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HYBRIDISATION OF A WAVE PROPAGATION MODEL (SWASH) AND A MESHFREE PARTICLE METHOD (SPH) FOR REAL APPLICATIONS

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Abstract

Numerical modelling is becoming an essential alternative and complementary tool to laboratory experiments for the assessment of wave transformation and wave-structure interaction phenomena. Grid-based models are not able to represent violent phenomena characterised by large deformations, where meshfree models can be still computationally too expensive. Boussinesq or nonlinear shallow water equation models have limitations due to the approximation of the governing equations: Navier-Stokes equations based methods can model properly the physics of fluid hydrodynamics but still require huge computational capacities. To overcome these drawbacks, a coupling between two different numerical models, characterized by different computational cost and different capabilities, can help to get a holistic representation of phenomenon of wave propagation, transformation and interaction with coastal structures. Therefore, a hybrid method is developed starting from the wave propagation model SWASH and the meshfree particle method DualSPHysics. The hybrid model has been validated with physical model data providing significantly improved predictions of wave heights, velocities, breaking points, overtopping information and forces exerted by waves onto the coastal structures.

Keywords: meshfree methods; SWASH; SPH; Hybridation

1. Introduction

Modelling the entire process of propagation, transformation and interaction of sea waves with coastline and coastal structures is a challenging task, both in physical and numerical models, but often required for a proper assessment of coastal flood risks. It is difficult to study the wave propagation from the deep ocean to the nearshore region using a single model due to the presence of multiple scales both in time and in space.

The present study considers the use of two different numerical models to generate and propagate the wave field from the offshore towards the nearshore. The purpose is to model and analyse the transformation of the sea waves due to the processes typical of the surf and swash zones, such as shoaling, wave breaking, uprush and backwash, run-up, overtopping, etc... A proper representation of the waves nearshore will make possible the proper modelling of the interaction between sea waves and coastal defences (sea dikes, breakwaters, embankments) with a particular focusing on the extreme storm conditions propagating from offshore and no-lineal wave transformation. Therefore two numerical models, SWASH and

DualSPHysics, with different but complementary characteristics have been chosen to develop a hybridisation strategy capable to handle the aforementioned issues.

The SWASH model is a time domain model for simulating non-hydrostatic, free-surface and rotational flow. Wave propagation models as SWASH have been proven to be able to simulate accurately surface wave and velocity field from deep water and with satisfactory results both in the open ocean and in nearshore but they are not suitable to deal with abrupt changes of shape of coastal structures.

DualSPHysics is an open-source numerical model based on the Smoothed Particle Hydrodynamics (SPH) method and can be freely downloaded from www.dual.sphysics.org. SPH model is used to simulate free-surface flows problems such as dam breaks, landslides, sloshing in tanks and wave impacts on structures. The expensive computational cost of SPH in comparison with other meshbased methods for CFD problems can be partially alleviated by general-purpose graphics processing unit (GPGPU) where a graphics processing unit (GPU card) is used to perform computations traditionally managed by big cluster machines with thousands of CPU cores. Thereby DualSPHysics was designed from the outset to use SPH for real engineering problems with software that can be run on either CPUs or GPUs and can simulate millions of particles at a reasonable computation time. Nevertheless, that is not enough if the goal is very demanding and if the purpose is to run the whole domain and for the whole duration of storm events.

For all the reasons mentioned above the development of a hybrid model becomes mandatory for coastal applications. The present work describes the implementation and validation phases of the hybridisation model. In particular the experimental data from SUSCO project (Hydralab III report, 2010) have been used to validate the goodness of the hybridisation technique.

2. Numerical models

A brief description of both numerical models of the hybridisation is reported in this section.

2.1 SWASH

The SWASH model is a time domain model for simulating non-hydrostatic, free-surface and rotational flow. The governing equations are the shallow water equations including a non-hydrostatic pressure term:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial hu}{\partial x} = 0 \qquad [1]$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \zeta}{\partial x} + \frac{1}{2}\frac{\partial q_b}{\partial x} + \frac{1}{2}\frac{q_b}{h}\frac{\partial(\zeta - d)}{\partial x} + c_f\frac{u|u|}{h} = \frac{1}{h}\frac{\partial}{\partial x}\left(hv_t\frac{\partial u}{\partial x}\right) \qquad [2]$$

$$\frac{\partial w_s}{\partial t} = \frac{2q_b}{h} - \frac{\partial w_b}{\partial t}, \; w_b = -u\frac{\partial d}{\partial x} \qquad [3]$$

$$\frac{\partial u}{\partial x} + \frac{w_s - w_b}{h} = 0 \qquad [4]$$

where t is time, x the horizontal coordinate, u the depth averaged velocity in x-direction, w_s and w_b the velocity in z-direction at the surface and at the bottom, respectively. ζ is the free-surface elevation from still water level, d is the still water depth and h the total depth. q_b is the non-hydrostatic pressure at the bottom, g the gravitational acceleration, c_f the dimensionless bottom friction coefficient and v_t the eddy viscosity.

