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NUMERICAL METHODS FOR DETERMINING LOCAL MOTIONS OF HUMAN KNEE JOINT

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1. INTRODUCTION

The Biomechanical Team of the Szent István University has been carrying on a comprehensive project of the human knee joint. The conception is oriented to the local motions and the phenomenon (and its influence) of wear. By establishing different experimental models, in order to verify and universalize the results, numerical models must be developed and applied. In these special fields, several numerical models have been introduced

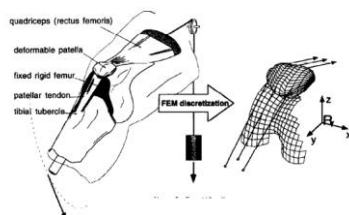


Figure 1.1.

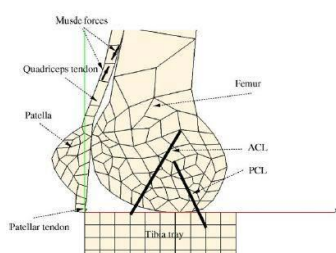


Fig. 9. Knee joint model.

Figure 1.2.

such as the classical J. Heegaard model [1]. Heegaard mostly investigated the stress and strain relation and the kinematics and dynamics of the patella, in the parameter of flexion. The above-mentioned model was investigated by the Finite Element method wherein the non-linear elastic material laws were taken into account (*Figure 1.1*). Besides the examination of the local motions, another important question is the wear of the contact surfaces. Among others, Franck Jourdan [2] dealt with the phenomenon of wear with his two-dimensional patello-femoral joint model (*Figure 1.2*). The third important effect on the connecting surfaces is the acting forces. Koehle and Hull [3] used in their research a commonly applied software program, called SIMM, which includes a complete musculoskeletal model of the lower limbs, allowing the reaction forces of the knee to be computed (*Figure 1.3*). The authors had only investigated the internal forces acting on the connecting points of the condyles during the flexion of the knee. The original SIMM 2D lower limb model was developed by Delp [4]. The knee model included a tibiofemoral and a patello-tibial joint. No patellar ligament was modeled, but an articulation was defined between the patella

and the tibia by the SIMM program. Since the knee joint is defined this way, the force generated by the quadriceps is transmitted to the tibia via a patellotibial joint reaction

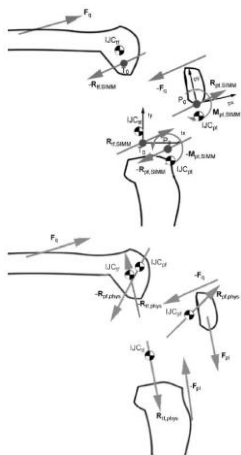


Figure 1.3.

force and moment, instead of by the patellar ligament. According to this effect, the physiologic knee joint reaction forces substantially differ from the results given by the SIMM. In this way, the authors revealed the weaknesses of the SIMM program, and suggested further modifications.

2. MAIN OBJECTIVES

Since the physiologic knee joint reaction forces are only calculated in certain angles, and there are no allusions about their distribution, one of the main objectives is to introduce the distribution of the contact forces between the tibiofemoral, and the patellofemoral relation during the flexion of the knee joint. The other objective is to investigate the local motions of the tibiofemoral connection, which are based on sliding and rolling features.

3. METHODS AND MODELS

3.1 Problems. Creation of models.

A numerical model was built in the MSC.ADAMS program. This system is able to analyze dynamical problems between flexible or rigid bodies. Since the motions of the 3D human knee are very difficult to model, a simpler, 2.5D model was created (2D model with a finite third extent). The geometrical model was mapped from MRI pictures, and used basic CAD methods to create the computational model (Figure 3.1.1).

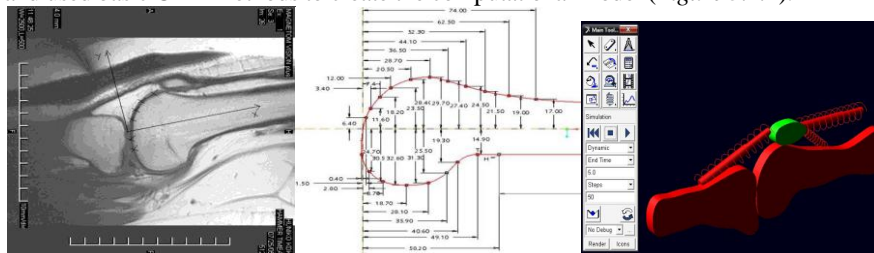


Figure 3.1.1

The properties of the model are the followings:

- 2.5D model,
- Continuum with mass,
- Rigid body.

