3D Quantitative Microwave Imaging of Inhomogeneous Dielectric Objects

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Abstract—**This paper deals with 3D quantitative microwave imaging or microwave tomography (MT), i.e. the reconstruction of inhomogeneous dielectric objects from measurements of the scattered field for different illuminations of those objects with microwaves. In contrast to qualitative microwave imaging methods, no approximations are made to the scattering model, thus MT tries to solve the full-vectorial 3D inverse scattering problem exactly. This is accomplished by casting the inverse problem into an optimization problem where a permittivity profile is sought that minimizes a properly chosen cost function, which measures the distance between the measured data and the simulated scattering from a given permittivity profile. In this work, a regularized Gauss-Newton method is employed to this end.**

Keywords—**Microwave Imaging, Inverse Problem, Optimization, Gauss-Newton, Regularization**

I. INTRODUCTION

THE aim of microwave imaging is the reconstruction of an object – or, more precisely, some characterizing physical parameter of an object – from measurements of the scattered electromagnetic fields that occur when that object is illuminated by a number of known incident microwaves. Apart from apparent applications in non-destructive testing and geophysical exploration, microwave imaging is a promising alternative to the existing imaging methods in the biomedical world. An interesting example is the detection of breast cancer tumors, for which microwaves are especially suited, because they easily penetrate the body without the risk that is attached to the ionizing X-rays that are commonly used for mammography these days.

Microwave imaging methods can be devided in two major classes. On one hand you have the qualitative reconstruction methods that provide approximate solutions, but linearized and therefore easy-to-solve problems, and on the other hand there is microwave tomography (MT) that attempts to solve the full-wave 3D inverse scattering problem in an exact manner. Despite improved image qualities, the latter class is known, however, for its non-linearity and its members are invariably burdened by computationally heavy iterative algorithms.

In this work, the 3D inverse scattering problem is solved using a regularized Gauss-Newton optimization algorithm [1]. Many efforts have been made to overcome computational burdens, which results in a robust and effective algorithm for quantitative microwave tomography.

II. THE FORWARD PROBLEM

The label "forward problem" in our field of research refers to the calculation of the electric field $\boldsymbol{E}^{\rm scat}(\boldsymbol{r})$ that is scattered from a known complex permittivity distribution $\epsilon(\boldsymbol{r})$ in an investigation domain $\mathcal{D}$ when a known incident field $\boldsymbol{E}^{\rm inc}(\boldsymbol{r})$ impinges on this domain (Figure 1).

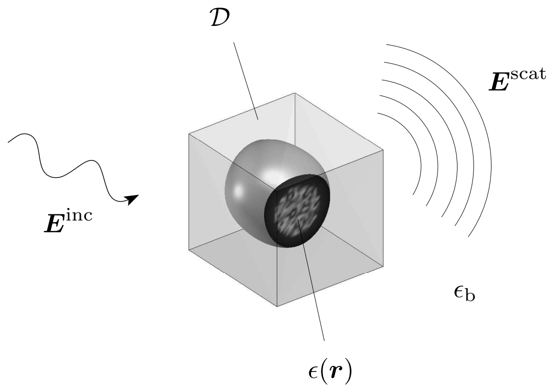

Fig. 1. The 3D scattering configuration.

A convenient way to solve the forward problem is by using the Volume Integral Equation (VIE) [2]. In this formulation, the scattered field is the solution to Maxwell's equations for a source current density $\boldsymbol{J}^{\rm scat}$ which is the product of the contrast in permittivity and the total electric field inside $\mathcal{D}$, i.e.

$$\boldsymbol{J}^{\rm scat}(\boldsymbol{r}) = (\epsilon(\boldsymbol{r}) - \epsilon_{\rm b})\boldsymbol{E}(\boldsymbol{r}), \tag{1}$$

where $\epsilon_{\rm b}$ is the permittivity of the background medium. Since the scattered field $\boldsymbol{E}^{\rm scat}$ can be expressed in terms of the contrast current $\boldsymbol{J}^{\rm scat}$ through an integral formulation and since the total field is the sum of the incident and the scattered fields, an integral equation for the total field can be obtained. Once this equation is solved, the total field – and consequently the current $\boldsymbol{J}^{\rm scat}$ – is known and the scattered field can be calculated.

The VIE is solved numerically using the Method of Moments (MoM). This means that the electric field inside the domain $\mathcal{D}$ is expanded in basis functions defined on a grid discretization of $\mathcal{D}$ and that a linear system for the expansion coefficients is obtained after testing the VIE with the same set of basis functions. Because of the large dimensions of this linear system, it is solved iteratively and an FFT technique [3] is used to speed up the matrix-vector multiplications that are needed by the iterative solver. In addition, an extrapolation procedure has been developed [4] to determine initial guesses that are already close to the solution vector, as a result of which the number of iterations is reduced.