The SWASH model uses sigma coordinates in the vertical direction and the number of the fluid layer can be changed in the calculation.

A full description of the numerical model, boundary conditions, numerical scheme and applications are given in Zijlema *et al.* (2011). Suzuki *et al.* (2011) demonstrated that this model produces satisfactory results for both wave transformation and wave overtopping for shallow foreshore topography in their one-dimensional calculation. This numerical model is a strong tool for the estimation of wave transformation since it is not demanding in terms of computation resources due to the depth averaged assumption and parallel computation capability even though it is a time-domain model.

2.2 Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics is a Lagrangian and meshless method where the fluid is discretised into a set of particles and each of these particles are a nodal point where physical quantities (such as position, velocity, density, pressure) are computed as an interpolation of the values of the neighbouring particles. The contribution of the nearest particles is weighted according to distance between particles and a kernel function (W) is used to measure this contribution depending on the inter-particle distance that is defining using a smoothing length (h). The smoothing length is a characteristic length used to define the area of influence of the kernel and the kernel presents compact support to not consider contributions with other particles beyond the smoothing length.

The mathematical fundamental of SPH is based on integral interpolants, therefore any function F can be computed by the integral approximation. This function F can be expressed in a discrete form based on the particles. Thus, the approximation of the function is interpolated at particle a and the summation is performed over all the particles within the region of compact support of the kernel:

$$F(\mathbf{r}) = \int F(\mathbf{r}')W(\mathbf{r}-\mathbf{r}',h)d\mathbf{r}' \qquad [5]$$

$$F(\mathbf{r}_a) \approx \sum_b F(\mathbf{r}_b)W(\mathbf{r}_a-\mathbf{r}_b,h)\frac{m_b}{\rho_b} \qquad [6]$$

where the volume associated to the neighbouring particle b is m_b/ρ_b, with m and ρ being the mass and the density, respectively.

The kernel functions W must fulfil several properties (Monaghan, 1992), such as positivity inside the area of interaction, compact support, normalization and monotonically decreasing with distance. One option is a quintic kernel where the weighting function vanishes for inter-particle distances greater than $2h$.

In the classical SPH formulation, the Navier-Stokes equations are solved and the fluid is treated as weakly compressible (e.g. see Gómez-Gesteira *et al.,* 2012). The conservation laws of continuum fluid dynamics, in the form of differential equations, are transformed into their particle forms by the use of the kernel functions.

The momentum equation proposed by Monaghan, 1992 has been used to determine the acceleration of a particle (a) as the result of the particle interaction with its neighbours (particles b):

$$\frac{d\mathbf{v}_a}{dt} = -\sum_b m_b\left(\frac{P_b}{\rho_b^2}+\frac{P_a}{\rho_a^2}+\Pi_{ab}\right)\nabla_a W_{ab}+\mathbf{g} \qquad [7]$$

being $\boldsymbol{v}$ velocity, P pressure, ρ density, m mass, $\boldsymbol{g}$=(0,0,-9.81) ms^{-2} the gravitational acceleration and W_{ab} the kernel function that depends on the distance between particle a and b. Π_{ab} is the viscous term according to the artificial viscosity proposed in Monaghan, 1992.

The mass of each particle is constant, so that changes in fluid density are computed by solving the conservation of mass or continuity equation in SPH form:

$$\frac{d\rho_a}{dt} = \sum_b m_b \mathbf{v}_{ab} \cdot \nabla_a W_{ab} \qquad [8]$$

In the weakly compressible approach, pressure is calculated starting from density values of the particle using Tait's equation of state.

The Symplectic time integration algorithm (Leimkuhler, 1996) was used in the present work. A variable time step was calculated, involving the CFL (Courant-Friedrich-Lewy) condition, the force terms and the viscous diffusion term.