In the case of any mechanical problem, one of the most important questions is the definition of the constraints. Since only ideal constraints are available in the ADAMS, the proper combination of the connecting constraints must be chosen in order to obtain a realistic motion.

The applied kinematical constraints of the model are the followings:

- All degrees of freedom of the femur were taken (*Fixed Joint*),
- The tibia is only allowed to carry out planar motions (*Planar Joint*),
- The patella is only allowed to carry out planar motions (*Planar Joint*),
- Patello-femoral and tibio-femoral contact relation (*Contact*).

In contrast with the SIMM model, the connecting ligaments such as the *patellar ligament*, the *quadriceps tendon*, and the *collateral medial ligament* were taken into account. The bones were attached to each other by springs which denote the connecting ligaments of the joint. These ligaments and the bones were coupled together according to functional anatomy. It is worth noting, that the patellar ligament is unable to elongate, thus its spring constant was set to infinite, while the collateral medial ligament and the quadriceps tendon was set to 5 [N/mm].

The applied load:

- Normal gravity field,
- $\vec{F}^T = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$ [N] force vector acting on the end of the tibia,
- Total applied weight of the tibia.

After setting the initial conditions ($\mu=0.11$ and $\mu_0=0.08$ [6]) and the solver algorithm (GSTIFF) the model was executed (*Figure 3.1.2-3.1.3*). Although the motion seemed to be proper, the given results (F_s/F_n ; the gradient of the friction and normal force quotient) could not be expounded upon (*Figure 3.1.4*).

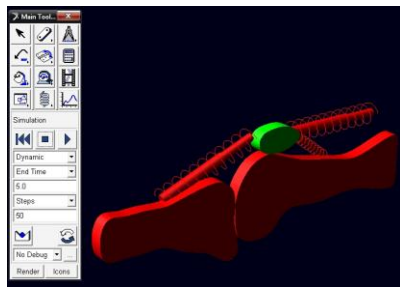


Figure 3.1.2.

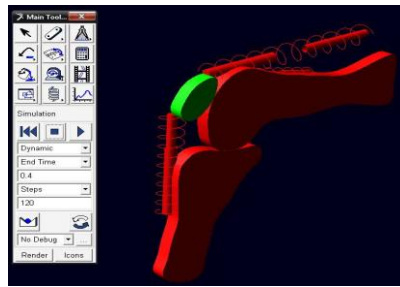


Figure 3.1.3.

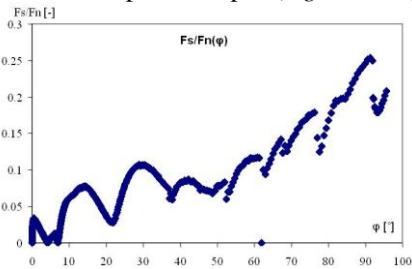


Figure 3.1.4

During the simulation the tibia started chattering as it was rolling and sliding along the condyles of the femur. The chattering caused the oscillation in the phenomena shown in Figure 3.1.4. This made it impossible to interpret. The problem was attributed to the default settings of the MSC.ADAMS. Since the program was not originally adjusted to handle contact problems, the simulation called forth false results. The solution can be obtained by setting the appropriate parameters, but to achieve that, a basic model must be developed and tested, where the analytical solution can be derived from the classical mechanical laws.

3.2 Validation by an elementary model

Let there be a cylinder with a radius denoted by R_1 , and a smaller cylinder denoted by R_0 . The R_0 cylinder starts rolling on the R_1 cylinder from a φ_0 position. The initial conditions are the followings:

- Initial velocity is zero.
- $|\overline{F}| = |\overline{G}|$
- $\varphi_0 = 30^\circ$
- Friction is defined between the two cylinders ($\mu = 0.3$).

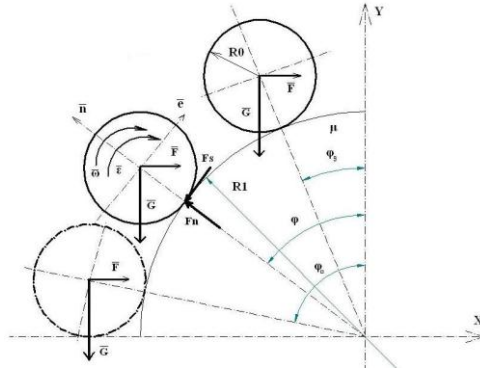


Figure 3.2.1.

