIGURE 11 THE MODEL FOR X_{BANK} PLOTTED TO THE FORCE X_{BANK} DERIVED FROM MODEL TESTS WITH THE KVLCC2 SHIP MODEL

CONCLUSION AND FUTURE WORK

Based upon an extensive set of model tests carried out in the shallow water towing tank a mathematical model is proposed for the increase of the ship resistance induced by the presence of banks. The relative position of the banks to the ship and the entire geometry of the cross section is taken into account without exaggerating changes in the bathymetry far away from the ship nor by underestimating changes nearby the vessel. A new dimensional speed, the Tuck number Tu, is introduced to take into account the critical speed in the cross section, the forward speed of the ship and the loading of the propeller.

The influence of the return flow and thus the increased water velocity along the hull on the viscosity and as a consequence, the scaling of the longitudinal force (resistance) has to be investigated further.

NOMENCLATURE

A_M	[m²]	midship area		W_h	[m]	canal width at full depth
B	[m]	breadth of the ship/tank		X	[N]	longitudinal force
D	[m]	propeller diameter		X_{BANK}	[N]	longitudinal bank effect force
F_{NR}	[N]	normal rudder force		X_H	[N]	hull force
Fr_h	[]	water depth dependent Froude number		X_R	[N]	rudder force
F_{TR}	[N]	tangential rudder force		Y	[N]	lateral force
g	[m/s²]	gravity of Earth		y	[m]	lateral position (earth fixed)
h	[m]	water depth		y_{infl}	[m]	influence width
K	[Nm]	roll moment		z	[m]	vertical position
L	[m]	length (ship/tank)		z_{VA}	[m]	running sinkage at the aft
m_{eq}	[]	equivalent blockage		z_{VF}	[m]	running sinkage at the fore
n	[1/s]	propeller rate				
Q_P	[Nm]	propeller torque		δ	[deg]	rudder angle
Q_R	[Nm]	torque on rudder stock		δV	[m/s]	return flow velocity
R_{OW}	[N]	open water resistance		Δ	[ton]	displacement weight
T	[m]	draft		ξ	[]	coefficient of the model
T_P	[m]	propeller thrust		χ	[]	weight value of a section
Tu	[]	Tuck number		Ω	[m²]	cross section area
V_{crit}	[m/s]	critical speed				

Subscripts:

V_{eq}	[m/s]	equivalent speed
V_{ship}	[m/s]	ship speed
w	[]	'weight' factor
W_0	[m]	canal width at free surface

lim	[]	taking into account the influence width
ocean	[]	infinite deep and wide
p	[]	at the port side
s	[]	at the starboard side

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