DualSPHysics is so capable of using the parallel processing power of either CPUs and/or GPUs making the study of real engineering problems possible. Crespo *et al.*, 2011 validated numerical results with experimental data in order to show how the technique combines the accuracy and the efficiency of GPU programming. Thus, this new technology makes the study of real-life engineering problems possible at a reasonable computational cost on a personal computer such as the numerical design of coastal breakwaters with SPH models (Altomare *et al.*, 2014).

3. Hybridisation technique

The hybridisation between DualSPHysics and SWASH has been obtained through a one-way coupling at this stage. The boundary between both models is set far enough from the region of interest. There is no much information that would be necessary to be transferred from SPH to the other model, so for this one-way coupling that information is neglected without loss of accuracy. The one-way coupling strategy used here is defined as "Moving boundary".

3.1 Moving boundary

The basic idea is to run SWASH for the whole domain to impose some boundary condition on a fictitious wall placed between both media. Each particle on that wall will experience a different movement to mimic the effect of the incoming wave. The time history of the displacement in each point or layer of the propagation model is reconstructed starting from the velocity information and interpolated along the vertical. The so-calculated movement is passed to the SPH particles that will form the wave paddle. Thus, the paddle is a set of points whose displacement is imposed by the wave propagated by SWASH and only exists for SPH.

3.2 Piston smoothing

SWASH gives values of velocity in different levels of depth. These values are used to move the piston particles. The displacement of each particle can be calculated using a lineal interpolation of velocity in the Z position of the particle. However, the lineal interpolation is not a good option because a small difference in velocity between two piston particles, which are very close in height, gives rise to an important difference in the accumulated displacement after several seconds of simulation. Furthermore, this problem is aggravated further because the height for the velocity measurements can vary in each instant depending on the height of water, which can result in a broken piston. In the top part of the Figure 1, it can be observed how the piston

is distorted after 200 and 400 seconds of simulation. The blue line represents the initial level of water and the red box shows the initial position of the piston. The solution is to get a smooth velocity, so that the velocity used in the particles does not change suddenly depending on the height. The result with this solution is depicted in the bottom part of the figure and shows the smaller distortion of the piston when the smoothing is applied.

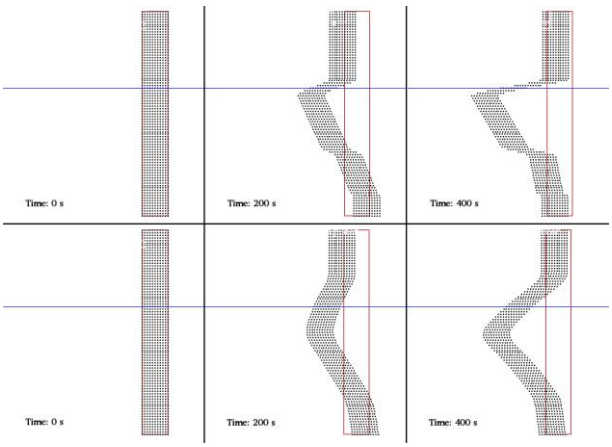

Figure 1. Piston smoothing sketch

4. Preliminary analysis

Prior to the hybridisation of the models, basic behaviour of the SWASH model is tested applying the physical model result from experiments carried out at the Maritime Engineering Laboratory of the Technical University of Catalonia (LIM-CIIRC/UPC). Four experiments, namely Test 65, 66, 115 and 116 of SUSCO data (Hydralab III report, 2010) have been used for the analysis. The basic wave properties of each test are shown in Table 1. The test 65 and test 66 use same wave condition: only bathymetry is different since test 65 and test 66 are the same test series to see how the bathymetry is changed after certain wave action. And same wave condition and different bathymetry also for test 115 and test 116.

Table 1. Wave conditions of test 65, 66, 115 and 116

Test No.	**65**	**66**	**115**	**116**
WAVE	Regular	Regular	Regular	Regular
H [m]	0.226	0.226	*0.37*	0.37
T [s]	6.0	6.0	*3.7*	3.7
h [m]	2.5	2.5	*2.5*	2.5
L [m]	28.3	28.3	*16.1*	16.1

Results from test 115 are reported in the present work and used to validate the hybridisation technique. The bathymetry of test 115 is shown in Figure 2.