As the R_0 cylinder starts rolling upon the R_1 cylinder (Figure 3.2.1), the motion can be characterized as pure rolling, since according to Coulomb's theorem the normal force is greater than the friction force. At φ_c position the

friction force and the normal turns to be equal, that is the point where the motion reaches the threshold limit, and beyond this position the moving object will have a sliding feature as well. If the threshold position could be calculated with instantaneous values between φ_c and φ_0 , then compared with a numerical model with same properties, the parameters of the contact module in the MSC.ADAMS could be set to obtain realistic results. As a first step to obtain φ_c , the principle of impulse for a rigid body must be applied;

$$\dot{\overline{I}} = \overline{F} \quad (1)$$

$$m \cdot \overline{a} = \overline{G} + \overline{F} + \overline{F}_K \quad (2)$$

$$m \cdot a_e = -G \cdot \sin \varphi + F \cdot \cos \varphi - F_s \rightarrow F_s = F \cdot \cos \varphi - G \cdot \sin \varphi - m \cdot R_0 \cdot \varepsilon(\varphi) \quad (3)$$

$$-m \cdot a_n = -G \cdot \cos \varphi - F \cdot \sin \varphi + F_N \rightarrow F_N = F \cdot \sin \varphi + G \cdot \cos \varphi - m \cdot \frac{v(\varphi)_0^2}{R_1 + R_0} \quad (4)$$

According to Coulomb's theorem:

$$F_s \leq \mu \cdot F_N \quad (5)$$

$$F \cdot \cos \varphi - G \cdot \sin \varphi - m \cdot R_0 \cdot \varepsilon(\varphi) \leq \mu \cdot \left(F \cdot \sin \varphi + G \cdot \cos \varphi - m \cdot \frac{v(\varphi)_0^2}{R_1 + R_0} \right) \quad (6)$$

If $\varphi = \varphi_0$ then the threshold position is reached!

Since both ε and v are the function of φ , two more equations must be used to solve the system to φ . From the principle of angular momentum for a rigid body around an axis:

$$\dot{\overline{\Pi}}_P = \overline{M}_P \quad (7)$$

$$J_{P'} \cdot \varepsilon_0 = R_0 \cdot F \cdot \cos \varphi - R_0 \cdot G \cdot \sin \varphi \rightarrow \varepsilon_0 = \frac{R_0 \cdot F}{J_{P'}} \cdot \cos \varphi - \frac{R_0 \cdot G}{J_{P'}} \cdot \sin \varphi \quad (8)$$

'P' is the connecting point of the two cylinders, and $J_{P'} = 3/2 \cdot m \cdot R_0^2$.

Now, the angular acceleration is obtained, by the use of the principle of work and energy for a rigid body, the last demanded parameter, the velocity can be easily derived:

$$E_{\varphi_g} - E_{\varphi_0} = W \quad (9)$$

$$\frac{1}{2} \cdot J_p \cdot \frac{v_g^2}{R^2} - 0 = F \cdot (R_1 + R_0) \cdot (\sin \varphi_0 - \sin \varphi_g) - G \cdot (R_1 + R_0) \cdot (\cos \varphi_g - \cos \varphi_0) \quad (10)$$

Substituting (8), (9) into (6), and carrying out a little algebraic manipulation, the (6) equation can be put in more convenient form:

$$\left[\frac{1}{3} \cdot \frac{F}{G} - \mu \right] \cos \varphi_g - \left[\frac{1}{3} + \mu \cdot \frac{F}{G} \right] \cdot \sin \varphi \leq \frac{4}{3} \cdot \left[\cos \varphi_g - \cos \varphi_0 - \frac{F}{G} \cdot (\sin \varphi_0 - \sin \varphi_g) \right] \quad (11)$$

