The role of index contrast in the efficiency of absorption and emission of a luminescent particle near a slab waveguide.

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Abstract—**The effect of waveguide index contrast on the absorption of the guided field by a luminescent particle near the core of a slab waveguide is analyzed, together with the Purcell-enhancement of the spontaneous luminescence and it's coupling back into the guided mode. The overall conversion efficiency is strongest for excitation by a TM-pump and coupling of fluorescent light to the TM-mode and depends approximately quadratically on the index contrast.**

***Keywords-** photonic sensors, nanoparticles, slab waveguide, Green function, spontaneous emission, Purcell effect*

I. INTRODUCTION

In the past years, sensors based on photonic integrated circuits have been developed as the backbone for the lab-on-a-chip integration platform [1]. Many of such sensors, such as fluorescence sensors, rely on the coupling of the fluorescence signal from molecules or fluorescent nanoparticles to the underlying guiding structures. Investigation for the optimum waveguide parameters for the best efficiency of such structures has been rarely done, although there is a rich literature on light emission by particles lying close to layered dielectric structures [2-4]. These articles mainly focused on the power radiated in the farfield, keeping in mind applications such as near-field microscopy and light extraction from LEDs. In [4], the enhancement of emission and total power coupled to waveguide has been investigated. Unfortunately, the methods to calculate power coupled to the individual modes, and how that power behaves as a function of various waveguide parameters have attracted little attention. The objective of this work is to analyse how the efficiency of conversion from a guided pump to spontaneous emission coupling to a guided mode depends on the index contrast (Δn) of the waveguide and to derive the optimum thickness. Emphasis is on two very popular waveguide systems: with silicon (Si) and silicon-nitride (Si_3N_4) core materials.

II. METHODOLOGY

Our goal is to investigate the overall efficiency of absorption of a guided mode by a luminescent particle at the core-cladding interface and the subsequent emission into one of the fundamental TE or TM guided modes, taking Purcell enhancement into account. Our methodology is applied to a common slab waveguide with a lower cladding of silica (n_s=1.45), an upper cladding of air (n_l=1), and variable core index n_c and core thickness d.

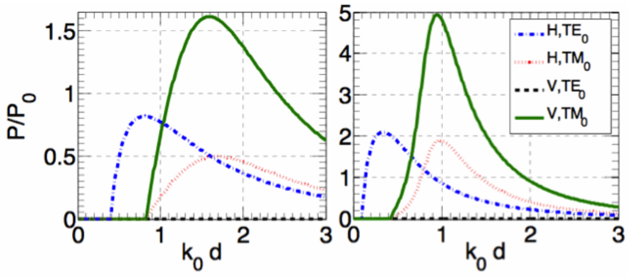

Figure 1: Power coupled to the fundamental modes, for dipoles oriented horizontal (H) and vertical (V) to the waveguide surfaces. The values are normalized to the total power that the dipole would radiate in free space. Left: Si_3N_4; Right: Si core.

The emission problem is modeled as a classical dipole emission problem in a layered medium. The total field at the location of the dipole can be calculated by expanding the dipole field into plane wave components, and applying generalized Fresnel reflectivity formulas [2-5]. The spontaneous decay rate, normalized to the free-space decay rate can then be calculated from the field at the dipole location. The total rate, normalized to free-space rate, P/P_0 turns out to be [4]:

$$\frac{P}{P_0} = 1 + \mathrm{Re}\left\{ \int_0^\infty dk_\rho \frac{{\mu_\rho}^2}{\mu^2} \frac{3}{4} \frac{k_\rho}{k_1^3 k_{1z}} [k_1^2 r_{TE} - {k_{1z}}^2 r_{TM}] + \frac{\mu_z^2}{\mu^2} \frac{3}{2} \frac{{k_\rho}^3}{k_1^3 k_{1z}} r_{TM} \right\} \qquad (1)$$

whereby, r_{TE} and r_{TM} are the reflectivity of the whole structure for TE and TM polarizations; μ and k_1 are the wave number in the upper cladding and the total dipole moment, μ_ρ,k_ρ, are the corresponding horizontal components and μ_z, k_{1z} are the corresponding vertical components. We note that each guided mode corresponds to a pole of the reflectivity r_{TE} and r_{TM} . An unappreciated fact is that the power coupled to a mode

