A Multiplicative Calderón-Based Preconditioner for the Combined Field Integral Equation

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Integral equation solvers are prime candidates for analyzing scattering from perfect electrically conducting (PEC) scatterers, because, compared to their differential equation counterparts, (i) they implicitly impose the radiation condition, (ii) they only require discretization on the scatterer's surface - thereby resulting in small interaction matrices, and (iii) they allow for efficient iterative solution of these matrix systems making use of the fast matrix-vector multiplication algorithms. Among all known boundary integral equations, the electric and magnetic field integral equations (EFIEs and MFIEs) are the most widely used. These equations, however, pose two problems: (i) EFIEs are often ill–posed and boundless due to the presence of compact and hypersingular terms. Consequently, the discretization of EFIEs results in ill-conditioned interaction matrices for dense discretizations. (ii) EFIEs and MFIEs are singular when applied to the analysis of wave interactions with closed surfaces at frequencies corresponding to eigenvalues of appropriately constructed interior Maxwell problems. Hence, the discretization of EFIEs and MFIEs at frequencies near these eigenvalues results in ill-conditioned interaction matrices. The ill-posedness due to the singular behaviour can be alleviated using nonsingular combined field integral equations (CFIEs), which comprise linear combinations the EFIEs and MFIEs. However, CFIEs remain boundless since their EFIE component still contains a hypersingular term. As a result, the matrices obtained upon discretizing CFIEs remain ill-conditioned for dense discretizations.

In the past, a variety of methods for regularizing EFIEs, i.e. for rendering them well-posed and bounded, have been proposed. These methods often use readily constructed approximations of the inverse of the EFIEs' hypersingular part, e.g. by inverting a static EFIE (G. Vecchi et al., Int. J. Microw. Millimet. Wave Comput. Aided Eng., 7(6), 1996, 410-431), by using quasi-Helmholtz decompositions (rearranged loop-star/tree) (J. S. Zhao and W. C. Chew, IEEE Trans. Antennas Propagat., 48(7), 2000, 1635 -1645) or wavelet-based source representations (F. P. Andriulli, et al, SIAM J. Scientific Computing, 29(1), 2007, 1-21), or by leveraging the Calderón identities (G. C. Hsiao and R. E. Kleinman, IEEE Trans. Antennas Propagat., 45(3), 1997, 316-328). Among these schemes, those that use Calderón-based preconditioners, i.e. that make use of the fact that the EFIE operator is self-regularizing, are the most effective at high frequencies and thus are preferred in dealing with CFIEs.

This work presents a regularized CFIE obtained by linearly combining the standard MFIE with an EFIE that is regularized using the Calderón Multiplicative Preconditioner (CMP) (F. P. Andriulli et al., accepted for publication in IEEE Trans. Antennas Propagat.). Particular care is exercised in constructing the Calderón-regularization since a classically Calderón-regularized EFIE cannot be combined directly with the MFIE as the "EFIE squared" acquires the resonances of the MFIE (G. C. Hsiao and R. E. Kleinman, IEEE Trans. Antennas Propagat., 45(3), 1997, 316-328) and therefore the resulting combined equation is not resonance-free. For this reason, a properly chosen localization technique has been adopted here. Additionally, well-conditioned Gram matrices, which link the domain and the range of the EFIE operator, are constructed using the method introduced in (F. P. Andriulli et al., accepted for publication in IEEE Trans. Antennas Propagat.). The resulting regularized CFIE is both free from internal resonances and well-conditioned regardless of the discretization density.

The effectiveness of the new preconditioner in alleviating the ill-posedness of the CFIE and the number of iterations required to solve the resulting matrix system will be demonstrated via its application to the analysis of wave interactions with densely discretized structures.

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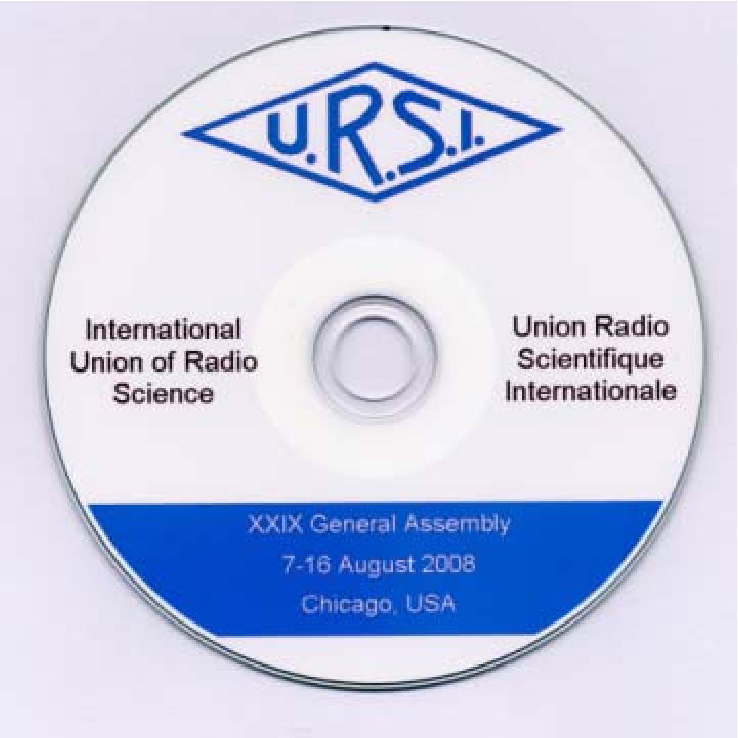

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