Now, the algebraic equation is ready to be solved, and the analytical solutions are:

$$\varphi_{c1} = 0.18066 \text{ [rad]}, \varphi_{c2} = 2.16089 \text{ [rad]}.$$

Obviously, the demanded solution is the φ_{c1} , which is cca. 10.35° . As the threshold position is obtained, the gradient of the F_s/F_n can be calculated. Again a little manipulation on the (11) equation:

$$\frac{F_s}{F_n} = \frac{F \cdot \cos \varphi - \sin \varphi - \frac{2}{3} \cdot \frac{F}{G} \cdot \cos \varphi + \frac{2}{3} \cdot \sin \varphi}{\cos \varphi + \frac{F}{G} \cdot \sin \varphi - \frac{4}{3} \cdot \frac{F}{G} \cdot (\sin \varphi_0 - \sin \varphi) + \frac{4}{3} \cdot (\cos \varphi - \cos \varphi_0)} \quad (12)$$

By substituting different φ into the (12) equation, the following diagram (Figure 3.2.2) can be created. One can see, that at φ_g the value is 0.3, which expresses the equilibrium of the friction and normal forces. Beyond that point, a sliding feature will be added to the motion. As for the next step, a numerical model must be created in the ADAMS (Figure 3.2.3) with the same properties given in the analytical model.

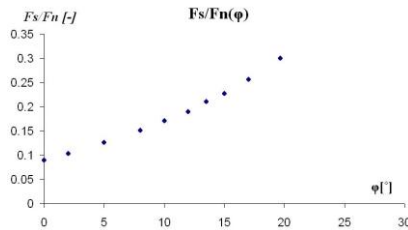


Figure 3.2.2.

The elementary model has the same kinematical constraints as the knee model. The following parameters could be set as CONTACT settings in the IMPACT type:

- Penetration depth,
- Force exponent,
- Stiffness and Damping.

After several trials, the system responded sensitively to the gradient of two parameters, which are the *penetration depth* and the *force exponent*. By setting the exponent to 10, and the penetration depth to 3, the program provides the demanded solution (Figure 3.2.4).

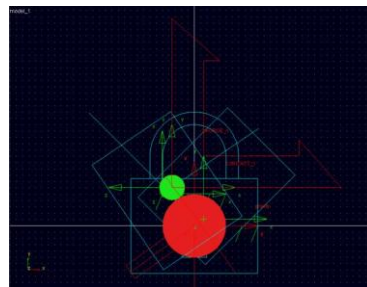


Figure 3.2.3.

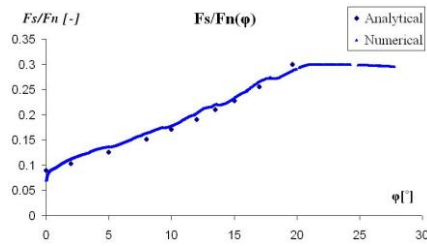


Figure 3.2.4.

4. RESULTS AND CONCLUSIONS

Now, the parameters are properly set to execute the simulation of the knee model as well. First, the gradient of the normal and friction unit forces have been estimated in the function of the unit length of the connecting curvature (Figure 4.1.).

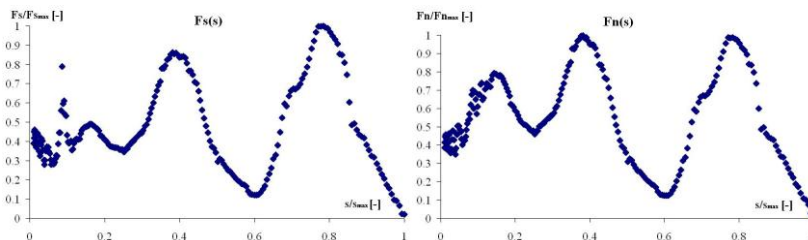


Figure 4.1.

Secondly, the gradient of the quotient of friction and normal forces have been calculated in the function of the unit length of the connecting curvature (Figure 4.2.). According to the solution of the elementary model the followings can be noted from this diagram:

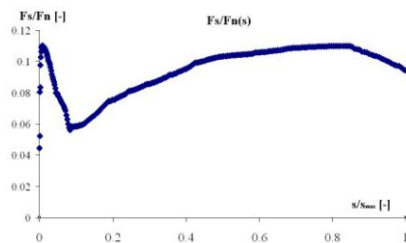


Figure 4.2.

During the initiative motion of the tibia along the femur the condyles rolls on the surface of the femur. Then, until cca. 8% of the unit length of the curvature is sliding and up to cca. 83% there is rolling. Over 83% sliding is dominant. Obviously, these relations are highly depended on the connections of the ligaments, the shape of the tibia plateau and the condyles.

To universalize the given results, more proper calculations and investigations must be done both numerically and experimentally, but according to the introduced methods the required examinations can be carried out.

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