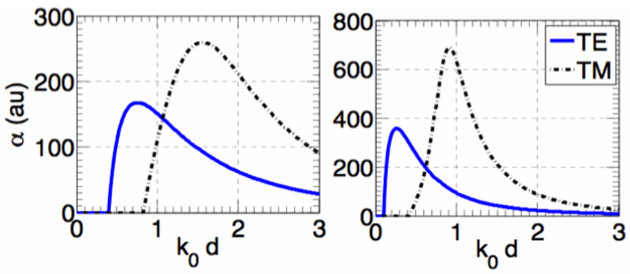

Figure 2: Absorption of the particle for fundamental TE and TM mode excitation as a function of k_0d of the slab waveguide. Left: Si_3N_4 core; Right: Si core

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can be determined by calculating only the integral in the real axis around the corresponding pole. We use the Gauss-Kronrod quadrature algorithm [6] to calculate the integral around the poles. Fig. 1 shows the power P/P_0 coupled to the two fundamental modes for Si and Si_3N_4 cores respectively for the two distinct dipole orientations.

The absorption of the guided mode by a particle is directly proportional to the total photon density at the location of the particle, which is further proportional to $|E|^2$ of the considered mode. Alternatively the absorption can be derived from the imaginary part of the effective index of the guided mode in a slab with a thin uniform layer of absorbing particles. Both methods yield the same results, at least in relative terms. Fig. 2 shows the absorption (in arbitrary units) of a particle for Si and Si_3N_4 cores for the two fundamental modes.

The overall efficiency of absorption and subsequent re-emission can be calculated, in arbitrary units, by multiplying absorption and emission rate of the corresponding modes. We assume a random orientation for the radiating dipole, implying that one third of the emitted power is due to vertical dipoles and two thirds due to horizontal ones. The Stokes shift between the pump light and the luminescence can be easily taken into account by shifting the k_0d as required. For simplicity, we ignore the Stokes shift here since it only has a weak impact on the results. The overall efficiencies for all the four pumping and coupling combinations (TE-TE, TE-TM, TM-TE and TM-TM) are plotted as a function of k_0d for Si and Si_3N_4 core in Fig. 3.

III. RESULTS AND DISCUSSION

Fig. 1 shows that as much as 1.6 and 5 times the power radiated by the particle in free space (in all possible directions) can be coupled into the waveguide modes, for Si_3N_4 and Si core respectively. The increase in emission rate can be attributed to the Purcell effect in the vicinity of the waveguide. The power coupled to a given mode is evidently dependent on dipole orientation, the cut-off and field confinement conditions of the waveguide. Dipoles oriented normal to the waveguide do not contribute to the TE mode, while dipoles oriented parallel to the waveguide contribute to both the TE and the TM mode. At k_0d values below cutoff, there is no power coupled to the waveguide mode and as k_0d is increased above the cutoff, the coupled power increases abruptly. However, as k_0d is increased further, increased confinement of the field in the core reduces the local field density at the dipole location, thus reducing the emission rate contribution from the mode and consequently the total power coupled to the mode. Keeping aside the obvious difference owing to dipole orientation, the power coupled to the waveguide roughly follows the trend of the field intensity at the dipole location and hence that of the absorption.

