2D-BOUNDARY INTEGRAL EQUATION WITH AN EXACT SURFACE ADMITTANCE OPERATOR FOR PERIODIC SCATTERERS IN A LAYERED MEDIUM

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***Abstract*—** A new boundary integral equation approach is proposed to solve two-dimensional periodic TM-scattering problems. The scatterers are described by a proper surface admittance operator, relating the equivalent electric surface currents to the electric fields on the scatterer's surface. This surface operator is determined analytically for a scatterer with rectangular cross-section. The new method allows for a considerable gain in CPU-time and memory requirements and remains very accurate even in the presence of very good conductors and this from DC to the high-frequency skin effect regime. The new approach is illustrated by studying the scattering of a plane wave by a periodic arrangement of lossy rectangles embedded in a dielectric slab.

1 INTRODUCTION

Boundary integral equation techniques, with suitable Green's functions [1] as kernel functions, are ideally suited for modelling piecewise homogeneous scatterers. These structures can be embedded in a layered medium or placed in a periodic arrangement. Typically, the equivalence principle is used to introduce unknown equivalent electric and magnetic surface currents on the scatterers' boundaries. These equivalent currents are then found by solving integral equations that enforce continuity of the total tangential electric and magnetic field at the scatterer's surface [2].
In this paper it is shown that it suffices to introduce an equivalent electric surface current $\mathbf{J}_s(\mathbf{r},\omega)$ on the scatterer's surface, provided a suitable surface admittance operator is introduced relating this current at each point $\mathbf{r}$ of the scatterer's surface to the tangential electric fields $\mathbf{E}_{tan}(\mathbf{r}',\omega)$ at every other point on the surface. This surface admittance operator allows to replace the medium of the scatterer by the medium of the surrounding background medium the scatterer is embedded in. The remaining field problem is solved by solely considering the interactions between the equivalent electric surface currents and the incident field in the sole presence of the background medium the scatterers were originally embedded in. It is shown that the admittance operator yields a highly accurate description of the behaviour of the scatterer for a wide range of electromagnetic material properties. This is in particular the case when the scatterer becomes highly conductive and for small skin depths. It is demonstrated that the surface admittance operator can be easily incorporated into an integral equation method, including the case of a layered background medium and/or for a periodic arrangement of the scatterers. In Section 2, the theoretical background of the surface admittance operator is briefly explained, together with the solution of the overall scattering problem. A general expression in terms of the Dirichlet eigenfunctions of the scatterer's cross-section is given. For the rectangular cross-section, application of the method of moments (MoM) leads to a discretised form of the surface admittance operator: the surface admittance matrix. The reader is referred to [3] for more details. In order to illustrate our new formalism we consider a periodic configuration consisting of rectangular dielectric scatterers, embedded in a dielectric slab. The incident wave is a plane TM-wave and both reflection and transmission coefficients are calculated, as well as the total dissipated power for varying losses in the scatterers.

2 INTEGRAL APPROACH AND SURFACE ADMITTANCE OPERATOR

Consider the cross-section S of an arbitrary non-magnetic scatterer embedded in a piecewise homogeneous non-magnetic background medium, as in Fig. 1. The homogeneous material of the layer the scatterer is embedded in, is characterised by the constitutive parameters ϵ, μ_0 and σ, whereas the constitutive parameters of the scatterer are given by ϵ_{sc}, μ_0 and σ_{sc}. In order to replace, in Fig. 1a, the material of the scatterer by the material of its surrounding layer, in this way undoing the discontinuity in permittivity and conductivity due to the scatterer's presence, we introduce an equivalent surface current density J_{sz} (Fig. 1b), related to the value of the electric field on the boundary c, by means of the differential surface admittance operator $\mathcal{Y}$ given by

$$J_{sz} = \mathcal{Y}\, E_z = \tau \sum_{m=1}^{\infty} \frac{\partial_n \xi_m \oint_c E_z\, \partial_n \xi_m\, dc}{(k_0^2 - \lambda_m)(k^2 - \lambda_m)}, \tag{1}$$