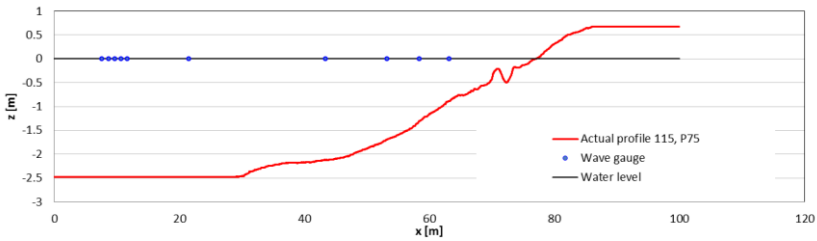

Figure 2. Bathymetry and wave gauge location (Test 115, P75)

Wave gauges positions are shown in Table 2. The horizontal distance x [m] starts from the wave paddle position. The type of the wave gauge is resistance type wave gauge. Incident waves used in the SWASH simulation are calculated based on the measurement of the time series of water surface in the offshore. Three wave gauges are used for the incident wave and reflection analysis (WG0, WG2 and WG3 for test 115). The incident waves are generated at the position of WG0 in the SWASH model: the SWASH computation domain starts at 7.7 m from the wave paddle location of CIEM flume.

Table 2. Wave gauge location in physical model (from the wave paddle)

Wave Gauge No.	**0**	**1**	**2**	**3**	**4**	**5**	**8**	**9**	**13**	**12**
Actual distance [m]	7.70	8.72	9.70	10.69	11.69	21.58	43.41	53.28	58.46	63.18

4.1 Numerical settings

Simulations are carried out with SWASH (version 1.10AB) using a grid size of 0.5 m in the horizontal direction with an initial time step of 0.05 s in prototype scale (same scale of physical model). The time series of the incident waves are prescribed at the wave boundary of the SWASH model. The length of the numerical flume is 100 m long with 200 grid cells. Note that the calculation time step is automatically adjusted in the calculation depends on the CFL condition. A maximum CFL value of 0.5 is used. The output time step of the SWASH model is 0.05 s. A weakly-reflective boundary is applied at the wave boundary. A Manning's value of 0.019 is used as a bottom friction in numerical model runs: this value represents a bottom friction in a sandy coast. The time duration of the numerical simulation was 23 minutes, as used in the physical model test. The number of layers tested in the SWASH was 8. Note that the result of wave propagation of 1 layer and 8 layers is not so different since the *kd* (*wave number · water depth*) value is less than 1 in this case.

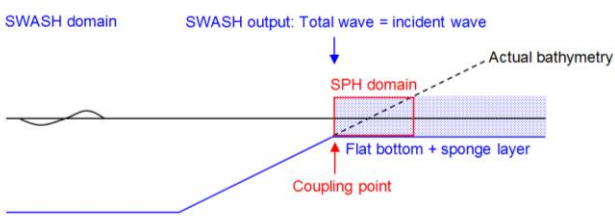

Figure 3. Sketch of the model domain and coupling point

SWASH has been firstly validated for the entire physical domain, showing high accuracy of the results in terms of wave height, wave period and wave setup. Once the model has proven to represent properly the wave propagation and transformation towards nearshore, the domain has been adjusted to obtain the input for DualSPHysics. In particular a calculation of SWASH with flat bottom and sponge layer behind the coupling point would be suitable for the boundary of SPH. In this way only incident wave characteristics are transferred from SWASH to SPH (Figure 3).

5. Hybridisation results

Test 115 of SUSCO data (Hydralab III report, 2010) has been used to validate the hybridisation strategy as defined in the previous section. SUSCO has been chosen as former validation case since its geometry results quite simple, the wave reflection very limited and because several measurements have been carried out with different wave conditions, so that a huge amount of data is available to be used.

Results from SWASH have been used as hydraulic boundary conditions (HBC) in DualSPHsyics corresponding to the position of WG8 in the physical model (43.41 m far from the physical wave paddle). A flat bottom is modelled from WG8 position backwards and the "Moving boundary" in DualSPHysics is finally slightly shifted back to reproduce properly the target wave conditions on horizontal bottom. The wave surface elevation has been measured in the DualSPHycics domain in WG8 and WG9 and compared with the experimental results.

