A low frequency stable formulation of the MLFMA in two dimensions

I. Bogaert* and F. Olyslager
Ghent University, Department of information technology (INTEC),
Belgium, http://www.ugent.be

In the past, many Fast Multipole Methods (FMMs) have been developed for iteratively solving integral equations containing the Green function of the Helmholtz equation as the integration kernel. Of these methods, the so-called Multilevel Fast Multipole Algorithm (MLFMA) is arguably the most widely used. It allows the simulation of very large scattering problems and as such it is useful in a plethora of applications. However, it has one big drawback. If the scattering geometry contains a lot of subwavelength geometrical detail, then the basis functions used to discretize this geometry are close to one another. If their distance is smaller than the wavelength, then the MLFMA suffers from a numerical instability, caused by the inevitable roundoff errors of a computer. This problem can be remedied in various ways, for example by handling the short-range interactions by means of a multipole based method. Another approach consists of eliminating the MLFMA altogether and using the spectral representation of the Green function instead. However, as stated in (H. Cheng et. al., J. Comput. Phys., 216, 300-325, 2006), the MLFMA is the most efficient method, and it should be used whenever possible.

Considerable effort has been devoted to stabilize the MLFMA at low frequencies (LF). In particular in (I. Bogaert, D. Pissoort and F. Olyslager, Micr. Opt. Tech. Lett., Vol 48, Issue 2, 237-243, Feb 2006), numerically stable translation operators were obtained. However, where the MLFMA needs L samples (ignoring the numerical instability) these translation operators need $2L$ sample points, so they are less efficient. A method for solving this problem, the two dimensional nondirective stable plane wave multilevel fast multipole algorithm (2DNSPWMLFMA), is presented in this contribution, which combines the LF-stability of the spectral methods with the efficiency of the MLFMA.

To reduce the number of sample points, a set of L wavevectors is selected, which have the property that the Fourier spectrum of a radiation pattern can be accurately reconstructed if the radiation pattern is sampled in these L wavevectors. This selection procedure is performed using the QR-algorithm. The stable translation operators are then transformed such that they can be used on the new, selected set of sample points. The result is a method in which the radiation patterns and translation operators have only L samples, while still being numerically stable. The only drawback is that the inter- and anterpolations must be done with full matrices, since the discretization is no longer uniform. This limits the method to LF-interactions, but this poses no problem since switching to the usual MLFMA once the box size exceeds a certain value is straightforward.

2008 IEEE AP-S International Symposium and USNC/URSI National Radio Science Meeting

Preliminary Version

July 5-12, 2008
San Diego, CA

Support: If you have problems or questions related to the installation of this disc, please contact the 3WAIsmen at FAX: (818) 952-0183 or e-mail: wais3men@yahoo.com

WAIs3men