Fast Fourier transforms for the evaluation of convolution products in electromagnetism: CPU versus GPU implementation

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To evaluate electromagnetic fields in space and time one often has to rely on numerical techniques to solve Maxwell's equations. In various cases, the observation points in which the fields are to be evaluated are densely distributed and coincide with the sources that generate the field. E.g. distributed charges that generate electrical fields, distributed currents and/or magnetic moments that generate magnetic fields, etc. In such cases, a common approach is to write Maxwell's equations in a volume integral formulation where the fields in all points $\mathbf{r}$ are expressed as a convolution product of a Green's function g over all sources s

$$f(\mathbf{r}) = \int_V g(\mathbf{r}-\mathbf{r}')s(\mathbf{r}')\,\mathrm{d}\mathbf{r}'. \qquad (1)$$

To solve this class of electromagnetic problems, the geometry is usually discretized using a uniform grid of finite difference cells. The evaluation of the convolution integral (1) in N finite difference cells requires then $\mathcal{O}(N^2)$ evaluations. The application of fast Fourier transforms (FFTs) reduces this computational burden to $\mathcal{O}(N\log(N))$ evaluations. In many computational schemes, convolution product(s) are to be evaluated a huge number of times pushing the need to further reduce the computation time. Recently, Graphics Processing Units (GPUs) became very popular to speed up numerical computations. While originally designed for purely graphical purposes, their huge parallel computation power can be exploited to speed-up simulations up to two orders of magnitude.

This paper describes how the application of FFT algorithms for the evaluation of convolution products can be yet optimized for finite 2D and 3D electromagnetic problems in which zero padding of the source matrices is required. Here, using general off-the-shelf 2D and 3D FFT algorithms results in many Fourier transforms on arrays containing only zeros. A non-execution of these Fourier transforms gives rise to non-negligible reductions in computation times. This paper describes the implementation of such an approach on GPU and compares the time gains on GPU with those on CPU. It is found that on CPU the speed-up corresponds with the theoretical limit, while in the GPU implementation the memory bandwidth limits the speed-up ratio.