An initial particle interspace of 0.02 m has been used to initially create the SPH particles. The resulting number of fluid particles is 96,048. The simulation runtime was about 2.5 hours using an Nvidia GeForce GTX 680 graphic card. The simulation runtime of SWASH was less than 5 minutes. A case with the entire physical domain was also simulated only with DualSPHysics to compare runtimes. The numerical wave paddle mimics the physical one (wedge type) and uses the same time series of displacement. The number of fluid particle is 356,706 and the computational time about 9.2 hours executed on the same graphics card. The comparison proves that the hybridisation is around 3.6 times less consuming in this case and the larger domains for SWASH are, higher speedups of the hybridisation execution will be obtained.

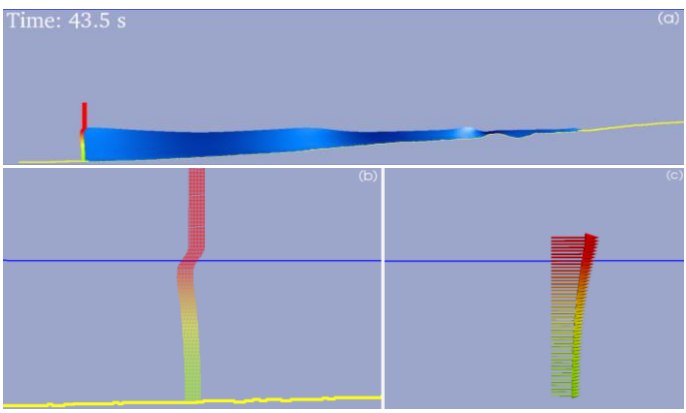

Figure 4. SPH simulation snapshot: (a) entire SPH domain; (b) Piston velocity detail; (c) input velocity from SWASH

Figure 4 shows a snapshot of the SPH simulation where it is possible to see the detail of the piston movement (the colours indicate different velocities of each part of the piston). The input velocities that have been used to calculate the piston displacement in time are plotted in the subfigures where the arrows (c) with different length and colour indicate different velocity along the vertical.

In total, 9 tests have been performed in DualSPHysics to investigate the sensitivity of the model to parameters related with DualSPHysics and with the piston smoothing. In detail, the attention has been focused on:

1. The k coefficient to calculate the smoothing length *h* from the assigned particle size, $h=k*(dx^2+dy^2+dz^2)^{0.5}$, testing values of k=[0.92,1.50].
2. The boundary viscosity: it means that the viscosity considered for the boundary particles has been set equal or not to the fluid viscosity (v=0 stays for no viscosity, v=1 means application of the same fluid viscosity).
3. The piston smoothing: it identifies the number of neighbour particles to be considered for the smoothing interpolation.

5.1 Wave surface elevation

The time series of the surface elevation in DualSPHysics coupled with SWASH (red line) are plotted together with the experimental ones (blue line) in Figure 5. The black line represents the error signal as the difference between experiment and hybridisation results).

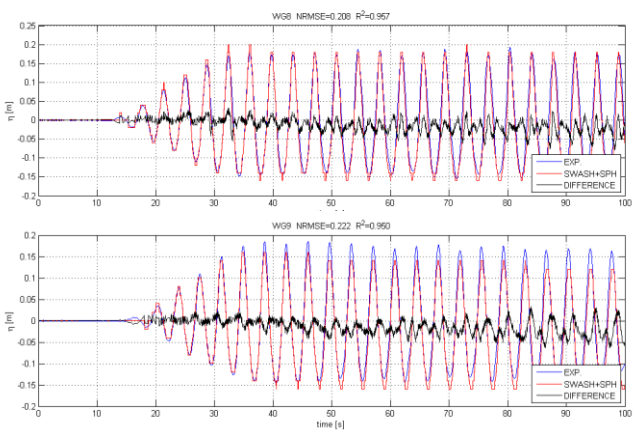

Figure 5. Time series of wave surface elevation and relative NMRSE and R^2 (test with k=1.50 and v=1)

The error in the DualSPHysics (coupled with SWASH) results has been quantified by the use of the coefficient of determination (R^2) and the Normalized Root-Mean-squared (NRMSE) error that is defined as follows:

$$NMRSE = \frac{RMSE}{\sqrt{\frac{\sum_{i=1}^{n}(y_i^{EXP})^2}{n}}} = \sqrt{\frac{\sum_{i=1}^{n}(y_i^{EXP} - y_i^{SPH})^2}{\sum_{i=1}^{n}(y_i^{EXP})^2}} \quad [9]$$

where y_i^{EXP} and y_i^{SPH} are respectively the experimental and numerical surface elevation at the time t_i RMSE is the Root-Mean-squared error and n is equal to the sample frequency multiplied for the simulate real time. The NRMSE is 0.21 and 0.22 and R^2 is 0.96 and 0.95 respectively for WG8 and WG9. The same values measured for the entire domain case result: NRMSE equal to 0.23 and 0.24 for WG8 and WG9 and R^2 equal to 0.93 in both positions.