with $\tau = [\sigma - \sigma_{sc} + j\omega(\epsilon - \epsilon_{sc})]$. The symbol k represents the wavenumber of the material of the scatterer, i.e. $k = \sqrt{-j\omega\mu_0(j\omega\epsilon_{sc} + \sigma_{sc})}$ and $k_0 = \sqrt{-j\omega\mu_0(j\omega\epsilon + \sigma)}$ is the wavenumber of the material the scatterer is embedded in. The ξ_m are the Dirichlet eigenfunctions of the cross-section S, bounded by the curve c, with corresponding eigenvalues λ_m. For a rectangular scatterer ($0 \le x \le a$ and $0 \le y \le b$), the Dirichlet eigenfunctions are $\xi_{mn} = \frac{2}{\sqrt{ab}} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$, with $\lambda_{mn} = ((m\pi)/a)^2 + ((n\pi)/b)^2$. In [3] it is shown that an analytical expression for J_{sz} can be obtained by expanding E_z on each side of the rectangle in an appropriate Fourier sine series. E.g. for $y = 0$ and $0 \le x \le a$ this series is $E_z = \sum_{m=1}^{P} A_m \sin\frac{m\pi x}{a}$. However, when solving the overall scattering problem, E_z and J_{sz} will be used in a Galerkin MoM. When applying pulse functions both as basis and test functions, we can now collect all the pulse amplitudes E_j, on all of the four sides, into a vector $\mathbf{E}$ and similarly all J_j's into a vector $\mathbf{J}$, in order to obtain the discretised form of $\mathcal{Y}$ as $\mathbf{J} = \mathbf{Y}_s \cdot \mathbf{E}$. $\mathbf{Y}_s$ is the $M \times M$ differential surface admittance matrix (all entries of $\mathbf{Y}_s$ have dimension Ω^{-1}). We again refer the reader to [3] for the detailed analytical expressions of the elements of $\mathbf{Y}_s$. The next step consists of coupling the surface admittance operator descriptions for the different scatterers to the description of the overall scattering problem based on

$$(E_z)_{scat}(\mathbf{r}) = \oint_c J_{sz}(\mathbf{r}')G(\mathbf{r}, \mathbf{r}')dc(\mathbf{r}'). \tag{2}$$

An appropriate Green's function $G(\mathbf{r}, \mathbf{r}')$ is to be used as integral kernel. For a periodic configuration of scatterers, we refer to reader to [1]. If we now consider (2) for $\mathbf{r}$ on the boundary c, an appropriate MoM discretisation of (2), again using pulse functions, leads to $\mathbf{E}_{scat} = \mathbf{G} \cdot \mathbf{J}$, where we have used the same set of pulse functions as for constructing the differential surface admittance matrix. The unknown currents $\mathbf{J}$, i.e. the amplitudes of the pulses representing the originally continuous current J_{sz}, can now simply be determined by enforcing the boundary condition $\mathbf{J} = \mathbf{Y}_s \cdot \mathbf{E} = \mathbf{Y}_s \cdot (\mathbf{E}_{inc} + \mathbf{E}_{scat}) = \mathbf{Y}_s \cdot (\mathbf{E}_{inc} + \mathbf{G} \cdot \mathbf{J})$, on c and solving for $\mathbf{J}$. The symbol $\mathbf{E}_{inc}$ stands for the pulse weighted field on c due to the incident field.