III. THE INVERSE PROBLEM

Since the contrast current (1) is the product of the permittivity and the total field, which in turn depends on the permittivity

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through the integral equation, the scattered field is a non-linear function of the permittivity. As a result, the inverse scattering problem is solved iteratively. It is formulated as an optimization problem in which the optimization variables are the values of $\epsilon(\boldsymbol{r})$ in the cells of a cubic grid that covers $\mathcal{D}$ and in which a cost function

$$\mathcal{F}(\boldsymbol{\varepsilon}) = \left[1 + \alpha \mathcal{F}^{\mathcal{R}}(\boldsymbol{\varepsilon})\right] \|\boldsymbol{e}^{\mathrm{scat}}(\boldsymbol{\varepsilon}) - \boldsymbol{e}^{\mathrm{meas}}\|^2, \qquad (2)$$

is minimized. In (2), the vectors $\boldsymbol{e}^{\mathrm{scat}}$ and $\boldsymbol{e}^{\mathrm{meas}}$ contain the simulated and measured scattered field data respectively, $\boldsymbol{\varepsilon}$ is a vector containing the optimization variables, α is a positive parameter and $\mathcal{F}^{\mathcal{R}}$ is a smoothing function that penalizes strong local variations in the permittivity. The presence of the smoothing function is required to put some constraints on the optimization process. Indeed, it is known that the inverse scattering problem is very ill-posed, i.e. large fluctuations on the permittivity profile can correspond to only small deviations on the scattered field and therefore noise on the data can be amplified to undesired levels in the reconstructions when no precautions are taken.

The optimization itself is carried out using a descent method with line search. The $(k+1)$-th iterate in the optimization process is calculated as

$$\boldsymbol{\varepsilon}_{k+1} = \boldsymbol{\varepsilon}_k + \beta_k \boldsymbol{s}_k, \qquad (3)$$

where the search direction $\boldsymbol{s}_k$ is the solution of the system

$$\left(\boldsymbol{J}_k^H \boldsymbol{J}_k + \lambda_k^2 \boldsymbol{\Sigma}\right) \boldsymbol{s}_k = -\left(\boldsymbol{J}_k^H \left[\boldsymbol{e}_k^{\mathrm{scat}} - \boldsymbol{e}^{\mathrm{meas}}\right] + \lambda_k^2 \boldsymbol{\Omega}_k^*\right), \qquad (4)$$

with $\lambda_k^2 = \alpha \|\boldsymbol{e}^{\mathrm{scat}}(\boldsymbol{\varepsilon}) - \boldsymbol{e}^{\mathrm{meas}}\|_k^2 / (1 + \alpha(\mathcal{F}^{\mathcal{R}})_k)$ and where the subscript k indicates quantities evaluated in $\boldsymbol{\varepsilon}_k$. The matrix $\boldsymbol{J}$ is the Jacobian matrix containing the first order derivatives of the simulated scattered field and the vector $\boldsymbol{\Omega}$ and the matrix $\boldsymbol{\Sigma}$ contain first and second order derivatives of the smoothing function, respectively. It can be shown [1, 5] that with this procedure the optimization process will end in a (local) minimum of the cost function.

This basic structure of the algorithm is implemented with great care for computational efficiency. For instance, the update systems (4) are ill-conditioned and therefore a subspace preconditioned LSQR algorithm [6] has been implemented to solve them iteratively in a limited number of iterations. Also, to have more control over the optimization process, upper and lower bounds on the real and imaginary parts of the permittivity can be imposed.

IV. Example

As an example, consider the test domain and antennaconfiguration of Figure 2. Three circular antenna-arrays, each with 30 antennas are placed on a cilinder around a test domain $\mathcal{D}$ in a water background. Inside the test domain a simplified leg structure is placed, consisting of a bone inside a cilinder of muscle material. The real part of the relative permittivity in the xy-plane is depicted in Figure 3(a) and the reconstruction with our microwave imaging algorithm is shown in Figure 3(b).

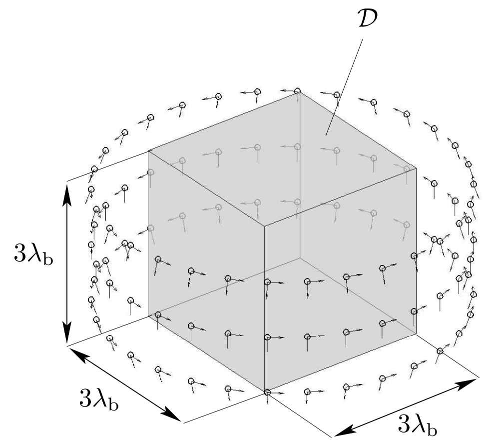
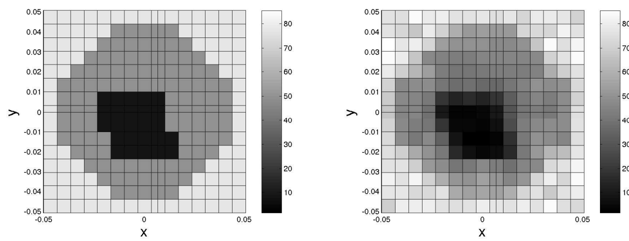


Fig. 2. The investigation domain and the antennaconfi guration for the biomedical example. The arrows indicate possible antenna polarizations. Sizes are expressed in background wavelengths λ_{b}.



(a) (b)

Fig. 3. Biomedical example: real part of the actual (a) and reconstructed (b) relative permittivity in the xy-plane.

V. Conclusions

The Gauss-Newton minimization method, combined with a proper regularization strategy offers an effective means to solve the inverse scattering problem and thus allows for quantitative microwave imaging, which is a promising technique in biomedical applications.

Acknowledgments

The authors would like to acknowledge the suggestions of many people.

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