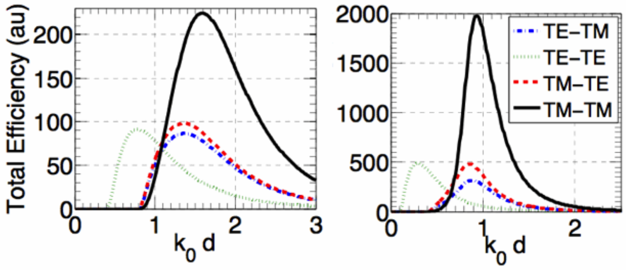

Figure 3: Overall efficiency for several mode excitation and coupling possibilities. (e.g. TE-TM means excited by fundamental TE mode and the emission coupled to fundamental TM mode) Left: Si_3N_4 core; Right: Si core

The trend that higher index contrast (Δn $=(n_c-n_s)$) enhances the emission and absorption efficiencies is evident in Fig. 2 - 4. It is apparent that among several pumping–coupling possibilities, TM pumping and the subsequent coupling to the TM mode has the highest overall efficiency. It is about 2 times stronger than the other polarization combinations in the case of nitride waveguides and about 4 times for silicon. Furthermore the best TM-TM efficiency for silicon is about 10 times stronger than for Si_3N_4.

In Fig. 4 we plot the optimal normalized slab thickness values k_0dn_c corresponding to the maximal efficiency as function of Δn. The corresponding optimal efficiency values are also plotted in Fig. 4, demonstrating that the TM-TM efficiency scales approximately quadratically with Δn. It is also interesting to note that the optimum k_0dn_c-values for higher index contrast for all the curves involving TM modes converges approximately to π (implying that $d=\lambda_0/2n_c$).

In conclusion, the overall efficiency of absorption and subsequent re-emission by a luminescent particle at the core cladding interface of a slab waveguide is most efficient for TM-pumping and subsequent coupling of the Stokes light into the TM-mode of the waveguide. This efficiency scales approximately quadratically with index contrast and is optimal for a slab thickness equal to half the wavelength in the core material. The present analysis can be extended to also describe spontaneous Raman scattering.

REFERENCES

[1] X. Fan, I. M. White et.al., "Sensitive optical biosensors for unlabeled targets: A review," Anal. Chim. Acta, vol. 620, pp. 8-26 (2008).

[2] W. Lukosz and R. E. Kunz, "Light emission by magnetic and electric dipoles close to plane interfaces", J. Opt. Soc. Am, vol. 67 , pp.1607-1619 (1977).

[3] R. R. Chance, A. Prock, and R. Silbey, ''Molecular fluorescence and energy transfer near interfaces,'' in Advances in Chemical Physics, I. Prigogine and S. A. Rice, eds., vol. 37, pp. 1–65, Wiley, New York, (1978),

[4] L. Novotny, ''Allowed and forbidden light in near-field optics. I. A single diplar light source,'' J. Opt. Soc. Am. A, vol. 14, pp. 91–104 (1997).

[5] M. Paulus, P. Gay-Balmaz, O. J. F. Martin., "Accurate and efficient computation of Green's tensor for stratified media", Phys Rev E, Vol 62, pp. 5797-5807 (2000)

[6] Calvetti, D.; Golub, G. H.; Gragg, W. B. and Reichel, L. "Computation of Gauss-Kronrod Quadrature Rules." *Math. Comput.* vol. 69, pp. 1035-1052 (2000).

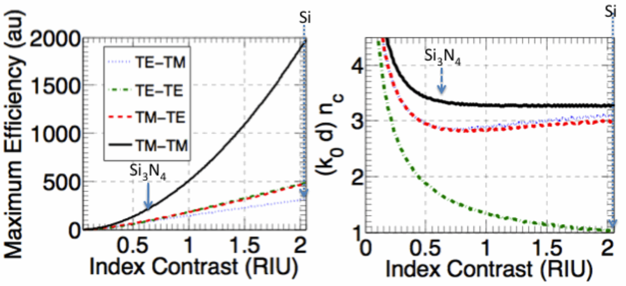

Figure 4: Left: Maximal overall efficiency as function of index contrast. Right: k_od n_c values corresponding to the maximum overall efficiency.