3 EXAMPLE

Consider a periodic grid of dielectric bars (Fig. 1(c)), buried in a non-magnetic and lossless dielectric slab with thickness $t = 18$ mm and a relative permittivity of $\epsilon_r = 3.0$. The dielectric slab is placed in free space. The bars (also non-magnetic and lossless) of size 6 mm $\times$ 3 mm have a complex permittivity $\epsilon_c = \epsilon_0\epsilon_r\,(1 - j\tan\delta)$, with $\epsilon_r = 3$, and are located 5 mm under the top surface of the dielectric slab. The centre-to-centre spacing between the bars is chosen to be 10 mm. The structure is excited by a TM incident plane wave $\mathbf{E}_i = E_i\mathbf{u}_z$, at a free-space wavelength $\lambda = 2$cm (i.e. about 15 GHz). In order to calculate the periodic Green's function [1], we place Perfectly Matched Layers (PMLs), backed by perfectly conducting (PEC) plates above and below the air-slab-air configuration and use the propagation characteristics of the resulting parallel-plate waveguide. The PMLs are placed at a distance $d_{\mathrm{air}} = 5$mm from the slab, with $d_{\mathrm{PML}} = 3.5$mm, $\kappa_0 = 15$, $\frac{\sigma_0}{\omega\epsilon_0} = 10$. In Figs. 2 and 3, the power reflection coefficient R and the power transmission coefficient T are shown as a function of the angle of incidence θ, for varying values of the losses in the bars acting as scatterers. $\tan\delta = 0.01$ corresponds to a skin-depth of 35mm, whereas $\tan\delta = 1000$ corresponds to a skin-depth of $142\mu m$. For small values of the loss tangent, the reflection coefficient decreases with increasing losses, because of the increasing power absorption in the bars. However, for large values of $\tan\delta$, the skin-effect comes into play and as fields are forced out of the bars, the reflection coefficient increases. The power transmission through the grid of lossy bars steadily decreases as losses in the bars increase.

References

[1] H. Rogier and D. De Zutter, "A fast converging series expansion for the 2D periodic Green's function based on Perfectly Matched Layers," *IEEE Trans. Microwave Theory Tech.*, vol. 52, pp. 1199–1206, Apr. 2004.

[2] F. Olyslager, D. De Zutter, and K. Blomme, "Rigorous analysis of the propagation characteristics of general lossless and lossy multiconductor transmission lines in multilayered media," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 79–88, Jan. 1993.

[3] D. De Zutter and L. Knockaert, "Skin effect modeling based on a differential surface admittance operator," *IEEE Trans. Microwave Theory Tech.*, vol. 53, pp. 1199–1206, Aug. 2005.

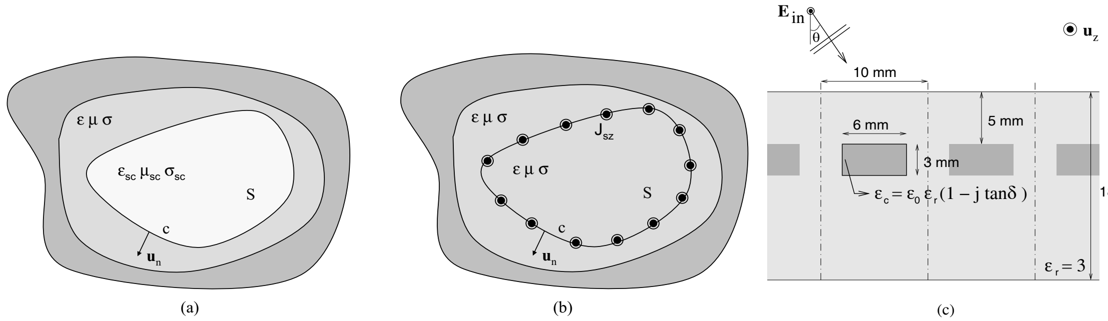

Figure 1: Problem geometry: (a) scatterer cross-section S with boundary c, embedded in a piecewise homogeneous background medium; (b) equivalent problem with surface currents J_{sz} and with the material of the scatterer replaced by that of the surrounding layer; (c) Periodic grid of lossy non-magnetic dielectric bars embedded in a non-magnetic dielectric slab and illuminated by an incident plane TM-wave.