5.2 Velocity profile

In the present section the velocities obtained in SWASH and SPH coupled with SWASH are computed in the position where the wave gauges are located. Remember that SWASH divide the fluid domain along the vertical in different layers (8 in these simulations) and the velocity is a sort of average value for each layer. Furthermore, at each time step, the layer has no the same height, because of the passage of the wave (the wet area is changing slightly). The velocity in DualSPHysics is computed referring to the centre of each SWASH layer in still water conditions (beginning of the simulation). However since the wave height is around 10 times smaller than the water depth at the toe of the beach and 8 layers is considered, a good discretisation of the depth can be assumed reliable to compare the velocities calculated by the two models (Figure 6).

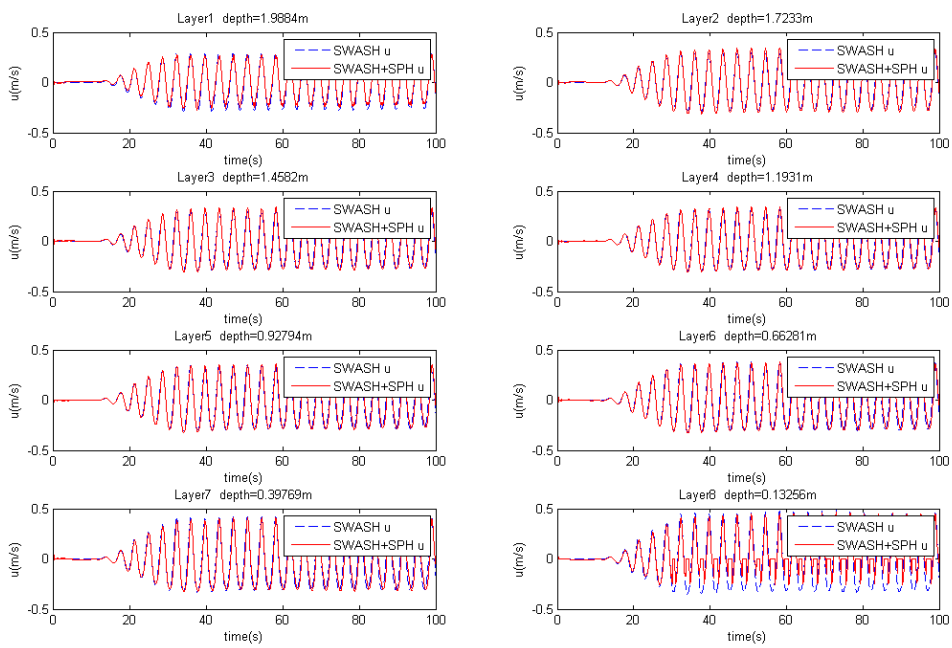

Figure 6. WG8 location – Example of Horizontal velocity for each SWASH layer. Comparison between SWASH and SPH results

Velocity values measured with the hybridisation model for layer 8 are zero when wave elevation is below that depth=0.13256m, since there is no water particles in that position.

The strategy can be finally considered successful since the results are more accurate and the runtime is shorter than the case simulated entirely in DualSPHysics.

6. Conclusions

A hybrid method is developed starting from the wave propagation model SWASH and the meshfree particle method DualSPHysics. The hybrid model has been validated with physical model data providing significantly improved predictions of wave heights and velocities, of breaking waves on sandy beaches. The cases described in the present work represent a first case of application of the new technique: physical model tests carried out at large scale (no scale effects and limited model effects) and a simple geometry have been chosen in this phase to in order to keep away more complexities making the analysis as simplest and most reliable as possible. It can be concluded that the implementation and application of hybridisation technique between two numerical models to wave propagation and transformation over a sandy beach, based on physical model tests, has proved to be a powerful strategy that overcome the drawbacks of the two models leading to a proper representation of the wave phenomena. Ongoing research aims to represent also irregular waves train and finally use the coupling to measure wave impacts on coastal structures.

Acknowledgments

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