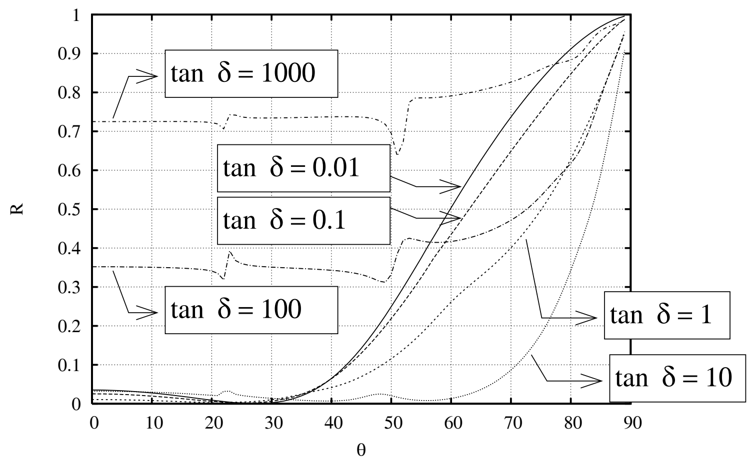

Figure 2: Power reflection coefficient R as a function of the angle of incidence θ.

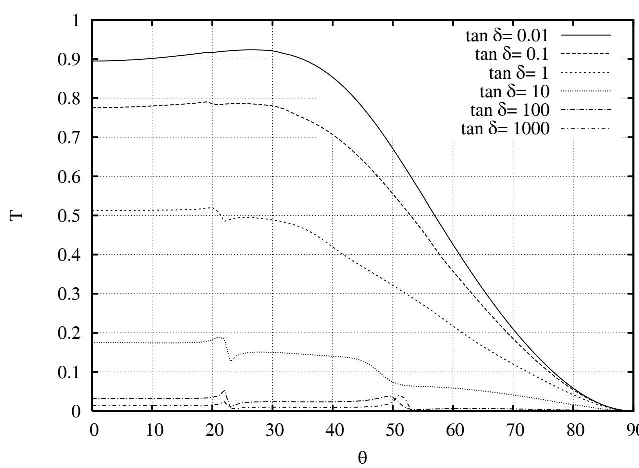

Figure 3: Power transmission coefficient T as a function of the angle of incidence θ.

O11-33: Novel Mathematical Methods in Electromagnetics II

Friday, July 27, 2007 • MacDonald Room

Organizers/Co-Chairs: K. Kobayashi, Chuo University, Japan
Y. Shestopalov, Karlstad University, Sweden

2:00 PM EMTS48: "2D-BOUNDARY INTEGRAL EQUATION WITH AN EXACT SURFACE ADMITTANCE OPERATOR FOR PERIODIC SCATTERERS IN A LAYERED MEDIUM"
H. Rogier, Ghent University, Belgium; D. De Zutter, Ghent University, Belgium;
L. Knockaert, Ghent University, Belgium
Presenter: Hendrik Rogier, Ghent University, Belgium

2:20 PM EMTS260: "Scattering of Electromagnetic Waves by Multilayered Inhomogeneous Columnar Dielectric Gratings Loaded with Dielectric Rectangular Cylinders"
R. Ozaki, Nihon University, Japan; T. Yamasaki, Nihon University, Japan;
T. Hinata, Nihon University, Japan
Presenter: Tsuneki Yamasaki, Nihon University, Japan

2:40 PM EMTS149: "DIFFRACTION COEFFICIENT OF AN IMPEDANCE WEDGE" (Invited)
A.V. Osipov, German Aerospace Center (DLR), Germany; T.B.A. Senior, University of Michigan, USA
Presenter: Andrey Osipov, German Aerospace Center (DLR), Germany

3:00 PM EMTS184: "New physical limitations in scattering and antenna problems" (Invited)
G. Kristensson, Lund University, Sweden; C. Sohl, Lund University, Sweden;
M. Gustafsson, Lund University, Sweden
Presenter: Gerhard Kristensson, Lund University, Sweden

3:20 PM EMTS238: "Scalar wave diffraction from arbitrarily shaped screens of revolution: a rigorous approach" (Invited)
P.D. Smith, Macquarie University, Australia; S.B. Panin, Macquarie University, Australia;
E.D. Vinogradova, Macquarie University, Australia; S.S. Vinogradov, CSIRO, Australia
Presenter: Paul Smith, Macquarie University, Australia

3:40 PM EMTS263: "Computational Techniques and Numerical Analysis in EM Wave Propagation and Scattering: An Overview and Trends" (Invited)
R. Talhi, University of Tours, France
Presenter: Rachid Talhi, University of Tours, France

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